Contents

Chapter 1

Column Generation

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{T} \lambda_t && && (1.1)\\
\text{s.t.} \quad & \sum_{t=1}^{T} (x_i^t)\lambda_t = 1 && i \in I && (1.2)\\
& \lambda_t \geq 0 && t = 1, \ldots, T. && (1.3)
\end{aligned}
$$

The pricing problem (finding a column with negative reduced cost) reduces to the knapsack problem

$$
\begin{aligned}
\min \quad & 1 - \sum_{i \in I} \pi_i x_i && && (1.4)\\
\text{s.t.} \quad & \sum_{i \in I} w_i x_i \leq W && && (1.5)\\
& x_i \in \{0, 1\} && i \in I && (1.6)
\end{aligned}
$$

1.0.1 Example

The complete formulation would be

$$
\begin{aligned}
\min \quad & \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 \\
& + \lambda_{10} + \lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_{14} \qquad (1.7)
\end{aligned}
$$

$$
\text{s.t.} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \\ \lambda_9 \\ \lambda_{10} \\ \lambda_{11} \\ \lambda_{12} \\ \lambda_{13} \\ \lambda_{14} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad (1.8)
$$

$$
\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{12}, \lambda_{13}, \lambda_{14} \geq 0 \qquad (1.9)
$$

However, we begin with only enough (artificial) columns. Let $M = 10^6$.

$$\min\ M\theta_1 + M\theta_2 + M\theta_3 + M\theta_4 \tag{1.10}$$

$$\text{s.t.}\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{1.11}$$

$$\theta_1, \theta_2, \theta_3, \theta_4 \geq 0 \tag{1.12}$$

The optimal solution is $(\theta_1, \theta_2, \theta_3, \theta_4) = (1, 1, 1, 1)$. The associated dual solution is $(\pi_1, \pi_2, \pi_3, \pi_4) = (M, M, M, M)$. To check for optimality, we must solve

$$\min\ 1 - Mx_1 - Mx_2 - Mx_3 - Mx_4 \tag{1.13}$$

$$\text{s.t.}\ w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 \leq W \tag{1.14}$$

$$x_1, x_2, x_3, x_4 \in \{0, 1\}. \tag{1.15}$$

Say the optimal solution is $(x_1, x_2, x_3, x_4) = (1, 1, 0, 1)$. Since $1 - M - M - 0 - M < 0$, the current basis is not optimal, and the column

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

is eligible for entering the basis. Thus, the master problem is updated to

$$\min\ M\theta_1 + M\theta_2 + M\theta_3 + M\theta_4 + \lambda_1 \tag{1.16}$$

$$\text{s.t.}\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{1.17}$$

$$\theta_1, \theta_2, \theta_3, \theta_4, \lambda_1 \geq 0 \tag{1.18}$$

We must continue until the optimal value of the pricing problem is greater or equal to zero.