Recommendation Systems

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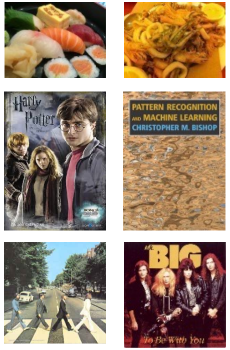
Problem formulation

Predicting movie ratings

User rates movies using zero to five stars



Training set

	Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)
Romance	Love at last	5	5	0	0
	Romance forever	5	4.5 ?	? 0	0
	Cute puppies of love	? 5	4	0	0 ?
Action	Nonstop car chases	0	0	5	4
	Swords vs. karate	0	0	5	4 ?

Nice

Boring

Recommendation Systems

Problem formulation

n_u = no. users = 4

n_m = no. movies = 5

$r(i,j)$ = 1 if user j has rated movie i

$y^{(i,j)}$ = rating given by user j to movie i (defined only if $r(i,j)=1$) ϵ {0,..,5}

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	Cute puppies of love	? 5	4	0	0 ?
Action	Nonstop car chases	0	0	5	4
	Swords vs. karate	0	0	5	4 ?

Nice

Boring

In our notation, $r(i,j)=1$ if user j has rated movie i, and $y^{(i,j)}$ is his rating on that movie. Consider the following example (no. of movies $n_m=1$, no. of users $n_u=3$):

	User 1	User 2	User 3
Movie 1	0	1	?
Movie 2	?	5	5

What is $r(2,1)$? How about $y^{(2,1)}$?

- $r(2,1)=0,\ y^{(2,1)}=1$
- $r(2,1)=1,\ y^{(2,1)}=1$
- $r(2,1)=0,\ y^{(2,1)}=\text{undefined}$
- $r(2,1)=1,\ y^{(2,1)}=\text{undefined}$

$n_u = 4 \ , \ n_m = 5$

Content-based recommender systems

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)
Love at last	5	5	0	0
Romance forever	5	?	?	0
Cute puppies of love	?	4	0	?
Nonstop car chases	0	0	5	4
Swords vs. karate	0	0	5	?

Content-based recommender systems

$n_u = 4$, $n_m = 5$

n=2

x_0=1

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)	x_1 (romance)	x_2 (action)
Love at last	5	5	0	0	0.9	0
Romance forever	5	?	?	0	1.0	0.01
Cute puppies of love	?	4	0	?	0.99	0
Nonstop car chases	0	0	5	4	0.1	1.0
Swords vs. karate	0	0	5	?	0	0.9

Suppose we have also a set of features for each movie (<u>related to the content!</u>):

x_1 measure the degree to be a romantic movie

x_2 measure the degree to be an action movie

"Love at last" as a degree of "romanticness" equal to 0.9 and it isn't an action movie

→ Each movie can be represented as feature vector $x^{(i)}=[x_0, x_1, x_2]^T=[1, x_1, x_2]^T$

→ $x^{(3)}$=[1,0.99,0] is the feature vector related to the film "Cute puppies of love"

Content-based recommender systems

$n_u = 4$, $n_m = 5$

n=2

$x_0=1$

Movie	Alice (1) $\theta^{(1)}$	Bob (2) $\theta^{(2)}$	Carol (3) $\theta^{(3)}$	Dave (4) $\theta^{(4)}$	x_1 (romance)	x_2 (action)
Love at last	5	5	0	0	0.9	0
Romance forever	5	?	?	0	1.0	0.01
Cute puppies of love	?	4	0	?	0.99	0
Nonstop car chases	0	0	5	4	0.1	1.0
Swords vs. karate	0	0	5	?	0	0.9

We can treat predicting the rating of each user as a linear regression problem.

For each user j, learn a parameter $\theta^{(j)} \in \mathbb{R}^3$. Predict user j as rating movie i with $(\theta^{(j)})^T x^{(i)}$ stars.

$\theta^{(j)} \in \mathbb{R}^{n+1}$

Content-based recommender systems

$n_u = 4$, $n_m = 5$

$n=2$

$\boxed{x_0=1}$

Movie	Alice (1) $\theta^{(1)}$	Bob (2) $\theta^{(2)}$	Carol (3) $\theta^{(3)}$	Dave (4) $\theta^{(4)}$	x_1 (romance)	x_2 (action)	
Love at last	5	5	0	0	0.9	0	$x^{(1)}$
Romance forever	5	?	?	0	1.0	0.01	$x^{(2)}$
Cute puppies of love	? 4.95	4	0	?	0.99	0	$x^{(3)}$
Nonstop car chases	0	0	5	4	0.1	1.0	$x^{(4)}$
Swords vs. karate	0	0	5	?	0	0.9	$x^{(5)}$

The main problem in this case is: How learn $\theta^{(1)} \dots \theta^{(4)}$?

Consider the following set of movie ratings:

Movie	Alice (1)	Bob (2)	Carol (3)	David (4)	(romance)	(action)
Love at last	5	5	0	0	0.9	0
Romance forever	5	?	?	0	1.0	0.01
Cute puppies of love	?	4	0	?	0.99	0
Nonstop car chases	0	0	5	4	0.1	1.0
Swords vs. karate	0	0	5	?	0	0.9

Which of the following is a reasonable value for $\theta^{(3)}$? Recall that $x_0 = 1$.

- $\theta^{(3)} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$
- $\theta^{(3)} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$
- $\theta^{(3)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
- $\theta^{(3)} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$

Submit

Skip

Problem formulation

$r(i,j) = 1$ if user j has rated movie i (0 otherwise)

$y^{(i,j)} =$ rating by user j on movie i (if defined)

$\theta^{(j)}$ = parameter vector for user j

$x^{(i)}$ = feature vector for movie i

For user j, movie i, predicted rating: $(\theta^{(j)})^T(x^{(i)})$ $\qquad \theta^{(j)} \in \mathbb{R}^{n+1}$

$m^{(j)}$ = no. of movies rated by user j

To learn $\theta^{(j)}$:

$$\min_{\theta^{(j)}} \frac{1}{2\cancel{m^{(j)}}} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T (x^{(i)}) - y^{(i,j)} \right)^2 + \frac{\lambda}{2\cancel{m^{(j)}}} \sum_{k=1}^{n} (\theta_k^{(j)})^2$$

Optimization objective:

To learn $\theta^{(j)}$ (parameter for user j):

$$\min_{\theta^{(j)}} \underbrace{\frac{1}{2} \sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2}_{} + \underbrace{\frac{\lambda}{2} \sum_{k=1}^{n} (\theta_k^{(j)})^2}_{}$$

To learn $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(n_u)}$:

$$\min_{\theta^{(1)},\ldots,\theta^{(n_u)}} \frac{1}{2} \boxed{\sum_{j=1}^{n_u}} \underbrace{\sum_{i:r(i,j)=1} \left((\theta^{(j)})^T x^{(i)} - y^{(i,j)} \right)^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^{n} (\theta_k^{(j)})^2}_{}$$

Optimization algorithm:

$$\min_{\theta^{(1)},\ldots,\theta^{(n_u)}} \frac{1}{2}\sum_{j=1}^{n_u}\sum_{i:r(i,j)=1}\left((\theta^{(j)})^T x^{(i)} - y^{(i,j)}\right)^2 + \frac{\lambda}{2}\sum_{j=1}^{n_u}\sum_{k=1}^{n}(\theta_k^{(j)})^2$$

Gradient descent update:

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \sum_{i:r(i,j)=1}((\theta^{(j)})^T x^{(i)} - y^{(i,j)})x_k^{(i)} \text{ (for } k = 0)$$

$$\theta_k^{(j)} := \theta_k^{(j)} - \alpha \left(\sum_{i:r(i,j)=1}((\theta^{(j)})^T x^{(i)} - y^{(i,j)})x_k^{(i)} + \lambda\theta_k^{(j)}\right) \text{ (for } k \neq 0)$$

Content Based Recommendation
Take Home Message

- We have features to capture content of the movies (e.g., given by analysis of the problem by expert or obtained through features extraction)
- We have some ratings (sparse information)
- We use regression on the feature to predict ratings. Parameters are learned considering the given ratings