Homework 2

Question 1

Find all of the zeros of $f(x) = x^3 - 4x^2 + x + 6$ **given that 2 is one of the roots of this equation.**
Solution

$$
\begin{array}{r|l}
 & x^2 - 2x - 3 \\
\hline
x - 2 & x^3 - 4x^2 + x + 6 \\
 & -\ (x^3 - 2x^2) \\
\hline
 & -2x^2 + x \\
 & -\ (-2x^2 + 4x) \\
\hline
 & 0 - 3x + 6 \\
 & -\ (-3x + 6) \\
\hline
 & 0
\end{array}
$$

$x^2 - 2x - 3$ factored is $(x-3)(x+1)$

So $(x-3)(x+1)(x-2)$ are facts meaning $x = 3, -1, 2$ are Roots.

Question 2

a) **Convert the exponential equation** $2^x = 8$ **to logarithmic form.**

Solution

Note: $log_a(x) = y \Leftrightarrow a^y = x$

$a = 2$
$x = 8$
$y = y$

Taking this into consideration, we get the log form of $log_2(8) = y$

b) **Convert the logarithmic equation** $log_2(2) + log_2(8x) = 4$ **into exponential form.**

Solution

Note: $log_2(2) = 1$

Taking this into consideration, we are able to rewrite the expression as $1 + log_2(8x) = 4$.

$1 + log_2(8x) = 4$

$log_2(8x) = 3$

So $a = 2$, $x = 8x$, and $y = 3$. Taking this into consideration, we can convert it to $2^3 = 8x$.
Taking it further, $2^3 = 8$ meaning that $x = 1$.

c) **Convert the logarithmic equation** $log_9(\sqrt[3]{27}) = \frac{1}{2}$ **to exponential form.**

Solution

Note: $\sqrt[3]{27} = 3$.

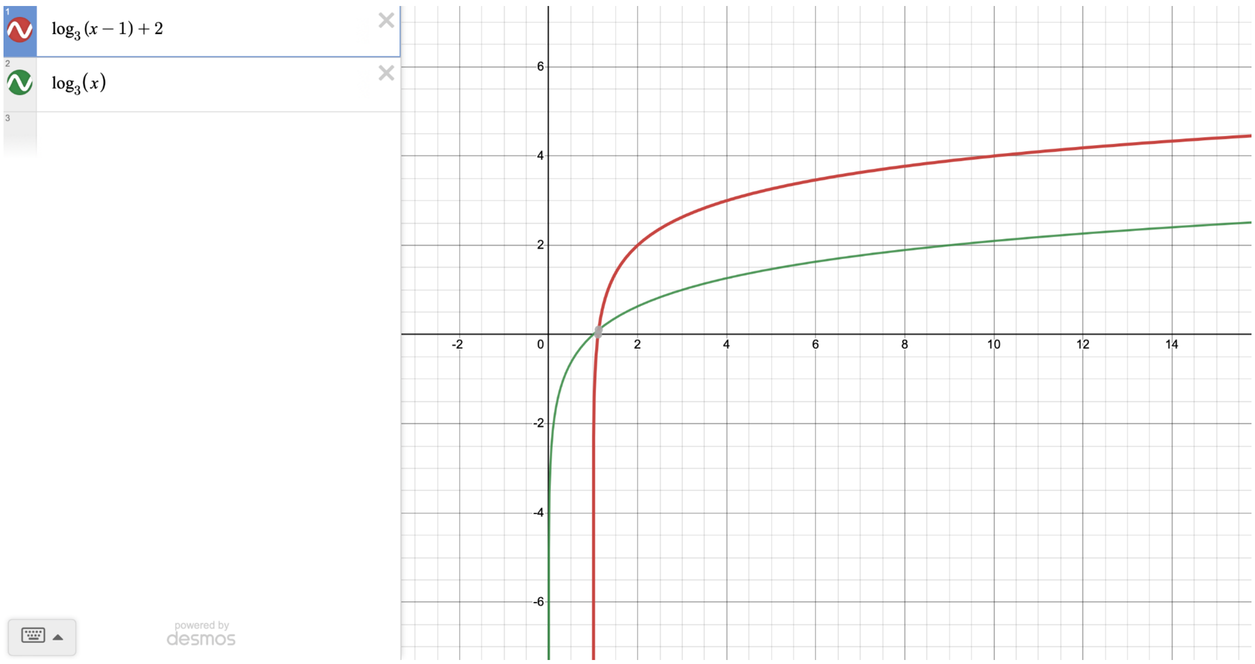
Homework 2

We are able to simplify this expression as $log_9(3) = \frac{1}{2}$. Using the formula above, we are able to get the exponential form of $9^{\frac{1}{2}} = 3$.

Question 3

Graph the function $f(x) = log_3(x - 1) + 2$. State the transformations of the function.

Solution



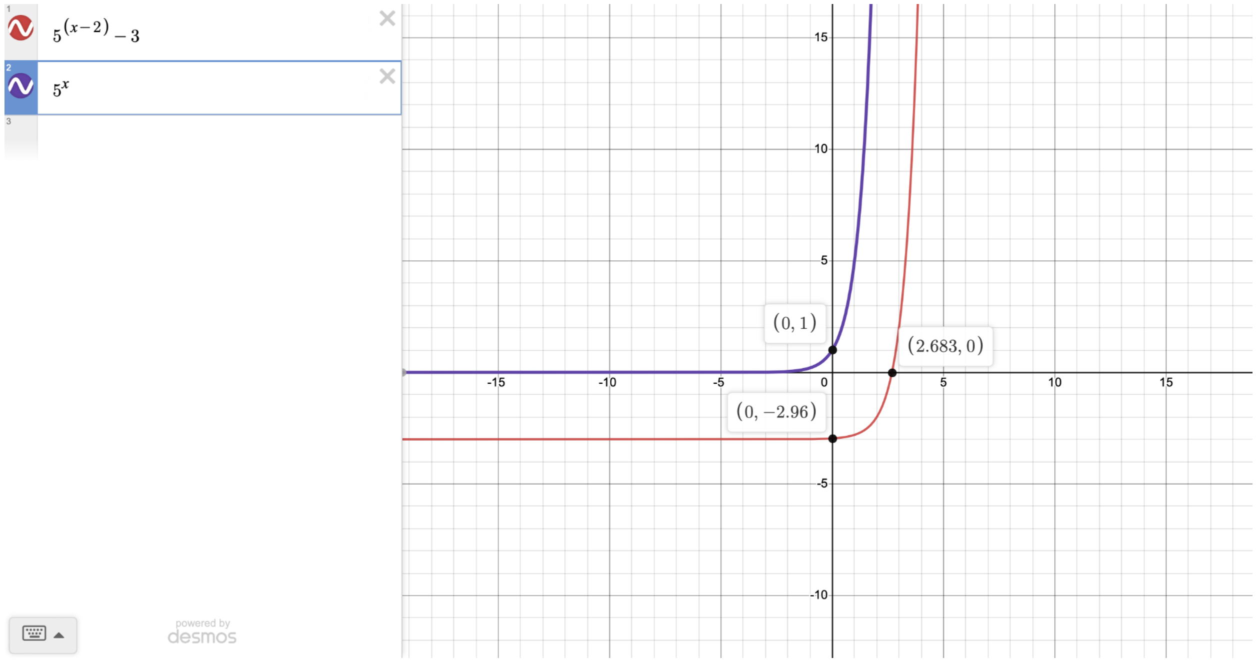
We can compare the transformed graph to the parent graph of $log_3(x)$. The transformed function will shift horizontally to the right one unit. It will then go up two units.

Homework 2

Question 4

Graph the exponential function $f(x) = 5^{x-2} - 3$. State the transformations compared to the parent function.

Solution



We can compared the transformed function to the parent function which is 5^x. The transformed function is going to move horizontally to the right 2 units. It will then go down three units.

Question 5

a) **Solve the equation $ln(5x - 1) + ln(3x + 2) = ln(56)$.**

Solution

We are able to use the rule that $ln(a) + ln(b) = ln(a * b)$ since the base is the same.

$ln((5x - 1)(3x + 2)) = ln(56)$

Note that e is the inverse of the natural log.

$e^{ln((5x-1)(3x+2))} = e^{ln(56)}$

$(5x - 1)(3x + 2) = 56$

$15x^2 + 10x - 3x - 2 = 56$

$15x^2 + 7x - 58 = 0$

This is now a root solving problem. I am going to use the quadratic equation.

$x = \frac{-7 \pm \sqrt{49-4(15)(-58)}}{30}$

So our roots are $x_1 = \frac{-7+\sqrt{3529}}{30}$ and $x_2 = \frac{-7-\sqrt{3529}}{30}$

Homework 2

b) **Solve the equation $log_{10}(2x - 3) - log_{10}(x + 4) = 1$.**

Solution

We are able to use the rule that $log_{10}(a) - log_{10}(b) = log_{10}(\frac{a}{b})$.

$log_{10}(\frac{2x-3}{x+4}) = 1$

We are then going to convert this expression into an exponential expression.

$a = 10$

$x = \frac{2x-3}{x+4}$

$y = 1$

Taking all of this into consideration, we get $10^1 = \frac{2x-3}{x+4}$

$10 = \frac{2x-3}{x+4}$

$10(x + 4) = 2x - 3$

$10x + 40 = 2x - 3$

$10x - 2x =- 3 - 40$ Combine like terms.

$8x =- 43$

$x = \frac{-43}{8}$

c) **Solve the equation $(\frac{1}{3})^{2x+1} = \frac{1}{9}$.**

Solution

Convert the expression into log form and get $log_{\frac{1}{3}}(\frac{1}{9}) = 2x + 1$

Note that $log_{\frac{1}{3}}(\frac{1}{9}) = 2$

So we are going to write the expression as $2 = 2x + 1$

$2 = 2x + 1$

$2 - 1 = 2x$

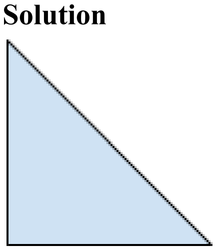
$1 = 2x$

$x = \frac{1}{2}$

Question 6

A right triangle has one length measuring 6 units and the hypotenuse measuring 10 units. What is the length of the remaining side?

Solution



$a^2 + b^2 = c^2$ is applicable because it is a right triangle.

Let $a = 6$ and $c = 10$

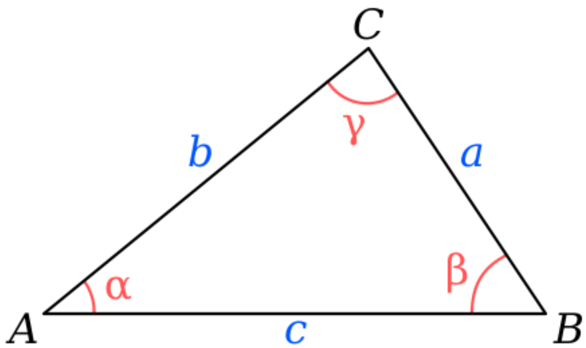
$b^2 = 10^2 - 6^2$

$b = \sqrt{10^2 - 6^2}$

$b = 8$

Question 7

Sarah, Sam, and Alex are camping in their own tents. The distance between Sarah and Sam is 153 ft, the distance between Sarah and Alex is 201 ft, and the distance between Sam and Alex is 175 ft. What is the angle between Sarah, Sam, and Alex?



Let length a be the distance between Sam and Sarah, b the distance between Sam and Alex, and c the distance between Sarah and Alex. We are going to find the angle gamma which corresponds to the length c.

Using the law of cosines, we get $201^2 = 153^2 + 175^2 - 2(153)(175)cos(\gamma)$.

Simplify by moving to the left hand side.

$201^2 - 153^2 - 175^2 =- 2(153)(175)cos(\gamma)$

$- 13633 =- 53550cos(\gamma)$

$\frac{-13633}{-53550} = cos(\gamma)$

Take the inverse cosine.

$cos^{-1}(\frac{13633}{53550}) = \gamma$

$\gamma = 75.25^\circ$

Question 8

a) Find the exact value of $tan(105^\circ)$ using one of the identities.

Solution

I am going to use the sum and differences identities.

We can rewrite the expression as $tan(60^\circ + 45^\circ)$.

Using this formula → $tan(a + b) = \frac{tan(a)+tan(b)}{1-tan(a)tan(b)}$

$tan(60 + 45) = \frac{tan(60)+tan(45)}{1-tan(60)tan(45)}$

Or we can use the half angle identities

Homework 2

$tan(\frac{\theta}{2}) \;=\; \pm\sqrt{\frac{1-cos(\theta)}{1+cos(\theta)}}$

$tan(\frac{210}{2}) \;=\; \sqrt{\frac{1-cos(210)}{1+cos(210)}}$ Positive value

b) Find the exact value of $sin(\frac{5\pi}{12})$ using one of the identities.

Solution

I am going to use the half angle identities.

$sin(\frac{\frac{5\pi}{6}}{2}) \;=\; \sqrt{\frac{1-cos(\frac{5\pi}{6})}{2}}$ Positive value