Engineering
Mathematics and Computing

Task 1: Coursework Assessment

Student Name: Sakariye Abiikar
KID: 2371673

Last Updated: October 19, 2024
Submission Deadline: November 7, 2024, 5pm

Git Repo : https://github.com/sakx7/mathcompuni

Chapter 1

Part A: Mathematics

Notes:

1.1 Q1

Use the quotient rule to differentiate the function $y = \frac{\ln 3x}{2x}$

The quotient rule states that if

$$y = \frac{u}{v} \quad \text{then} \quad y' = \frac{u'v - uv'}{v^2}$$

Here:

$$u = \ln 3x \qquad\qquad v = 2x$$

First, find the derivatives:

$$u = \ln(3x)$$

Let:

$$\delta = 3x \qquad u = \ln(\delta)$$

$$\frac{du}{d\delta} = \frac{1}{\delta} \qquad \frac{d\delta}{dx} = 3$$

Using the chain rule, we can find:

$$\frac{du}{dx} = \frac{du}{d\delta} \cdot \frac{d\delta}{dx}$$

$$\frac{du}{dx} = \frac{3}{\delta} = \frac{3}{3x} = \frac{1}{x}$$

General rule for composite/nested functions is solved via chain rule it's:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

$$v = 2x$$

$$v' = \frac{d}{dx}(2x) = 2$$

Now we can apply the quotient rule:

$$y' = \frac{\left(\frac{1}{x}\right)(2x) - (\ln 3x)(2)}{(2x)^2} = \frac{2 - 2\ln 3x}{4x^2}$$

$$\boxed{y' = \frac{1 - \ln 3x}{2x^2}}$$

1.2 Q2

Find the angle between the vectors $2i - 11j - 10k$ and $5i + 8j + 7k$

The angle θ between two vectors **a** and **b** is given by:

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|}$$

Calculate the dot product:

$$\mathbf{a} \cdot \mathbf{b} = \underbrace{(2)(5)}_{i} + \underbrace{(-11)(8)}_{j} + \underbrace{(-10)(7)}_{k} = -148$$

Calculate the magnitudes:

$$\|\mathbf{a}\| = \sqrt{2^2 + (-11)^2 + (-10)^2} = \sqrt{225} = 15$$

$$\|\mathbf{b}\| = \sqrt{5^2 + 8^2 + 7^2} = \sqrt{138}$$

Find $\cos\theta$:

$$\cos\theta = \frac{-148}{15 \times \sqrt{138}}$$

Thus, the angle θ is:

$$\boxed{\theta = \cos^{-1}\left(\frac{-148}{15 \times \sqrt{138}}\right) \approx 147.1^\circ}$$

1.3 Q3

Find the rate of change of $y = \ln(16t^2 + 19)$ at the specified point $t = 9$

Differentiate y with respect to t:

Note prior (q1) that the general rule for composite/nested functions is:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

Here for $y = \ln(16t^2 + 19)$

$$\frac{dy}{dt} = \frac{1}{16t^2 + 19} \cdot \frac{d}{dt}(16t^2 + 19)$$

$$\frac{d}{dt}(16t^2 + 19) = 32t$$

$$\frac{dy}{dt} = \frac{1}{16t^2 + 19} \cdot 32t = \frac{32t}{16t^2 + 19}$$

Evaluate at $t = 9$:

$$\boxed{\left.\frac{dy}{dt}\right|_{t=9} = \frac{32(9)}{16(9)^2 + 19} = \frac{288}{1315} \approx 0.219}$$

1.4 Q4

Express $\cos t - 8\sin t$ in the form $A\cos(\omega t + \alpha)$, where $\alpha \geq 0$

you see the thing is the left side, $\cos t - 8\sin t$, is standard, but the right side introduces a different frequency, ω, in $A\cos(\omega t + \alpha)$.

Since the left side has a frequency of 1, im just gonna assume $\omega = 1$ for simplicity. If ω were different, you would need to rewrite the left side in terms of ωt, but this isn't specified in the problem.

The angle subtraction formula is:

$$A\cos(t - \phi) = A\cos(\phi)\cos(t) + A\sin(\phi)\sin(t)$$

By comparing coefficients from both sides, we have:

$$a = A\cos(\phi)$$
$$b = A\sin(\phi)$$

To find A we use $A = \sqrt{a^2 + b^2}$

This arises from squaring both equations $a = A\cos(\phi)$ and $b = A\sin(\phi)$:

$$a^2 + b^2 = (A\cos(\phi))^2 + (A\sin(\phi))^2 = A^2(\cos^2(\phi) + \sin^2(\phi)) = A^2$$

To find the phase shift ϕ, we use $\tan\phi = \frac{b}{a}$

This comes from the definitions of sine and cosine:

$$\tan\phi = \frac{A\sin(\phi)}{A\cos(\phi)} = \frac{b}{a}$$

In so we derive and make use of:

$$A\cos(t - \phi) = a\cos(t) + b\sin(t)$$

Where $A = \sqrt{a^2 + b^2}$ and $\tan\phi = \frac{b}{a}$

---

for $b = -8$ and $a = 2$

$$A = \sqrt{1^2 + (-8)^2} = \sqrt{65}$$

$$\tan\phi = \frac{-8}{1} = -8 \qquad \phi = \arctan(-8) \approx -82.87°$$

In so plugging in gives

$$\begin{aligned}\cos t - 8\sin t &= A\cos(t - \phi)\\ &= \sqrt{65}\cos(t - \arctan(-8))\\ &= \sqrt{65}\cos(t - (-82.87°))\end{aligned}$$

$$\boxed{\cos t - 8\sin t \approx 8.06\cos(t + 82.87°)}$$

1.5 Q5

Solve the following system of three linear equations using Cramer's rule

$$\begin{cases} 11v_1 - v_2 + v_3 = 31.4 \\ v_1 + \frac{v_2}{2} - v_3 = 1.9 \\ -9v_1 + 11v_3 = -12 \end{cases}$$

The system can be written in matrix form $A\mathbf{v} = \mathbf{b}$, specifically because of the variable distributions where:

$$A = \begin{bmatrix} 11 & -1 & 1 \\ 1 & \frac{1}{2} & -1 \\ -9 & 0 & 11 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 31.4 \\ 1.9 \\ -12 \end{bmatrix}$$

Calculate the determinant $\det(A)$.

$$\det(A) = 67$$

Now solve for each variable using Cramer's rule:

$$A_1 = \begin{bmatrix} 31.4 & -1 & 1 \\ 1.9 & \frac{1}{2} & -1 \\ -12 & 0 & 11 \end{bmatrix} \quad A_2 = \begin{bmatrix} 11 & 31.4 & 1 \\ 1 & 1.9 & -1 \\ -9 & -12 & 11 \end{bmatrix} \quad A_3 = \begin{bmatrix} 11 & -1 & 31.4 \\ 1 & \frac{1}{2} & 1.9 \\ -9 & 0 & -12 \end{bmatrix}$$

Calculate the determinants:

$$\det(A_1) \approx 187.6 \quad \det(A_2) \approx 40.2 \quad \det(A_3) \approx 80.4$$

$\mathbf{v}$ can be found as:

$$v_1 = \frac{\det(A_1)}{\det(A)}, \quad v_2 = \frac{\det(A_2)}{\det(A)}, \quad v_3 = \frac{\det(A_3)}{\det(A)}$$

$$\boxed{\begin{aligned} v_1 &= \tfrac{187.6}{67} = 2.8 \\ v_2 &= \tfrac{40.2}{67} = 0.6 \\ v_3 &= \tfrac{80.4}{67} = 1.2 \end{aligned}}$$

There are a few ways to calculate the determinants, but especially in this working out i chose not show the method i used since it's not the main focus of the question.

If you're curious about how I did it, the Sarrus method for simplicity.

1.6 Q6

Transpose $z = d + a\sqrt{y}$ to make y the subject.

Starting with:

$$z = d + a\sqrt{y}$$

$$z - d = a\sqrt{y}$$

$$\frac{z-d}{a} = \sqrt{y}$$

$$\left(\frac{z-d}{a}\right)^2 = y$$

Thus, the expression for y is:

$$\boxed{y = \frac{(z-d)^2}{a^2}}$$

i generally consider this the most concise and general expression for y.
im aware you could expand $(z-d)^2$:

$$y = \frac{z^2 - 2zd + d^2}{a^2}$$

and additionally take into account of both possible factorisations:

$$y = \frac{z^2 + d(d - 2z)}{a^2}$$

$$y = \frac{d^2 + z(z - 2d)}{a^2}$$

1.7 Q7

Find a vector that is perpendicular to both of the vectors

$$\boldsymbol{a} = 4\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$$

$$\boldsymbol{b} = 3\boldsymbol{i} + 4\boldsymbol{j} - 6\boldsymbol{k}$$

Hence find a unit vector that is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.

The unit vector is

$$\hat{\mathbf{r}} = \frac{\vec{r}}{||\vec{r}||}$$

Here we want to define $\vec{r}$ as a vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, to do that we take there cross product $\mathbf{a} \times \mathbf{b}$

$$\vec{r} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 5 \\ 3 & 4 & -6 \end{vmatrix} = -38\mathbf{i} + 39\mathbf{j} + 7\mathbf{k}$$

Find the magnitude:

$$||\vec{r}|| = \|\mathbf{a} \times \mathbf{b}\| = \sqrt{(-38)^2 + 39^2 + 7^2} = \sqrt{3014}$$

The unit vector is:

$$\boxed{\frac{-38\mathbf{i} + 39\mathbf{j} + 7\mathbf{k}}{\sqrt{3014}}}$$

1.8 Q8

If $M = \begin{pmatrix} 7 & 9 \\ 1 & -2 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 1 \\ -2 & 6 \end{pmatrix}$ find MN and NM

Calculate MN:

$$MN = \begin{bmatrix} 7 & 9 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \cdot 2 + 9 \cdot (-2) & 7 \cdot 1 + 9 \cdot 6 \\ 1 \cdot 2 + (-2) \cdot (-2) & 1 \cdot 1 + (-2) \cdot 6 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} -4 & 61 \\ 6 & -11 \end{bmatrix}}$$

Calculate NM:

$$NM = \begin{bmatrix} 2 & 1 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 7 & 9 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \cdot 7 + 1 \cdot 1 & 2 \cdot 9 + 1 \cdot (-2) \\ -2 \cdot 7 + 6 \cdot 1 & -2 \cdot 9 + 6 \cdot (-2) \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 15 & 16 \\ -8 & -30 \end{bmatrix}}$$

1.9 Q9

If $y = x^4 - 4x^3 - 90x^2$, find the values of x for which $y'' = 0$

First, find the first derivative:

$$y' = \frac{d}{dx}(x^4 - 4x^3 - 90x^2) = 4x^3 - 12x^2 - 180x$$

Find the second derivative:

$$y'' = \frac{d}{dx}(4x^3 - 12x^2 - 180x) = 12x^2 - 24x - 180$$

Set $y'' = 0$:

$$12x^2 - 24x - 180 = 0$$

Divide by 12:

$$x^2 - 2x - 15 = 0$$

Factor:

$$(x - 5)(x + 3) = 0$$

$$\boxed{x = 5,\ x = -3}$$

1.10 Q10

Transpose $b = g + t(a - 3)$ to make a the subject

Starting with:

$$b = g + t(a - 3)$$

this is easily rearranged to isolate a:

$$b - g = t(a - 3)$$

$$\frac{b - g}{t} = a - 3$$

$$\boxed{a = \frac{b - g}{t} + 3}$$

Chapter 2

Part B: Computing

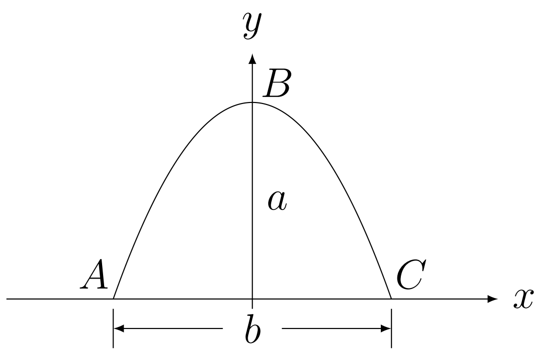
Notes:

- All graphs and images were created using TikZ PGF.
- Code implementations are original and written in MATLAB and to strictly follow guidelines with some python also. In order to copy the code from my Github please follow the relevant links provided.
  - Honestly, relying only on MATLAB feels a bit short-sighted. its a temporary student license, and no point getting used to it. focusing solely on MATLAB in this course means missing out on essential programming skills, assets and applicability. While I **have to use MATLAB** for this module, it's not fully aligned with the long-term goals that it states.
- Please consider the answers as a whole.

2.1 Q1

The arc length of a segment of a parabola ABC of an ellipse with semi-minor axes a and b is given approximately by:

$$L_{ABC} = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a} \ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$



Write a universal, user-friendly code, test your programme and determine L_{ABC} if $a = 11$ cm and $b = 9$ cm.

Since the goal is utilize Matlab, this question is fairly easy this version should suffice if a widely used and intuitive code is required.

matlab scripts/q1_easy.m

```

%make a function to call for the calculation

% In MATLAB, when defining a function that returns a value,
% you must specify an output argument in the function declaration
% and assign the result to this output argument,
% rather than using return %

function L_ABC = calculate_arc_length(a, b)
    %write in ascci math using matlab built in functions
    L_ABC = (1/2) * sqrt(b^2 + 16 * a^2) + (b^2 / (8 * a)) * log((4 * a + sqrt(b^2 +
       16 * a^2)) / b);
end

%prompt the user for input values
a = input('Enter the value of a (height): ');
b = input('Enter the value of b (width): ');

%calculate the arc length by calling the function
L_ABC = calculate_arc_length(a, b);

%display the result by printing
fprintf('The arc length L_ABC is: %.2f\n', L_ABC);

%realistically not much error handling is inputed but i dont think that is the gaol
   here
```

2.1.1 Advanced version

We are tasked with analyzing a parabola where:

- a represents the height of the parabola
- b represents the width of the parabola

My goal is to plot the parabola, first by formulating an equation in terms of these factors. Initially, we begin with a simple parabola in the general shape (x^2) similar to shown in the graph. Our goal is to select a reasonable function, with variables associated with the x scale and y location of the parabola:

$$y = -(\beta x)^2 + \gamma$$

Given our problem statement:

- γ represents the height, so $\gamma = a$
- The width is related to the roots of the equation when $y = 0$

To find β, we solve:

$$0 = -(\beta x)^2 + \gamma$$

$$\beta = \frac{2\sqrt{\gamma}}{b}$$

since we know γ:

$$\beta = \frac{2\sqrt{a}}{b}$$

Note: This requires $b \neq 0$ and $\sqrt{a} \neq 0$ (i.e., $a > 0$).
Substituting γ and β into our original equation:

$$\boxed{y = -\left(\frac{2\sqrt{a}}{b}x\right)^2 + a}$$

This is our final parabola equation in terms of a and b. The arc length of the parabola from $x = -\frac{b}{2}$ to $x = \frac{b}{2}$ is given by:

$$L_{ABC} = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a}\ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$

When implementing this in a program:

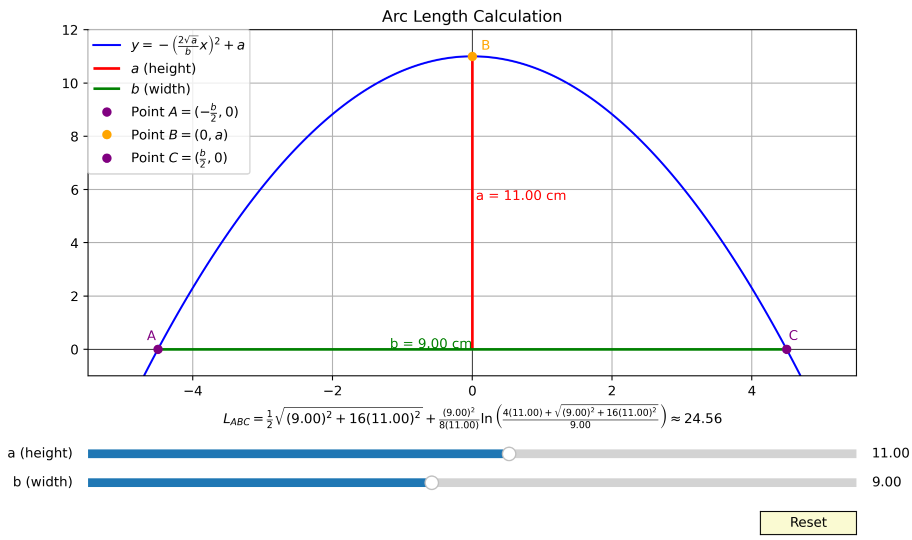
- Ensure that when there are no limit parameters for the inputs a and b, the conditions $a > 0$ and $b \neq 0$ are appropriately handled.
- Update the plot of $y = -\left(\frac{2\sqrt{a}}{b}x\right)^2 + a$ so that the sliders for a and b directly control the height and width, respectively.
- Calculate the arc length of the parabola using a function with inputs a and b, and display the result in the plot.

The rest is just nitpicky presentation, all just preference i tried to do this in MATLAB but it don't really feel nice just finicky. ao i did in py, it is possible in MATLAB though.

py scripts/q1_advanced.py

```
"""
Requirements to run:
    - Python 3.x (https://www.python.org/)
    - NumPy and Matplotlib (install with cmd prmp: pip install numpy matplotlib)
"""

import numpy as np
import matplotlib.pyplot as plt
from matplotlib.widgets import Slider, Button

def arc_length_parabola(a, b):
    L_ABC=(1/2)*np.sqrt(b**2+16*a**2)+(b**2/(8*a))*np.log((4*a+np.sqrt(b**2+16*a**2))/b)
    return L_ABC

fig, ax = plt.subplots(figsize=(10, 6))
plt.subplots_adjust(left=0.1, bottom=0.3, right=0.9, top=0.9)
initial_a = 11.0
initial_b = 9.0

ax_slider_a = plt.axes([0.1, 0.15, 0.8, 0.03])
ax_slider_b = plt.axes([0.1, 0.1, 0.8, 0.03])

slider_a = Slider(ax_slider_a, 'a (height)', 0.1, 20, valinit=initial_a, valfmt='%0.2f')
slider_b = Slider(ax_slider_b, 'b (width)', 0.1, 20, valinit=initial_b, valfmt='%0.2f')

x = np.linspace(-10, 10, 1000)

def update(val):
    a = slider_a.val
    b = slider_b.val
    L_ABC = arc_length_parabola(a, b)

    ax.clear()
    # should add if a > 0 and b != 0: but im not going to since i set the slider limits accordingly
    y = -((2*np.sqrt(a))*x/b)**2 + a
    ax.plot(x, y, label=r'$y = -\left(\frac{2\sqrt{a}}{b}x\right)^2 + a$', color='blue')

    ax.vlines(0, 0, a, color='red',linewidth=2 , zorder=10, label=r'$a$ (height)')
    ax.hlines(0, -b/2, b/2, color='green',linewidth=2, zorder=10, label=r'$b$ (width)')

    ax.text(0.05, a/2, f'a = {a:.2f} cm', color='red', fontsize=10, verticalalignment='bottom')
    ax.text(0, 0.05, f'b = {b:.2f} cm', color='green', fontsize=10, horizontalalignment='right')

    A_x = -b/2
    C_x = b/2

    ax.plot(A_x, 0, 'o', zorder=10, color='purple', label=r'Point $A = (-\frac{b}{2}, 0)$')
    ax.plot(0, a, 'o',zorder=10,  color='orange', label=r'Point $B = (0, a)$')
    ax.plot(C_x, 0, 'o',zorder=10, color='purple', label=r'Point $C = (\frac{b}{2}, 0)$')

```

```
    ax.annotate('A', (A_x, 0), textcoords="offset points", xytext=(-5,7), ha='center', color='purple')
    ax.annotate('B', (0, a), textcoords="offset points", xytext=(10,5), ha='center', color='orange')
    ax.annotate('C', (C_x, 0), textcoords="offset points", xytext=(5,7), ha='center', color='purple')

    ax.set_title(f'Arc Length Calculation')
    ax.set_xlabel(r'$L_{ABC} = \frac{1}{2}\sqrt{' + f'({b:.2f})^2 + 16({a:.2f})^2' + r'} + \frac{' + f'({b:.2f})^2' + r'}{8(' + f'{a:.2f}' + r')}\ln\left(\frac{4(' + f'{a:.2f}' + r') + \sqrt{' + f'({b:.2f})^2 + 16({a:.2f})^2' + r'}}{' + f'{b:.2f}' + r'}\right) \approx ' + f'{L_ABC:.2f}$')

    ax.set_ylim(-1, a + 1)
    ax.set_xlim(-b/2 - 1, b/2 + 1)

    ax.legend(bbox_to_anchor=(0, 1), loc='upper left', borderaxespad=0.)

    ax.grid(True)
    ax.axhline(y=0, color='k', linestyle='-', linewidth=0.5)
    ax.axvline(x=0, color='k', linestyle='-', linewidth=0.5)
    plt.draw()

slider_a.on_changed(update)
slider_b.on_changed(update)

resetax = plt.axes([0.8, 0.025, 0.1, 0.04])
button = Button(resetax, 'Reset', color='lightgoldenrodyellow', hovercolor='0.975')

def reset(event):
    slider_a.reset()
    slider_b.reset()
    update(None)

button.on_clicked(reset)

update(None)

plt.savefig('figures/Figure_1.png',dpi=400)
plt.show()
```



With the sliders, you can interactively update the plot and change a and b values in real time!

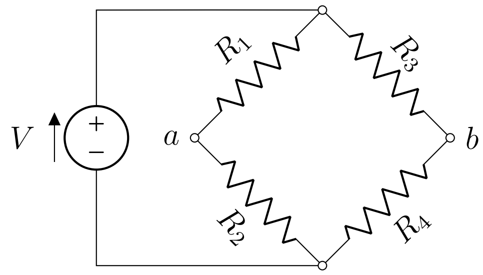
2.2 Q1

The voltage difference V_{ab} between points a and b in the Wheatstone bridge circuit is:

$$V_{ab} = V\left(\frac{R_1R_3 - R_2R_4}{(R_1 + R_2)(R_3 + R_4)}\right)$$

Write a universal, user-friendly program that calculates the voltage difference V_{ab}. Test your program using the following values:

- V = 14 volts
- R_1 = 120.6 Ω
- R_2 = 119.3 Ω
- R_3 = 121.2 Ω
- R_4 = 118.8 Ω



Using MATLAB is the aim, thus answering this issue is not too difficult. I don't see a necessity for an interactive or complicated presentation for this specific question. I could include user interface components that would let users enter numbers by clicking resistors or something similar, but given the limitations, I'll go with a more cautious strategy. It really just depends on how fascinating or even marginally intriguing I find the question.

matlab scripts/q2.m

```matlab
function wheatstone_bridge_ui()
    % create a UI figure
    fig = uifigure('Name', 'Wheatstone Bridge Circuit Calculator', 'Position', [100, 100, 600, 400]);

    % image stuff dont worry bout this
    imgURL = 'https://raw.githubusercontent.com/sakx7/mathcompuni/main/figures/wheatstone_bridge.jpg';
    localImgPath = fullfile(pwd, 'wheatstone_bridge.jpg');
    if ~isfile(localImgPath)
        try
            websave(localImgPath, imgURL);
        catch
            uialert(fig, 'Failed to download the image.', 'Download Error');
            return;
        end
    end
    img = uiimage(fig);
    img.Position = [25, 75, 250, 250];
    img.ImageSource = localImgPath;

    % add input labels and fields on the right side
    voltageLabel = uilabel(fig, 'Text', '$V$ ($V$/volts):', 'Position', [333, 300, 100, 30], 'Interpreter', 'latex');
    voltageField = uieditfield(fig, 'numeric', 'Position', [405, 300, 100, 30]);

    r1Label = uilabel(fig, 'Text', '$R_1$ ($\Omega$/ohms):', 'Position', [325, 250, 100, 30], 'Interpreter', 'latex');
    r1Field = uieditfield(fig, 'numeric', 'Position', [405, 250, 100, 30]);

```

```matlab
    r2Label = uilabel(fig, 'Text', '$R_2$ ($\Omega$/ohms):', 'Position', [325, 200, 100, 30], 'Interpreter', 'latex');
    r2Field = uieditfield(fig, 'numeric', 'Position', [405, 200, 100, 30]);

    r3Label = uilabel(fig, 'Text', '$R_3$ ($\Omega$/ohms):', 'Position', [325, 150, 100, 30], 'Interpreter', 'latex');
    r3Field = uieditfield(fig, 'numeric', 'Position', [405, 150, 100, 30]);

    r4Label = uilabel(fig, 'Text', '$R_4$ ($\Omega$/ohms):', 'Position', [325, 100, 100, 30], 'Interpreter', 'latex');
    r4Field = uieditfield(fig, 'numeric', 'Position', [405, 100, 100, 30]);

    % add label for result
    outputLabel = uilabel(fig, 'Position', [25, 20, 250, 50], 'Text', 'Output will be displayed here.', 'FontSize', 14, 'Interpreter', 'latex');

    % add button
    calcButton = uibutton(fig, 'push', ...
        'Text', 'Calculate $V_{ab}$', ...
        'Position', [375, 50, 150, 30], ...
        'Interpreter', 'latex', ...
        'ButtonPushedFcn', @(calcButton,event) calculate_V_ab(fig, voltageField, r1Field, r2Field, r3Field, r4Field, outputLabel)); % Use outputLabel
end

% Main gunction to calculate weat (V_ab) and display the result
function calculate_V_ab(fig, voltageField, r1Field, r2Field, r3Field, r4Field, outputLabel)
    % get values from the input fields
    V = voltageField.Value;
    R_1 = r1Field.Value;
    R_2 = r2Field.Value;
    R_3 = r3Field.Value;
    R_4 = r4Field.Value;

    % input validation error handles
    inputs = [V, R_1, R_2, R_3, R_4];
    if any(isnan(inputs)) || any(inputs <= 0)
        uialert(fig, 'enter valid values.', 'Input Error');
        return;
    end

    % the weat
    V_ab = V * (R_1 * R_3 - R_2 * R_4) / ((R_1 + R_2) * (R_3 + R_4));

    % final display result in the label
    msg = sprintf('The output voltage $V_{ab}$ is: %.4f V', V_ab);
    outputLabel.Text = msg;  % update the label with the output message
end
```