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Mini project #1 2025:Design and Implementation of a Biquad

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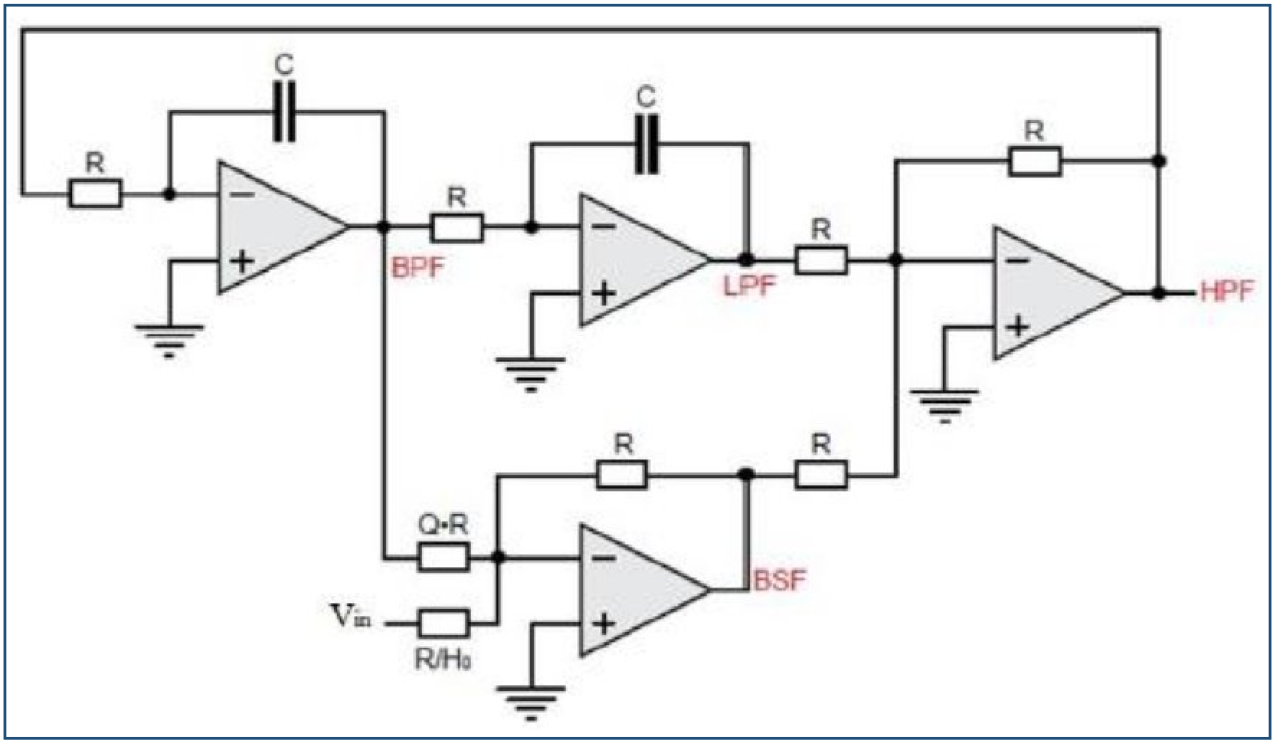
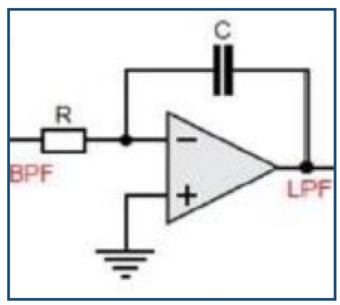


Figure 1: Universal Biquadratic Filter Circuit

➢ Circuits of BPF & LPF are Integrators Then:

Derivation for Integrator:

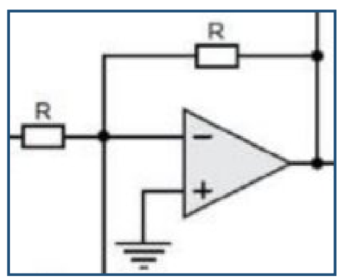
$$\frac{V_i - 0}{R} = (0 - V_o)\,SC \Longrightarrow \frac{V_o}{V_i} = \frac{-1}{SRC}$$



➢ Circuits of BSF & HPF are Inverting Amplifiers:

Derivation for Inverting Amplifier:

$$\frac{V_i - 0}{R_1} = \frac{0 - V_0}{R_2} \Longrightarrow \frac{V_0}{V_i} = -\frac{R_2}{R_1}$$



- $V_{BPF} = -\frac{V_{HPF}}{SRC}$ $\Rightarrow$ (1)
- $V_{LPF} = -\frac{V_{BPF}}{SRC}$ $\Rightarrow$ (2)
- $V_{HPF} = -\frac{R}{R}(V_{LPF} + V_{BSF}) = -(V_{LPF} + V_{BSF})$ $\Rightarrow$ (3)
- $V_{BSF} = -\frac{R}{R/H_0} V_{in} - \frac{R}{QR} V_{BPF} = -\left(H_0 V_{\text{in}} + \frac{1}{Q} V_{BPF}\right)$ $\Rightarrow$ (4)

[1] For BPF

From (3) $\Rightarrow$ $V_{BSF} = -V_{LPF} - V_{HPF}$

From (1), (2) $\Rightarrow$ $V_{BSF} = \frac{V_{BPF}}{SRC} + V_{BPF}SRC$ $\Rightarrow$ (5)

From (4):

$$-\left(H_0 V_{\text{in}} + \frac{1}{Q} V_{BPF}\right) = \frac{V_{BPF}}{SRC} + V_{BPF}SRC$$

$$V_{BPF}\left[\frac{1}{SRC} + SRC + \frac{1}{Q}\right] = -H_0 V_{\text{in}}$$

$$\frac{V_{\text{BPF}}}{V_{\text{in}}} = \frac{-H_0}{SRC + \frac{1}{SRC} + \frac{1}{Q}} * \frac{S/RC}{S/RC}$$

$$\boxed{\therefore \frac{V_{BPF}}{V_{\text{in}}} = \frac{-H_0 \frac{S}{RC}}{S^2 + \frac{1}{QRC}S + \frac{1}{R^2C^2}}}$$

- Verify function of BPF:

$$@\ \omega = 0 \Rightarrow \frac{V_{\text{BPF}}}{V_{\text{in}}} = 0$$

$$@\ \omega = \infty \Rightarrow \frac{V_{\text{BPF}}}{V_{\text{in}}} = \lim_{\omega \to \infty} \left(\frac{V_{\text{BPF}}}{V_{\text{in}}}\right) = 0$$

$$@\ \omega = \omega_0 = \frac{1}{RC} \Rightarrow \frac{V_{BPF}}{V_{\text{in}}} = \frac{-H_0 \frac{1}{RC}\frac{j}{RC}}{\frac{-1}{R^2C^2} + \frac{j}{QR^2C^2} + \frac{1}{R^2C^2}} = -H_0 Q$$

$\therefore$ This verifies the function of Band Pass as it attenuates both low and high frequencies and only allow a certain frequency band to Pass.

- Note:

$\because$ We use Ideal op Amps.

$\therefore$ Each stage should ideally have very high input impedance and very low output impedance. So, no loading effect between the stages and we can use cascade relations between stages.

[2] For LPF

$$\frac{V_{LPF}}{V_{\text{in}}} = \frac{V_{LPF}}{V_{BPF}} * \frac{V_{BPF}}{V_{\text{in}}} = \frac{-1}{SRC} * \frac{-H_0 \frac{S}{RC}}{S^2 + \frac{1}{QRC}S + \frac{1}{R^2C^2}}$$

$$\boxed{\therefore \frac{V_{LPF}}{V_{\text{in}}} = \frac{H_0 \frac{1}{R^2C^2}}{S^2 + \frac{1}{QRC}S + \frac{1}{R^2C^2}}}$$

- Verify function of LPF:

$@\ \omega = 0 \Longrightarrow \frac{V_{\text{LPF}}}{V_{\text{in}}} = H_0$

$@\ \omega = \infty \Rightarrow \frac{V_{\text{LPF}}}{V_{\text{in}}} = \lim_{\omega \to \infty} \left(\frac{V_{\text{LPF}}}{V_{\text{in}}} \right) = 0$

∴ This verifies the function of low-pass as it attenuates higher frequencies and only allows lower frequencies to pass.

[3] For HPF:

$$\frac{V_{HPF}}{V_{\text{in}}} = \frac{V_{HPF}}{V_{BPF}} * \frac{V_{BPF}}{V_{\text{in}}} = -V_{BPF} * SRC * \frac{-H_0 \frac{S}{RC}}{S^2 + \frac{1}{QRC}S + \frac{1}{R^2C^2}}$$

$$\therefore \frac{V_{\text{HPF}}}{V_{\text{in}}} = \frac{H_0 S^2}{S^2 + \frac{1}{QRC}S + \frac{1}{R^2C^2}}$$

- Verify function of HPF:

$@\ \omega = 0 \Rightarrow \frac{V_{HPF}}{V_{\text{in}}} = 0$

$@\ \omega = \infty \Rightarrow \frac{V_{\text{HPF}}}{V_{\text{in}}} = \lim_{\omega \to \infty} \left(\frac{V_{\text{HPF}}}{V_{\text{in}}} \right) = H_0$

∴ This verifies the function of high-pass as, it attenuates lower frequencies and only allow higher frequencies to pass.

[4] For BSF:

From (5) $\Rightarrow \frac{V_{BSF}}{V_{\text{BPF}}} = \frac{1}{SRC} + SRC$

$$\frac{V_{BSF}}{V_{\text{in}}} = \frac{V_{BSF}}{V_{BPF}} * \frac{V_{BPF}}{V_{\text{in}}} = \left(\frac{1}{SRC} + SRC \right) * \frac{-H_0 \frac{S}{RC}}{S^2 + \frac{1}{QRC}S + \frac{1}{R^2C^2}}$$

$$\boxed{\therefore \frac{V_{BSF}}{V_{\text{in}}} = \frac{-H_0 \left(S^2 + \frac{1}{R^2C^2} \right)}{S^2 + \frac{1}{QRC}S + \frac{1}{R^2C^2}}}$$

➢ Verify function of BSF:

$$@\ \omega = 0 \Longrightarrow \frac{V_{BSF}}{V_{\text{in}}} = -H_0$$

$$@\ \omega = \infty \Rightarrow \frac{V_{BSF}}{V_{\text{in}}} = \lim_{\omega\to\infty}\left(\frac{V_{BSF}}{V_{\text{in}}}\right) = -H_0$$

$$@\ \omega = \omega_0 = \frac{1}{RC} \Rightarrow \frac{V_{BSF}}{V_{\text{in}}} = \frac{-H_0\left[\left(\frac{j}{RC}\right)^2 + \frac{1}{R^2C^2}\right]}{\left(\frac{j}{RC}\right)^2 + \frac{1}{QRC} * \frac{j}{RC} + \frac{1}{R^2C^2}} = 0$$

∴ This verifies the function of Band Stop, as it attenuates only a certain band of frequency and allows both low and high frequencies to pass.

Design Parameters

❖ Design R,C to obtain $f_0 = 5\text{MHz}, \text{Q} = 1.7, H_0 = 1$

$$\omega_0 = 2\pi f_0 = \frac{1}{RC} = 31415926.54\text{rad/sec}$$

⇒ Capacitor selection is often the first step because resistors have a wide practical range.

⇒ our optimal choice for Capacitor is : $C = 10pf$

- for smaller than $C = 10$ PF:
  - (1) it will be more sensitive to parasitic capacitance.
  - (2) it requires large value of R which leading to increase noise as , $V_n^2 = 4KTR$.
- for larger than $C = 10$ PF:
  - (1) it will Consume large Area.
  - (2) it requires small value of R which consume more current and more power.

$$\text{C} = 10\text{PF} \Rightarrow \text{which is a Practical value}$$

$$\omega_0 = \frac{1}{RC} \Rightarrow R = 3.183\text{k}\Omega$$

Schematic

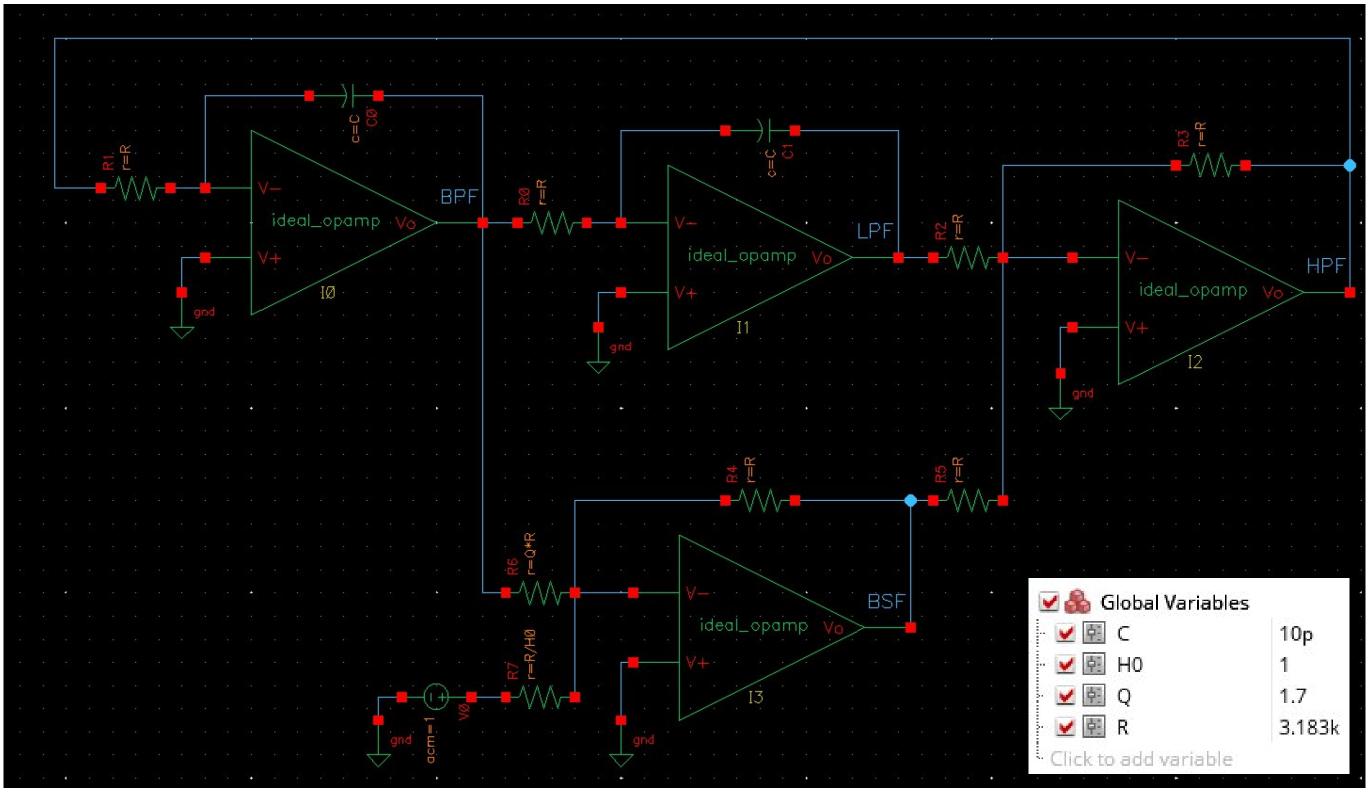


Figure 2: Schematic of the Biquadratic filter showing values of R & C

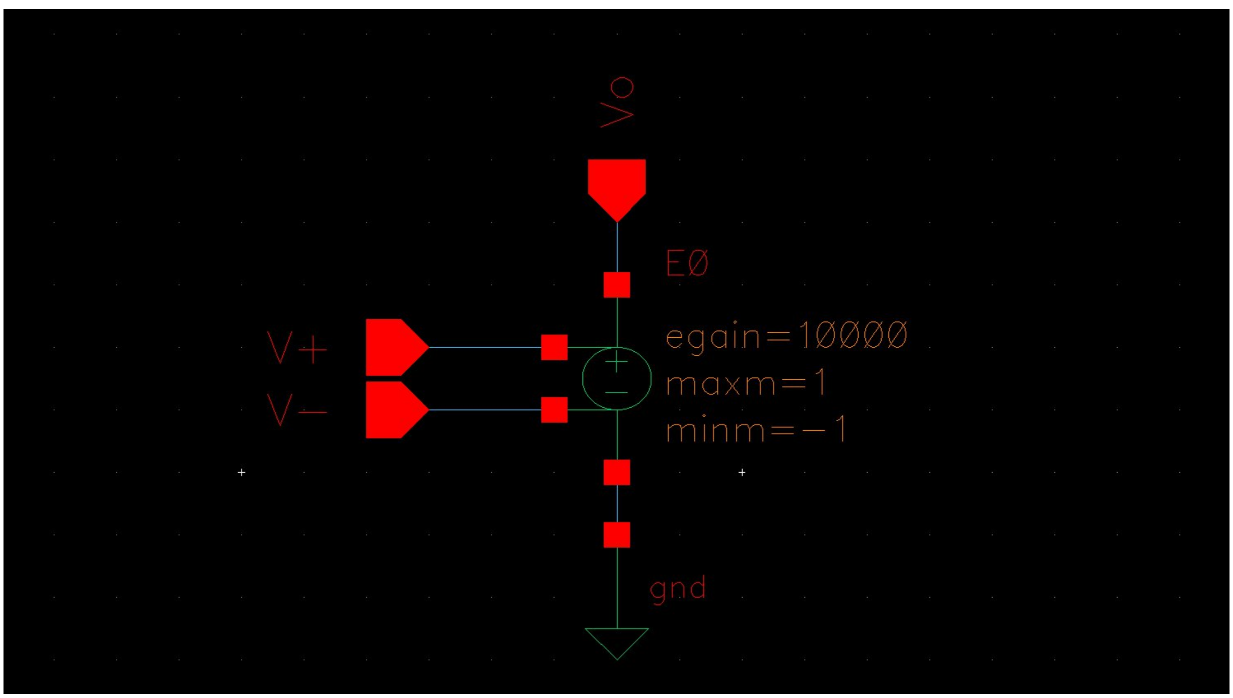


Figure 3: Schematic of the ideal op-amp

Figure 3 represents the model of the ideal op-amp as voltage-controlled voltage source VCVS with a gain of 10000 and V_{Max} = 1V, V_{Min} = -1V.

Frequency Response

1. LPF Frequency Response

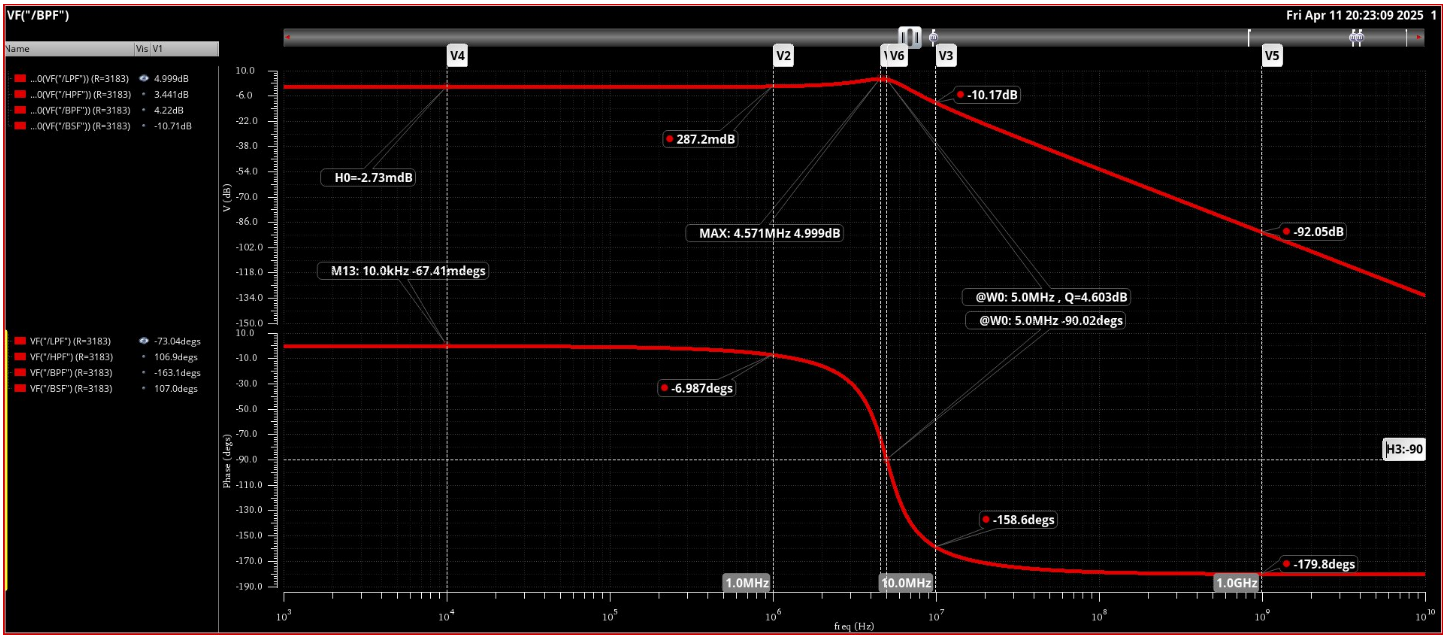


Figure 4: LPF Frequency Response (Magnitude & Phase)

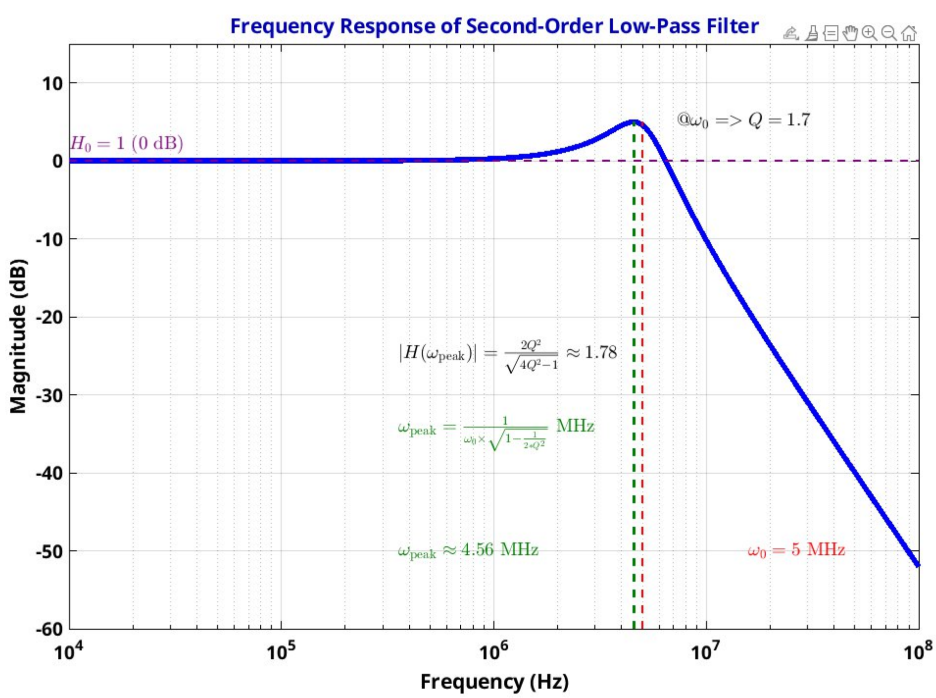


Figure 5 LPF Theoretical Plot

As shown from Figure 5 and 4 we could calculate the parameters by getting them from the corresponding locations and evaluating the equations:

- H_0 at low frequencies = -2.73 mdB which is near to **1** in linear scale. (as calculated in hand analysis)
- Get peak value and evaluate $Q \approx 1.7$ also get $\omega_0 \approx$ 5 MHz.
- Check at ω0 we get $Q \approx 1.7$.

2. HPF Frequency Response

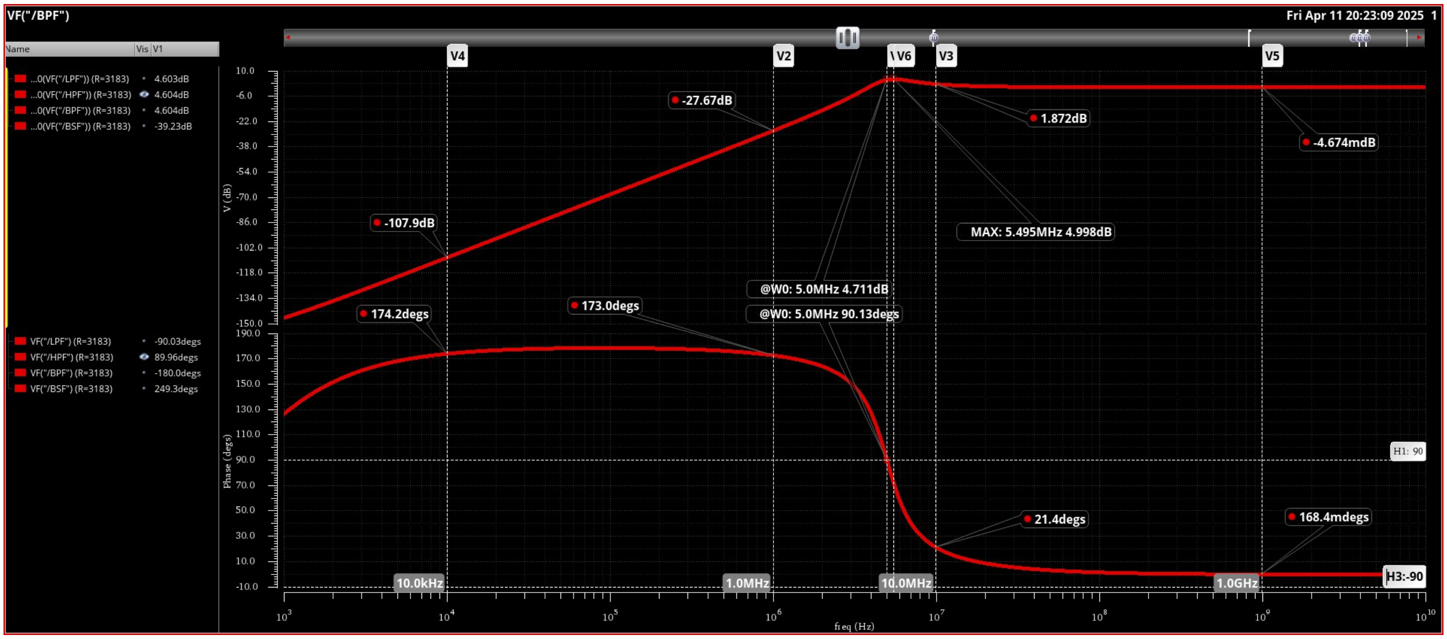


Figure 6: HPF Frequency Response (Magnitude & Phase)

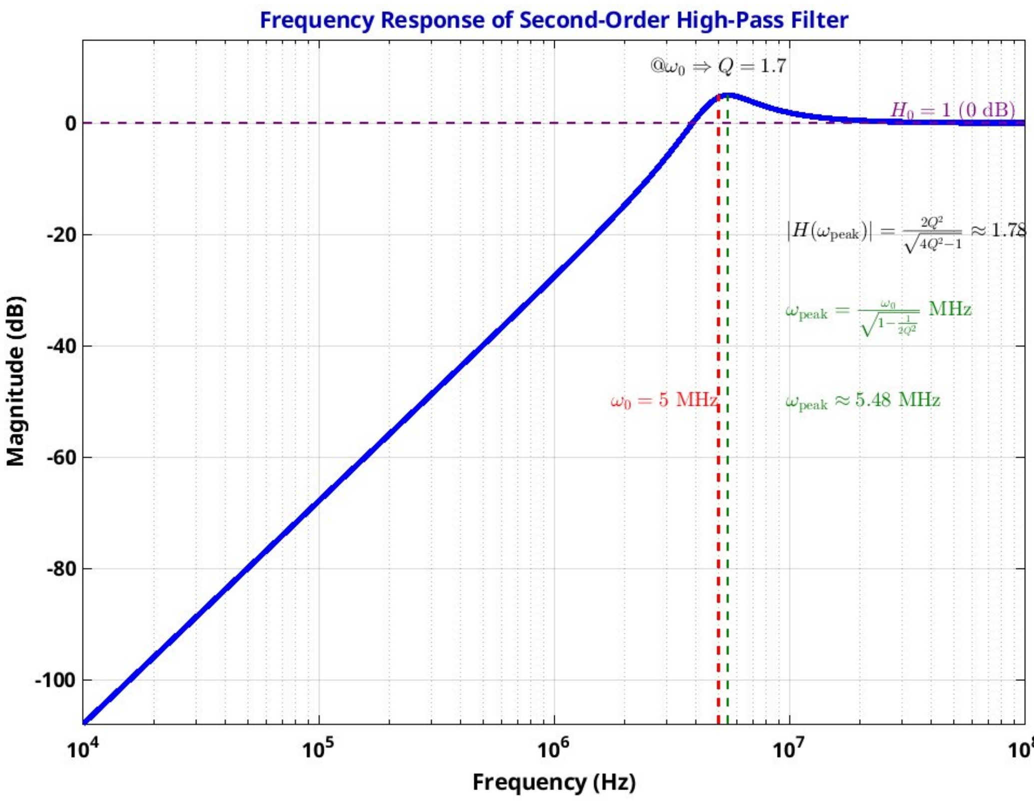


Figure 7 HPF Theoretical Plot

As shown from Figure 7 and 6 we could calculate the parameters by getting them from the corresponding locations and evaluating the equations:

- H_0 at High frequencies = -4.674 mdB which is near to **1** in linear scale. (as calculated in hand analysis)
- Get peak value and evaluate Q ≈ 1.7 also get $\omega_0 \approx$ 5 MHz.
- Check at ω0 we get Q ≈ 1.7.

3. BPF Frequency Response

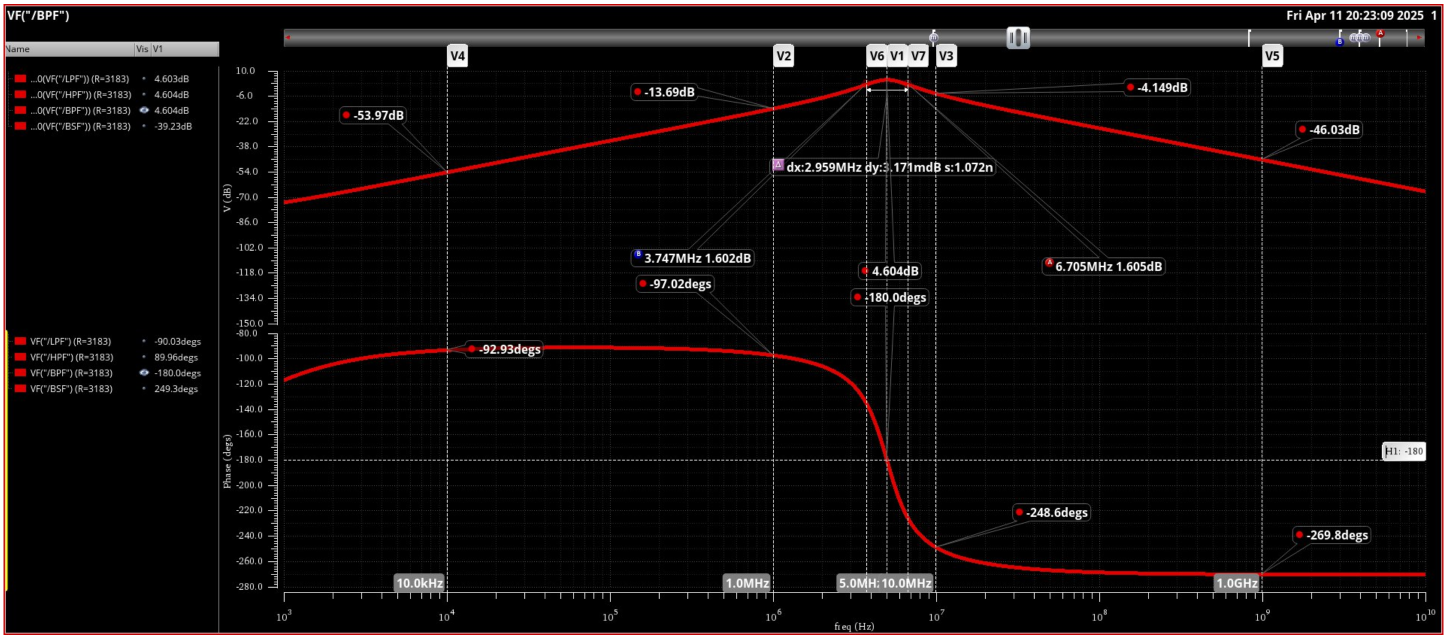


Figure 8: BPF Frequency Response (Magnitude & Phase)

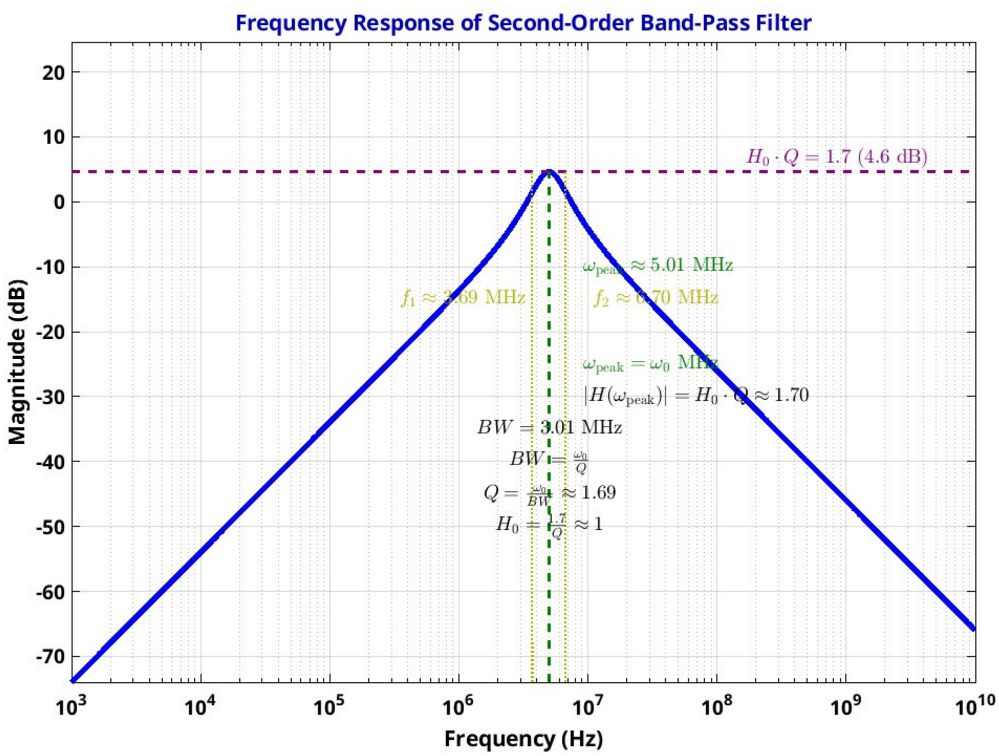


Figure 9 BPF Theoretical Plot

As shown from Figure 9 and 8 we could calculate the parameters by getting them from the corresponding locations and evaluating the equations:

- $H_0 * Q = 1.7$ (peak value) @ $\omega 0 \approx$ 5 MHz.
- Get Q from BW, $Q \approx 1.7$.
- Then evaluate $H_0 \approx 1$.

4. BSF Frequency Response

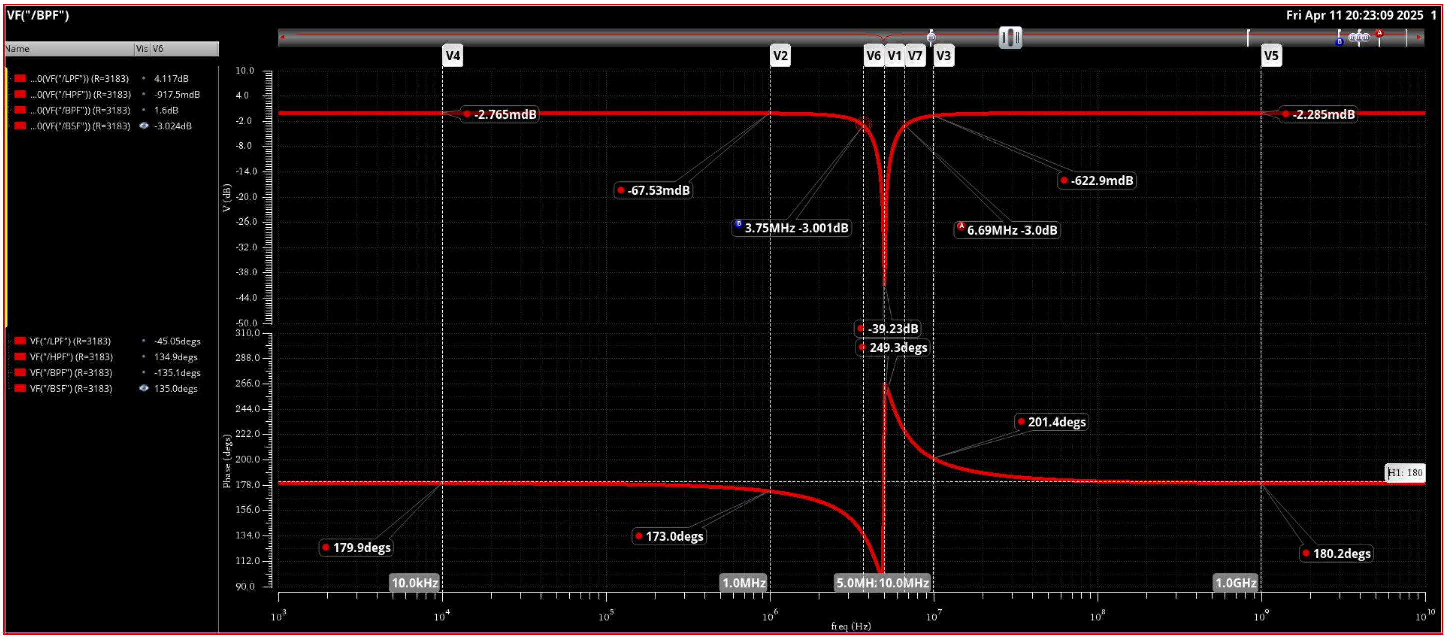


Figure 10: BSF Frequency Response (Magnitude & Phase)

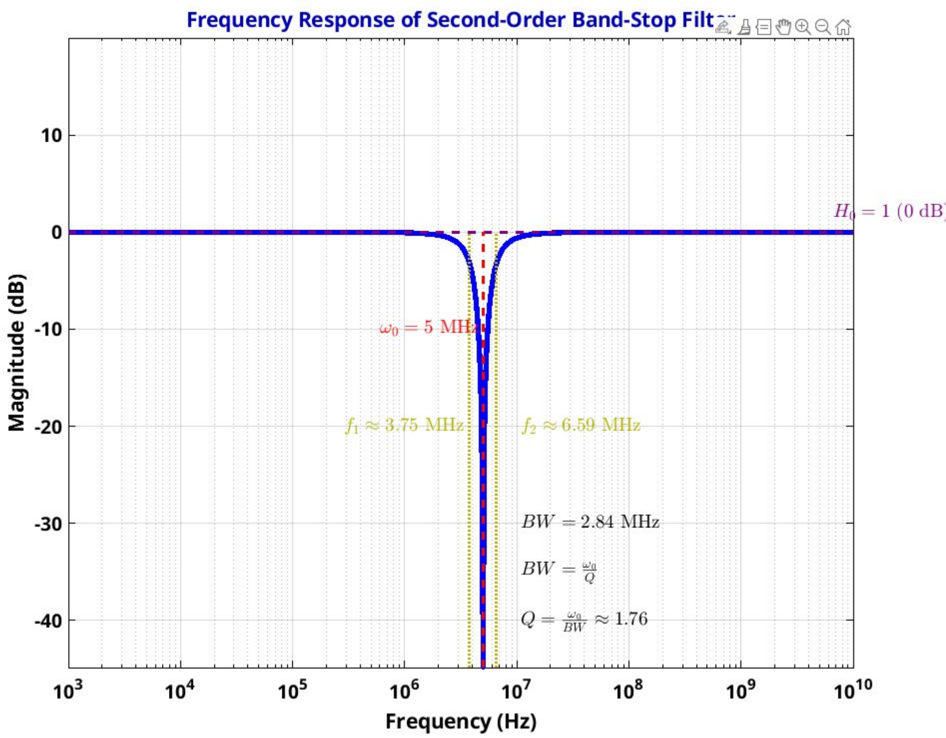


Figure 11 BSF Theoretical Plot

As shown from Figure 11 and 10 we could calculate the parameters by getting them from the corresponding locations and evaluating the equations:

- H_0 at top = -2.73 mdB which is near to **1** in linear scale.
- At ω_0, the gain is a minimum at F_0 = 5.01187 MHz. ≈ 5 MHz
- As for Q we get using BW like the BPF.

Transient LPF and HPF outputs

1. Apply an input sine wave at 1MHz & 5MHz & 10MHz for LPF

- ➢ Apply a Sine wave V_{in} With
- ❖ Amplitude = 500 mv
- ❖ Frequencies $f_1 = 1$ MHz, $f_2 = 5$ MHz & $f_3 = 10$ MHz:
  - $V_{in1} = 500 \sin(2\pi f_1)$ mV
  - $V_{in2} = 500 \sin(2\pi f_2)$ mV
  - $V_{in3} = 500 \sin(2\pi f_3)$ mV

$$|V_{out}| = |V_{in}| * |H(\omega_0)| \quad \&\& \quad \angle V_{out} = \angle V_{in} + \angle H(\omega_0)$$

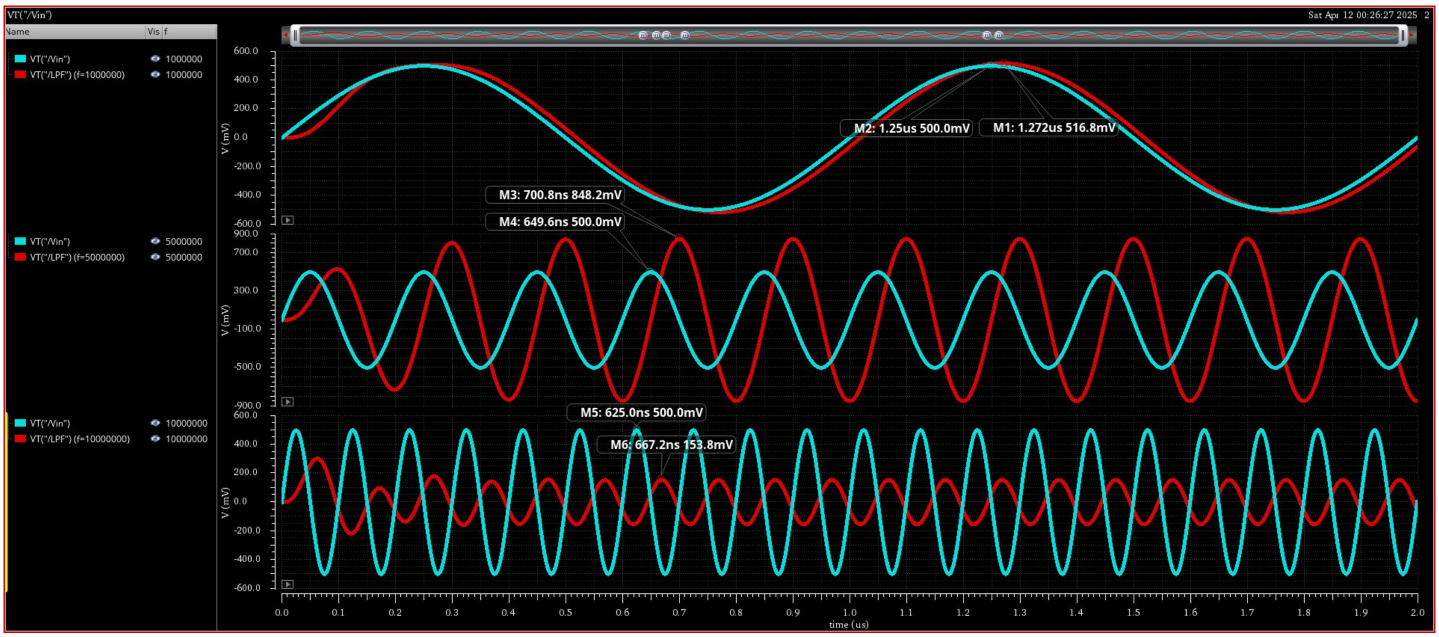


Figure 12: LPF Transient Response for input sine waves at 1 MHz & 5 MHz & 10 MHz

As shown in Figure 12:

Input Sine Wave Frequency	1 MHz	5 MHz	10 MHz
Input Peak	500 mV	500 mV	500 mV
Output Peak	516.8 mV	848.2 mV	153.8 mV
Gain (from i/p & o/p)	$20\log\left(\frac{516.8}{500}\right) = 287.05 mdB$	$20\log\left(\frac{848.2}{500}\right) = 4.59 dB$	$20\log\left(\frac{153.8}{500}\right) = -10.24 dB$
Gain (from magnitude response)	287.2 mdB	4.603 dB	-10.17 dB
Phase Change (from phase response)	$-6.987°$	$-90.02°$	$-158.6°$

Shown from the previous table, we can find that the gain added to the sine waves is **nearly equal to** the gain of the frequency response of the LPF.

Calculating the phase change between the input and output sine waves with the following formula:

$$pasechange = \frac{T_1 - T_2}{period} * 360$$

T_1 = the time of the input peak & T_2 = the time of the output peak

Hence, the phase change of each sine wave is:

- @ 1 MHz à $\Delta phase = \frac{(1.25-1.272)*10^{-6}}{\frac{1}{1*10^6}} * 360 = -7.92°$
- @ 5 MHz à $\Delta phase = \frac{(649.6-700.8)*10^{-9}}{\frac{1}{5*10^6}} * 360 = -92.16°$
- @ 10 MHz à $\Delta phase = \frac{(625-667.2)*10^{-9}}{\frac{1}{10*10^6}} * 360 = -151.92°$

Comparing the phase change of the sine waves to the phase change response of the LPF (shown in the table), the phase change of the sine waves is **nearly equal** to the phase response of the LPF.

2. Apply an input sine wave at 1MHz & 5MHz & 10MHz for HPF

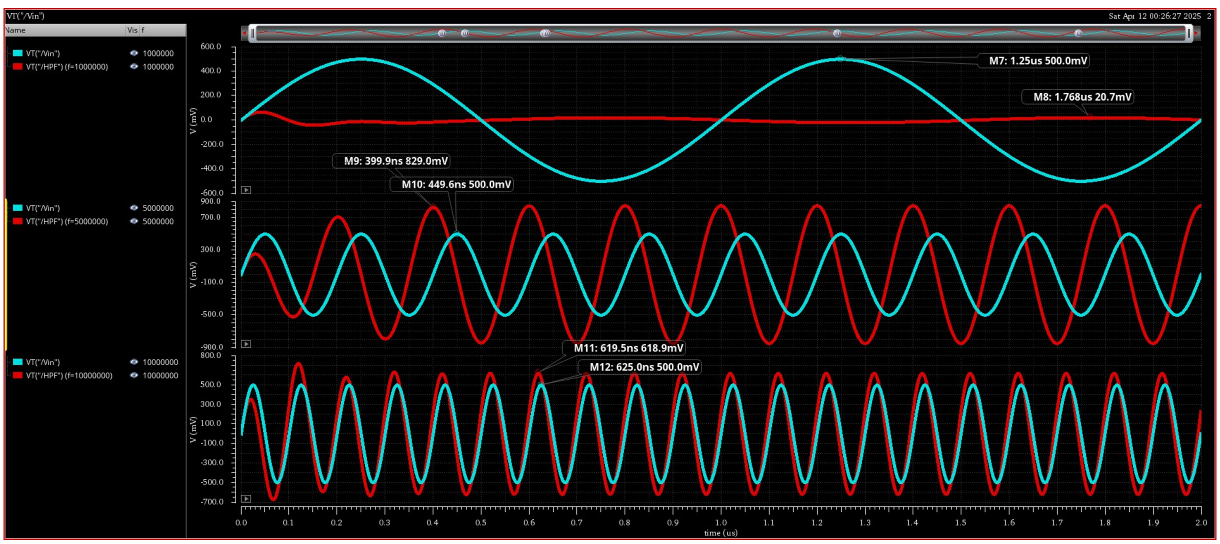


Figure 13: HPF Transient Response for input sine waves at 1 MHz & 5 MHz & 10 MHz

As shown in Figure 13:

Input Sine Wave Frequency	1 MHz	5 MHz	10 MHz
Input Peak	500 mV	500 mV	500 mV
Output Peak	20.7 mV	829 mV	618.9 mV
Gain (from i/p & o/p)	$20\log\left(\frac{20.7}{500}\right) = -27.66dB$	$20\log\left(\frac{829}{500}\right) = 4.39dB$	$20\log\left(\frac{618.9}{500}\right) = 1.853dB$
Gain (from magnitude response)	-27.67 dB	4.711 dB	1.872 dB
Phase Change (from phase response)	173°	90.13°	21.4°

Shown from the previous table, we can find that the gain added to the sine waves is **nearly equal to** the gain of the frequency response of the HPF.

Calculating the phase change between the input and output sine waves with the following formula:

$$p\,asech\,ange = \frac{T_1 - T_2}{period} * 360$$

T_1 = the time of the input peak & T_2 = the time of the output peak

Hence, the phase change of each sine wave is:

- @ 1 MHz à $\Delta phase = \frac{(1.768-1.25)*10^{-6}}{\frac{1}{1*10^6}} * 360 = 186.48°$
- @ 5 MHz à $\Delta phase = \frac{(449.6-399.9)*10^{-9}}{\frac{1}{5*10^6}} * 360 = 89.46°$
- @ 10 MHz à $\Delta phase = \frac{(625-619.5)*10^{-9}}{\frac{1}{10*10^6}} * 360 = 19.8°$

Comparing the phase change of the sine waves to the phase change response of the HPF (shown in the table), the phase change of the sine waves is **nearly equal** to the phase response of the HPF.

Transient BPF and BSF outputs

1. Apply an input square wave at 1MHz & 5MHz & 10MHz for BPF

- Apply a square wave V_{in} With
- $V_{high} = 500$ mv & $V_{low} = -500$ mv
- Frequencies: $f_1 = 1$ MHz, $f_2 = 5$ MHz & $f_3 = 10$ MHz

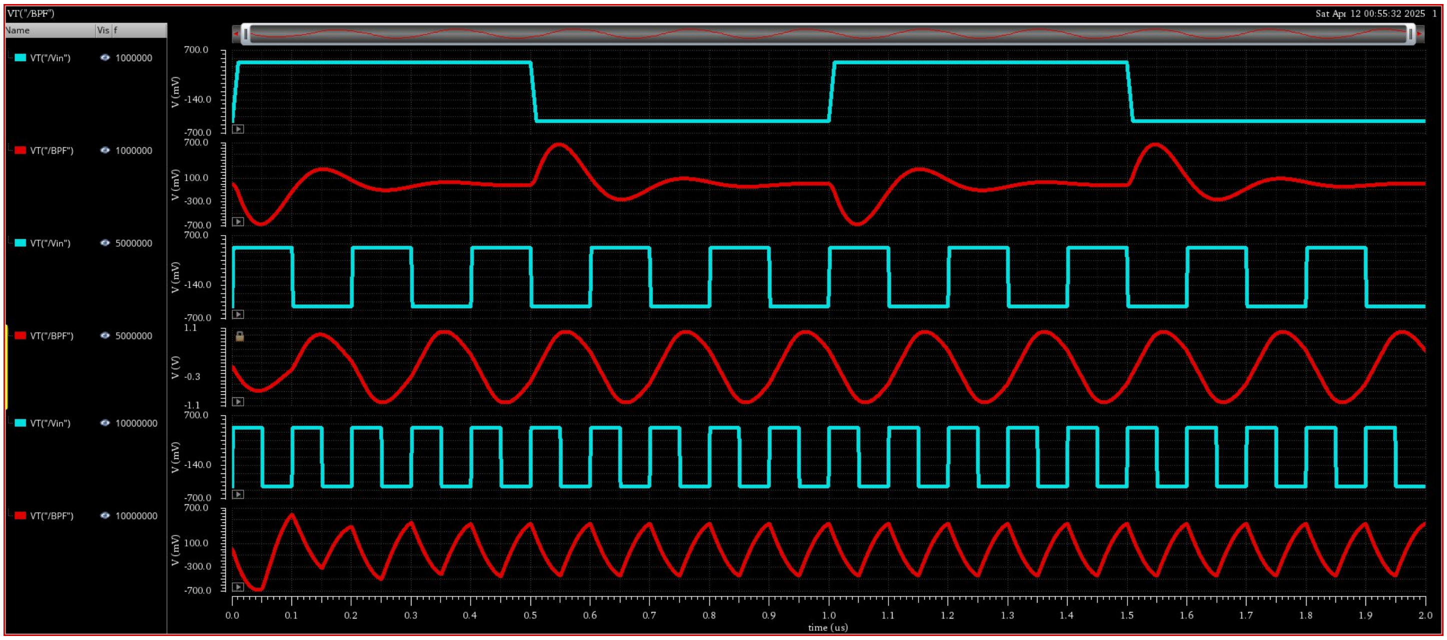


Figure 14: BPF Transient Response for input square waves at 1 MHz & 5 MHz & 10 MHz

Comment:

- BPF (Band-Pass Filter) Analysis:
  - 1 MHz Input:

    The output resembles the response of a second-order control system to a unit step input. This is due to the BPF's attenuation of frequencies far from its cut-off frequency ω_0=5 MHz, resulting in a transient response similar to an under-damped system.

  - 5 MHz Input:

    The output primarily contains the main component of the Fourier series at 5 MHz, which is the cut-off frequency ω_0. This component is passed with the largest amplitude, but due to the filter's bandwidth, other nearby frequency components are also present, resulting in a slightly distorted sine wave.

  - 10 MHz Input:

    The output shows significant attenuation as 10 MHz is outside the pass-band. The waveform is heavily distorted, with minimal amplitude, as the BPF effectively rejects frequencies far from ω_0.

2. Apply an input square wave at 1MHz & 5MHz & 10MHz for BSF

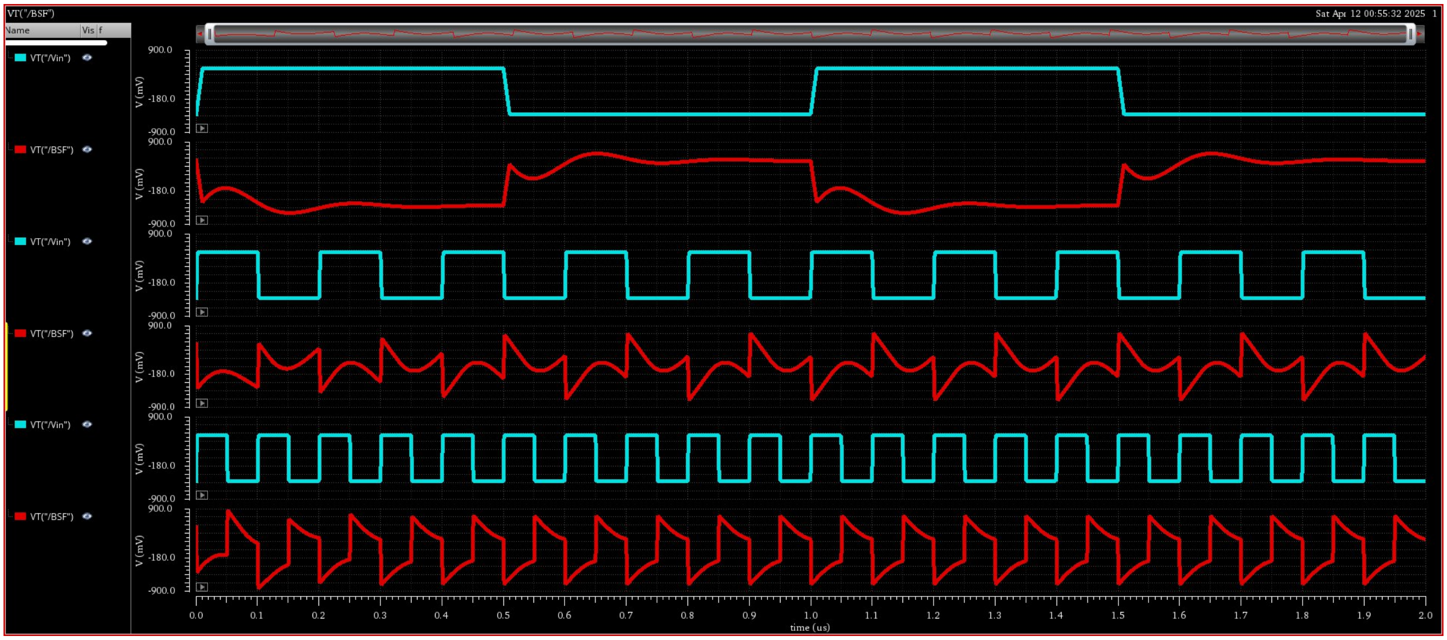


Figure 15: BSF Transient Response for input square waves at 1 MHz & 5 MHz & 10 MHz

Comment:

- BSF (Band-Stop Filter) Analysis:
  - 1 MHz Input:

    The output retains most of the square wave characteristics since 1 MHz is outside the stop-band centered at $\omega_0 = 5$ MHz. The waveform is largely unaffected, showing minimal distortion.
  - 5 MHz Input:

    The output shows significant attenuation of the 5 MHz component, as it lies within the stop-band. The waveform appears as a subtraction of the 5 MHz sine wave from the input, resulting in a distorted output.
  - 10 MHz Input:

    The output retains most of the square wave characteristics, similar to the 1 MHz input, as 10 MHz is outside the stop-band. The waveform is largely unaffected, with minimal distortion.