Engineering Mathematics and Computing

Task 2: Coursework Assessment

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Git Repo : https://github.com/sakx7/mathcompuni2

Part A

Mathematics

A.1 Q1

1. Find $\int \frac{1}{7x+6}\,dx$

$$\int \frac{1}{7x+6}\,dx$$

Use substitution, Let

$$u = 7x + 6$$

$$\frac{du}{dx} = 7 \quad \Rightarrow \quad dx = \frac{du}{7}$$

Substituting into the integral, we get

$$\int \frac{1}{7x+6}\,dx = \frac{1}{7}\int \frac{1}{u}\,du$$

This is a standard integral of the reciprocal

$$\frac{1}{7}\int \frac{1}{u}\,du = \frac{1}{7}\ln|u| + C$$

Finally, substituting u back in, we get

$$\boxed{\int \frac{1}{7x+6}\,dx = \frac{1}{7}\ln|7x+6| + C}$$

A.2 Q2

2. Find $\int \frac{x}{\sqrt{4-x^2}}\,dx$

$$\int \frac{x}{\sqrt{4-x^2}}\,dx$$

Use substitution, Let

$$u = 4 - x^2$$

$$\frac{du}{dx} = -2x \quad \Rightarrow \quad dx = -\frac{du}{2x}$$

Substituting into integral, we get

$$\int \frac{x}{\sqrt{4-x^2}}\,dx = \int \frac{x}{\sqrt{u}} \cdot \left(-\frac{du}{2x}\right) = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$

This is a standard integral of the power

$$-\frac{1}{2}\int u^{-\frac{1}{2}}\,du = -\frac{1}{2} \cdot 2\sqrt{u} + C = -\sqrt{u} + C$$

Finally, substituting back $u = 4 - x^2$, we get

$$\boxed{\int \frac{x}{\sqrt{4-x^2}}\,dx = -\sqrt{4-x^2} + C}$$

A.3 Q3

3. Obtain the general solution of the equation $\frac{d^2y}{dx^2} - 18\frac{dy}{dx} + 81y = 0$

$$\frac{d^2y}{dx^2} - 18\frac{dy}{dx} + 81y = 0$$

We use the ansatz $y = e^{rx}$, substituting it, along with its corresponding derivatives:

$$r^2e^{rx} - 18re^{rx} + 81e^{rx} = 0$$

Dividing through by e^{rx} (which is never zero), we get the characteristic equation:

$$r^2 - 18r + 81 = 0$$

We can factorize the quadratic as:

$$(r - 9)(r - 9) = 0$$

This gives us a repeated root:

$$r_1 = r_2 = 9$$

For a second-order linear homogeneous differential equation with constant coefficients and a repeated root r, the general solution is:

$$y(x) = c_1e^{rx} + c_2xe^{rx}$$

Substituting $r = 9$ into the general solution, we get:

$$y(x) = c_1e^{9x} + c_2xe^{9x}$$

Factorising we get the final solution as:

$$\boxed{y(x) = e^{9x}(c_1 + c_2x)}$$

Interval of validity: Nothing really stands out, no singularities or undefined behaviours. Therefore, the interval of validity is:

$$(-\infty, \infty) = \{x \in \mathbb{R} \mid -\infty < x < \infty\}$$

This means the solution is valid for all real values of x. The constant C does not effect the solutions validity regardless of its value.

A.4 Q4

4. Find the particular solution of the differential equation $\frac{dy}{dx} + 3yx^3 = 0$, given $y(0) = 1$

$$\frac{dy}{dx} + 3yx^3 = 0$$

Separate the variables:

$$\frac{1}{y}\,dy = -3x^3\,dx$$

Integrating both sides:

$$\int \frac{1}{y}\,dy = \int -3x^3\,dx$$

$$\ln|y| = -\frac{3x^4}{4} + C$$

Exponentiating both sides:

$$y = \exp\left(-\frac{3x^4}{4} + C\right)$$
$$y = C\exp\left(-\frac{3x^4}{4}\right)$$

Apply the initial condition $y(0) = 1$:

$$1 = C\exp\left(-\frac{3(0)^4}{4}\right)$$

$$C = 1$$

Thus, the particular solution is:

$$\boxed{y(x) = e^{-\frac{3x^4}{4}}}$$

Interval of validity: Nothing really stands out, no singularities or undefined behaviours. Therefore, the interval of validity is:

$$(-\infty, \infty) = \{x \in \mathbb{R} \mid -\infty < x < \infty\}$$

This means the solution is valid for all real values of x. The constant C does not effect the solutions validity regardless of its value

A.5 Q5

5. If $z = \frac{11+10j}{9-3j}$, express both $\frac{1}{z}$ and $z + \frac{1}{z}$ in the standard form $\alpha + \beta j$

$$z = \frac{11+10j}{9-3j}$$

$$z = \frac{(11+10j)(9-3j)}{(9-3j)(9-3j)}$$

Simplify the denominator (difference of squares):

$$\begin{aligned}(9-3j)(9+3j) &= 9^2 - (3j)^2 \\ &= 81 - (-9) \\ &= 81 + 9 = 90\end{aligned}$$

Simplify the numerator:

$$\begin{aligned}(11+10j)(9+3j) &= (11 \cdot 9) + (11 \cdot 3j) \\ &\quad + (10j \cdot 9) + (10j \cdot 3j) \\ &= 99 + 33j + 90j + 30j^2 \\ &= 99 + 123j + 30(-1) \\ &= 99 + 123j - 30 \\ &= 69 + 123j\end{aligned}$$

So:

$$\begin{aligned}z &= \frac{69+123j}{90} \\ &= \frac{69}{90} + \frac{123}{90}j\end{aligned}$$

$$z = 0.7\dot{6} + 1.3\dot{6}j$$

$$\frac{1}{z} = \frac{9-3j}{11+10j}$$

$$\frac{1}{z} = \frac{(9-3j)(11-10j)}{(11+10j)(11-10j)}$$

Simplify the denominator (difference of squares):

$$\begin{aligned}(11+10j)(11-10j) &= 11^2 - (10j)^2 \\ &= 121 - (-100) \\ &= 121 + 100 = 221\end{aligned}$$

The numerator is the conjugate of the previously calculated numerator, so:

$$(9-3j)(11-10j) = 69 - 123j$$

Thus:

$$\begin{aligned}\frac{1}{z} &= \frac{69-123j}{221} \\ &= \frac{69}{221} + \frac{-123}{221}j\end{aligned}$$

$$\boxed{\frac{1}{z} \approx 0.31 - 0.56j}$$

now we can calculate $z + \frac{1}{z}$ easily

$$\begin{aligned}z + \frac{1}{z} &= \frac{69}{90} + \frac{123}{90}j + \frac{69}{221} + \frac{-123}{221}j \\ &= \left(\frac{69}{90} + \frac{69}{221}\right) + \left(\frac{123}{90} - \frac{123}{221}\right)j\end{aligned}$$

$$\boxed{z + \frac{1}{z} \approx 1.08 + 0.81j}$$

A.6 Q6

6. Obtain the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 48y = 5$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 48y = 5$$

Since this is **non-homogenous** we need to solve for the **complementary solution (y_c)** and then the **particular solution (y_p)** the general form of the solution the addition of these

$$y(x) = y_c(x) + y_p(x)$$

First lets solve the complementary solution, which solves the associated homogeneous equation:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 48y = 0$$

We use the ansatz $y = e^{rx}$, substituting it, along with its corresponding derivatives:

$$r^2e^{rx} - 2re^{rx} - 48e^{rx} = 0$$

Dividing through by e^{rx} (which is never zero), we obtain the characteristic equation:

$$r^2 - 2r - 48 = 0$$

We can factorize the quadratic as:

$$(r-8)(r+6) = 0$$

This gives us the roots:

$$r_1 = 8, \quad r_2 = -6$$

The general solution to a second-order linear homogeneous equation is given by:

$$y(x) = c_1e^{r_1x} + c_2e^{r_2x}$$

In so for our solution this is

$$y_c(x) = c_1e^{8x} + c_2e^{-6x}$$

Since the non-homogeneous term is a constant 5, we use the ansatz $y = A$, where A is a constant. Since A is a constant, its derivatives are zero, so:

$$-48A = 5$$

$$A = -\frac{5}{48}$$

Therefore, the particular solution is:

$$y_p(x) = -\frac{5}{48}$$

Now we have complementary solution (y_c) and the particular solution (y_p), The general solution to the non-homogeneous equation can be written as so:

$$\boxed{y(x) = c_1e^{8x} + c_2e^{-6x} - \frac{5}{48}}$$

Interval of validity: Nothing really stands out, no singularities or undefined behaviours. Therefore, the interval of validity is:

$$(-\infty, \infty) = \{x \in \mathbb{R} \mid -\infty < x < \infty\}$$

This means the solution is valid for all real values of x. The constant C does not effect the solutions validity regardless of its value.

A.7 Q7

7. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $z = xy^4e^{2x}$

$$z = xy^4e^{2x}$$

$$\frac{\partial z}{\partial x} = \frac{\partial\left(xy^4e^{2x}\right)}{\partial x}$$

Use the product rule:

$$\frac{\partial(u(x)\cdot v(x))}{\partial x} = v\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial x}$$

let $u(x) = x$ and $v(x) = y^4e^{2x}$

$$\frac{\partial z}{\partial x} = \frac{\partial\left(xy^4e^{2x}\right)}{\partial x} = y^4e^{2x}\frac{\partial(x)}{\partial x} + x\frac{\partial\left(y^4e^{2x}\right)}{\partial x}$$

Easy to solve, just treat y as a constant and differentiate with respect to x:

$$\frac{\partial(x)}{\partial x} = 1, \qquad \frac{\partial\left(y^4e^{2x}\right)}{\partial x} = 2e^{2x}$$

now plug in we get

$$\begin{aligned}\frac{\partial z}{\partial x} &= y^4e^{2x} + xy^4\left(2e^{2x}\right)\\ &= e^{2x}y^4(2x+1)\end{aligned}$$

Thus:

$$\boxed{\frac{\partial z}{\partial x} = e^{2x}y^4(2x+1)}$$

$$z = xy^4e^{2x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial\left(xy^4e^{2x}\right)}{\partial y}$$

Treat x as a constant, we only differentiate y^4:

$$\frac{\partial z}{\partial y} = \frac{\partial\left(xy^4e^{2x}\right)}{\partial y} = xe^{2x}\frac{\partial\left(y^4\right)}{\partial y}$$

$$\frac{\partial\left(y^4\right)}{\partial y} = 4y^3$$

Thus:

$$\boxed{\frac{\partial z}{\partial y} = 4xy^3e^{2x}}$$

A.8 Q8

8. Integrate the function $\int \frac{x+9}{x(x+5)}\,dx$

$$\int \frac{x+9}{x(x+5)}\,dx$$

Split the integral:

$$\int \frac{x+9}{x(x+5)}\,dx = \int \frac{(x+5)+4}{x(x+5)} = \int \frac{1}{x}\,dx + 4\int \frac{1}{x(x+5)}\,dx$$

The first integral is straightforward:

$$\int \frac{1}{x}\,dx = \ln|x| + C_1$$

For the second integral, rewrite:

$$\int \frac{1}{x(x+5)}\,dx = \int \frac{1}{x^2\left(\frac{5}{x}+1\right)}\,dx$$

Now use sub

$$u = \frac{5}{x} + 1$$

$$\frac{du}{dx} = -\frac{5}{x^2} \quad \Rightarrow \quad dx = -\frac{x^2}{5}\,du$$

Substituting into the integral, we get:

$$\int -\frac{x^2}{5x^2u}\,du = -\frac{1}{5}\int \frac{1}{u}\,du = -\frac{1}{5}\ln|u| + C_2$$

Substituting $u = \frac{5}{x} + 1 = \frac{5+x}{x}$ and then combing results yields:

$$\begin{aligned}\int \frac{x+9}{x(x+5)}\,dx &= \ln|x| + 4\left(-\frac{1}{5}\ln\left|\frac{x+5}{x}\right|\right) + C \\ &= \ln|x| - \frac{4}{5}\left(\ln|x+5| - \ln|x|\right) + C \\ &= -\frac{4\ln|x+5| - 9\ln|x|}{5} + C\end{aligned}$$

Thus, the solution is:

$$\boxed{\int \frac{x+9}{x(x+5)}\,dx = \frac{9\ln|x| - 4\ln|x+5|}{5} + C}$$

A.9 Q9

9. Solve the equation $\frac{dy}{dt} + y \cot t = 5 \sin t$

$$\frac{dy}{dt} + y \cot t = 5 \sin t$$

Use integrating factor $\mu(t)$:

$$\mu(t) = e^{\int \cot t \, dt}$$

Since this is a conventional integral, I shouldn't actually include it in the calculation because it should be widely recognised as $\ln|\sin t|$. Regardless, I'll demonstrate how to solve it quickly:

$$\int \cot t \, dt = \int \frac{1}{\tan t} \, dt = \int \frac{\cos t}{\sin t} \, dt$$

Now use sub

$$u = \sin t$$

$$\frac{du}{dt} = \cos t \quad \Rightarrow \quad dx = \frac{1}{\cos t} \, dt$$

$$\int \frac{\cos t}{\sin t} \, dt = \int \frac{1}{u} du = \ln|\sin t| + C$$

so the int factor $\mu(t)$ is

$$\mu(t) = e^{\ln|\sin t|} = |\sin t|$$

Multiply both sides of the differential equation by $\mu(t)$, we get:

$$\frac{dy}{dt} \sin t + y \sin t \cot t = 5 \sin^2 t$$

$$\frac{dy}{dt} \sin t + y \cos t = 5 \sin^2 t$$

The left side of the equation is nothing more than the product rule:

$$\frac{d}{dt}(y \sin t) = 5 \sin^2 t$$

Integrating both sides, we obtain:

$$\begin{aligned} y \sin t &= \int 5 \sin^2 t \, dt \\ &= \int 5 \cdot \frac{1 - \cos 2t}{2} \, dt \\ &= \frac{5}{2} \left(\int 1 \, dt - \int \cos 2t \, dt \right) \\ &= \frac{5}{2} \left(t - \frac{\sin 2t}{2} \right) + C \end{aligned}$$

In so y as an explicit solution is:

$$\boxed{y = \frac{5}{2 \sin t} \left(t - \frac{\sin 2t}{2} \right) + \frac{C}{\sin t}}$$

Interval of validity: $\sin(t)$ is undefined where $\sin(t) = 0$, which occurs at $t = n\pi$, where n is any integer. Therefore, the interval of validity of the solution is:

$$\{t \in \mathbb{R} : t \neq n\pi, n \in \mathbb{Z}\}$$

This means the solution is valid for all t being real, except for those values where $\frac{t}{\pi}$ is an integer. The constant C does not effect the solutions validity regardless of its value.

A.10 Q10

10. Evaluate the integral $\int_1^3 4xe^{4x}\,dx$

$$\int_1^3 4xe^{4x}\,dx$$

Use substitution, Let

$$u = 4x$$

$$\frac{du}{dx} = 4 \quad \Rightarrow \quad dx = \frac{1}{4}\,du$$

Now change of limits

$$4(3) = 12, \qquad 4(1) = 4$$

The integral becomes

$$\frac{1}{4}\int_4^{12} ue^u\,dx$$

Use integration by parts $\int_b^a uv = [uv]_b^a - \int_b^a vu'$

$$u = x \qquad dv = e^u$$
$$du = 1 \qquad v = e^u$$

$$\frac{1}{4}\left([ue^u]_4^{12} - \int_4^{12} e^u\right)$$

$$\frac{1}{4}\left([ue^u - e^u]_4^{12}\right)$$

$$\frac{1}{4}\left(\left(12e^{12} - e^{12}\right) - \left(4e^4 - e^4\right)\right) \approx 4.4753 \times 10^5$$

$$\boxed{\int_1^3 4xe^{4x}\,dx \approx 4.4753 \times 10^5}$$

Part B

Computing

B.1 Q1

The boiling temperature of water, $\mathbf{T_B}$, at various altitudes $\mathbf{h}$, is given in the following table:

Altitude (m)	Boiling Temperature (°C)
0	100
2300	98.8
3000	95.1
6100	92.2
7900	90.0
10000	81.2
12000	75.6

Determine a linear equation in the form:

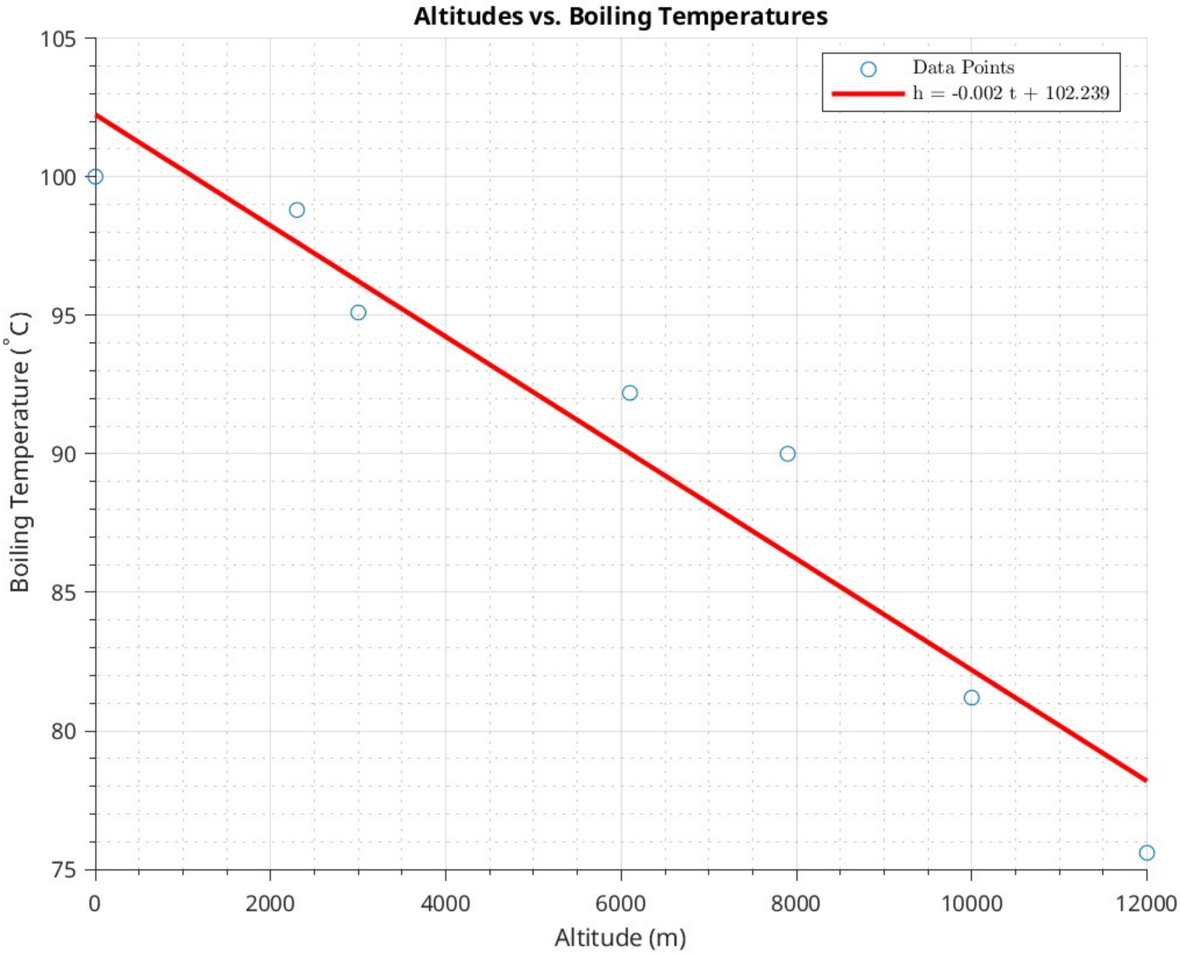
$$T_B = mh + b$$

that best fits the data. Use this equation to calculate the boiling temperature at 5000 m. Additionally, make a plot of the data points and the equation.

main/q1.m

```
function applyPlotFormatting(ax, fig, titleText, xlabelText, ylabelText, legendText,
    filename)
    if ~isempty(titleText)
        title(ax, titleText);
    end
    if ~isempty(xlabelText)
        xlabel(ax, xlabelText);
    end
    if ~isempty(ylabelText)
        ylabel(ax, ylabelText);
    end
    fig.Color = 'w';
    ax.XGrid = 'on';
    ax.YGrid = 'on';
    ax.XMinorGrid = 'on';
    ax.YMinorGrid = 'on';
    ax.XMinorTick = 'on';
    ax.YMinorTick = 'on';
    ax.TickDir = 'out';
    ax.FontName = 'Calibri';
    ax.FontSize = 9;
    if ~isempty(legendText) && ~isequal(legendText, 'None')
        legend(ax, legendText, 'Location', 'best', 'Interpreter', 'latex');
    end
    if ~isempty(filename) && ~strcmp(filename, 'None')
        outputDir = 'graphs_images';
        savePath = fullfile(outputDir,[filename,'.jpeg']);
        if ~exist(outputDir, 'dir')
            [status, msg] = mkdir(outputDir);
            if ~status
                error('Unable to create the directory "%s": %s', outputDir, msg);
            end
        end
        try
            print(fig, savePath, '-djpeg');
```

```
        catch ME
            warning('MATLAB:%s', ME.identifier, ...
                ['Error saving image to "%s".\n' ...
                 'Attempting to save in the current directory instead.\n' ...
                 'Error details: %s'], savePath, ME.message);
            savePath = fullfile(pwd, [filename, '.jpeg']);
            print(fig, savePath, '-djpeg');
        end
    end
end


% Data
h = [0, 2300, 3000, 6100, 7900, 10000, 12000]; % Altitudes (m)
t_b = [100, 98.8, 95.1, 92.2, 90, 81.2, 75.6]; % Boiling temperatures (degrees)

% Perform linear regression
p = polyfit(h, t_b, 1);

% Extract slope and intercept
slope = p(1);
intercept = p(2);

% Display results
fprintf('The slope of the line (rate of change) is: %.4f\n', slope);
fprintf('The y-intercept of the line is: %.4f\n', intercept);
disp('-------------Linear Function-----------------')
fprintf('T_b = %.4f h + %.4f \n', slope, intercept);

% Generate line of best fit
x_fit = linspace(min(h), max(h), 100);
y_fit = polyval(p, x_fit);

% Plot data and line of best fit
figure;
ax = gca;
fig = gcf;

scatter(h, t_b, 'o'); % Plot data points
hold on;
plot(x_fit, y_fit, '-r', 'LineWidth', 2); % Plot line of best fit

applyPlotFormatting( ...
    ax, ...
    fig, ...
    'Altitudes vs. Boiling Temperatures', ... % Title
    'Altitude (m)', ... % X-axis label
    'Boiling Temperature (^\circ{C})', ... % Y-axis label
    {'Data Points', sprintf('h = %.3f t + %.3f', slope, intercept)}, ... % Legend
       entries
    'altitudes_vs_boiling_temperatures'); % Save filename
```




```
The slope of the line (rate of change) is: -0.0020
The y-intercept of the line is: 102.2386
-------------Linear Function----------------
T_b = -0.0020 h + 102.2386
```

It's hard to provide input for this type of data-related question. You could consider adding altitude (h) as an input to determine the resulting boiling temperature (T_b).

B.2 Q2

The standard air density, D, at different heights, h, from sea level up to a height of 33 km is given below:

Height (km)	**Density (kg/m^3)**
0	1.2
3	0.91
6	0.66
9	0.47
12	0.31
15	0.19
18	0.12
21	0.075
24	0.046
27	0.029
30	0.018
33	0.011

Make the following four plots of the data points (D as a function of h) on the same figure:

1. Both axes with linear scale.
2. h with log axis, D with linear axis.
3. h with linear axis, D with log axis.
4. Both axes with log scale.

Based on the plots, choose a function (linear, power, exponential, or logarithmic) that best fits the data points and determine its coefficients. Plot the function and the points using linear axes.

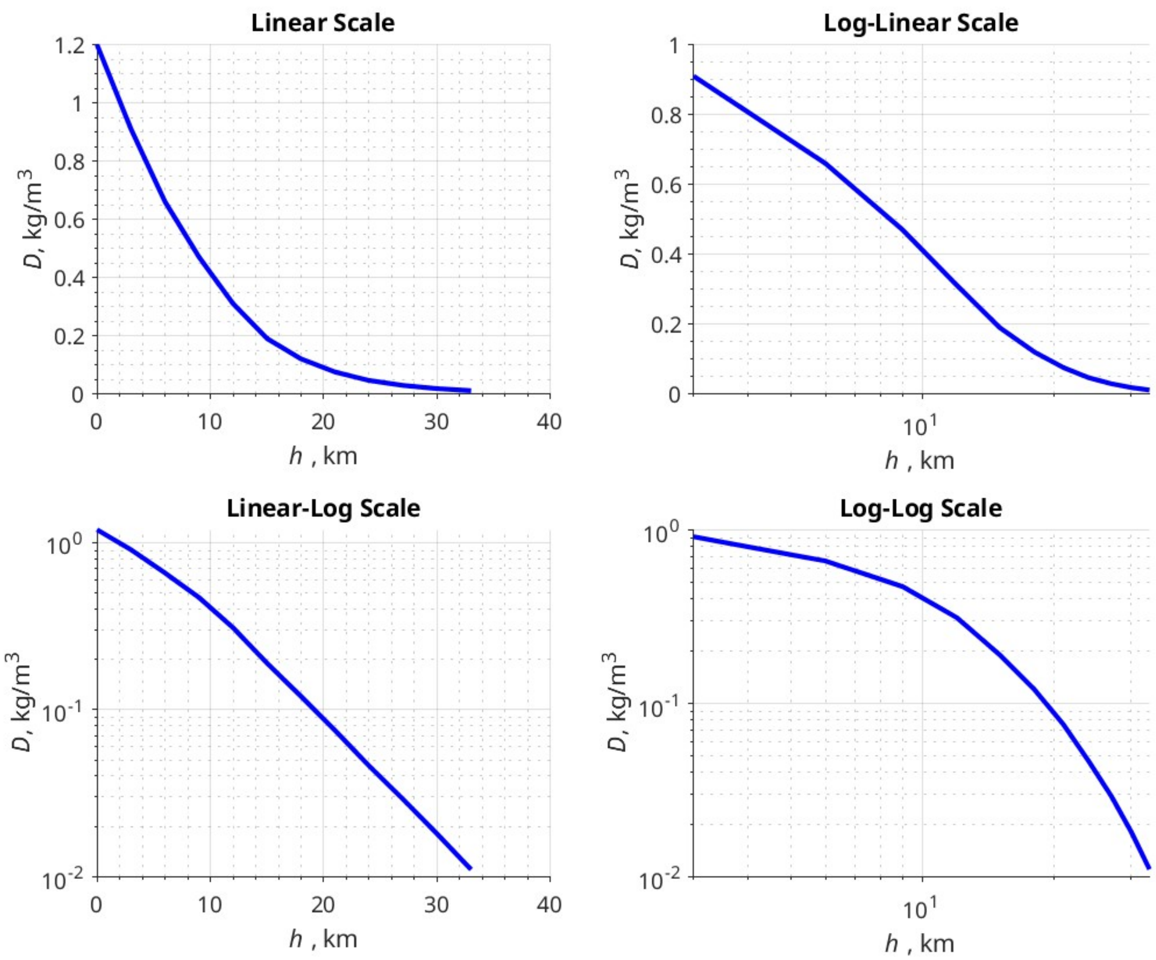
main/q2.m

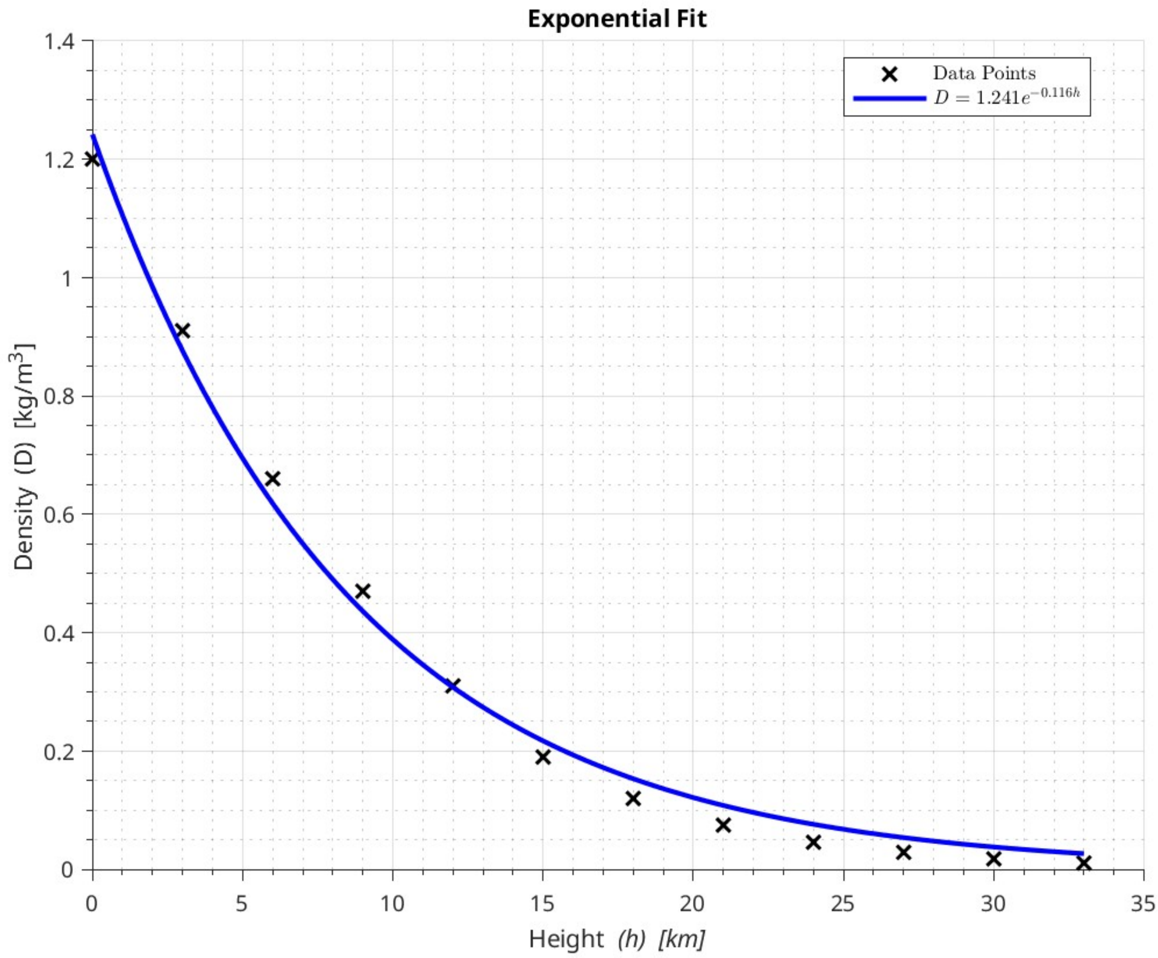
```
function applyPlotFormatting(ax, fig, titleText, xlabelText, ylabelText, legendText,
    filename)
    if ~isempty(titleText)
        title(ax, titleText);
    end
    if ~isempty(xlabelText)
        xlabel(ax, xlabelText);
    end
    if ~isempty(ylabelText)
        ylabel(ax, ylabelText);
    end
    fig.Color = 'w';
    ax.XGrid = 'on';
    ax.YGrid = 'on';
    ax.XMinorGrid = 'on';
    ax.YMinorGrid = 'on';
    ax.XMinorTick = 'on';
    ax.YMinorTick = 'on';
    ax.TickDir = 'out';
    ax.FontName = 'Calibri';
```

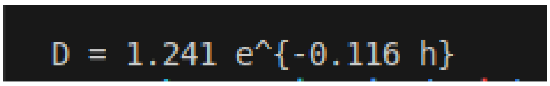
```
    ax.FontSize = 9;
    if ~isempty(legendText) && ~isequal(legendText, 'None')
        legend(ax, legendText, 'Location', 'best', 'Interpreter', 'latex');
    end
    if ~isempty(filename) && ~strcmp(filename, 'None')
        outputDir = 'graphs_images';
        savePath = fullfile(outputDir,[filename,'.jpeg']);
        if ~exist(outputDir, 'dir')
            [status, msg] = mkdir(outputDir);
            if ~status
                error('Unable to create the directory "%s": %s', outputDir, msg);
            end
        end
        try
            print(fig, savePath, '-djpeg');
        catch ME
            warning('MATLAB:%s', ME.identifier, ...
                ['Error saving image to "%s".\n' ...
                'Attempting to save in the current directory instead.\n' ...
                'Error details: %s'], savePath, ME.message);
            savePath = fullfile(pwd, [filename, '.jpeg']);
            print(fig, savePath, '-djpeg');
        end
    end
end

% Data
h = [0 3 6 9 12 15 18 21 24 27 30 33];
D = [1.2 0.91 0.66 0.47 0.31 0.19 0.12 0.075 0.046 0.029 0.018 0.011];

% Combine the arrays into a 2xN matrix also known as zipping
data = [h; D];

% Create first figure
fig1 = figure;
fig1.Color = [1, 1, 1];

% Loop for subplots
for i = 1:4
    ax(i) = subplot(2, 2, i);
    L(i) = line(data(1, :), data(2, :), 'Parent', ax(i));

    % Configure line properties
    L(i).Color = 'b';
    L(i).LineWidth = 2;
    L(i).LineStyle = '-';

    % Apply axis transformations based on subplot
    switch i
        case 1
            ax(i).XScale = 'linear';
            ax(i).YScale = 'linear';
            title = 'Linear Scale';  % Legend for linear axes
        case 2
            ax(i).XScale = 'log';
            ax(i).YScale = 'linear';
            title = 'Log-Linear Scale';  % Legend for log-linear axes
        case 3
            ax(i).XScale = 'linear';
            ax(i).YScale = 'log';
            title = 'Linear-Log Scale';  % Legend for linear-log axes
```

```
        case 4
            ax(i).XScale = 'log';
            ax(i).YScale = 'log';
            title = 'Log-Log Scale';  % Legend for log-log axes
    end
    if i==5
        applyPlotFormatting(ax(i), fig1,title,'\ith \rm, km','\itD\rm, kg/m^{3}','',''
           ');
    else
        applyPlotFormatting(ax(i), fig1,title,'\ith \rm, km','\itD\rm, kg/m^{3}','',
           'subplts');
    end
end

% Adjust scales for specific subplots
ax(2).XScale = 'log';
ax(3).YScale = 'log';
ax(4).XScale = 'log';
ax(4).YScale = 'log';

% Fit data to exponential model

res = fit(data(1, :)', data(2, :)', 'exp1');

% fit requires one of the following:
%   Curve Fitting Toolbox
%   Model-Based Calibration Toolbox
%   Predictive Maintenance Toolbox
%   SimBiology
%   Statistics and Machine Learning Toolbox


% Create second figure for the fitted curve
fig2 = figure;
fig2.Color = [1, 1, 1];

% Generate fitted line data
hplot = linspace(min(data(1, :)), max(data(1, :)), 1000);
Dplot = res.a * exp(res.b * hplot);

% Plot data and fitted curve
ax1 = subplot(1, 1, 1);

% Plot original data points
Y1 = line(data(1, :), data(2, :), 'Parent', ax1);
Y1.Color = 'k';
Y1.LineStyle = 'none';
Y1.LineWidth = 1.5;
Y1.Marker = 'x';
Y1.MarkerSize = 8;

% Plot fitted curve
Y2 = line(hplot, Dplot, 'Parent', ax1);
Y2.Color = 'b';
Y2.LineStyle = '-';
Y2.LineWidth = 2;

fprintf('D = %.3f e^{%.3f h}\n', res.a, res.b)

% Add title and labels using the applyPlotFormatting function
applyPlotFormatting(ax1, fig2, 'Exponential Fit', 'Height  \it(h)  [km]', 'Density  (
```

```
    D)  [kg/m^{3}]', ...
     {'Data Points', sprintf('$\\mathit{D} = %.3f e^{%.3f \\mathit{h}}$', res.a, res.b
        )}, 'fitted_curve');
```



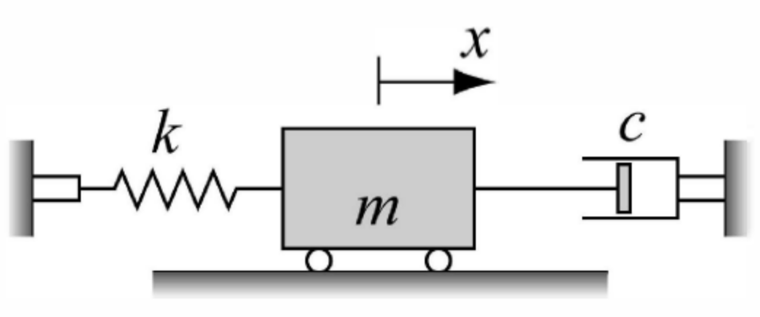




Again it's hard to provide input for this type of data-related question. You could consider adding height (h) as an input to determine the resulting density (D).

B.3 Q3

Damped free vibrations can be modeled by a block of mass (m) attached to a spring and a dashpot as shown.



From Newton's second law of motion, the displacement **x** of the mass as a function of time can be determined by solving the differential equation:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where k is the spring constant and c is the damping coefficient. If the mass is displaced from its equilibrium position and released, it will oscillate back and forth. The nature of the oscillations depends on the values of m, k, and c.

For the system shown, $m = 10\,\text{kg}$ and $k = 28\,\text{N/m}$. At $t = 0$, the mass is displaced to $x = 0.18\,\text{m}$ and released from rest. Derive expressions for the displacement $x(t)$ and velocity $v(t)$, considering the following cases:

1. $c = 3\,\text{Ns/m}$, $0 \le t \le 20\,\text{s}$,
2. $c = 50\,\text{Ns/m}$, $0 \le t \le 10\,\text{s}$.

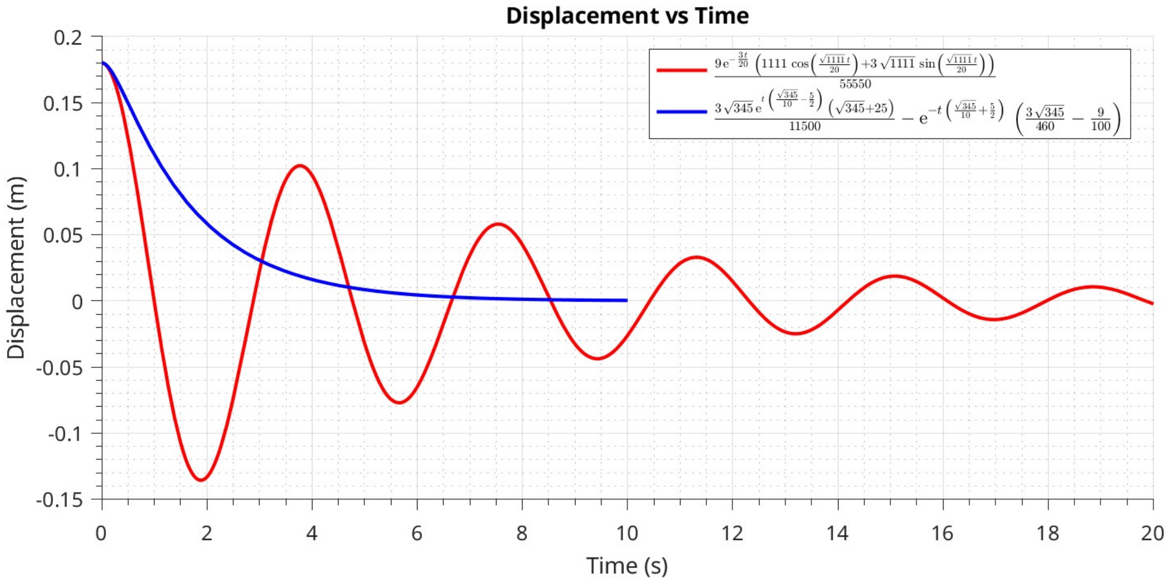
For each case, plot $x(t)$ and $v(t)$ versus t (two plots on one page).

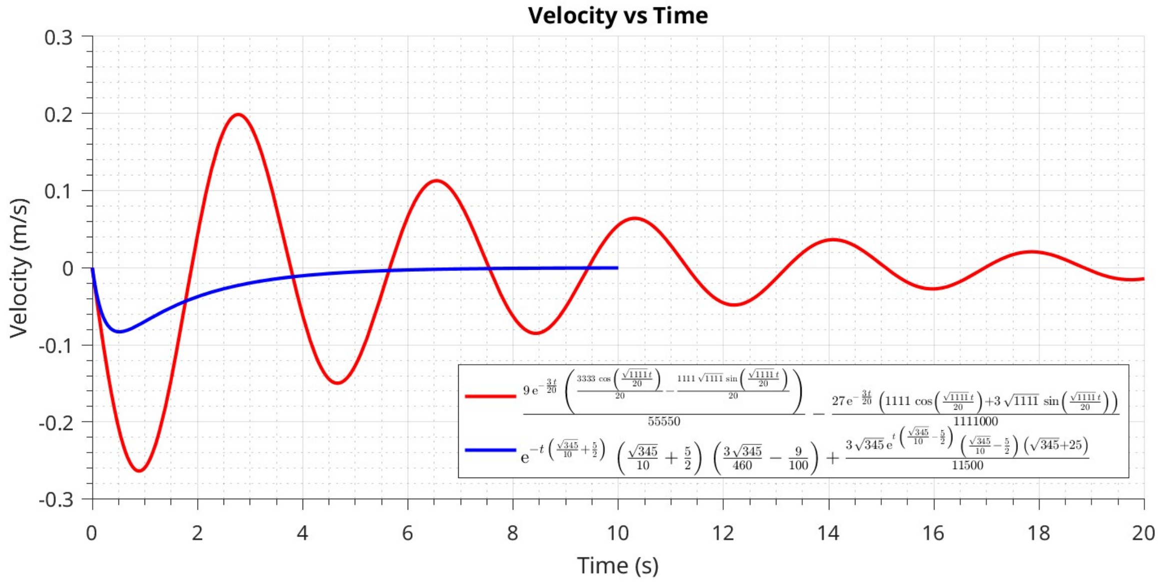
main/q3.m

```
function applyPlotFormatting(ax, fig, titleText, xlabelText, ylabelText, legendText,
    filename)
    if ~isempty(titleText)
        title(ax, titleText);
    end
    if ~isempty(xlabelText)
        xlabel(ax, xlabelText);
    end
    if ~isempty(ylabelText)
        ylabel(ax, ylabelText);
    end
    fig.Color = 'w';
    ax.XGrid = 'on';
    ax.YGrid = 'on';
    ax.XMinorGrid = 'on';
    ax.YMinorGrid = 'on';
    ax.XMinorTick = 'on';
    ax.YMinorTick = 'on';
    ax.TickDir = 'out';
```

```
    ax.FontName = 'Calibri';
    ax.FontSize = 12;
    if ~isempty(legendText) && ~isequal(legendText, 'None')
        legend(ax, legendText, 'Location', 'best', 'Interpreter', 'latex');
    end
    if ~isempty(filename) && ~strcmp(filename, 'None')
        outputDir = 'graphs_images';
        savePath = fullfile(outputDir,[filename,'.jpeg']);
        if ~exist(outputDir, 'dir')
            [status, msg] = mkdir(outputDir);
            if ~status
                error('Unable to create the directory "%s": %s', outputDir, msg);
            end
        end
        try
            print(fig, savePath, '-djpeg');
        catch ME
            warning('MATLAB:%s', ME.identifier, ...
                ['Error saving image to "%s".\n' ...
                'Attempting to save in the current directory instead.\n' ...
                'Error details: %s'], savePath, ME.message);
            savePath = fullfile(pwd, [filename, '.jpeg']);
            print(fig, savePath, '-djpeg');
        end
    end
end

% Prompt user for parameters
disp('--------- Input Parameters --------');
m = input('Enter mass (m) in kg: ');
k = input('Enter spring constant (k) in N/m: ');
x0 = input('Enter initial displacement (x0) in m: ');
v0 = input('Enter initial velocity (v(0)) IVP in m/s: ');
c1 = input('Case 1: Enter first damping coefficient (c1) in Ns/m: ');
c2 = input('Case 2: Enter second damping coefficient (c2) in Ns/m: ');
disp('---------------------------------');

% Define symbolic variables
syms x(t) c; % Define symbolic variables for displacement and damping

% Differential Equation
eq = m * diff(x, t, 2) + c * diff(x, t) + k * x == 0;
disp('Differential Equations:');
fprintf('For c1 : %.2f * d^2x/dt^2 + %.2f * dx/dt + %.2f * x = 0\n', m, c1, k);
fprintf('For c2 : %.2f * d^2x/dt^2 + %.2f * dx/dt + %.2f * x = 0\n', m, c2, k);

% Velocity
vel = diff(x, t);

% Initial Conditions
cond1 = x(0) == x0;
cond2 = vel(0) == v0;

% Solve for both damping coefficients
disp('Solving differential equations...');
Sol1 = dsolve(subs(eq, c, c1), [cond1, cond2]);
Sol2 = dsolve(subs(eq, c, c2), [cond1, cond2]);

% Display the displacement solutions
disp('Solution for c1 (x(t)):');
disp(Sol1);
```

```
disp('Solution for c2 (x(t)):');
disp(Sol2);

% Compute and display the velocity (v(t)) as the derivative of the solutions
Vel1 = diff(Sol1, t);
Vel2 = diff(Sol2, t);

disp('Velocity for c1 (v(t)):');
disp(Vel1);
disp('Velocity for c2 (v(t)):');
disp(Vel2);

% Time ranges
tmin = 0;
tmax1 = 20;
tmax2 = 10;

% Plotting
fig = figure('Position', [100, 100, 800, 800]);

% Create LaTeX strings for legend
legendStr1_disp = latex(Sol1);
legendStr2_disp = latex(Sol2);
legendStr1_vel = latex(Vel1);
legendStr2_vel = latex(Vel2);

% Escape backslashes in the LaTeX string

% Displacement plot
ax1 = subplot(2,1,1);
hold on;
fplot(Sol1, [tmin, tmax1], 'r', 'LineWidth', 2);
fplot(Sol2, [tmin, tmax2], 'b', 'LineWidth', 2);
hold off;
applyPlotFormatting(ax1, fig, 'Displacement vs Time', 'Time (s)', 'Displacement (m)',
     ...
    {['$' legendStr1_disp '$'], ['$' legendStr2_disp '$']}, '');

% Velocity plot
ax2 = subplot(2,1,2);
hold on;
fplot(Vel1, [tmin, tmax1], 'r', 'LineWidth', 2);
fplot(Vel2, [tmin, tmax2], 'b', 'LineWidth', 2);
hold off;
applyPlotFormatting(ax2, fig, 'Velocity vs Time', 'Time (s)', 'Velocity (m/s)', ...
    {['$' legendStr1_vel '$'], ['$' legendStr2_vel '$']}, 'damped_oscillation');
```






```
--------- Input Parameters --------
Enter mass (m) in kg: >>10
Enter spring constant (k) in N/m: >>28
Enter initial displacement (x0) in m: >>0.18
Enter initial velocity (v(0)) IVP in m/s: >>0
Case 1: Enter first damping coefficient (c1) in Ns/m: >>3
Case 2: Enter second damping coefficient (c2) in Ns/m: >>50
--------------------------------
Differential Equations:
For c1 : 10.00 * d^2x/dt^2 + 3.00 * dx/dt + 28.00 * x = 0
For c2 : 10.00 * d^2x/dt^2 + 50.00 * dx/dt + 28.00 * x = 0
Solving differential equations...
Solution for c1 (x(t)):
(9*exp(-(3*t)/20)*(1111*cos((1111^(1/2)*t)/20) + 3*1111^(1/2)*sin((1111^(1/2)*t)/20)))/55550

Solution for c2 (x(t)):
(3*345^(1/2)*exp(t*(345^(1/2)/10 - 5/2))*(345^(1/2) + 25))/11500 - exp(-t*(345^(1/2)/10 + 5/2))*((3*345^(1/2))/460 - 9/100)

Velocity for c1 (v(t)):
(9*exp(-(3*t)/20)*((3333*cos((1111^(1/2)*t)/20))/20 - (1111*1111^(1/2)*sin((1111^(1/2)*t)/20))/20))/55550 - (27*exp(-(3*t)/20)*(1111*cos((1111^(1/2)*t)/20) + 3*1111^(1/2)*sin((1111^(1/2)*t)/20)))/1
111000

Velocity for c2 (v(t)):
exp(-t*(345^(1/2)/10 + 5/2))*(345^(1/2)/10 + 5/2)*((3*345^(1/2))/460 - 9/100) + (3*345^(1/2)*exp(t*(345^(1/2)/10 - 5/2))*(345^(1/2)/10 - 5/2)*(345^(1/2) + 25))/11500
```

1. **$c = 3\,\mathrm{Ns/m}$, $0 \le t \le 20\,\mathrm{s}$,**

$$x(t) = \frac{9e^{-\frac{3t}{20}}\left(1111\cos\left(\frac{\sqrt{1111}t}{20}\right) + 3\sqrt{1111}\sin\left(\frac{\sqrt{1111}t}{20}\right)\right)}{55550}$$

$$v(t) = \frac{9e^{-\frac{3t}{20}}\left(\frac{3333\cos\left(\frac{\sqrt{1111}t}{20}\right)}{20} - \frac{1111\sqrt{1111}\sin\left(\frac{\sqrt{1111}t}{20}\right)}{20}\right)}{55550} - \frac{27e^{-\frac{3t}{20}}\left(1111\cos\left(\frac{\sqrt{1111}t}{20}\right) + 3\sqrt{1111}\sin\left(\frac{\sqrt{1111}t}{20}\right)\right)}{1111000}$$

This has been plotted on the graph within the respective range $0 \le t \le 20\,\mathrm{s}$

2. **$c = 50\,\mathrm{Ns/m}$, $0 \le t \le 10\,\mathrm{s}$.**

$$x(t) = \frac{3\sqrt{345}e^{t\left(\frac{\sqrt{345}}{10}-\frac{5}{2}\right)}\left(\sqrt{345}+25\right)}{11500} - e^{-t\left(\frac{\sqrt{345}}{10}+\frac{5}{2}\right)}\left(\frac{3\sqrt{345}}{460} - \frac{9}{100}\right)$$

$$v(t) = e^{-t\left(\frac{\sqrt{345}}{10}+\frac{5}{2}\right)}\left(\frac{\sqrt{345}}{10} + \frac{5}{2}\right)\left(\frac{3\sqrt{345}}{460} - \frac{9}{100}\right) + \frac{3\sqrt{345}e^{t\left(\frac{\sqrt{345}}{10}-\frac{5}{2}\right)}\left(\frac{\sqrt{345}}{10} - \frac{5}{2}\right)\left(345^{1/2}+25\right)}{11500}$$

This has been plotted on the graph within the respective range $0 \le t \le 10\,\mathrm{s}$

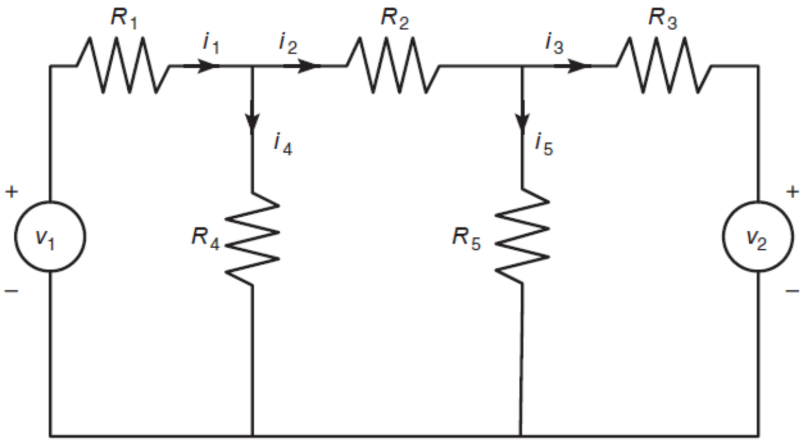
B.4 Q4

The currents i_1, i_2, and i_3 in the circuit are described by the equation set:

$$\begin{pmatrix} 2R & -R & 0 \\ -R & 3R & -R \\ 0 & -R & 2R \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} v_1 \\ 0 \\ v_2 \end{pmatrix}$$

Here, v_1 and v_2 are applied voltages. The other two currents can be found as:

$$i_4 = i_1 - i_2, \quad i_5 = i_2 - i_3.$$



1. Use both the matrix inverse and division methods to solve for the currents in terms of R, v_1, and v_2.
2. Calculate the numerical values for the currents when $R = 1000\,\Omega$, $v_1 = 100\,\text{V}$, and $v_2 = 25\,\text{V}$.

When i hear division i think of Cramer's rule, to calculate the currents (i_n) in a system of linear equations using Cramer's Rule, I used the following method in MATLAB. Cramer's Rule states:

$$i_n = \frac{\det(A_n)}{\det(A)}$$

Here:

- A is the coefficient matrix representing the system of linear equations.
- A_n is the matrix obtained by replacing the n-th column of A with the constant vector b, which represents the applied voltages or external influences.

To compute this programmatically:

1. I first calculated the determinant of the original matrix A (denoted as Δ_0) using the `det` function in MATLAB.
2. Then, I used a `for` loop to iterate through each column of the matrix:
   - For each iteration k, I created a modified version of A (denoted as `mNAM`) where the k-th column was replaced with the constant vector b.
   - I computed the determinant of the modified matrix (Δ_k).
   - Using Cramer's Rule, I calculated the current $i_k = \frac{\Delta_k}{\Delta_0}$ and stored it in the array `I`.

For the inverse method, I interpret the solution as:

$$i_n = A^{-1} \cdot b$$

In this case, I utilize MATLAB's built-in function `inv` to compute the inverse of matrix (A^{-1}). It's important to note that I define these matrices explicitly in my calculations before proceeding both methods.

main/q4.m

```
% 1. Finding currents in terms of voltage and resistance


% Define symbolic variables for resistance and voltages
syms R v_1 v_2 % syms requires Symbolic Math Toolbox.

% Create the node admittance matrix (NAM) for the circuit
NAM = [2*R -R 0; -R 3*R -R; 0 -R 2*R];
b = [v_1; 0; v_2];
I = sym('I', [3, 1]);

% Method 1: Solve system using  division (Cramer''s Rule)

delta_0 = det(NAM);
% Replace columns with b vector
for k = 1:3
    mNAM = NAM;
    mNAM(:,k) = b;
    delta_k = det(mNAM);
    I(k) = delta_k / delta_0;
end

% Add branch currents i4 and i5
I = [I;I(1)-I(2);I(2)-I(3)];

% Display results
disp('Currents using division (Cramer''s Rule):')
for k = 1:length(I)
    fprintf('i%d = %s\n', k, char(I(k)))
end

% Method 2: Solve system using matrix inverse

% Use inverse function
NAM_inv = inv(NAM);
% Solution bro
I2 = NAM_inv*b;
% Add branch currents i4 and i5
I2 = [I2;I2(1)-I2(2);I2(2)-I2(3)];

% Display results from matrix inverse method
disp('Currents using matrix inverse:')
for k = 1:length(I2)
    fprintf('i%d = %s\n', k, char(I2(k)))
end

% 2. Numerical values for the inputed resistance and voltages to the derived currents

disp('-----------------------------------------------------------')

% Get numerical values from user for resistance and voltages
```

```
Rs = input('Give me the R in ohms: ');
v1s = input('Give me the v_1 in volts: ');
v2s = input('Give me the v_2 in volts: ');

% Substitute numerical values into symbolic solutions and compute currents

I_num_c = vpa(subs(I, {R, v_1, v_2}, {Rs, v1s, v2s}), 6);
disp('Currents with given R and v_1, v_2 using Cramer''s Rule:')
for k = 1:5
    fprintf('i%d = %.6f\n', k, double(I_num_c(k)))
end

I_num_inv = vpa(subs(I2, {R, v_1, v_2}, {Rs, v1s, v2s}), 6);
disp('Currents with given R and v_1, v_2 using Matrix Inverse:')
for k = 1:length(I2)
    fprintf('i%d = %.6f\n', k, double(I_num_inv(k)))
end
```

```
Currents using division (Cramer's Rule):
i1 = (5*R^2*v_1 + R^2*v_2)/(8*R^3)
i2 = (2*R^2*v_1 + 2*R^2*v_2)/(8*R^3)
i3 = (R^2*v_1 + 5*R^2*v_2)/(8*R^3)
i4 = (5*R^2*v_1 + R^2*v_2)/(8*R^3) - (2*R^2*v_1 + 2*R^2*v_2)/(8*R^3)
i5 = (2*R^2*v_1 + 2*R^2*v_2)/(8*R^3) - (R^2*v_1 + 5*R^2*v_2)/(8*R^3)
Currents using matrix inverse:
i1 = (5*v_1)/(8*R) + v_2/(8*R)
i2 = v_1/(4*R) + v_2/(4*R)
i3 = v_1/(8*R) + (5*v_2)/(8*R)
i4 = (3*v_1)/(8*R) - v_2/(8*R)
i5 = v_1/(8*R) - (3*v_2)/(8*R)
----------------------------------------------------------
Give me the R in ohms: >>1000
Give me the v_1 in volts: >>100
Give me the v_2 in volts: >>25
Currents with given R and v_1, v_2 using Cramer's Rule:
i1 = 0.065625
i2 = 0.031250
i3 = 0.028125
i4 = 0.034375
i5 = 0.003125
Currents with given R and v_1, v_2 using Matrix Inverse:
i1 = 0.065625
i2 = 0.031250
i3 = 0.028125
i4 = 0.034375
i5 = 0.003125
```

Currents using division (Cramer's Rule):

$$i_1 = \frac{5R^2v_1 + R^2v_2}{8R^3}$$

$$i_2 = \frac{2R^2v_1 + 2R^2v_2}{8R^3}$$

$$i_3 = \frac{R^2v_1 + 5R^2v_2}{8R^3}$$

$$i_4 = \frac{5R^2v_1 + R^2v_2}{8R^3} - \frac{2R^2v_1 + 2R^2v_2}{8R^3}$$

$$i_5 = \frac{2R^2v_1 + 2R^2v_2}{8R^3} - \frac{R^2v_1 + 5R^2v_2}{8R^3}$$

Currents using matrix inverse:

$$i_1 = \frac{5v_1}{8R} + \frac{v_2}{8R}$$

$$i_2 = \frac{v_1}{4R} + \frac{v_2}{4R}$$

$$i_3 = \frac{v_1}{8R} + \frac{5v_2}{8R}$$

$$i_4 = \frac{3v_1}{8R} - \frac{v_2}{8R}$$

$$i_5 = \frac{v_1}{8R} - \frac{3v_2}{8R}$$

Given the following values: , $R = 1000\,\Omega$, $v_1 = 100\,\text{V}$, $v_2 = 25\,\text{V}$

$$i_1 = 0.065625$$
$$i_2 = 0.031250$$
$$i_3 = 0.028125$$
$$i_4 = 0.034375$$
$$i_5 = 0.003125$$

$$i_1 = 0.065625$$
$$i_2 = 0.031250$$
$$i_3 = 0.028125$$
$$i_4 = 0.034375$$
$$i_5 = 0.003125$$

The numerical values are identical, as can also be observed from the equations, which are equivalent in each case.