<u>Lecture 1 + 2</u>

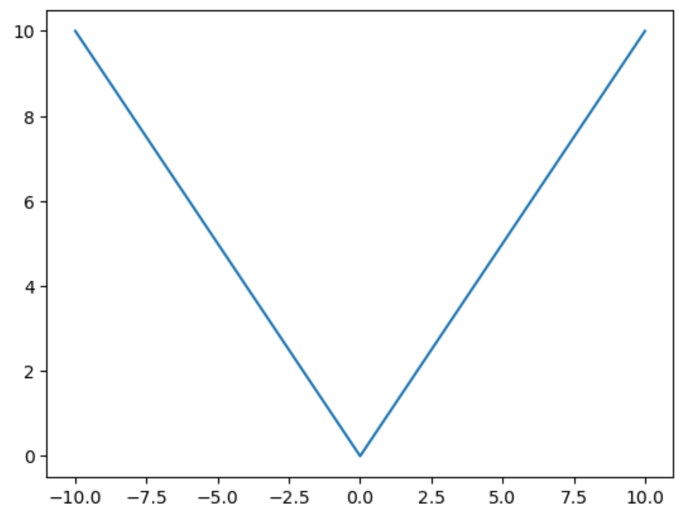
<u>Absolute Value</u>

Definition: The absolute value of $x \in \Re$ is the non-negative value of x.

The absolute value has fundamental properties listed below given that $a, b \in \Re$.

1) $|a| \geq 0$ (Non Negativity)
2) $|a| = 0$ means that $a = 0$ (Positive Definiteness)
3) $|ab| = |a|\,|b|$ (Multiplicativity)
4) $|a + b| \leq |a| + |b|$ (Triangle Inequality)

When we plot the absolute value function, we get the following graph below.



We can also write the absolute value in piecewise notation defined below.

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0. \end{cases}$$

Problems

1) Solve the following equation.

$|5x - 10| = 4$

NOTE : $|a| = b$ implies $|a| = b$ or $|a| = -b$

$|5x-10| = 4$ or $|5x-10| = -4$

$5x = 4+10$ $\quad$ $5x = -4+10$

$\frac{5x}{5} = \frac{14}{5}$ $\quad$ $\frac{5x}{5} = \frac{6}{5}$

Case 1 : $x = \frac{14}{5}$ $\quad$ Case 2: $x = \frac{6}{5}$

Check by Plugging in! $\longrightarrow$ $|\cancel{5}(\frac{14}{\cancel{5}})-10| = 4$

$|14-10| = 4$

Case 1: $|4| = 4$ ✓

$|\cancel{5}(\frac{6}{\cancel{5}})-10| = 4$

$|6-10| = 4$

$|-4| = 4$

Case 2: $4 = 4$ ✓

2) Solve the following equation.

$|5 + 6x| = 7x$

$|5+6x| = 7x$

$5 = 7x-6x$

$5 = x$

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3) Find the solution to the equation below.

$$\left|x^2 + 2x\right| = 15$$

$|x^2 + 2x| = 15$

$x^2 + 2x - 15$

$\frac{-2 \pm \sqrt{2^2 - 4(1)(-15)}}{2} = \frac{-2 \pm \sqrt{64}}{2}$

$|x^2 + 2x| = -15$

$x^2 + 2x + 15$

$\frac{-2 \pm \sqrt{2^2 - 4(1)(15)}}{2} = \frac{-2 \pm \sqrt{-56}}{2} = -1 \pm i\frac{\sqrt{56}}{2}$

Solution: $x = -5$, $x = 3$

Absolute Value Inequalities

Key Rules: If you multiply or divide the inequality by a negative number, the inequality sign becomes reversed.

1) $|y| < a$ means that $-a < y < a$
2) $|y| \leq a$ means that $-a \leq y \leq a$
3) $|y| > a$ means that $y > a$ or $y < -a$
4) $|y| \geq a$ means that $y \geq a$ or $y \leq -a$

Problems

1) Solve the following equation.

$$|5y + 10| < 5$$

$|y| < a$ means $-a < y < a$

$-5 < 5y + 10 < 5$

$-5 - 10 < 5y < 5 - 10 \rightarrow \frac{-15}{5} < \frac{5y}{5} < \frac{-5}{5}$

$\boxed{-\frac{15}{5} < y < -1 \checkmark \quad \downarrow \quad \left(-\frac{15}{5}, -1\right)}$

2) Solve the following equation

$$|4 - 13z| > 8$$

$|y| > a$ means $y > a$ or $y < -a$

$4 - 13z > 8$

$-13z > 8 - 4$

$-13z > 4$

$z < \frac{4}{-13}$

$\downarrow$

$\left(-\infty ; \frac{-4}{13}\right)$

or

$4 - 13z < -8$

$-13z < -8 - 4$

$\frac{-13z}{-13} < \frac{-12}{-13}$

$z > \frac{12}{13}$

$\downarrow$

$\left(\frac{12}{13}, \infty\right)$

Polynomials

Definition: A polynomial can be written in the form of $\alpha_0 x^n + \alpha_1 x^{n-1} + \alpha_2 x^{n-2} + \ldots\ldots + \alpha_n$.

We are able to build polynomial functions via addition, multiplication, and exponentiation.

Adding and Subtracting Polynomials

We are able to add polynomials using the associative law of addition and the commutative law. Essentially, we are going to group all expressions into a single summation. Then, we are going to reorder and perform mathematical operations for like terms.

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Problems

1) Add the below two equations.

$$f(x) = 10x^5 + x^3 + 6x^2 - x - 10$$
$$g(x) = 6x^4 - 5x^2 - 10x - 20$$

$f(x) + g(x) = 10x^5 + x^3 + 6x^2 - x - 10 + (6x^4 - 5x^2 - 10x - 20)$
$= 10x^5 + x^3 + 6x^2 - x - 10 + 6x^4 + 5x^2 + 10x - 20$
$= 10x^5 + 6x^4 + x^3 + (6x^2 + 5x^2) + (-x + 10x) + (-10 - 20)$
$= 10x^5 + 6x^4 + x^3 + 11x^2 + 9x - 30$

FOIL Method and Multiplication of Polynomials

When multiplying two binomials, we are able to use the FOIL method. We can express this mathematically as : $(a + b)(c + d) = ac + ad + bc + bd$.

The FOIL method is a generalization of the distributive law and the above expression can only be applied to that particular case.

Definition: The distributive given elements $x, y, z \in S$ is defined as

$x * (y + z) = (x * y) + (x * z)$.

Generally, we are able to state that the multiplication property distributes over the addition expressions.

Problems

1) Simplify the following expression.

$$\underbrace{(a + b)(c + d)}_{= f} + \underbrace{(a + 3b)(5c - d)}_{= g}$$

$f = (a+b)(c+d)$
$f = ac + ad + bc + bd$

$g = (a+3b)(5c-d)$
$g = 5ac - ad + 15bc - 3db$

$f+g = ac + \cancel{ad} + bc + bd + 5ac \cancel{- ad} + 15bc - 3db$

$f+g = 6ac + 16bc - 2bd$

2) Simplify the following expression.

$$x(y - 1)(x - 3)$$

$(y-1)(x-3) = yx - 3y - x + 3$

$x(yx - 3y\ 3x +$ $= yx^2 - 3xy - x^2 + 3x$

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Factoring

Greatest Common Factor: Determine if we are able to factor out any common terms. Note the distributive law of $x(y + z) = xy + xz$. In this case, x is the common factor.

Problem

1) Factor the following expression.

$10x^5 - 20x^4 + 2x^2$

x^2 is lowest exponent so pull out

$2x^2(5x^3 - 10x^2 + 1)$

Factor By Grouping: Group the expression so we can find a common factor. The factored polynomial can be written as $(x + a)(x + b) = x^2 + (a + b)x + ab$.

Problem

1) Use factor by grouping for the following expression.

$3x^2 - 2x + 12x - 8$

$(3x^2 - 2x) + (12x - 8)$

$x(3x - 2) + 4(3x - 2)$

$f(x) = (x + 4)(3x - 2)$

$f(x) = 3x^2 - 2x + 12x - 8$

$= 3x^2 + 10x - 8 = 3x^2 - 2x + 12x - 8$ ✓

Factoring Second Order Polynomials: We are able to write our polynomial as $ax^2 + bx + c$ where a, b, c represent the coefficients of our polynomial.

Problem

1) Factor the following second order polynomial and find the roots.

$x^2 - 5x + 6$

3 2

What multiply to 6 and adds to -5?

$3 \times 2 = 6$ and $3 + 2 = 5$

$-3 \times 2 = -6$ and $-3 + 2 = -1$

$3 \times -2 = -6$ and $3 - 2 = 1$

$-3 \times -2 = 6$ and $-3 - 2 = -5$

$(x - 3)(x - 2)$

Check → $(x - 3)(x - 2)$ $x^2 - 2x - 3x + 6$

$x^2 - 5x + 6$ ✓

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2) Factor the following second order polynomial and find the roots.

$3x^2 - 12x + 9$

$a \times c = 27 \rightarrow$ $-9 \times -3 = 27$

$b = -12$ $-9 - 3 = -12$

$3x^2 - 9x - 3x + 9$

$3x(x-3) - 3(x-3)$

So $\underline{(x-3)(3x-3)}$

Check $(x-3)(3x-3)$

$3x^2 - 3x - 9x + 9$

$3x^2 - 12x + 9$ ✓

3) Determine the values of a and b in the following expression:

$x^2 + (a - 5)x + b = (x - 3)(x + 4)$

$x^2 + 4x - 3x - 12$

$x^2 + x - 12$ So note $\rightarrow$ $x^2 + (a-5)x + b = x^2 + x - 12$

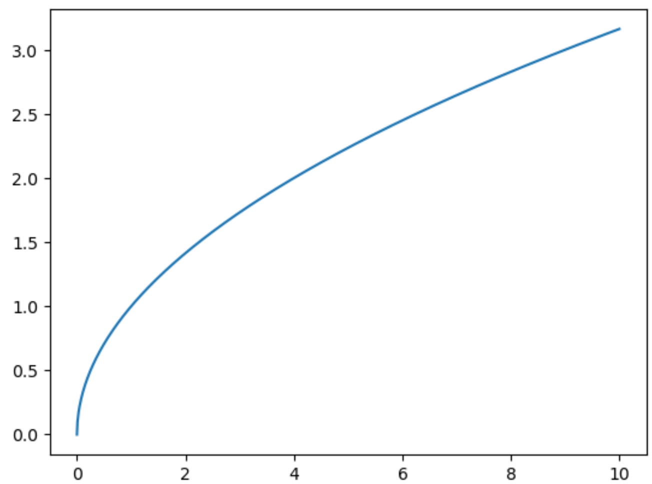
implying $b = -12$ and $a - 5 = 1$

$a - 5 = 1$
$+5 \quad +5$

$\underline{a = 6 \text{ and } b = -12}$

Square Root

Definition: The square root can be written as $f(x) = \sqrt{x}$. Every positive number has two square roots. One is the positive square root which is written as $\sqrt{x}$. The other is the negative square root which is written as $-\sqrt{x}$.



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Key Properties

1) $\sqrt{a} * \sqrt{b} = \sqrt{a * b}$
2) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ given that $b \neq 0$
3) $\sqrt{a} = a^{\frac{1}{2}}$

Problems

1) Solve the equation

$\sqrt{x + 3} = 5$ $\longrightarrow$ Rewrite as $(x+3)^{1/2} = 5$

$(x+3)^{1/2} = 5$

$((x+3)^{1/2})^2 = 5^2$

$x+3 = 25$

$x = 25-3$

$x = 22$

2) Solve the inequality

$\sqrt{x + 2} > 3$

$(x+2)^{1/2} > 3$

$((x+2)^{1/2})^2 > (3)^2$

$x+2 > 9$
$\;\;-2 \;\;\; -2$

$x > 7$

3) Simplify

$(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$

$(\sqrt{5}\sqrt{5}) + \cancel{(\sqrt{5}\sqrt{3})} - \cancel{(\sqrt{3}\sqrt{5})} - (\sqrt{3}\sqrt{3})$

$(\sqrt{5})^2 - (\sqrt{3})^2 = 2$

4) Simplify

$\sqrt{8x^2}$

$(\sqrt{8})(\sqrt{x^2})$ or $(8)^{1/2}(x^2)^{1/2}$

$\sqrt{8}\ x \longrightarrow \sqrt{(4)(2)}\ x$ notice $\sqrt{4} = 2$

$\boxed{\text{so } 2\sqrt{2}\,x}$

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Integer Exponents Properties

Definition: Given the expression x^n, x is the base and n is the exponent. Here are some properties of exponents.

1) $a^m a^n = a^{m+n}$
2) $(a^m)^n = a^{m*n}$
3) $a^{-m} = \frac{1}{a^m}$
4) $\frac{1}{a^{-m}} = a^m$
5) $\frac{a^m}{a^n} = a^{m-n}$ $if\ m > n$
6) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ $if\ n > m$
7) $a^0 = 1$
8) $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$
9) $(ab)^n = a^n b^n$
10) $(\frac{a}{b})^m = (\frac{a^m}{b^m})$, $b \neq 0$

Problems

1) Simplify the following expression.

$$(\frac{z^2 y^{-10} x^{-3}}{x^{-8} z^{-6} y^4})^{-4}$$

$\left(\frac{z^2 x^8 z^6}{y^{10} x^3 y^4}\right)^{-4}$ Apply Rule 3 + 4

$\left(\frac{z^8 x^8}{y^{14} x^3}\right)^{-4}$ Apply Rule 1

$\left(\frac{z^8 x^5}{y^{14}}\right)^{-4}$ Apply Rule 5

$\frac{1}{(\frac{z^8 x^5}{y^{14}})^4}$ Apply Rule 3

$\frac{1}{\frac{(z^8 x^5)^4}{(y^{14})^4}}$ Apply Rule 10

$\frac{(y^{14})^4}{(z^8 x^5)^4}$ Apply Rule 2 + 9

$\frac{y^{56}}{(z^8)^4 (x^5)^4}$

$\frac{y^{56}}{z^{32} x^{20}}$

2) Simplify the following expression.

$$\frac{(p^2)^{-3} q^{-4}}{(6q)^{-1} p^{-7}}$$

$= \frac{p^{-6} q^{-4}}{(6q)^{-1} p^{-7}}$

$= \frac{6q\, p^7}{p^6 q^4}$

$= \frac{6p}{q^3}$

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