Engineering
Mathematics and Computing

Task 2: Coursework Assessment

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Git Repo : https://github.com/sakx7/mathcompuni2

Part A

Mathematics

A.1 Q1

1. Find $\int \frac{1}{7x+6}\,dx$

$$\int \frac{1}{7x+6}\,dx$$

Use substitution, Let

$$u = 7x + 6$$

$$\frac{du}{dx} = 7 \quad \Rightarrow \quad dx = \frac{du}{7}$$

Substituting into the integral, we get

$$\int \frac{1}{7x+6}\,dx = \frac{1}{7}\int \frac{1}{u}\,du$$

This is a standard integral of the reciprocal

$$\frac{1}{7}\int \frac{1}{u}\,du = \frac{1}{7}\ln|u| + C$$

Finally, substituting back $u = 7x + 6$, we get

$$\boxed{\int \frac{1}{7x+6}\,dx = \frac{1}{7}\ln|7x+6| + C}$$

A.2 Q2

2. Find $\int \frac{x}{\sqrt{4-x^2}}\,dx$

$$\int \frac{x}{\sqrt{4-x^2}}\,dx$$

Use substitution, Let

$$u = 4 - x^2$$

$$\frac{du}{dx} = -2x \quad \Rightarrow \quad dx = -\frac{du}{2x}$$

Substituting into integral, we get

$$\int \frac{x}{\sqrt{4-x^2}}\,dx = \int \frac{x}{\sqrt{u}} \cdot \left(-\frac{du}{2x}\right) = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$

This is a standard integral of the exponent/power

$$-\frac{1}{2}\int u^{-\frac{1}{2}}\,du = -\frac{1}{2} \cdot 2\sqrt{u} + C = -\sqrt{u} + C$$

Finally, substituting back $u = 4 - x^2$, we get

$$\boxed{\int \frac{x}{\sqrt{4-x^2}}\,dx = -\sqrt{4-x^2} + C}$$

A.3 Q3

3. Obtain the general solution of the equation $\frac{d^2y}{dx^2} - 18\frac{dy}{dx} + 81y = 0$

$$\frac{d^2y}{dx^2} - 18\frac{dy}{dx} + 81y = 0$$

We use the ansatz $y = e^{rx}$, substituting it, along with its corresponding derivatives:

$$r^2e^{rx} - 18re^{rx} + 81e^{rx} = 0$$

Dividing through by e^{rx} (which is never zero), we get the characteristic equation:

$$r^2 - 18r + 81 = 0$$

We can factorize the quadratic as:

$$(r-9)(r-9) = 0$$

This gives us a repeated root:

$$r_1 = r_2 = 9$$

For a second-order linear homogeneous differential equation with constant coefficients and a repeated root r, the general solution is:

$$y(x) = c_1e^{rx} + c_2xe^{rx}$$

Substituting $r = 9$ into the general solution, we get:

$$y(x) = c_1e^{9x} + c_2xe^{9x}$$

Factorising we get the final solution as:

$$\boxed{y(x) = e^{9x}(c_1 + c_2x)}$$

Interval of validity: Nothing really stands out, no singularities or undefined behaviours. Therefore, the interval of validity is:

$$x \in \mathbb{R}$$

This means the solution is valid for all real values of x. The constant C does not effect the solutions validity regardless of its value.

A.4 Q4

4. Find the particular solution of the differential equation $\frac{dy}{dx} + 3yx^3 = 0$, given $y(0) = 1$

$$\frac{dy}{dx} + 3yx^3 = 0$$

Separate the variables:

$$\frac{1}{y}\,dy = -3x^3\,dx$$

Integrating both sides:

$$\int \frac{1}{y}\,dy = \int -3x^3\,dx$$

$$\ln|y| = -\frac{3x^4}{4} + C$$

Exponentiating both sides:

$$\begin{aligned} y &= \exp\left(-\frac{3x^4}{4} + C\right) \\ &= C\exp\left(-\frac{3x^4}{4}\right) \end{aligned}$$

Apply the initial condition $y(0) = 1$:

$$1 = C\exp\left(-\frac{3(0)^4}{4}\right)$$

$$C = 1$$

Thus, the particular solution is:

$$\boxed{y(x) = e^{-\frac{3x^4}{4}}}$$

Interval of validity: Nothing really stands out, no singularities or undefined behaviours. Therefore, the interval of validity is:

$$x \in \mathbb{R}$$

This means the solution is valid for all real values of x. The constant C does not effect the solutions validity regardless of its value

A.5 Q5

5. If $z = \frac{11+10j}{9-3j}$, express both $\frac{1}{z}$ and $z + \frac{1}{z}$ in the standard form $\alpha + \beta j$

$$z = \frac{11+10j}{9-3j}$$

$$z = \frac{(11+10j)(9-3j)}{(9-3j)(9-3j)}$$

Simplify the denominator:

$$\begin{aligned}(9-3j)(9+3j) &= 9^2 - (3j)^2 \\ &= 81 - (-9) \\ &= 81 + 9 = 90\end{aligned}$$

Simplify the numerator:

$$\begin{aligned}(11+10j)(9+3j) &= (11 \cdot 9) + (11 \cdot 3j) \\ &\quad + (10j \cdot 9) + (10j \cdot 3j) \\ &= 99 + 33j + 90j + 30j^2 \\ &= 99 + 123j + 30(-1) \\ &= 99 + 123j - 30 \\ &= 69 + 123j\end{aligned}$$

So:

$$\begin{aligned}z &= \frac{69+123j}{90} \\ &= \frac{69}{90} + \frac{123}{90}j \\ &= 0.7\dot{6} + 1.3\dot{6}j\end{aligned}$$

$$\frac{1}{z} = \frac{11+10j}{9-3j}$$

$$\frac{1}{z} = \frac{(9-3j)(11-10j)}{(11+10j)(11-10j)}$$

Simplify the denominator:

$$\begin{aligned}(11+10j)(11-10j) &= 11^2 - (10j)^2 \\ &= 121 - (-100) \\ &= 121 + 100 = 221\end{aligned}$$

The numerator is the conjugate of the previously calculated numerator, so:

$$(9-3j)(11-10j) = 69 - 123j$$

Thus:

$$\begin{aligned}\frac{1}{z} &= \frac{69-123j}{221} \\ &= \frac{69}{221} + \frac{-123}{221}j \\ &\approx \boxed{0.31 - 0.56j}\end{aligned}$$

now we can calculate $z + \frac{1}{z}$ easily

$$\begin{aligned}z + \frac{1}{z} &= \frac{69}{90} + \frac{123}{90}j + \frac{69}{221} + \frac{-123}{221}j \\ &= \left(\frac{69}{90} + \frac{69}{221}\right) + \left(\frac{123}{90} - \frac{123}{221}\right)j\end{aligned}$$

$$\boxed{\approx 1.08 + 0.81j}$$

in non approximate form this is $\frac{7153}{6630} + \frac{5371}{6630}j$

A.6 Q6

6. Obtain the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 48y = 5$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 48y = 5$$

Since this is **non-homogenous** we need to solve for the **complementary solution ($\boldsymbol{y_c}$)** and then the **particular solution ($\boldsymbol{y_p}$)** the general form of the solution the addition of these

$$y(t) = y_c(t) + y_p(t)$$

First lets solve the complementary solution, which solves the associated homogeneous equation:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 48y = 0$$

We use the ansatz $y = e^{rx}$, substituting it, along with its corresponding derivatives:

$$r^2e^{rx} - 2re^{rx} - 48e^{rx} = 0$$

Dividing through by e^{rx} (which is never zero), we obtain the characteristic equation:

$$r^2 - 2r - 48 = 0$$

We can factorize the quadratic as:

$$(r - 8)(r + 6) = 0$$

This gives us the roots:

$$r_1 = 8, \quad r_2 = -6$$

The general solution to a second-order linear homogeneous equation is given by:

$$y(x) = c_1e^{r_1x} + c_2e^{r_2x}$$

In so for our solution this is

$$y_c(x) = c_1e^{8x} + c_2e^{-6x}$$

$$z = xy^4e^{2x}$$

$$\frac{\partial z}{\partial x} = \frac{\partial\left(xy^4e^{2x}\right)}{\partial x}$$

Use the product rule:

$$\frac{\partial(u(x)\cdot v(x))}{\partial x} = v\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial x}$$

let $u(x) = x$ and $v(x) = y^4e^{2x}$

$$\frac{\partial z}{\partial x} = \frac{\partial\left(xy^4e^{2x}\right)}{\partial x} = y^4e^{2x}\frac{\partial(x)}{\partial x} + x\frac{\partial\left(y^4e^{2x}\right)}{\partial x}$$

Easy to solve, just treat y as a constant and differentiate with respect to x:

$$\frac{\partial(x)}{\partial x} = 1, \qquad \frac{\partial\left(y^4e^{2x}\right)}{\partial x} = 2e^{2x}$$

now plug in we get

$$\begin{aligned}\frac{\partial z}{\partial x} &= y^4e^{2x} + xy^4\left(2e^{2x}\right)\\ &= e^{2x}y^4(2x+1)\end{aligned}$$

Thus:

$$\boxed{\frac{\partial z}{\partial x} = e^{2x}y^4(2x+1)}$$

$$z = xy^4e^{2x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial\left(xy^4e^{2x}\right)}{\partial y}$$

Treat x as a constant, we only differentiate y^4:

$$\frac{\partial z}{\partial y} = \frac{\partial\left(xy^4e^{2x}\right)}{\partial y} = xe^{2x}\frac{\partial\left(y^4\right)}{\partial y}$$

$$\frac{\partial\left(y^4\right)}{\partial y} = 4y^3$$

Thus:

$$\boxed{\frac{\partial z}{\partial y} = 4xy^3e^{2x}}$$

$$\int \frac{x+9}{x(x+5)}\,dx$$

Split the integral:

$$\int \frac{x+9}{x(x+5)}\,dx = \int \frac{(x+5)+4}{x(x+5)} = \int \frac{1}{x}\,dx + 4\int \frac{1}{x(x+5)}\,dx$$

The first integral is straightforward:

$$\int \frac{1}{x}\,dx = \ln|x| + C_1$$

For the second integral, rewrite:

$$\int \frac{1}{x(x+5)}\,dx = \int \frac{1}{x^2\left(\frac{5}{x}+1\right)}\,dx$$

Now use sub

$$u = \frac{5}{x} + 1$$

$$\frac{du}{dx} = -\frac{5}{x^2} \quad \Rightarrow \quad dx = -\frac{x^2}{5}\,du$$

Substituting into the integral, we get:

$$\int -\frac{x^2}{5x^2u}\,du = -\frac{1}{5}\int \frac{1}{u}\,du = -\frac{1}{5}\ln|u| + C_2$$

Substituting $u = \frac{5}{x} + 1 = \frac{5+x}{x}$ and then combing results yields:

$$\begin{aligned}\int \frac{x+9}{x(x+5)}\,dx &= \ln|x| + 4\left(-\frac{1}{5}\ln\left|\frac{x+5}{x}\right|\right) + C \\ &= \ln|x| - \frac{4}{5}\left(\ln|x+5| - \ln|x|\right) + C \\ &= -\frac{4\ln|x+5| - 9\ln|x|}{5} + C\end{aligned}$$

Thus, the solution is:

$$\boxed{\int \frac{x+9}{x(x+5)}\,dx = \frac{9\ln|x| - 4\ln|x+5|}{5} + C}$$

tcb@saveboxU se integrating factor $\mu(t)$: $\mu(t) = e^{\int \cot t \, dt}$Since this

A.10 Q10

10. Evaluate the integral $\int_1^3 4xe^{4x}\,dx$

$$\int_1^3 4xe^{4x}\,dx$$

$$\text{let } u = 4x \quad dx = \tfrac{1}{4}$$

$$\frac{1}{4}\int_4^{12} ue^u\,dx$$

$$\frac{1}{4}\left([ue^u - e^u]_4^{12}\right)$$

$$\frac{1}{4}\left(\left(12e^{12} - e^{12}\right) - \left(4e^4 - e^4\right)\right)$$

$$\boxed{\int_1^3 4xe^{4x}\,dx = \frac{11e^{12} - 3e^4}{4} \approx 4.4753 \times 10^5}$$