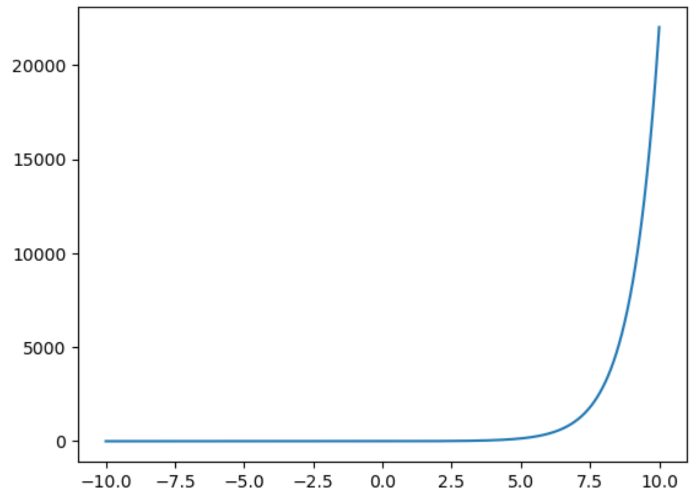
Lecture 5

Exponential Functions

Definition: The exponential function is written as $f(x) = a^x$ where $x \in \Re$ is a variable and a is a constant that is $a > 0$ and not $a \neq 1$.
Most of the time, we use the exponential base e. Note that e is a constant which is approximately 2.71828.
The exponential function can be written as $f(x) = e^x$ or sometimes we write $f(x) = exp(x)$.



The graph of the exponential function is upward and increases exponentially as x increases. The exponential function graph passes through the point $(0, 1)$. The x-axis is the horizontal asymptote.

Given the function $f(x) = a^x$ if $a < 1$ the function is decreasing. If $a > 1$ the function is increasing.

Exponential Function Rules

Note that $a > 0$, $b > 0$, $x \in \Re$, and $y \in \Re$.

1) $a^x a^y = a^{x+y}$
2) $\frac{a^x}{a^y} = a^{x-y}$
3) $(a^x)^y = a^{x*y}$
4) $a^x b^x = (ab)^x$
5) $(\frac{a}{b})^x = \frac{a^x}{b^x}$
6) $a^0 = 1$
7) $a^{-x} = \frac{1}{a^x}$

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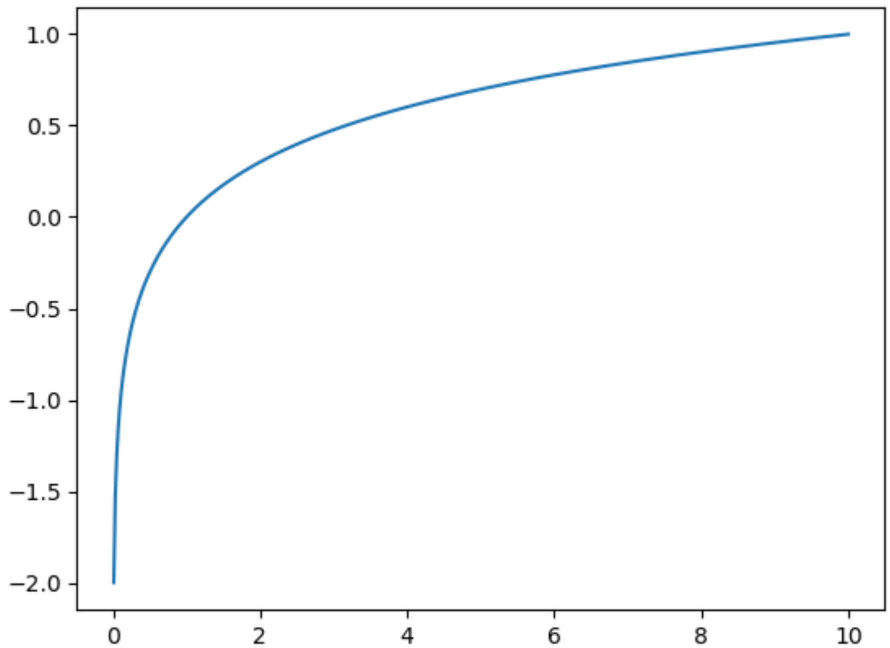
Logarithmic Functions

Definition: The logarithm is the inverse function to exponentiation. We are able to write the function as $f(x) = log_b(x)$ where b represents the base. The base $b > 0$ and cannot equal one and $x > 0$.

Note: We can also write $f(x) = log_b(x)$ is equivalent to $x = b^y$.



Here is a graph of the log function with base 10. The graph has a y- axis asymptote.

Given the function $f(x) = log_a(x)$ *if* $a > 1$ *the function is increasing . If* $0 < a < 1$ *the function is decreasing.*

Another important concept to understand is the natural log. We are able to write the natural log function as $f(x) = ln(x)$. The main difference between the log and natural log function is that natural log refers to the log function with a base e.

Logarithmic Function Rules

1) $log_b(mn) = log_b(m) + log_b(n)$
2) $log_b(\frac{m}{n}) = log_b(m) - log_b(n)$
3) $log_b(m^n) = nlog_b(m)$
4) $log_b(m) = \frac{log_a(m)}{log_a(b)}$

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Converting Logarithmic and Exponential Functions

For any number a, x, y with $a, x > 0$ $(a \neq 1)$, $log_a(x) = y$ if and only if $a^y = x$.

Problems

1) Rewrite the expression $log_4(16) = 2$ into an exponential expression

$\log_a(x) = y \longrightarrow a^y = x$

$a = 4$, $x = 16$, $y = 2$ $\rightarrow$ $4^2 = 16$ Exponential form $\rightarrow$ Is this expression true? Yes, cause $4 \times 4 = 16$

$\log_4(16) = 4$ logarithmic form

2) Rewrite the expression $4^y = 256$ as a logarithmic expression

$\log_a(x) = y \longrightarrow a^y = x$

$a = 4$, $y = y$, $x = 256$ $\rightarrow$ $\log_4(256) = y$ logarithmic expression.

using calculator, $\log_4(256) =$

3) Rewrite the expression $log_4(x^2 + 1) = 2log_4(x)$

LH RH

LH: $\log_4(x^2+1)$

RH: $2\log_4(x)$

$a = 4$, $y = 2\log_4(x)$, $x = x^2+1$ $\rightarrow$ $4^{2\log_4(x)} = x^2+1$

$4^{\log_4(x^2)} = x^2+1$

Graphing Exponential Functions

Characteristics of the Exponential Function Graph

1) One to one function
2) The graph will pass through the point $(0, 1)$ if the function is in format $f(x) = b^x$
3) If $b > 1 \rightarrow$ Graph is increasing
4) If $b < 1 \rightarrow$ Graph is decreasing

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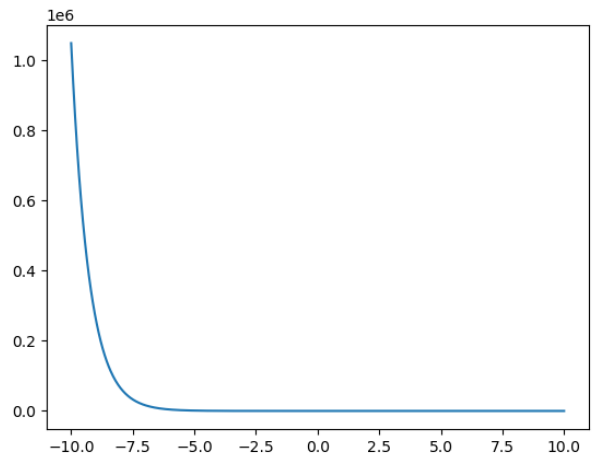
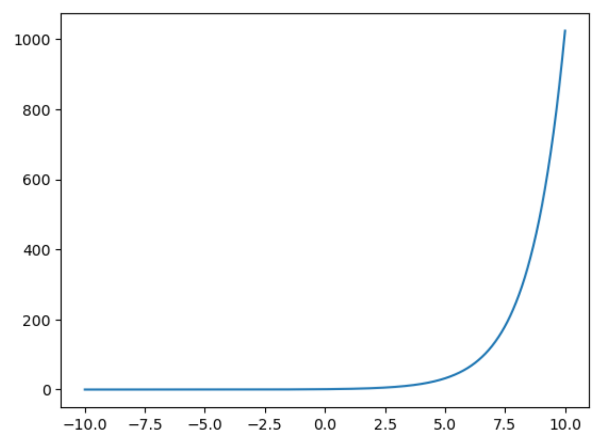
Function Formula	**Action**
$f(x) = b^{x+c} + d$	1) Graph shifts the function d units in the vertical direction. Keep in mind the sign of d to determine the up or down shift. 2) Graph shifts c units horizontally. Note the sign of c for left or right movement.
$f(x) = ab^{x}$	1) Graph will stretch vertically by factor a if $a > 1$. 2) Graph will be compressed vertically by factor a if $a < 1$.





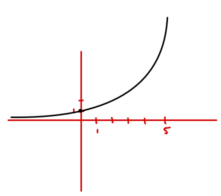
The graph on the left is decreasing and the graph on the right is increasing.

Example

Plot the function $f(x) = 2^{x}$

Given $f(x)=a^x$ note $a=2$ and $2>1$ So graph is increasing

x	y
0	1
1	2
5	32



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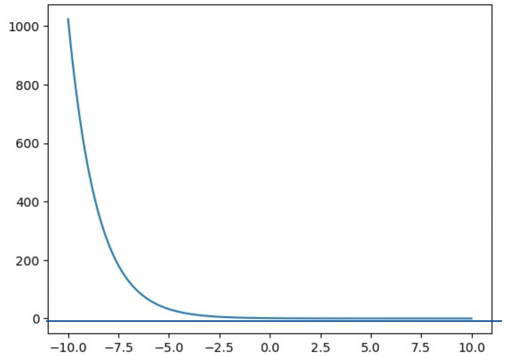
Problems

1) Plot the function $f(x) = 0.5^x$

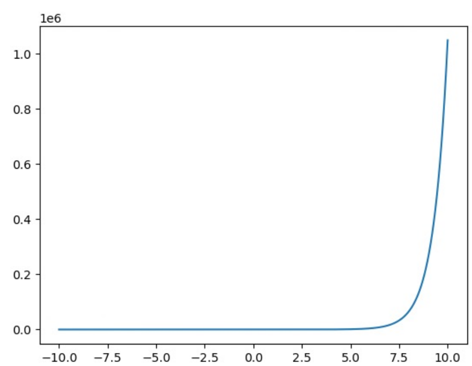


$a = 0.5 < 1$ So decreasing

x	y
0	$1 = 0.5^0$
1	$0.5 = 0.5^1$
10	$0.00097 = 0.5^{10}$

asymptotes at zero

2) Plot the function $f(x) = 4^x$



$a = 4 > 1$ So increasing

Graphing Logarithmic Functions

Characteristics of the Logarithmic Function Graph

1) One to one function
2) Given the function format $f(x) = log_b(x)$, the graph will increase if $b > 1$.
3) Given the function format $f(x) = log_b(x)$, the graph will decrease if $0 < b < 1$.

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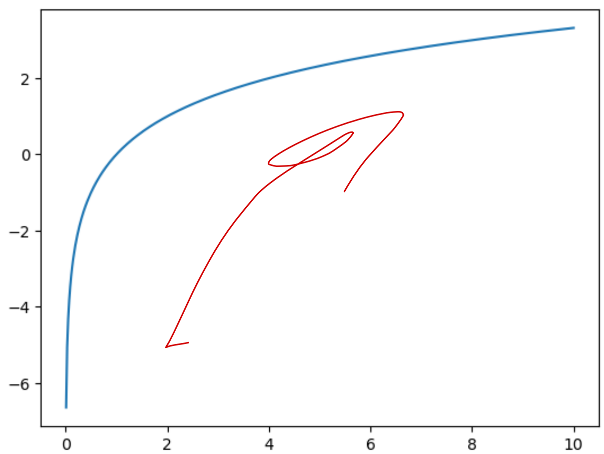
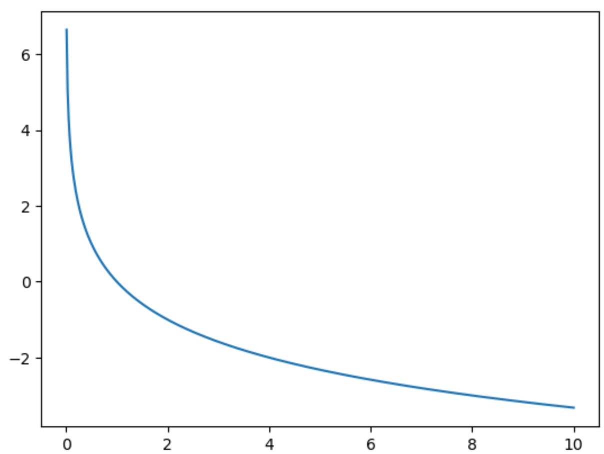
Function Format	Action
$f(x) = log_b(x + c)$	The graph will shift horizontally to the left c units.
$f(x) = log_b(x - c)$	The graph will shift horizontally to the right c units.
$f(x) = log_b(x) + d$	The graph will shift vertically up d units
$f(x) = log_b(x) - d$	The graph will shift vertically down d units.
$f(x) = alog_b(x)$ and $a > 1$	The graph will stretch vertically
$f(x) = alog_b(x)$ and $a < 1$	The graph will compress vertically.

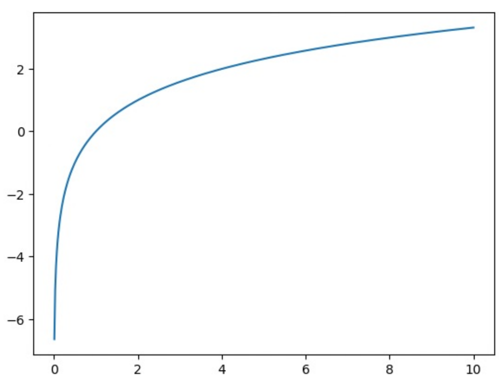




The graph on the left is an increasing graph and the right is a decreasing graph.

Example

Graph the following function $f(x) = log_2(x)$



$b = 2 > 1$ so increasing

$x = 0$ is the VA

$x = 1 \longrightarrow 0$

$x = 2 \longrightarrow \log_2(2) = 1$

$(1,0)$

$(2,1)$

$\log_3(3) = 1$

If $\log_2(x+1)+2$

we find $x+1=0 \rightarrow x=-1$ this is VA

$x+1=1 \rightarrow 0$

$x+1=2 \rightarrow 1$

$\boxed{\begin{array}{l} x=0 \rightarrow \log_2(1)+2 = 2 \\ x=1 \rightarrow \log_2(2)+2 = 1+2 = 3 \end{array}}$

Plot these and VA $= -1$

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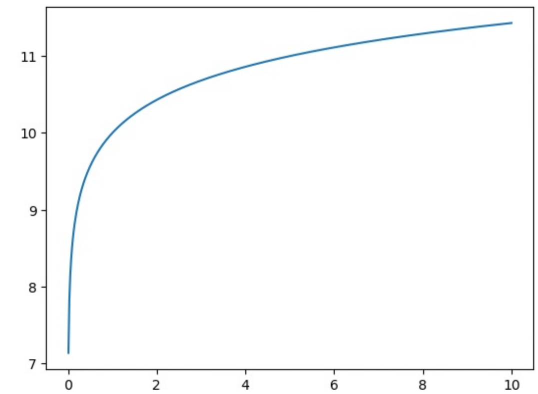
Problem

Graph the following function $f(x) = log_5(x) + 10$



Solve Exponential Equations

Example

Solve for x given the expression $9^{x+4} = 27^{1-x}$

$(3^2)^{(x+4)} = (3^3)^{(1-x)}$

$(3)^{2(x+4)} = (3)^{3(1-x)}$ this is only true if $2(x+4) = 3(1-x)$

$2x+8 = 3-3x$

$2x+3x = 3-8$

$5x = -5$

$x = -5/5 = -1$

Problems

1) Solve the following exponential equation $2^{3x} = 16$

$2^{3x} = 2^4 \rightarrow 3x = 4$ or $x = \frac{3}{4}$

2) Solve the following exponential equation $e^{2x+3} = 7$

$\ln(e^{2x+3}) = \ln(7)$

$2x+3 = \ln(7)$

$2x = \ln(7) - 3$

$x = \frac{\ln(7)-3}{2}$

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3) Solve the following exponential equation $4^{x^2-1} = \frac{1}{64}$

$\log_a(x) = y \iff a^y = x$

$\log_4(\frac{1}{64}) = x^2 - 1 \longrightarrow x^2 = \log_4(\frac{1}{64}) + 1$

Note: $\log_4(\frac{1}{64})$ is -3

meaning $-3+1=-2$

Solving Logarithmic Equations

Solve for x given the expression $8 + log_5(x + 4) = 9$

$\log_5(x+4) = 1$

$5^1 = x+4$

$5 = x+4 \rightarrow x = 1$

Problems

1) Solve the following logarithmic equation $log_2(x) = 3$

$2^3 = x$

2) Solve the following logarithmic equation $2log_3(x) = log_3(9)$

$\log_3(x^2) = \log_3(9)$

$3^{\log_3(9)} = x^2$

$3^2 = x^2 \rightarrow x = \pm 3$

3) Solve the following logarithmic equation $3log_3(x + 1) - log_2(4) = log_2(8)$

$3\log_3((x+1)) - \log_2(4)$

$3\log_3((x+1)) = \log_2(8) + \log_2(4)$

$\log_3(x+1) = \frac{3+2}{3}$

$\log_3(x+1) = \frac{5}{3}$

$3^{5/3} = x+1$

$x = 3^{5/3} - 1$