Engineering Mathematics and Computing

Task 2: Coursework Assessment

Student Name: Sakariye Abiikar
KID: 2371673

Last Updated: November 6, 2024
Submission Deadline: December 12, 2024, 5pm

Git Repo : https://github.com/sakx7/mathcompuni2

Part A

Mathematics

A.1 Q1

1. Find $\int \frac{1}{7x+6}\, dx$

To evaluate the integral

$$\int \frac{1}{7x+6}\, dx$$

use substitution, Let

$$u = 7x + 6$$

then

$$\frac{du}{dx} = 7 \quad \Rightarrow \quad dx = \frac{du}{7}$$

Substituting into the integral, we get

$$\int \frac{1}{7x+6}\, dx = \frac{1}{7} \int \frac{1}{u}\, du$$

this is easy

$$\frac{1}{7} \int \frac{1}{u}\, du = \frac{1}{7} \ln |u| + C$$

Finally, substituting back $u = 7x + 6$, we get

$$\boxed{\int \frac{1}{7x+6}\, dx = \frac{1}{7} \ln |7x+6| + C}$$

A.2 Q2

2. Find $\int \frac{x}{\sqrt{4-x^2}}\,dx$

To evaluate the integral

$$\int \frac{x}{\sqrt{4-x^2}}\,dx$$

we can use a substitution method.
Let

$$u = 4 - x^2$$

then

$$\frac{du}{dx} = -2x \quad \Rightarrow \quad dx = -\frac{du}{2x}$$

Substituting into integral, we get

$$\int \frac{x}{\sqrt{4-x^2}}\,dx = \int \frac{x}{\sqrt{u}} \cdot \left(-\frac{du}{2x}\right) = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$

this is easy

$$-\frac{1}{2}\int u^{-\frac{1}{2}}\,du = -\frac{1}{2} \cdot 2\sqrt{u} + C = -\sqrt{u} + C$$

Finally, substituting back $u = 4 - x^2$, we get

$$\boxed{\int \frac{x}{\sqrt{4-x^2}}\,dx = -\sqrt{4-x^2} + C}$$

A.3 Q3

3. Obtain the general solution of the equation $\frac{d^2y}{dx^2} - 18\frac{dy}{dx} + 81y = 0$

$$\frac{d^2y}{dx^2} - 18\frac{dy}{dx} + 81y = 0$$

We use the ansatz $y = e^{rx}$, with the respective derivative:

$$r^2e^{rx} - 18re^{rx} + 81e^{rx} = 0$$

Dividing through by e^{rx} (which is never zero), we get the characteristic equation:

$$r^2 - 18r + 81 = 0$$

solve this quadratic equation, relatively easy:

$$r^2 - 18r + 81 = (r - 9)(r - 9) = 0$$

This gives us a repeated root:

$$r_1 = r_2 = 9$$

For a second-order linear homogeneous differential equation with constant coefficients and a repeated root r, the general solution is:

$$y(x) = c_1e^{rx} + c_2xe^{rx}$$

Substituting $r = 9$ into the general solution, we get:

$$y(x) = c_1e^{9x} + c_2xe^{9x}$$

So the correct form of the solution is:

$$\boxed{y(x) = e^{9x}(c_1 + c_2x)}$$

A.4 Q4

4. Find the particular solution of the differential equation $\frac{dy}{dx} + 3yx^3 = 0$, given $y(0) = 1$

$$\frac{dy}{dx} + 3yx^3 = 0$$

Separate the variables:

$$\frac{1}{y}\,dy = -3x^3\,dx$$

Integrating both sides:

$$\int \frac{1}{y}\,dy = \int -3x^3\,dx$$

$$\ln|y| = -\frac{3x^4}{4} + C$$

Exponentiating both sides:

$$\begin{aligned} y =& \exp\left(-\frac{3x^4}{4} + C\right) \\ =& C\exp\left(-\frac{3x^4}{4}\right) \end{aligned}$$

Apply the initial condition $y(0) = 1$:

$$1 = C\exp\left(-\frac{3(0)^4}{4}\right)$$

$$C = 1$$

Thus, the particular solution is:

$$\boxed{y(x) = e^{-\frac{3x^4}{4}}}$$

A.5 Q5

5. If $z = \frac{11+10j}{9-3j}$, express both $\frac{1}{z}$ and $z + \frac{1}{z}$ in the standard form $\alpha + \beta j$

$$z = \frac{11+10j}{9-3j} \quad \Rightarrow \quad \frac{1}{z} = \frac{9-3j}{11+10j}$$

rationalize the denominator:

$$\frac{1}{z} = \frac{9-3j}{11+10j} = \frac{(9-3j)(11-10j)}{(11+10j)(11-10j)}$$

First, simplify the denominator:

$$(11+10j)(11-10j) = 11^2 - (10j)^2 = 121 - (-100) = 121 + 100 = 221$$

Now simplify the numerator:

$$(9-3j)(11-10j) = (9 \cdot 11) + (9 \cdot -10j) + (-3j \cdot 11) + (-3j \cdot -10j)$$

$$= 99 - 90j - 33j - 30 = 69 - 123j$$

Thus:

$$\frac{1}{z} = \frac{69-123j}{221}$$

Now, break it down:

$$\boxed{\frac{1}{z} = \frac{69}{221} + \frac{-123}{221}j \approx 0.31 - 0.56j}$$

Now, calculate $z + \frac{1}{z}$: First, let's simplify z:

$$z = \frac{11+10j}{9-3j} \cdot \frac{9+3j}{9+3j} = \frac{(11+10j)(9+3j)}{(9-3j)(9+3j)}$$

Calculate the denominator:

$$(9-3j)(9+3j) = 9^2 - (3j)^2 = 81 + 9 = 90$$

Calculate the numerator:

$$(11+10j)(9+3j) = (11 \cdot 9) + (11 \cdot 3j) + (10j \cdot 9) + (10j \cdot 3j) = 99 + 33j + 90j + 30j^2$$

$$= 99 + 123j + 30(-1) = 99 + 123j - 30 = 69 + 123j$$

So:

$$z = \frac{69+123j}{90} = \frac{69}{90} + \frac{123}{90}j = \frac{23}{30} + \frac{41}{30}j$$

$$z + \frac{1}{z} = \left(\frac{23}{30} + \frac{41}{30}j\right) + \left(\frac{69}{221} - \frac{123}{221}j\right)$$

Find a common denominator for the fractions:

$$z + \frac{1}{z} = \frac{23 \cdot 221 + 69 \cdot 30}{30 \cdot 221} + \frac{41 \cdot 221 - 123 \cdot 30}{30 \cdot 221}j$$

$$= \frac{7153}{6630} + \frac{5371}{6630}j$$

Thus:

$$\boxed{z + \frac{1}{z} = \frac{7153}{6630} + \frac{5371}{6630}j \approx 1.08 + 0.81j}$$

A.6 Q6

6. Obtain the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 48y = 5$

To obtain the general solution of the non-homogeneous differential equation:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 48y = 5$$

we follow these steps:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 48y = 0$$

We use the ansatz $y = e^{rx}$. Substituting this into the homogeneous equation, we get:

$$r^2e^{rx} - 2re^{rx} - 48e^{rx} = 0$$

Dividing through by e^{rx} (which is never zero), we obtain the characteristic equation:

$$r^2 - 2r - 48 = 0$$

Solving this quadratic equation:

$$r^2 - 2r - 48 = (r - 8)(r + 6) = 0$$

This gives us the roots:

$$r_1 = 8, \quad r_2 = -6$$

Therefore, the general solution to the homogeneous equation is:

$$y_h(x) = c_1e^{8x} + c_2e^{-6x}$$

Since the non-homogeneous term is a constant (5), we use the ansatz $y_p = A$, where A is a constant. Substituting $y_p = A$ into the non-homogeneous equation:

$$\frac{d^2A}{dx^2} - 2\frac{dA}{dx} - 48A = 5$$

Since A is a constant, its derivatives are zero:

$$-48A = 5$$

Solving for A:

$$A = -\frac{5}{48}$$

Therefore, the particular solution is:

$$y_p(x) = -\frac{5}{48}$$

The general solution to the non-homogeneous equation is the sum of the homogeneous solution and the particular solution:

$$y(x) = y_h(x) + y_p(x)$$

$$y(x) = c_1e^{8x} + c_2e^{-6x} - \frac{5}{48}$$

A.7 Q7

7. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = xy^4e^{2x}$

To find $\frac{\partial z}{\partial x}$, treat y as a constant:

$$z = xy^4e^{2x}$$

Differentiate with respect to x:

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(xy^4e^{2x}\right)$$

Use the product rule. For the terms x and y^4e^{2x}, we get:

$$\frac{\partial z}{\partial x} = \left(\frac{\partial}{\partial x}x\right)y^4e^{2x} + x\frac{\partial}{\partial x}\left(y^4e^{2x}\right)$$

The derivative of x with respect to x is 1, and the derivative of y^4e^{2x} with respect to x is $y^4 \cdot 2e^{2x}$. Therefore:

$$\frac{\partial z}{\partial x} = y^4e^{2x} + xy^4 \cdot 2e^{2x} = e^{2x}(2x+1)y^4$$

Thus:

$$\boxed{\frac{\partial z}{\partial x} = e^{2x}(2x+1)y^4}$$

To find $\frac{\partial z}{\partial y}$, treat x as a constant:

$$z = xy^4e^{2x}$$

Differentiate with respect to y:

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(xy^4e^{2x}\right)$$

Since x and e^{2x} are constants with respect to y, we only differentiate y^4:

$$\frac{\partial z}{\partial y} = xe^{2x} \cdot \frac{\partial}{\partial y}y^4 = xe^{2x} \cdot 4y^3$$

Thus:

$$\boxed{\frac{\partial z}{\partial y} = 4xy^3e^{2x}}$$

A.8 Q8

8. Integrate the function $\int \frac{x+9}{x(x+5)}\,dx$

$$\int \frac{x+9}{x(x+5)}\,dx$$

split the integral:

$$\int \frac{x+9}{x(x+5)}\,dx = \int \frac{1}{x}\,dx + 4\int \frac{1}{x(x+5)}\,dx$$

The first integral is straightforward:

$$\int \frac{1}{x}\,dx = \ln|x| + C_1$$

For the second integral, rewrite:

$$\int \frac{1}{x(x+5)}\,dx = \int \frac{1}{x^2\left(\frac{5}{x}+1\right)}\,dx$$

Let $u = \frac{5}{x} + 1$, then $dx = -\frac{x^2}{5}\,du$. Substituting into the integral, we get:

$$-\frac{1}{5}\int \frac{1}{u}\,du = -\frac{1}{5}\ln|u| + C_2$$

Combing results and Substituting back $u = \frac{5}{x} + 1$:

$$\ln|x| + 4\left(-\frac{1}{5}\ln|x+5| + \frac{1}{5}\ln|x|\right) + C = -\frac{4\ln|x+5| - 9\ln|x|}{5} + C$$

Thus, the solution is:

$$\boxed{\int \frac{x+9}{x(x+5)}\,dx = -\frac{4\ln|x+5| - 9\ln|x|}{5} + C}$$

A.9 Q9

9. Solve the equation $\frac{dy}{dt} + y \cot t = 5 \sin t$

$$\frac{dy}{dt} + y \cot t = 5 \sin t$$

First, we identify the integrating factor $\mu(t)$:

$$\mu(t) = e^{\int \cot t \, dt} = e^{\ln|\sin t|} = |\sin t|$$

Multiplying both sides of the differential equation by $\mu(t)$, we get:

$$\sin t \frac{dy}{dt} + y \sin t \cot t = 5 \sin^2 t$$

$$\frac{dy}{dt} \sin t + y \cos(t) = 5 \sin^2 t$$

$$u'v + uv'$$

using the product rule, the derivative of $y \sin(t)$ with respect to t is the left hand side.

$$\frac{d}{dt}(y \sin t) = 5 \sin^2 t$$

Integrating both sides, we obtain:

$$y \sin t = \int 5 \sin^2 t \, dt$$

Using the identity $\sin^2 t = \frac{1 - \cos 2t}{2}$, we get:

$$y \sin t = \int 5 \cdot \frac{1 - \cos 2t}{2} \, dt = \frac{5}{2} \left(\int 1 \, dt - \int \cos 2t \, dt \right)$$

$$= \frac{5}{2} \left(t - \frac{\sin 2t}{2} \right) + C$$

Solving for y, we get:

$$y = \frac{5}{2 \sin t} \left(t - \frac{\sin 2t}{2} \right) + \frac{C}{\sin t}$$

Thus, the solution is:

$$\boxed{y = C \csc(t) + \frac{5}{2}(t \csc(t) - \cos(t))}$$

A.10 Q10

10. Evaluate the integral $\int_1^3 4xe^{4x}\,dx$

$$\int_1^3 4xe^{4x}\,dx$$

$$\text{let } u = 4x \quad dx = \tfrac{1}{4}$$

$$\frac{1}{4}\int_4^{12} ue^u\,dx$$

$$\frac{1}{4}\left([ue^u - e^u]_4^{12}\right)$$

$$\frac{1}{4}\left(\left(12e^{12} - e^{12}\right) - \left(4e^4 - e^4\right)\right)$$

$$\boxed{\int_1^3 4xe^{4x}\,dx = \frac{11e^{12} - 3e^4}{4} \approx 4.4753 \times 10^5}$$