Homework 1 Solutions

Question 1

a) Determine the solution set for the absolute value inequality $|2x + 5| \leq 3x - 2$.

Solution

Note that we have functions on both sides.

$2x + 5 \geq -3x + 2$

$5x \geq -3$

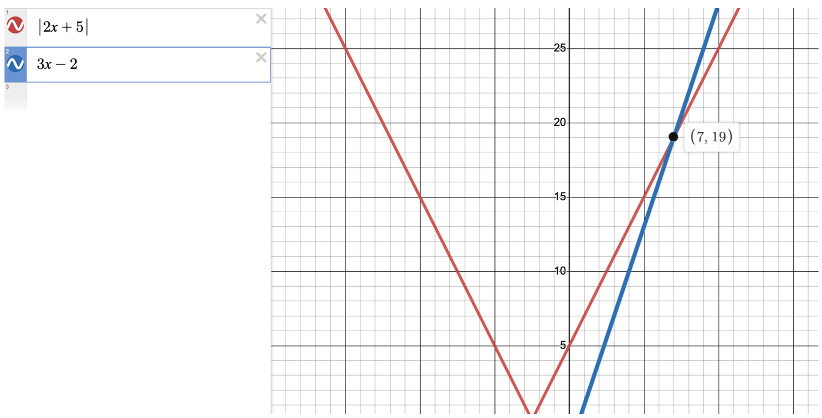
$x \geq \frac{-3}{5}$

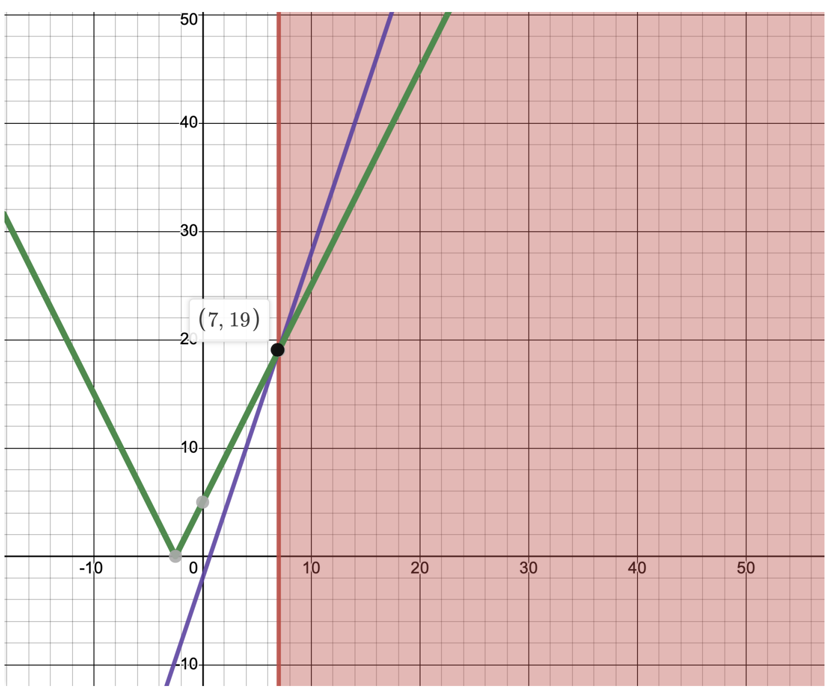
$2x + 5 \leq 3x - 2$

$-x \leq -7$

$x \geq 7$

The answer is $x \geq 7$. Note that $x \geq \frac{-3}{5}$ is not true and we are able to graphically visualize the results.





Homework 1 Solutions

b) Solve the absolute value inequality $|x^2 - 4| \geq 3$.

Solution

Using the properties listed in question 1, we are able to write the expression below.

Case 1: $x^2 - 4 \geq 3$ or Case 2: $x^2 - 4 \leq -3$

Case 1 : $x^2 - 4 \geq 3$

$x^2 \geq 7$

$x \geq 7$

$x \leq -7$

Important to notice the square root properties.

Case 2 : $x^2 - 4 \leq -3$

$x^2 \leq 1$

$x \leq 1$

$x \geq -1$

So the solutions are $x \geq 7$ or $x \leq -7$ or $x \leq 1$ or $x \geq -1$.

c) Solve the absolute value inequality $|x - 2| + 3 > 9$.

Solution

Moving over the 3 to the right hand side, we get the following expression.

$|x - 2| > 6$

Using property 3, we are able to get the following expression.

Case 1 : $x - 2 > 6$ or Case 2: $x - 2 < -6$

Case 1: $x - 2 > 6$

$x > 8$

Case 2 : $x - 2 < -6$

$x < -4$

So the solution is $x > 8$ or $x < -4$.

Question 2

a) Completely factor the expression $2x^3 - 8x^2 - 12x$.

Solution

$2x(x^2 - 4x - 6)$

b) Factor the quadratic expression $2x^2 + 11x + 15$

Solution

Since we cannot immediately factor out a number, we are going to use the AC Method.

$A * C = 30$

$B = 11$

Find numbers that multiply to 30 and add to 11.

We only have one possibility which is 6 and 5 since $6 * 5 = 30$ and $6 + 5 = 11$.

Then, we will use factor by grouping and rewrite it as the expression below.

$2x^2 + 6x + 5x + 15$

Homework 1 Solutions

$2x(x + 3) + 5(x + 3)$
Since our inner expression $(x + 3)$ is the same for both groups, we will proceed with the factor by grouping method.

Our final answer is $(2x + 5)(x + 3)$.
Check our answer by foiling which will give $2x^2 + 6x + 5x + 15$ which is the same expression we generated above.

Question 3

a) Evaluate the value of $\frac{(-3a)^4(-3a)^3}{(-3a)^2(-3a)^1}$.

Solution

$\frac{(-3a)^7}{(-3a)^3}$

$(-3a)^4$

Final Answer : $81a^4$

b) Simplify the expression $\frac{(3x^{-4}y^{-3}-3x^{-3}y^{-4})}{(3x^{-2}y-3xy^{-2})}$.

Solution
We can rewrite the above expression into the formula below.

$\frac{\frac{3}{x^4y^3}-\frac{3}{x^3y^4}}{\frac{3y}{x^2}-\frac{3x}{y^2}}$

$\frac{\frac{3x^3y^4-3x^4y^3}{x^7y^7}}{\frac{3y^3-3x^3}{x^2y^2}}$

$\frac{3x^3y^4-3x^4y^3}{x^7y^7}\left(\frac{x^2y^2}{3y^3-3x^3}\right)$

$\frac{3(x^3y^4-x^4y^3)}{x^5y^5}\left(\frac{1}{(y^3-x^3)3}\right)$

$\frac{x^3y^4-x^4y^3}{x^5y^5(y^3-x^3)}$

$\frac{x^3y^3(y-x)}{x^5y^5(y^3-x^3)}$

$\frac{(y-x)}{x^2y^2(y^3-x^3)}$

$\frac{(y-x)}{x^2y^2(y-x)(x^2+xy+y^2)}$

Final Answer : $\frac{1}{x^2y^2(x^2+xy+y^2)}$

<u>Homework 1 Solutions</u>

Question 4

a) Simplify $\sqrt{18} - \sqrt{8}$.

Solution

We can rewrite the expression below.

$\sqrt{9 * 2} - \sqrt{4 * 2}$

$\sqrt{9}\sqrt{2} - \sqrt{4}\sqrt{2}$

Note that $\sqrt{9} = 3$ and $\sqrt{4} = 2$

$3\sqrt{2} - 2\sqrt{2}$

$\sqrt{2}(3 - 2)$

Final Answer : $\sqrt{2}$

b) Evaluate $\sqrt{4x^2 + 9y^2}$ when $x = 3$ and $y = 2$.

Solution

Plug in $x = 3$ and $y = 2$ into the function

$\sqrt{4(3)^2 + 9(2)^2}$

$\sqrt{36 + 36}$

$\sqrt{72}$

$\sqrt{9 * 8}$

$\sqrt{9}\sqrt{8}$

$3\sqrt{8}$

$3\sqrt{4 * 2}$

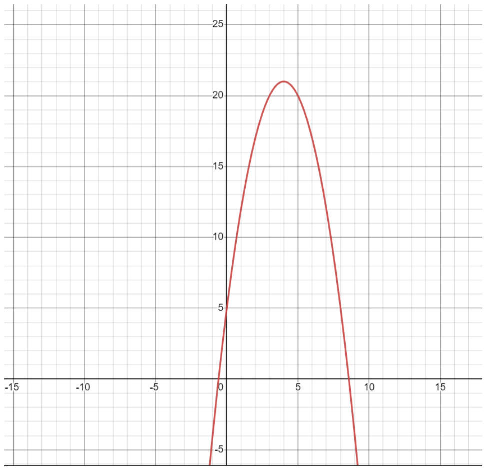
$3\sqrt{4}\sqrt{2}$

Final Answer : $6\sqrt{2}$

Question 5

A ball is launched from the ground. Its trajectory can be modeled by the following equation: $f(x) = -x^2 + 8x + 5$ where $f(x)$ denotes the ball's position. At what x-coordinate does the ball land? How far does the ball travel? (Hint: for the total distance, consider both roots.)

Homework 1 Solutions



Consider the graph of the equation when solving this problem. When determining at which x-coordinate the ball lands, we want to find the larger root of the equation if we assume that the ball is launched from the smaller root. This makes sense – when f(x) = 0, this means that the ball is on the ground. To find the roots, we use the quadratic formula where a = -1, b = 8, and c = 5. This gives us $x_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4(-1)(5)}}{2(-1)} = \frac{-8 \pm \sqrt{84}}{-2}$. Because $\frac{-8-\sqrt{84}}{-2} > \frac{-8+\sqrt{84}}{-2}$ and we are assuming that the ball was launched from the smaller root, the x-coordinate of the ball upon hitting the ground after being launched is given by

$$x_{land} = \frac{-8-\sqrt{84}}{-2}.$$

To calculate the total distance, we must note that the ball is not launched from the $x = 0$ coordinate, but rather from the $x_{launch} = \frac{-8+\sqrt{84}}{-2}$. To find the total distance traveled from launch to land, we simply subtract the launch coordinate from the land coordinate i.e.,

$$\text{Total x-distance} = \Delta x = x_{land} - x_{launch} = \frac{-8-\sqrt{84}}{-2} - \frac{-8+\sqrt{84}}{-2} = \sqrt{84} \approx 9.165\ units.$$

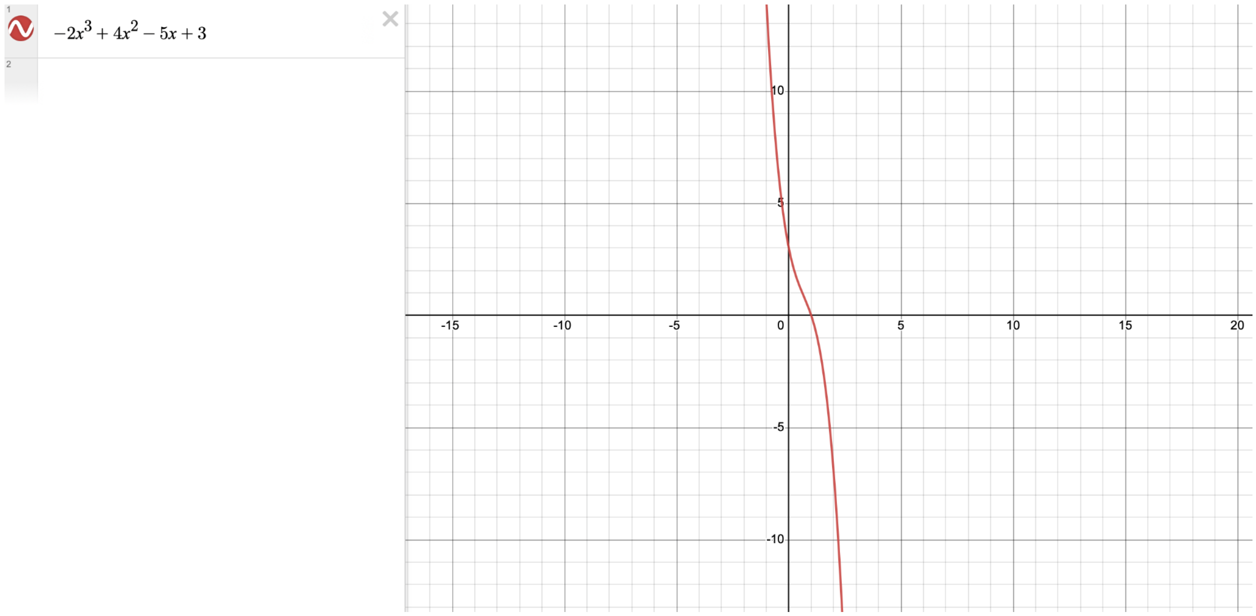
Question 6

Determine the end behavior of the polynomial $f(x) = -2x^3 + 4x^2 - 5x + 3$. Plot the polynomial. Show the results

Solutions

The highest polynomial degree is three and the coefficient is negative. So based on the table, the polynomial will fall to the right and rise to the left. We can represent it in limit form as $\lim_{x \to +\infty} f(x) = -\infty$ and $\lim_{x \to -\infty} f(x) = +\infty$.

Homework 1 Solutions



Question 7

a) **Find the domain of the rational function:** $f(x) = \frac{x^2-4}{x+2}$**.**

Solution

The domain of the function is $x > -2$ and $x < -2$. Note that $x = -2$ is a singularity point and doesn't exist. It is a hole in the function.

b) **Determine the vertical asymptote(s) of the rational function:** $f(x) = \frac{3x-1}{x^2-4}$**.**

Solution

The vertical asymptote is found by setting the denominator to zero.

$x^2 - 4 = 0$

$x^2 = 4$

$x = -2$ and $x = 2$

c) **Find the horizontal asymptote(s) of the rational function:** $f(x) = \frac{5x^3+2x^2-3}{3x^3+x+1}$**.**

Solution

Since the highest power for the numerator and denominator is the same, the horizontal asymptote is going to be the coefficient ratio.

The answer is $\frac{5}{3}$

d) **Find the x-coordinate(s) of any holes in the rational function:** $f(x) = \frac{x^2-9}{x^2-4}$**.**

Solution

We can factor the rational function as follows.

$f(x) = \frac{(x+3)(x-3)}{(x-2)(x+2)}$

Since none of the factors cancel out, we do not have any holes in the function.

Question 8

Divide the polynomial $4x^3 + 2x^2 - 3x + 1$ by $2x - 1$

Solution

$$\begin{array}{r l}
 & 2x^2 + 2x - \tfrac{1}{2} \\
2x-1 & \overline{)\,4x^3 + 2x^2 - 3x + 1} \\
 & -\ 4x^3 - 2x^2 \\ \hline
 & 0 \quad 4x^2 - 3x \\
 & -\quad 4x^2 - 2x \\ \hline
 & \qquad -x + 1 \\
 & \quad -\ \ -x + \tfrac{1}{2} \\ \hline
 & \qquad\quad \tfrac{1}{2}
\end{array}$$

Remainder: $\tfrac{1}{2}$

So Answer: $2x^2 + 2x - \frac{1}{2} + \frac{1/2}{2x-1}$