Review for Pre-Calculus

This is a review worksheet to re-familiarize yourself with concepts learned prior to pre-calculus that may be relevant within topics we will study over the course of the program. This assignment is not required: it will not be turned in nor will it be graded, but you may find it useful to work through some of these problems, particularly those in areas you may not remember as well.

1 Factoring

Recall the following methods of factoring/root finding:

- **Complete the Square**: When given a polynomial of the form $ax^2 + bx$, we can write this as $\left(ax + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$.
- **Difference of Squares**: When given a polynomial of the form $a^2x^2 - b^2$, we can write this as $(ax - b)(ax + b)$.
- **Quadratic Formula**: A polynomial of the form $ax^2 + bx + c$ has roots of the form: $x_{1,2} = \frac{-(b) \pm \sqrt{b^2 - 4(a)(c)}}{2(a)}$.

With this, answer the following:

1. What is the discriminant? What can it tell us? (Think about the sign)
2. In what cases can we use the quadratic formula?
3. Solve the following equations using different methods:

   (a) $x^2 + 6x + 9 = 0$ (b) $7x^2 + 3x + 4 = 0$ (c) $x^2 + 6x - 16 = 0$ (d) $64x^2 - 56 = 0$

2 Radicals

Recall the following rules regarding simplifying radicals:

- No radicand contains a factor to a power greater than or equal to the index of the radical i.e., for $\sqrt[n]{b^m}$, then $m \ngeq n$.
- No power of the radicand and the index of the radical have a common factor other than 1 i.e., given the form above, n and m have no common factors.
- No radical appears in the denominator.
- No fraction appears within a radical.

1. How do you simplify an expression with a radical in the denominator?

2. Simplify the following expressions if they are not already fully simplified:

   (a) $\sqrt[8]{x^4}$ (b) $\sqrt[9]{x^{13}}$ (c) $\frac{\sqrt{\frac{5}{3}}}{\sqrt{2}}$

3 Complex Numbers

A complex number has both a real and an imaginary part. If of the form $a+ib$, a is the real component and ib is the imaginary component of the complex number. Recall the following regarding the rules of imaginary and complex numbers:

- We define $i=\sqrt{-1}$.
- Therefore: $i^2=-1$, $i^3=-i$, and $i^4=1$.
- The standard form of a complex number is expressed as $a+ib$.
- The complex conjugate of a complex number is found by negating the imaginary component of the complex number. The complex conjugate of $a+ib$ is $a-ib$.
- To rationalize expressions with a complex or imaginary number in the denominator, multiply both the denominator and the numerator of the expression by the complex conjugate of the denominator. If we have $\frac{1}{a+ib}$, we multiply the expression by $\frac{a-ib}{a-ib}$ to get $\frac{1}{a+ib} * \frac{a-ib}{a-ib} = \frac{a-ib}{a^2+b^2} = \frac{a}{a^2+b^2} - \frac{ib}{a^2+b^2}$.

Answer the following questions regarding complex and imaginary numbers:

1. When do we encounter complex numbers?

2. What is i^6 equivalent to? (Hint: rewrite the expression with two exponents)

3. Write the following expressions in standard complex form:

   (a) $\frac{\sqrt{-49}}{2+3i}$ (b) $83+16i-(3+4i)(1-2i)$

4 Sets of Numbers

We introduce the definitions for various sets of real numbers to allow for familiarity with the language.

- We can think of the set of **real numbers** as containing all points on the number line. It is often denoted $\mathcal{R}$.
- The set of **integers** contains all positive and negative integers (whole numbers) as well as 0 and it is often denoted with $\mathcal{Z}$.
- The set of **natural numbers** contains all integers greater than and equal to 1. It is often denoted $\mathcal{N}$.

- The set of **rational numbers** is often denoted with $\mathcal{Q}$ and contains all numbers that can be expressed as a quotient of two integers. $\frac{1}{3}$, 5, and $\frac{37}{43}$ are rational numbers, while π and $\sqrt{2}$ are not.

- The set of **irrational numbers** contains all numbers that are not rational.

5 Word Problems

1. A ball is launched from the ground. Its trajectory can be modelled by the following equation: $f(x) = -x^2 + 8x + 5$ where $f(x)$ denotes the ball's position. At what x-coordinate does the ball land? How far does the ball travel? (Hint: for the total distance, consider both roots.)

2. Olivia owns a flower shop that has $2,000$ dollars in the bank. Each arrangement sells for 20 dollars. The flower shop's bills total $3,000$, and Olivia would like to have *more than* $1,500$ dollars left over. How many arrangements does she need to sell to make this happen?

3. The width of a particular lawn is equal to 5 times its length. If the area must be *less than* 25 acres, what's the maximum value that the width can be?