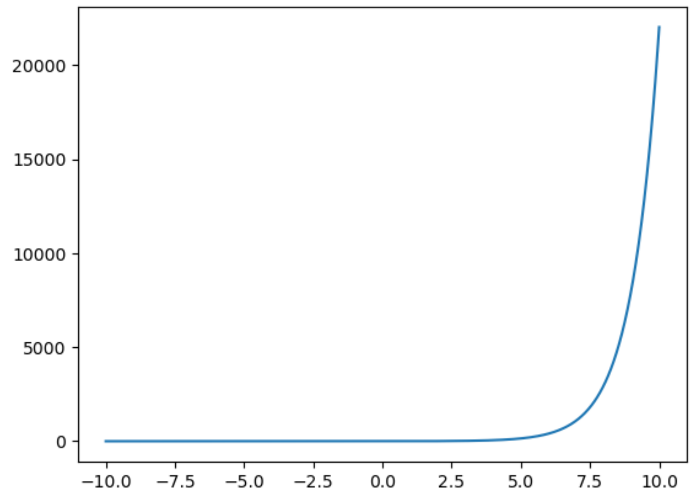
Lecture 5

Exponential Functions

Definition: The exponential function is written as $f(x) = a^x$ where $x \in \Re$ is a variable and a is a constant that is $a > 0$ and not $a \neq 1$.
Most of the time, we use the exponential base e. Note that e is a constant which is approximately 2.71828.
The exponential function can be written as $f(x) = e^x$ or sometimes we write $f(x) = exp(x)$.



The graph of the exponential function is upward and increases exponentially as x increases. The exponential function graph passes through the point $(0, 1)$. The x-axis is the horizontal asymptote.

Given the function $f(x) = a^x$ if $a < 1$ the function is decreasing. If $a > 1$ the function is increasing.

Exponential Function Rules

Note that $a > 0$, $b > 0$, $x \in \Re$, and $y \in \Re$.

1) $a^x a^y = a^{x+y}$
2) $\frac{a^x}{a^y} = a^{x-y}$
3) $(a^x)^y = a^{x*y}$
4) $a^x b^x = (ab)^x$
5) $(\frac{a}{b})^x = \frac{a^x}{b^x}$
6) $a^0 = 1$
7) $a^{-x} = \frac{1}{a^x}$

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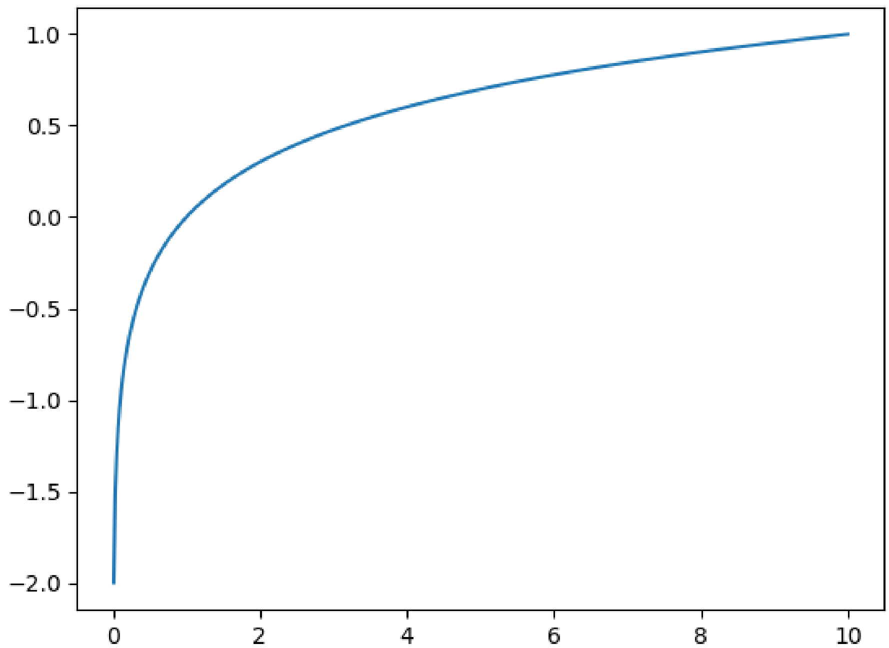
Logarithmic Functions

Definition: The logarithm is the inverse function to exponentiation. We are able to write the function as $f(x) = log_b(x)$ where b represents the base. The base $b > 0$ and cannot equal one and $x > 0$.

Note: We can also write $f(x) = log_b(x)$ is equivalent to $x = b^y$.



Here is a graph of the log function with base 10. The graph has a y- axis asymptote.

Given the function $f(x) = log_a(x)$ *if* $a > 1$ *the function is increasing . If* $0 < a < 1$ *the function is decreasing.*

Another important concept to understand is the natural log. We are able to write the natural log function as $f(x) = ln(x)$. The main difference between the log and natural log function is that natural log refers to the log function with a base e.

Logarithmic Function Rules

1) $log_b(mn) = log_b(m) + log_b(n)$
2) $log_b(\frac{m}{n}) = log_b(m) - log_b(n)$
3) $log_b(m^n) = nlog_b(m)$
4) $log_b(m) = \frac{log_a(m)}{log_a(b)}$

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Converting Logarithmic and Exponential Functions

For any number a, x, y *with* $a, x > 0$ $(a \neq 1)$, $log_a(x) = y$ *if and only if* $a^y = x$.

Problems

1) Rewrite the expression $log_4(16) = 2$ into an exponential expression

2) Rewrite the expression $4^y = 256$ as a logarithmic expression

3) Rewrite the expression $log_4(x^2 + 1) = 2log_4(x)$

Graphing Exponential Functions

Characteristics of the Exponential Function Graph

1) One to one function
2) The graph will pass through the point $(0, 1)$ if the function is in format $f(x) = b^x$
3) If $b > 1$→ Graph is increasing
4) If $b < 1$→ Graph is decreasing

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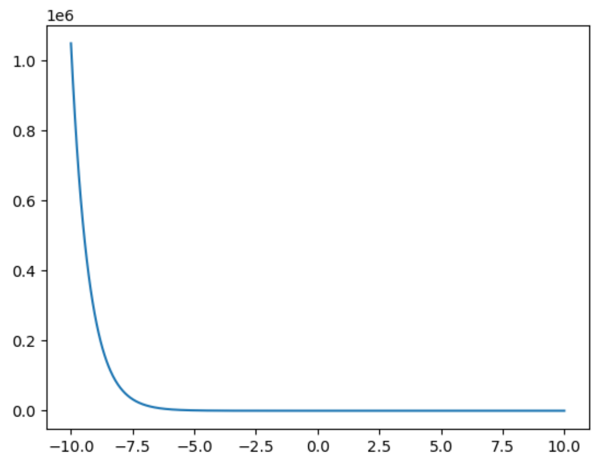
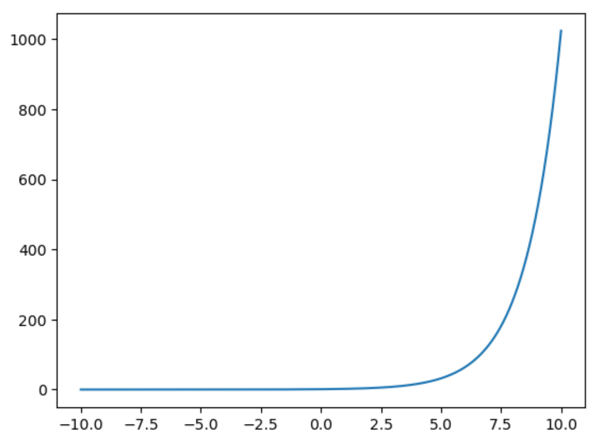
Function Formula	**Action**
$f(x) = b^{x+c} + d$	1) Graph shifts the function d units in the vertical direction. Keep in mind the sign of d to determine the up or down shift. 2) Graph shifts c units horizontally. Note the sign of c for left or right movement.
$f(x) = ab^{x}$	1) Graph will stretch vertically by factor a if $a > 1$. 2) Graph will be compressed vertically by factor a if $a < 1$.





The graph on the left is decreasing and the graph on the right is increasing.

Example

Plot the function $f(x) = 2^{x}$

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Problems

1) Plot the function $f(x) = 0.5^x$

2) Plot the function $f(x) = 4^x$

Graphing Logarithmic Functions

Characteristics of the Logarithmic Function Graph

1) One to one function
2) Given the function format $f(x) = log_b(x)$, the graph will increase if $b > 1$.
3) Given the function format $f(x) = log_b(x)$, the graph will decrease if $0 < b < 1$.

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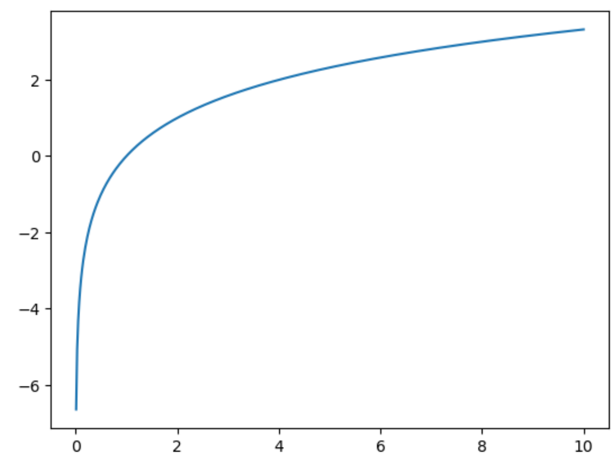
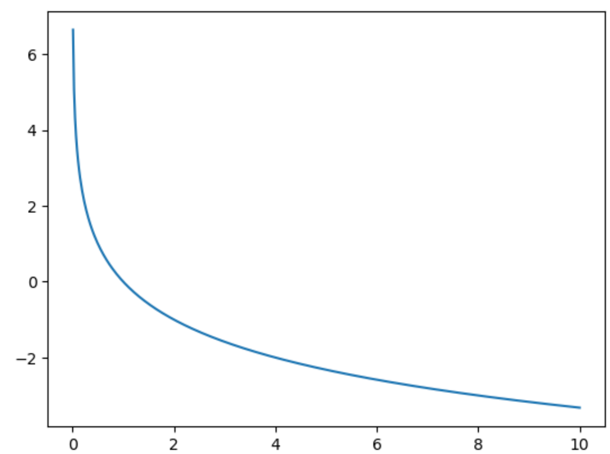
Function Format	**Action**
$f(x) = log_b(x + c)$	The graph will shift horizontally to the left c units.
$f(x) = log_b(x - c)$	The graph will shift horizontally to the right c units.
$f(x) = log_b(x) + d$	The graph will shift vertically up d units
$f(x) = log_b(x) - d$	The graph will shift vertically down d units.
$f(x) = alog_b(x)$ and $a > 1$	The graph will stretch vertically
$f(x) = alog_b(x)$ and $a < 1$	The graph will compress vertically.





The graph on the left is an increasing graph and the right is a decreasing graph.

Example
Graph the following function $f(x) = log_2(x)$

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Problem

Graph the following function $f(x) = log_5(x) + 10$

Solve Exponential Equations

Example

Solve for x given the expression $9^{x+4} = 27^{1-x}$

Problems

1) Solve the following exponential equation $2^{3x} = 16$

2) Solve the following exponential equation $e^{2x+3} = 7$

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3) Solve the following exponential equation $4^{x^2-1} = \frac{1}{64}$

Solving Logarithmic Equations

Solve for x given the expression $8 + log_5(x + 4) = 9$

Problems

1) Solve the following logarithmic equation $log_2(x) = 3$

2) Solve the following logarithmic equation $2log_3(x) = log_3(9)$

3) Solve the following logarithmic equation $3log_3(x + 1) - log_2(4) = log_2(8)$