# Engineering
# Mathematics and Computing

Task 1: Coursework Assessment

Student Name: Sakariye Abiikar
KID: 2371673

Last Updated: October 19, 2024
Submission Deadline: November 7, 2024, 5pm

# Chapter 1

# Part A: Mathematics

---

**Notes:**

## 1.1 Q1

Use the quotient rule to differentiate the function $y = \frac{\ln 3x}{2x}$

The quotient rule states that if

$$y = \frac{u}{v} \quad \text{then} \quad y' = \frac{u'v - uv'}{v^2}$$

Here:

$$u = \ln 3x \qquad\qquad v = 2x$$

First, find the derivatives:

$$u = \ln(3x)$$

Let:

$$\delta = 3x \qquad u = \ln(\delta)$$

$$\frac{du}{d\delta} = \frac{1}{\delta} \qquad \frac{d\delta}{dx} = 3$$

Using the chain rule, we can find:

$$\frac{du}{dx} = \frac{du}{d\delta} \cdot \frac{d\delta}{dx}$$

$$\frac{du}{dx} = \frac{3}{\delta} = \frac{3}{3x} = \frac{1}{x}$$

General rule for composite/nested functions is solved via chain rule it's:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

$$v = 2x$$

$$v' = \frac{d}{dx}(2x) = 2$$

Now we can apply the quotient rule:

$$y' = \frac{\left(\frac{1}{x}\right)(2x) - (\ln 3x)(2)}{(2x)^2} = \frac{2 - 2\ln 3x}{4x^2}$$

$$\boxed{y' = \frac{1 - \ln 3x}{2x^2}}$$

## 1.2 Q2

Find the angle between the vectors $2i - 11j - 10k$ and $5i + 8j + 7k$

The angle $\theta$ between two vectors **a** and **b** is given by:

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}$$

Calculate the dot product:

$$\mathbf{a}\cdot\mathbf{b} = \underbrace{(2)(5)}_{i} + \underbrace{(-11)(8)}_{j} + \underbrace{(-10)(7)}_{k} = -148$$

Calculate the magnitudes:

$$\|\mathbf{a}\| = \sqrt{2^2 + (-11)^2 + (-10)^2} = \sqrt{225} = 15$$

$$\|\mathbf{b}\| = \sqrt{5^2 + 8^2 + 7^2} = \sqrt{138}$$

Find $\cos\theta$:

$$\cos\theta = \frac{-148}{15 \times \sqrt{138}}$$

Thus, the angle $\theta$ is:

$$\boxed{\theta = \cos^{-1}\left(\frac{-148}{15 \times \sqrt{138}}\right) \approx 147.1^\circ}$$

## 1.3 Q3

Find the rate of change of $y = \ln(16t^2 + 19)$ at the specified point $t = 9$

Differentiate $y$ with respect to $t$:

Note prior (q1) that the general rule for composite/nested functions is:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

Here for $y = \ln(16t^2 + 19)$

$$\frac{dy}{dt} = \frac{1}{16t^2 + 19} \cdot \frac{d}{dt}(16t^2 + 19)$$

$$\frac{d}{dt}(16t^2 + 19) = 32t$$

$$\frac{dy}{dt} = \frac{1}{16t^2 + 19} \cdot 32t = \frac{32t}{16t^2 + 19}$$

Evaluate at $t = 9$:

$$\boxed{\left.\frac{dy}{dt}\right|_{t=9} = \frac{32(9)}{16(9)^2 + 19} = \frac{288}{1315} \approx 0.219}$$

## 1.4 Q4

Express $\cos t - 8\sin t$ in the form $A\cos(\omega t + \alpha)$, where $\alpha \geq 0$

you see the thing is the left side, $\cos t - 8\sin t$, is standard, but the right side introduces a different frequency, $\omega$, in $A\cos(\omega t + \alpha)$.

Since the left side has a frequency of 1, im just gonna assume $\omega = 1$ for simplicity. If $\omega$ were different, you would need to rewrite the left side in terms of $\omega t$, but this isn't specified in the problem.

---

The angle subtraction formula is:

$$A\cos(t - \phi) = A\cos(\phi)\cos(t) + A\sin(\phi)\sin(t)$$

By comparing coefficients from both sides, we have:

$$a = A\cos(\phi)$$
$$b = A\sin(\phi)$$

To find $A$ we use $A = \sqrt{a^2 + b^2}$

This arises from squaring both equations $a = A\cos(\phi)$ and $b = A\sin(\phi)$:

$$a^2 + b^2 = (A\cos(\phi))^2 + (A\sin(\phi))^2 = A^2(\cos^2(\phi) + \sin^2(\phi)) = A^2$$

To find the phase shift $\phi$, we use $\tan\phi = \frac{b}{a}$

This comes from the definitions of sine and cosine:

$$\tan\phi = \frac{A\sin(\phi)}{A\cos(\phi)} = \frac{b}{a}$$

In so we derive and make use of:

$$A\cos(t - \phi) = a\cos(t) + b\sin(t)$$

Where $A = \sqrt{a^2 + b^2}$ and $\tan\phi = \frac{b}{a}$

---

for $b = -8$ and $a = 2$

$$A = \sqrt{1^2 + (-8)^2} = \sqrt{65}$$

$$\tan\phi = \frac{-8}{1} = -8 \qquad \phi = \arctan(-8) \approx -82.87°$$

In so plugging in gives

$$\begin{aligned}\cos t - 8\sin t &= A\cos(t - \phi)\\ &= \sqrt{65}\cos(t - \arctan(-8))\\ &= \sqrt{65}\cos(t - (-82.87°))\end{aligned}$$

$$\boxed{\cos t - 8\sin t \approx 8.06\cos(t + 82.87°)}$$

## 1.5 Q5

Solve the following system of three linear equations using Cramer's rule

$$\begin{cases} 11v_1 - v_2 + v_3 = 31.4 \\ v_1 + \frac{v_2}{2} - v_3 = 1.9 \\ -9v_1 + 11v_3 = -12 \end{cases}$$

The system can be written in matrix form $A\mathbf{v} = \mathbf{b}$, specifically because of the variable distributions where:

$$A = \begin{bmatrix} 11 & -1 & 1 \\ 1 & \frac{1}{2} & -1 \\ -9 & 0 & 11 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 31.4 \\ 1.9 \\ -12 \end{bmatrix}$$

Calculate the determinant $\det(A)$.

$$\det(A) = 67$$

Now solve for each variable using Cramer's rule:

$$A_1 = \begin{bmatrix} 31.4 & -1 & 1 \\ 1.9 & \frac{1}{2} & -1 \\ -12 & 0 & 11 \end{bmatrix} \quad A_2 = \begin{bmatrix} 11 & 31.4 & 1 \\ 1 & 1.9 & -1 \\ -9 & -12 & 11 \end{bmatrix} \quad A_3 = \begin{bmatrix} 11 & -1 & 31.4 \\ 1 & \frac{1}{2} & 1.9 \\ -9 & 0 & -12 \end{bmatrix}$$

Calculate the determinants:

$$\det(A_1) \approx 187.6 \quad \det(A_2) \approx 40.2 \quad \det(A_3) \approx 80.4$$

$\mathbf{v}$ can be found as:

$$v_1 = \frac{\det(A_1)}{\det(A)}, \quad v_2 = \frac{\det(A_2)}{\det(A)}, \quad v_3 = \frac{\det(A_3)}{\det(A)}$$

$$\boxed{\begin{aligned} v_1 &= \tfrac{187.6}{67} = 2.8 \\ v_2 &= \tfrac{40.2}{67} = 0.6 \\ v_3 &= \tfrac{80.4}{67} = 1.2 \end{aligned}}$$

There are a few ways to calculate the determinants, but especially in this working out i chose not show the method i used since it's not the main focus of the question.

If you're curious about how I did it, the Sarrus method for simplicity.

## 1.6 Q6

Transpose $z = d + a\sqrt{y}$ to make $y$ the subject.

Starting with:

$$z = d + a\sqrt{y}$$

$$z - d = a\sqrt{y}$$

$$\frac{z-d}{a} = \sqrt{y}$$

$$\left(\frac{z-d}{a}\right)^2 = y$$

Thus, the expression for $y$ is:

$$\boxed{y = \frac{(z-d)^2}{a^2}}$$

i generally consider this the most concise and general expression for $y$.
im aware you could expand $(z-d)^2$:

$$y = \frac{z^2 - 2zd + d^2}{a^2}$$

and additionally take into account of both possible factorisations:

$$y = \frac{z^2 + d(d - 2z)}{a^2}$$

$$y = \frac{d^2 + z(z - 2d)}{a^2}$$

## 1.7 Q7

Find a vector that is perpendicular to both of the vectors

$$\boldsymbol{a} = 4\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$$

$$\boldsymbol{b} = 3\boldsymbol{i} + 4\boldsymbol{j} - 6\boldsymbol{k}$$

Hence find a unit vector that is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.

The unit vector is

$$\hat{\mathbf{r}} = \frac{\vec{r}}{||\vec{r}||}$$

Here we want to define $\vec{r}$ as a vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, to do that we take there cross product $\mathbf{a} \times \mathbf{b}$

$$\vec{r} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 5 \\ 3 & 4 & -6 \end{vmatrix} = -38\mathbf{i} + 39\mathbf{j} + 7\mathbf{k}$$

Find the magnitude:

$$||\vec{r}|| = \|\mathbf{a} \times \mathbf{b}\| = \sqrt{(-38)^2 + 39^2 + 7^2} = \sqrt{3014}$$

The unit vector is:

$$\boxed{\frac{-38\mathbf{i} + 39\mathbf{j} + 7\mathbf{k}}{\sqrt{3014}}}$$

## 1.8 Q8

If $M = \begin{pmatrix} 7 & 9 \\ 1 & -2 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 1 \\ -2 & 6 \end{pmatrix}$ find $MN$ and $NM$

Calculate $MN$:

$$MN = \begin{bmatrix} 7 & 9 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \cdot 2 + 9 \cdot (-2) & 7 \cdot 1 + 9 \cdot 6 \\ 1 \cdot 2 + (-2) \cdot (-2) & 1 \cdot 1 + (-2) \cdot 6 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} -4 & 61 \\ 6 & -11 \end{bmatrix}}$$

Calculate $NM$:

$$NM = \begin{bmatrix} 2 & 1 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 7 & 9 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \cdot 7 + 1 \cdot 1 & 2 \cdot 9 + 1 \cdot (-2) \\ -2 \cdot 7 + 6 \cdot 1 & -2 \cdot 9 + 6 \cdot (-2) \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 15 & 16 \\ -8 & -30 \end{bmatrix}}$$

## 1.9 Q9

If $y = x^4 - 4x^3 - 90x^2$, find the values of $x$ for which $y'' = 0$

First, find the first derivative:

$$y' = \frac{d}{dx}(x^4 - 4x^3 - 90x^2) = 4x^3 - 12x^2 - 180x$$

Find the second derivative:

$$y'' = \frac{d}{dx}(4x^3 - 12x^2 - 180x) = 12x^2 - 24x - 180$$

Set $y'' = 0$:

$$12x^2 - 24x - 180 = 0$$

Divide by 12:

$$x^2 - 2x - 15 = 0$$

Factor:

$$(x - 5)(x + 3) = 0$$

$$\boxed{x = 5,\ x = -3}$$

## 1.10 Q10

Transpose $b = g + t(a - 3)$ to make $a$ the subject

Starting with:

$$b = g + t(a - 3)$$

this is easily rearranged to isolate $a$:

$$b - g = t(a - 3)$$

$$\frac{b - g}{t} = a - 3$$

$$\boxed{a = \frac{b - g}{t} + 3}$$

# Chapter 2

# Part B: Computing

---

## Notes:

- All graphs and images have been coded in TikZ PGF by yours truly.
- Code implementations are also original work.
- Please evaluate the answer to the questions in its whole.

## Interpretation of Module Learning Outcomes

The module learning outcomes are seen as flexible guidelines that promote mastery of mathematical concepts alongside creative problem-solving approaches. This interpretation is supported by the following perspectives:

1. **Diverse Language Utilization:** While **MATLAB** is specifically mentioned, the focus seems to be on demonstrating mathematical proficiency rather than adhering to a particular programming language. This opens the essentially lets me use tools like Python or R to effectively illustrate an understanding of the concepts. I will strive to align my work with MATLAB where possible.

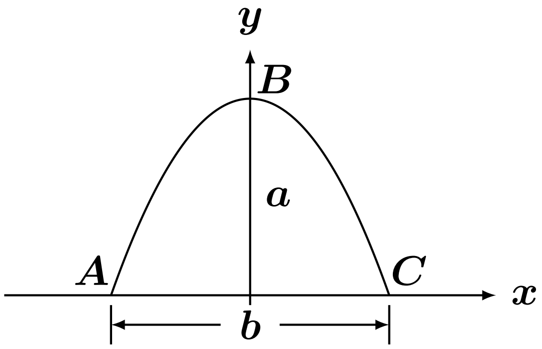
   But because of MATLAB's drawbacks—for instance, it's not free—I frequently find myself adopting alternatives. These come with difficulties including restricted access and a lack of software expertise. In general, I find it to be rather annoying.

2. **Understanding Over Language Proficiency:** The assessment values comprehension of mathematical methods and their application more than expertise in a specific software environment. This perspective enables the presentation of clear, well-documented solutions that reflect a solid understanding of the material, regardless of the medium employed.

## 2.1 Q1

The arc length of a segment of a parabola $ABC$ of an ellipse with semi-minor axes $a$ and $b$ is given approximately by:

$$L_{ABC} = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a} \ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$




Write a universal, user-friendly code, test your programme and determine $L_{ABC}$ if $a = 11$ cm and $b = 9$ cm.

We are tasked with analyzing a parabola where:

- $a$ represents the height of the parabola
- $b$ represents the width of the parabola

To plot the parabola, we must first formulate an equation in terms of these factors. Initially, we begin with a simple parabola in the general shape shown in the graph. Our goal is to select a reasonable function, with variables associated with the $x$ scale and $y$ location of the parabola:

$$y = -(\beta x)^2 + \gamma$$

Given our problem statement:

- $\gamma$ represents the height, so $\gamma = a$
- The width is related to the roots of the equation when $y = 0$

To find $\beta$, we solve:

$$0 = -(\beta x)^2 + \gamma$$

$$\beta = \frac{2\sqrt{\gamma}}{b}$$

since we know $\gamma$:

$$\beta = \frac{2\sqrt{a}}{b}$$

Note: This requires $b \neq 0$ and $\sqrt{a} \neq 0$ (i.e., $a > 0$).
Substituting $\gamma$ and $\beta$ into our original equation:

$$\boxed{y = -\left(\frac{2\sqrt{a}}{b}x\right)^2 + a}$$

This is our final parabola equation in terms of $a$ and $b$.

The arc length of the parabola from $x = -\frac{b}{2}$ to $x = \frac{b}{2}$ is given by:

$$L_{ABC} = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a}\ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$

When implementing this in a program:

- Ensure that when there are no limit parameters for the inputs $a$ and $b$, the conditions $a > 0$ and $b \neq 0$ are appropriately handled.
- Update the plot of $y = -\left(\frac{2\sqrt{a}}{b}x\right)^2 + a$ so that the sliders for $a$ and $b$ directly control the height and width, respectively.
- Calculate the arc length of the parabola using a function with inputs $a$ and $b$, and display the result in the plot.

The rest is just nitpicky presentation, all just preference.

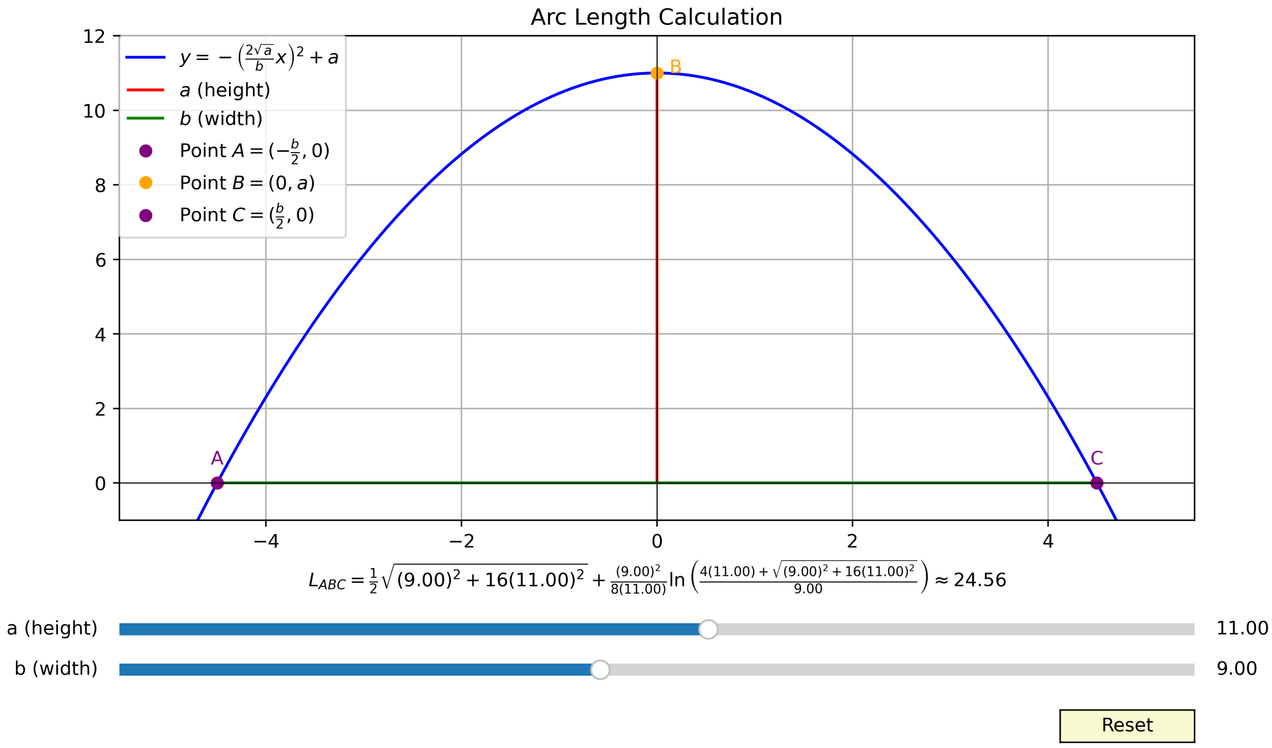
**Python Script: q1.py**

```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.widgets import Slider, Button

def arc_length_parabola(a, b):
    L_ABC=(1/2)*np.sqrt(b**2+16*a**2)+(b**2/(8*a))*np.log((4*a+np.sqrt(b**2+16*a**2))/b)
    return L_ABC

fig, ax = plt.subplots(figsize=(10, 6))
plt.subplots_adjust(left=0.1, bottom=0.3, right=0.9, top=0.9)
initial_a = 11.0
initial_b = 9.0

ax_slider_a = plt.axes([0.1, 0.15, 0.8, 0.03])
ax_slider_b = plt.axes([0.1, 0.1, 0.8, 0.03])

slider_a = Slider(ax_slider_a, 'a (height)', 0.1, 20, valinit=initial_a, valfmt='%0.2f')
slider_b = Slider(ax_slider_b, 'b (width)', 0.1, 20, valinit=initial_b, valfmt='%0.2f')

x = np.linspace(-10, 10, 400)

def update(val):
    a = slider_a.val
    b = slider_b.val
    L_ABC = arc_length_parabola(a, b)

    ax.clear()
    # should add if a > 0 and b != 0: but im not going to since i set the slider limits accordingly
    y = -((2*np.sqrt(a))*x/b)**2 + a
    ax.plot(x, y, label=r'$y = -\left(\frac{2\sqrt{a}}{b}x\right)^2 + a$', color='blue')
    ax.vlines(0, 0, a, color='red', label=r'$a$ (height)')
    ax.hlines(0, -b/2, b/2, color='green', label=r'$b$ (width)')

    A_x = -b/2
    C_x = b/2

```

```python
    ax.plot(A_x, 0, 'o', color='purple', label=r'Point $A = (-\frac{b}{2}, 0)$')
    ax.plot(0, a, 'o', color='orange', label=r'Point $B = (0, a)$')
    ax.plot(C_x, 0, 'o', color='purple', label=r'Point $C = (\frac{b}{2}, 0)$')

    ax.annotate('A', (A_x, 0), textcoords="offset points", xytext=(0,10), ha='center', color='purple')
    ax.annotate('B', (0, a), textcoords="offset points", xytext=(10,0), ha='center', color='orange')
    ax.annotate('C', (C_x, 0), textcoords="offset points", xytext=(0,10), ha='center', color='purple')

    ax.set_title(f'Arc Length Calculation')
    ax.set_xlabel(r'$L_{ABC} = \frac{1}{2}\sqrt{' + f'({b:.2f})^2 + 16({a:.2f})^2' + r'} + \frac{' + f'({b:.2f})^2' + r'}{8(' + f'{a:.2f}' + r')}\ln\left(\frac{4(' + f'{a:.2f}' + r') + \sqrt{' + f'({b:.2f})^2 + 16({a:.2f})^2' + r'}}{' + f'{b:.2f}' + r'}\right) \approx ' + f'{L_ABC:.2f}$')

    ax.set_ylim(-1, a + 1)
    ax.set_xlim(-b/2 - 1, b/2 + 1)

    ax.legend(bbox_to_anchor=(0, 1), loc='upper left', borderaxespad=0.)

    ax.grid(True)
    ax.axhline(y=0, color='k', linestyle='-', linewidth=0.5)
    ax.axvline(x=0, color='k', linestyle='-', linewidth=0.5)
    plt.draw()

slider_a.on_changed(update)
slider_b.on_changed(update)

resetax = plt.axes([0.8, 0.025, 0.1, 0.04])
button = Button(resetax, 'Reset', color='lightgoldenrodyellow', hovercolor='0.975')

def reset(event):
    slider_a.reset()
    slider_b.reset()
    update(None)

button.on_clicked(reset)

update(None)

plt.savefig('Figure_1.png',dpi=400)
plt.show()
```

- **Environment Setup:**
  - Install Python 3.x from `https://www.python.org/downloads/`
  - Verify installation: `python --version`
- **Required Packages:**
  - Install NumPy and Matplotlib: `pip install numpy matplotlib`
  - For permission issues: Run command prompt as administrator
- **Script Execution:**
  - Install `q1.py` from my GitHub
  - Run script

**Enjoy the Interactivity:** Follow prompts and explore the output!




Since the goal was to utilize Matlab, I opted for this approach.
However, if a truly universal and user-friendly code is required, then this py simplified version should be suitable:

```
import numpy as np

def arc_length_parabola(a, b):
    L_ABC=(1/2)*np.sqrt(b**2+16*a**2)+(b**2/(8*a))*np.log((4*a+np.sqrt(b**2+16*a**2))
        /b)
    return L_ABC

a = float(input("Enter the value of a (height): "))
b = float(input("Enter the value of b (width): "))

L_ABC = arc_length_parabola(a, b)

print(f"The arc length L_ABC is: {L_ABC:.2f}")
```

The voltage difference $V_{ab}$ between points $a$ and $b$ in the Wheatstone bridge circuit is: