Задание 1.

$$Q(w) = (y - xw)^T (y - xw) + \lambda w^2$$

1. $$Q(w) = y^T y - 2w(x^T y) + w^2 (x^T x) + \lambda w^2$$

$$\nabla_w Q(w) = \frac{\partial Q(w)}{\partial w} = -2x^T y + 2w(x^T x) + 2\lambda w =$$

$$= 2w(x^T x + \lambda) - 2x^T y = 0$$

$$2w(x^T x + \lambda) = 2x^T y$$

$$w = \frac{\cancel{2} \cdot x^T y}{\cancel{2}(x^T x + \lambda)} = \frac{x^T y}{x^T x + \lambda}$$

2. $$\frac{\partial^2 Q(w)}{\partial w^2} = 2(\underbrace{x^T x}_{\geq 0} + \lambda) \quad \rightarrow >0 \text{ по условию}$$

$$\Rightarrow \frac{\partial^2 Q(w)}{\partial w^2} > 0$$

3. $$\nabla_w Q(w) = 2w(x^T x + \lambda) - 2x^T y$$

$$w_{t+1} = w_t - \eta \nabla_w Q(w_t)$$

$$w_{t+1} = w_t - \eta(2w_t(x^T x + \lambda) - 2x^T y)$$

$$\boxed{w_{t+1} = w_t(1 - 2\eta(x^T x + \lambda)) + 2\eta x^T y}$$