# Engineering
# Mathematics and Computing

Task 1: Coursework Assessment

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Git Repo : https://github.com/sakx7/mathcompuni

# Part A

# Mathematics

## A.1 Q1

Use the quotient rule to differentiate the function $y = \frac{\ln 3x}{2x}$

The quotient rule states that if

$$y = \frac{u}{v} \quad \text{then} \quad y' = \frac{u'v - uv'}{v^2}$$

Here:

$$u = \ln 3x \qquad\qquad v = 2x$$

First, find the derivatives:

$$u = \ln(3x)$$

Let:

$$\delta = 3x \qquad u = \ln(\delta)$$

$$\frac{du}{d\delta} = \frac{1}{\delta} \qquad \frac{d\delta}{dx} = 3$$

Using the chain rule, we can find:

$$\frac{du}{dx} = \frac{du}{d\delta} \cdot \frac{d\delta}{dx}$$

$$\frac{du}{dx} = \frac{3}{\delta} = \frac{3}{3x} = \frac{1}{x}$$

General rule for composite/nested functions is solved via chain rule it's:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

$$v = 2x$$

$$v' = \frac{d}{dx}(2x) = 2$$

Now we can apply the quotient rule:

$$y' = \frac{\left(\frac{1}{x}\right)(2x) - (\ln 3x)(2)}{(2x)^2} = \frac{2 - 2\ln 3x}{4x^2}$$

$$\boxed{y' = \frac{1 - \ln 3x}{2x^2}}$$

## A.2 Q2

Find the angle between the vectors $2i - 11j - 10k$ and $5i + 8j + 7k$

The angle $\theta$ between two vectors **a** and **b** is given by:

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|}$$

Calculate the dot product:

$$\mathbf{a} \cdot \mathbf{b} = \underbrace{(2)(5)}_{i} + \underbrace{(-11)(8)}_{j} + \underbrace{(-10)(7)}_{k} = -148$$

Calculate the magnitudes:

$$\|\mathbf{a}\| = \sqrt{2^2 + (-11)^2 + (-10)^2} = \sqrt{225} = 15$$

$$\|\mathbf{b}\| = \sqrt{5^2 + 8^2 + 7^2} = \sqrt{138}$$

Find $\cos\theta$:

$$\cos\theta = \frac{-148}{15 \times \sqrt{138}}$$

Thus, the angle $\theta$ is:

$$\boxed{\theta = \cos^{-1}\left(\frac{-148}{15 \times \sqrt{138}}\right) \approx 147.1^\circ}$$

## A.3 Q3

Find the rate of change of $y = \ln(16t^2 + 19)$ at the specified point $t = 9$

Differentiate $y$ with respect to $t$:

Note prior (q1) that the general rule for composite/nested functions is:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

Here for $y = \ln(16t^2 + 19)$

$$\frac{dy}{dt} = \frac{1}{16t^2 + 19} \cdot \frac{d}{dt}(16t^2 + 19)$$

$$\frac{d}{dt}(16t^2 + 19) = 32t$$

$$\frac{dy}{dt} = \frac{1}{16t^2 + 19} \cdot 32t = \frac{32t}{16t^2 + 19}$$

Evaluate at $t = 9$:

$$\boxed{\left.\frac{dy}{dt}\right|_{t=9} = \frac{32(9)}{16(9)^2 + 19} = \frac{288}{1315} \approx 0.219}$$

## A.4 Q4

Express $\cos t - 8 \sin t$ in the form $A \cos(\omega t + \alpha)$, where $\alpha \geq 0$

you see the thing is the left side, $\cos t - 8 \sin t$, is standard, but the right side introduces a different frequency, $\omega$, in $A \cos(\omega t + \alpha)$.

Since the left side has a frequency of 1, im just gonna assume $\omega = 1$ for simplicity. If $\omega$ were different, you would need to rewrite the left side in terms of $\omega t$, but this isn't specified in the problem.

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The angle subtraction formula is:

$$A \cos(t - \phi) = A \cos(\phi) \cos(t) + A \sin(\phi) \sin(t)$$

By comparing coefficients from both sides, we have:

$$a = A \cos(\phi)$$
$$b = A \sin(\phi)$$

To find $A$ we use $A = \sqrt{a^2 + b^2}$

This arises from squaring both equations $a = A \cos(\phi)$ and $b = A \sin(\phi)$:

$$a^2 + b^2 = (A \cos(\phi))^2 + (A \sin(\phi))^2 = A^2(\cos^2(\phi) + \sin^2(\phi)) = A^2$$

To find the phase shift $\phi$, we use $\tan \phi = \frac{b}{a}$

This comes from the definitions of sine and cosine:

$$\tan \phi = \frac{A \sin(\phi)}{A \cos(\phi)} = \frac{b}{a}$$

In so we derive and make use of:

$$A \cos(t - \phi) = a \cos(t) + b \sin(t)$$

Where $A = \sqrt{a^2 + b^2}$ and $\tan \phi = \frac{b}{a}$

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for $b = -8$ and $a = 2$

$$A = \sqrt{1^2 + (-8)^2} = \sqrt{65}$$

$$\tan \phi = \frac{-8}{1} = -8 \qquad \phi = \arctan(-8) \approx -82.87°$$

In so plugging in gives

$$\begin{aligned} \cos t - 8 \sin t &= A \cos(t - \phi) \\ &= \sqrt{65} \cos(t - \arctan(-8)) \\ &= \sqrt{65} \cos(t - (-82.87°)) \end{aligned}$$

$$\boxed{\cos t - 8 \sin t \approx 8.06 \cos(t + 82.87°)}$$

## A.5 Q5

Solve the following system of three linear equations using Cramer's rule

$$\begin{cases} 11v_1 - v_2 + v_3 = 31.4 \\ v_1 + \frac{v_2}{2} - v_3 = 1.9 \\ -9v_1 + 11v_3 = -12 \end{cases}$$

The system can be written in matrix form $A\mathbf{v} = \mathbf{b}$, specifically because of the variable distributions where:

$$A = \begin{bmatrix} 11 & -1 & 1 \\ 1 & \frac{1}{2} & -1 \\ -9 & 0 & 11 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 31.4 \\ 1.9 \\ -12 \end{bmatrix}$$

Calculate the determinant $\det(A)$.

$$\det(A) = 67$$

Now solve for each variable using Cramer's rule:

$$A_1 = \begin{bmatrix} 31.4 & -1 & 1 \\ 1.9 & \frac{1}{2} & -1 \\ -12 & 0 & 11 \end{bmatrix} \quad A_2 = \begin{bmatrix} 11 & 31.4 & 1 \\ 1 & 1.9 & -1 \\ -9 & -12 & 11 \end{bmatrix} \quad A_3 = \begin{bmatrix} 11 & -1 & 31.4 \\ 1 & \frac{1}{2} & 1.9 \\ -9 & 0 & -12 \end{bmatrix}$$

Calculate the determinants:

$$\det(A_1) \approx 187.6 \quad \det(A_2) \approx 40.2 \quad \det(A_3) \approx 80.4$$

$\mathbf{v}$ can be found as:

$$v_1 = \frac{\det(A_1)}{\det(A)}, \quad v_2 = \frac{\det(A_2)}{\det(A)}, \quad v_3 = \frac{\det(A_3)}{\det(A)}$$

$$\boxed{\begin{aligned} v_1 &= \tfrac{187.6}{67} = 2.8 \\ v_2 &= \tfrac{40.2}{67} = 0.6 \\ v_3 &= \tfrac{80.4}{67} = 1.2 \end{aligned}}$$

There are a few ways to calculate the determinants, but especially in this working out i chose not show the method i used since it's not the main focus of the question.

If you're curious about how I did it, the Sarrus method for simplicity.

## A.6 Q6

Transpose $z = d + a\sqrt{y}$ to make $y$ the subject.

Starting with:

$$z = d + a\sqrt{y}$$

$$z - d = a\sqrt{y}$$

$$\frac{z-d}{a} = \sqrt{y}$$

$$\left(\frac{z-d}{a}\right)^2 = y$$

Thus, the expression for $y$ is:

$$\boxed{y = \frac{(z-d)^2}{a^2}}$$

i generally consider this the most concise and general expression for $y$.
im aware you could expand $(z-d)^2$:

$$y = \frac{z^2 - 2zd + d^2}{a^2}$$

and additionally take into account of both possible factorisations:

$$y = \frac{z^2 + d(d - 2z)}{a^2}$$

$$y = \frac{d^2 + z(z - 2d)}{a^2}$$

## A.7 Q7

Find a vector that is perpendicular to both of the vectors

$$\boldsymbol{a} = 4\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$$

$$\boldsymbol{b} = 3\boldsymbol{i} + 4\boldsymbol{j} - 6\boldsymbol{k}$$

Hence find a unit vector that is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.

The unit vector is

$$\hat{\mathbf{r}} = \frac{\vec{r}}{||\vec{r}||}$$

Here we want to define $\vec{r}$ as a vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$, to do that we take there cross product $\mathbf{a} \times \mathbf{b}$

$$\vec{r} = \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 5 \\ 3 & 4 & -6 \end{vmatrix} = -38\mathbf{i} + 39\mathbf{j} + 7\mathbf{k}$$

Find the magnitude:

$$||\vec{r}|| = \|\mathbf{a} \times \mathbf{b}\| = \sqrt{(-38)^2 + 39^2 + 7^2} = \sqrt{3014}$$

The unit vector is:

$$\boxed{\frac{-38\mathbf{i} + 39\mathbf{j} + 7\mathbf{k}}{\sqrt{3014}}}$$

## A.8 Q8

If $M = \begin{pmatrix} 7 & 9 \\ 1 & -2 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 1 \\ -2 & 6 \end{pmatrix}$ find $MN$ and $NM$

Calculate $MN$:

$$MN = \begin{bmatrix} 7 & 9 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \cdot 2 + 9 \cdot (-2) & 7 \cdot 1 + 9 \cdot 6 \\ 1 \cdot 2 + (-2) \cdot (-2) & 1 \cdot 1 + (-2) \cdot 6 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} -4 & 61 \\ 6 & -11 \end{bmatrix}}$$

Calculate $NM$:

$$NM = \begin{bmatrix} 2 & 1 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 7 & 9 \\ 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \cdot 7 + 1 \cdot 1 & 2 \cdot 9 + 1 \cdot (-2) \\ -2 \cdot 7 + 6 \cdot 1 & -2 \cdot 9 + 6 \cdot (-2) \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 15 & 16 \\ -8 & -30 \end{bmatrix}}$$

## A.9 Q9

If $y = x^4 - 4x^3 - 90x^2$, find the values of $x$ for which $y'' = 0$

First, find the first derivative:

$$y' = \frac{d}{dx}(x^4 - 4x^3 - 90x^2) = 4x^3 - 12x^2 - 180x$$

Find the second derivative:

$$y'' = \frac{d}{dx}(4x^3 - 12x^2 - 180x) = 12x^2 - 24x - 180$$

Set $y'' = 0$:

$$12x^2 - 24x - 180 = 0$$

Divide by 12:

$$x^2 - 2x - 15 = 0$$

Factor:

$$(x - 5)(x + 3) = 0$$

$$\boxed{x = 5,\ x = -3}$$

## A.10 Q10

Transpose $b = g + t(a - 3)$ to make $a$ the subject

Starting with:

$$b = g + t(a - 3)$$

this is easily rearranged to isolate $a$:

$$b - g = t(a - 3)$$

$$\frac{b - g}{t} = a - 3$$

$$\boxed{a = \frac{b - g}{t} + 3}$$

# Part B

# Computing

Part B should include commented MATLAB code and screenshots of the command window and graphs. All graph formatting must be done by code. All programmes should have user-friendly input and formatted output.

## Notes:

- All graphs and images in the questions are created using TikZ PGF by yours truly.
- Code implementations are original and written primarily in MATLAB to adhere to guidelines, with some sections in Python. For easy code access, please refer to my GitHub repository.
  - While I am required to use MATLAB for this module, relying solely on it feels somewhat short-sighted. It is a temporary student license, and focusing exclusively on MATLAB means missing out on essential programming skills and their broader applicability. Although I must use MATLAB for this course, it does not fully align with my long-term goals.
- To develop a robust, universal, and user-friendly code for this section, it is crucial to consider both usability and accessibility across a broad spectrum of potential users. Whether users are seasoned developers or novices, the approach taken should cater to their needs in an intuitive and efficient manner. Two primary strategies can be employed to achieve this:
  1. **Approach 1: Enhancing User Experience through Clear and Visual Representation of Mathematical Solutions** When solving complex mathematical problems using software such as MATLAB, presenting the results in a clear and visually engaging format is crucial for enhancing the user's comprehension and interaction with the solution. Whether dealing with calculus, linear algebra, vector analysis, geometry, trigonometry, or statistical data, effective output presentation is key. The following strategies can be used to achieve this:
     - *Graphical Representation of Mathematical Solutions*: Solutions involving differentiation, integration, matrix operations, or vector analysis can often be more intuitively understood when visualized. Plotting graphs of functions (e.g., derivatives, integrals), 3D surfaces, vector fields, and matrix visualizations helps users grasp complex concepts quickly. MATLAB's built-in plotting functions such as `plot`, `surf`, and `quiver` can be utilized to generate these visualizations.
     - *Interactive Graphical Interfaces for Mathematical Exploration*: Creating interactive UI elements, such as sliders for parameter adjustment, allows users to dynamically explore how changes in inputs affect the solution. For example, users can visualize the effects of altering coefficients in a system of differential equations or observe real-time changes in vector fields with interactive 3D plots.

  - *Formatted Output of Analytical Results*: For problems involving algebraic solutions (e.g., symbolic differentiation, integration, matrix factorizations), presenting the results in a neatly formatted output, using LaTeX-style expressions or equation editors, ensures clarity. MATLAB's `live scripts` and symbolic math toolbox can display these outputs in a way that is both readable and professional.
  - *Data Analysis and Statistical Visualization*: When evaluating statistical data, probability distributions, or regression models, it's essential to represent results in charts such as histograms, scatter plots, box plots, or pie charts. Visual tools for analyzing statistical variance, probability distributions, or hypothesis testing can make data-driven insights more accessible.

2. **Approach 2: Providing an Interactive Programming Environment** For users more comfortable with programming, but still seeking a level of simplicity, an interactive coding environment can serve as an excellent middle ground. This environment would provide:

  - *Clear and Accessible Code Snippets*: Providing well-commented, modular code blocks that users can easily adapt and integrate into their own projects encourages experimentation without requiring deep understanding of the entire system.
  - *Interactive Command-Line Interface (CLI)*: An interactive CLI can guide users through the execution of the code by providing prompts, options, and feedback, making it easier to follow steps and understand what each part of the code accomplishes.
  - *Jupyter Notebooks or Similar Tools*: Environments like Jupyter Notebooks allow users to experiment with the code interactively, running small sections, visualizing results, and receiving immediate feedback, all while maintaining clarity and structure in the workflow.

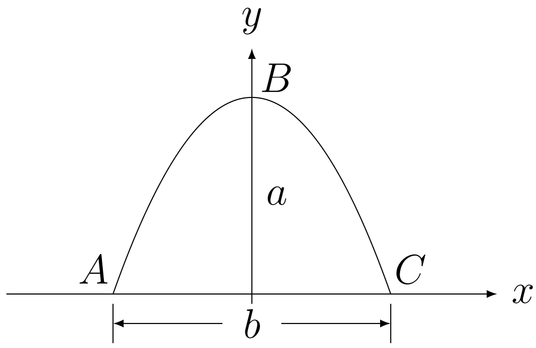
In summary, these two approaches collectively cater to both ends of the user spectrum, ensuring that the code is accessible, user-friendly, and adaptable. For each query or task, I will provide at least one refined solution, allowing flexibility for users to select the one that best meets their individual preferences and expertise.

- Finally, please consider the answers as a whole.

## B.1 Q1

The arc length of a segment of a parabola $ABC$ of an ellipse with semi-minor axes $a$ and $b$ is given approximately by:

$$L_{ABC} = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a}\ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$

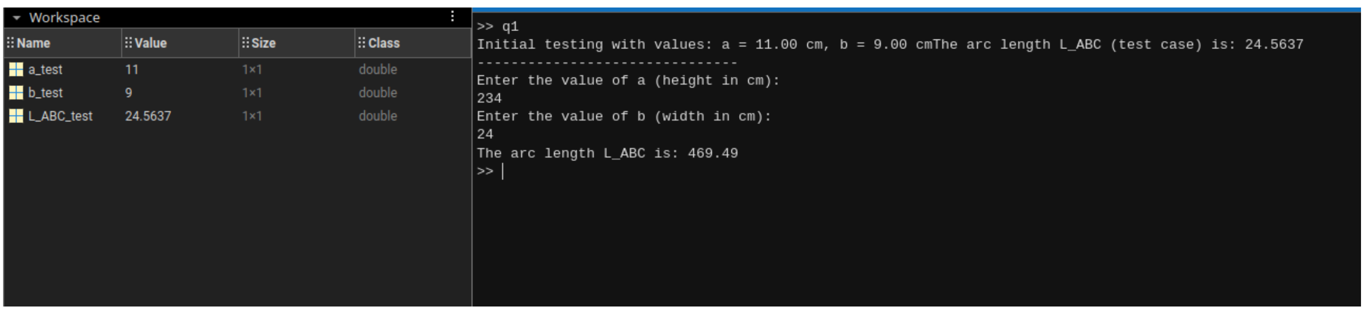



Write a universal, user-friendly code, test your programme and determine $L_{ABC}$ if $a = 11$ cm and $b = 9$ cm.

**matlab scripts/q1.m**

```matlab

function L_ABC = l_acb(a,b)
    L_ABC=((1/2) * sqrt(b^2 + 16 * a^2) + (b^2 / (8 * a)) * log((4 * a + sqrt(b^2 + 16 * a^2)) / b));
end
%make a function to call for the calculation
function calculate_arc_length()
    %prompt the user for input values
    a = input('Enter the value of a (height in cm): ');
    b = input('Enter the value of b (width in cm): ');
    L_ABC = l_acb(a,b);%realistically not much error handling is inputed but i dont think that is the gaol here
    fprintf('The arc length L_ABC is: %.2f\n', L_ABC);
end
a_test=11;
b_test=9;
% calculate using the test values
L_ABC_test=l_acb(a_test,b_test);
fprintf('Initial testing with values: a = %.2f cm, b = %.2f cm', ...
    a_test, b_test);
fprintf('The arc length L_ABC (test case) is: %.4f\n', L_ABC_test);
fprintf('-------------------------------\n');
%calculate the arc length by calling the function
calculate_arc_length();
```



### B.1.1 Advanced version

We are tasked with analyzing a parabola where:

- $a$ represents the height of the parabola
- $b$ represents the width of the parabola

My goal is to plot the parabola, first by formulating an equation in terms of these factors. Initially, we begin with a simple parabola in the general shape ($x^2$) similar to shown in the question. Our goal is to select a reasonable function, with variables associated with the $x$ scale and $y$ location of the parabola:

$$y = -(\beta x)^2 + \gamma$$

Given our problem statement:

- $\gamma$ represents the height, so $\gamma = a$
- The width is related to the roots of the equation when $y = 0$

To find $\beta$, we solve:

$$0 = -(\beta x)^2 + \gamma$$

$$\beta = \frac{2\sqrt{\gamma}}{b}$$

since we know $\gamma$:

$$\beta = \frac{2\sqrt{a}}{b}$$

Note: This requires $b \neq 0$ and $\sqrt{a} \neq 0$ (i.e., $a > 0$).
Substituting $\gamma$ and $\beta$ into our original equation:

$$\boxed{y = -\left(\frac{2\sqrt{a}}{b}x\right)^2 + a}$$

This is our final parabola equation in terms of $a$ and $b$.
The approximation arc length of the parabola from $x = -\frac{b}{2}$ to $x = \frac{b}{2}$ is given by:

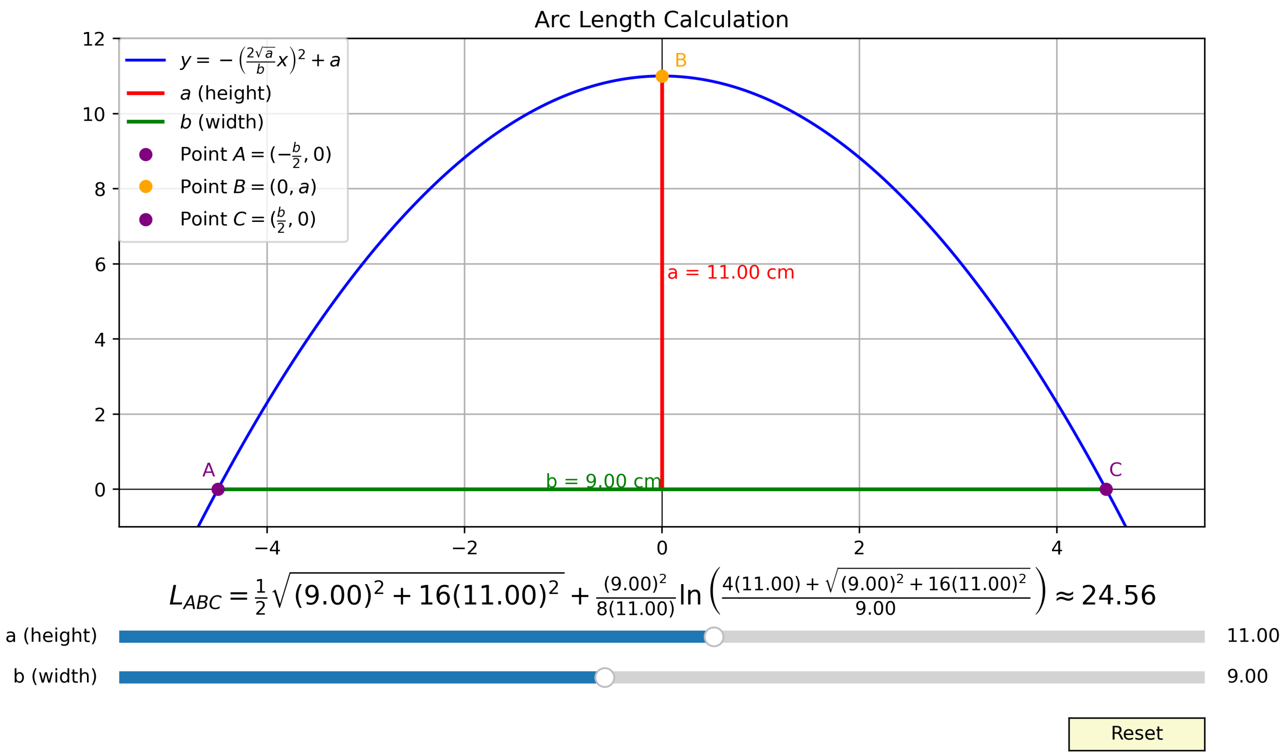
$$L_{ABC} = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a}\ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$

This result is close to evaluating the integral $\int \sqrt{1 + \left(\frac{\partial}{\partial x}\left(-\left(\frac{2\sqrt{a}}{b}x\right)^2 + a\right)\right)^2}\,dx$. When implementing this in a program:

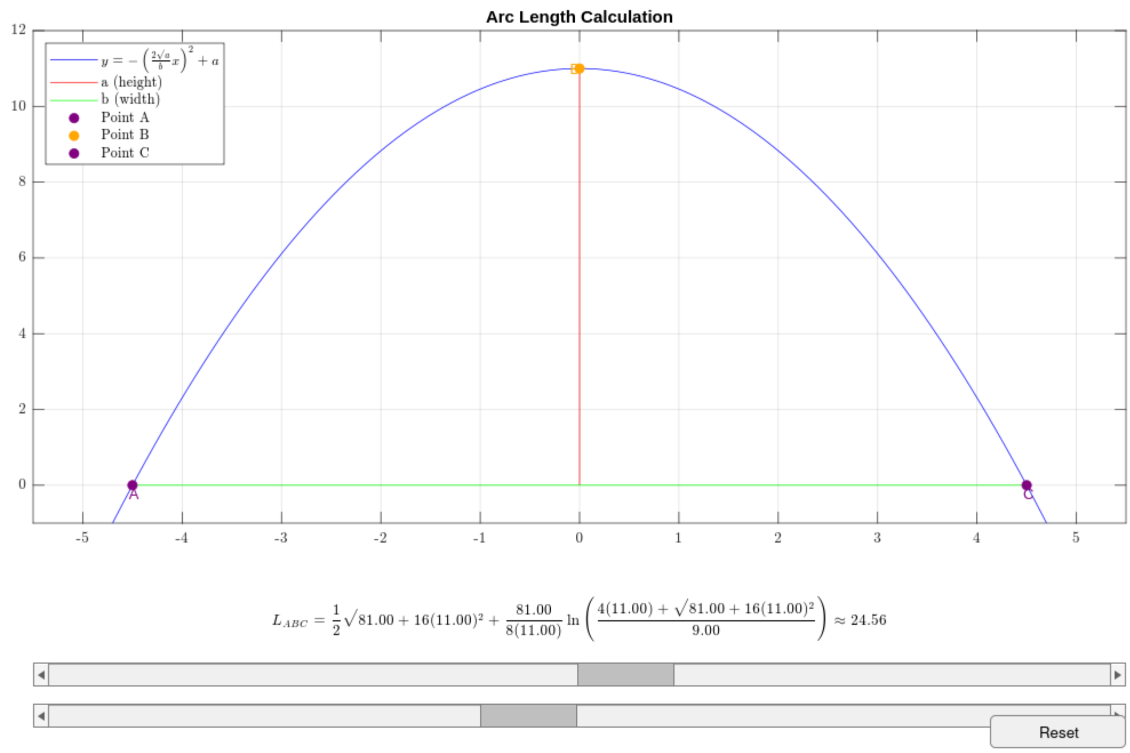
- Ensure that when there are no limit parameters for the inputs $a$ and $b$, the conditions $a > 0$ and $b \neq 0$ are appropriately handled.
- Update the plot of $y = -\left(\frac{2\sqrt{a}}{b}x\right)^2 + a$ so that the sliders for $a$ and $b$ directly control the height and width, respectively.
- Calculate the arc length of the parabola using the function given the question with inputs $a$ and $b$, and display the result in the plot.

For the remaining portion, it just boils down to presentation and flavour. I tried using MATLAB to do this, but it seemed really awkward and unreliable. Although its fine `.m` version still could require development, but I find that the `.py` version functions better.

## py scripts/q1_advanced.py




## matlab scripts/q1_advanced.m



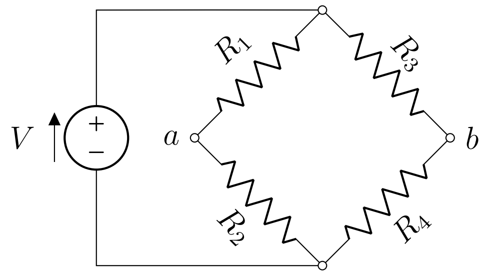
## B.2 Q2

The voltage difference $V_{ab}$ between points $a$ and $b$ in the Wheatstone bridge circuit is:

$$V_{ab} = V\left(\frac{R_1R_3 - R_2R_4}{(R_1 + R_2)(R_3 + R_4)}\right)$$

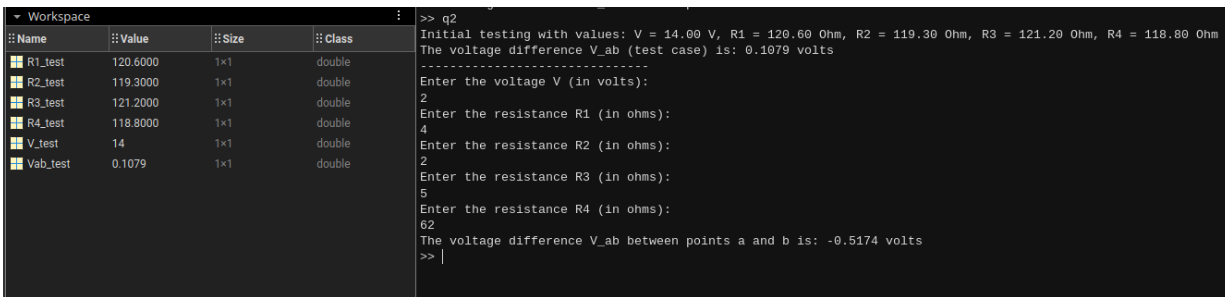
Write a universal, user-friendly program that calculates the voltage difference $V_{ab}$. Test your program using the following values:

- $V = 14$ volts
- $R_1 = 120.6\ \Omega$
- $R_2 = 119.3\ \Omega$
- $R_3 = 121.2\ \Omega$
- $R_4 = 118.8\ \Omega$




**matlab scripts/q2_.m**

```
function calculate_voltage_difference()
    % request user input for voltage and resistances
    V = input('Enter the voltage V (in volts): ');
    R1 = input('Enter the resistance R1 (in ohms): ');
    R2 = input('Enter the resistance R2 (in ohms): ');
    R3 = input('Enter the resistance R3 (in ohms): ');
    R4 = input('Enter the resistance R4 (in ohms): ');
    Vab = V * (R1 * R3 - R2 * R4) / ((R1 + R2) * (R3 + R4));
    fprintf('The voltage difference V_ab between points a and b is: %.4f volts\n',
        Vab);
end
% test the program with given values
V_test = 14;
R1_test = 120.6;
R2_test = 119.3;
R3_test = 121.2;
R4_test = 118.8;
% calculate using the test values
Vab_test = V_test * (R1_test * R3_test - R2_test * R4_test) / ((R1_test + R2_test) *
    (R3_test + R4_test));
fprintf('Initial testing with values: V = %.2f V, R1 = %.2f Ohm, R2 = %.2f Ohm, R3 =
    %.2f Ohm, R4 = %.2f Ohm\n', ...
        V_test, R1_test, R2_test, R3_test, R4_test);
fprintf('The voltage difference V_ab (test case) is: %.4f volts\n', Vab_test);
fprintf('------------------------------\n');
calculate_voltage_difference()
```

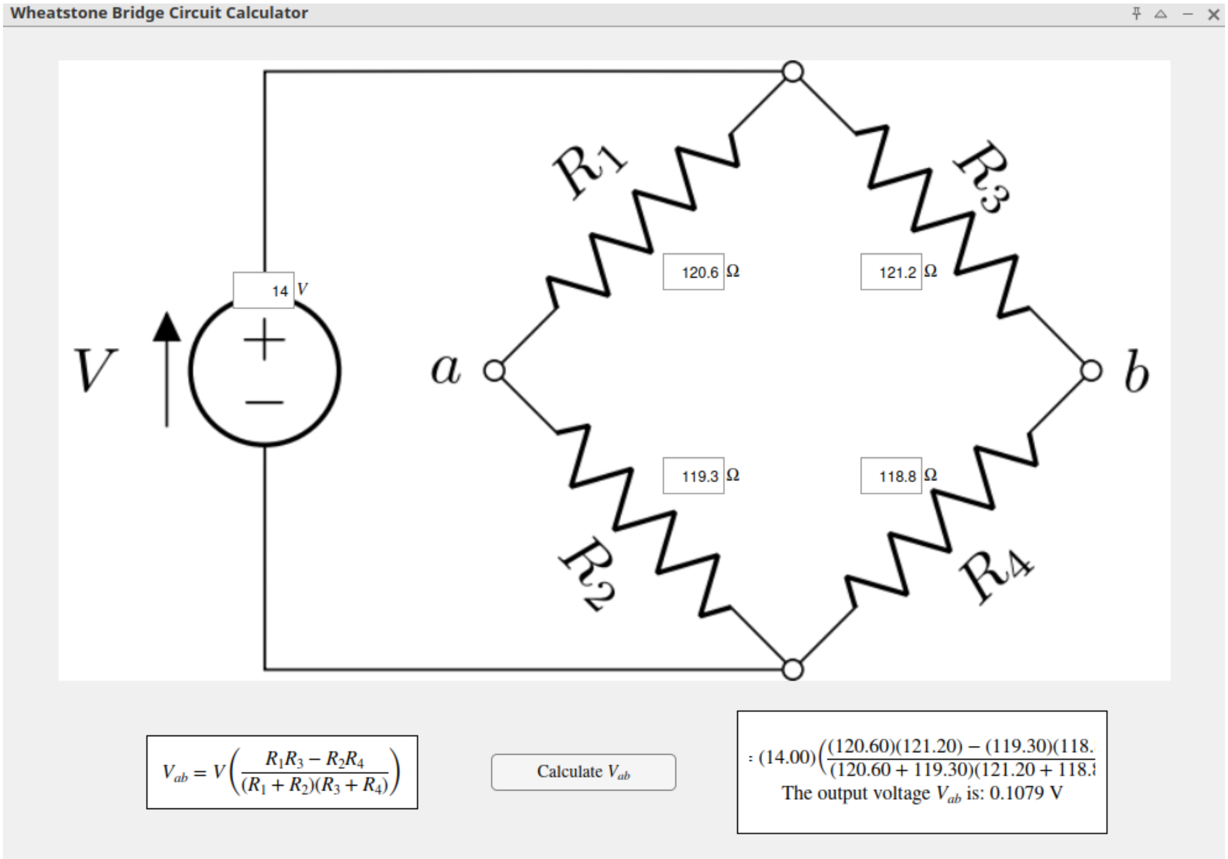


### B.2.1 Somewhat Advanced Version

I say "somewhat" because this isn't all that advanced. One way to improve this project is by creating an animation that demonstrates how the voltage changes over time, perhaps using Manim. You could also incorporate real-time elements, such as allowing users to adjust the resistors with sliders, making it interactive. Ultimately, it's all about how creative you want to be with this.

For now, I have developed a nice UI interface in MATLAB, which should suffice. If I have enough time and motivation, I may revisit this and create the animation.

**matlab scripts/q2_advanced.m**




please don't take about wrapping i will fix if u care that much.

## B.3 Q2

Newton's law of cooling gives the temperature $T(t)$ of an object at time $t$ in terms of $T_0$, its temperature at $t = 0$, and $T_s$, the temperature of the surroundings:

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

A police officer arrives at a crime scene in a hotel room at 9:18 PM, where he finds a dead body. He immediately measures the body's temperature and finds it to be $26.4°C$. Exactly one hour later, he measures the temperature again and finds it to be $25.5°C$.

Determine the time of death, assuming that the victim's body temperature was normal ($36.6°C$) prior to death, and that the room temperature was constant at $20.5°C$.

To solve this problem, i need to fully understand it first before actually importing computing aspects. First i will use the formula given along with the given information to find the time of death.

Let's break it down step by step:

1. **Finding the cooling constant** $k$:
- At 9:18 PM ($t = 0$): $T_0 = 26.4°C$
- At 10:18 PM ($t = 1$ hour): $T(1) = 25.5°C$
- Room temperature ($T_s$) = $20.5°C$

Using the equation:

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

Plugging in the values:

$$25.5 = 20.5 + (26.4 - 20.5)e^{-k\cdot 1}$$

This simplifies to:

$$5 = 5.9e^{-k}$$

Rearranging gives:

$$e^{-k} = \frac{5}{5.9}$$

Taking the natural logarithm:

$$-k = \ln\left(\frac{5}{5.9}\right)$$

Thus,

$$k \approx -\ln\left(\frac{5}{5.9}\right) \approx 0.1656 \text{ per hour}$$

2. **Finding the time when** $T_0 = 36.6°C$:
Now that we have $k$, we can use the original equation to find $t$:

$$26.4 = 20.5 + (36.6 - 20.5)e^{-0.1656t}$$

Simplifying this:

$$5.9 = 16.1e^{-0.1656t}$$

Taking the natural logarithm gives:

$$e^{-0.1656t} = \frac{5.9}{16.1}$$

Taking the natural logarithm again:

$$-0.1656t = \ln\left(\frac{5.9}{16.1}\right)$$

Finally, solving for $t$:

$$t = -\frac{\ln\left(\frac{5.9}{16.1}\right)}{0.1656} \approx 6.24 \text{ hours}$$

3. **Calculating the time of death**:
This means the body had been cooling for about 6.24 hours when the officer arrived at $9:18$ PM. To find the time of death, we subtract 6.24 hours from $9:18$ PM:

$$9:18 \text{ PM} - 6.24 \text{ hours} \approx 3:04 \text{ PM}$$

Therefore, the estimated time of death is around **3:04 PM** on the same day.

**Note:** This is an estimate based on the model and assumptions given. In real forensic work, many other factors would be considered for a more accurate determination of time of death.