Control

Part 3

Examples on Voltage series feedback + Gain Stability with Feedback + Nyquist Criterion + Gain and Phase Margins (BODE plot)

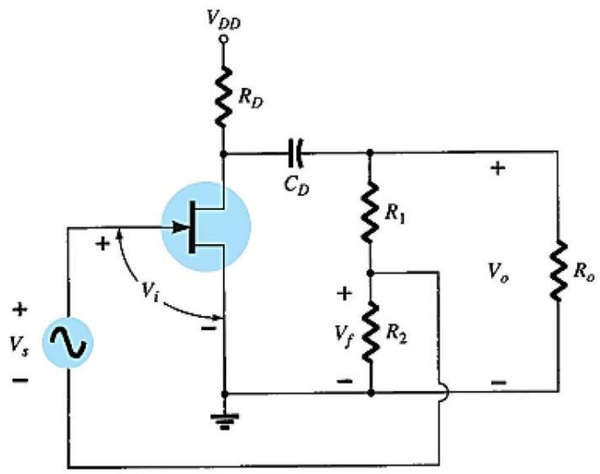
Examples on the voltage series Feedback

Example 1:

For the circuit shown in fig (1)
calculate the gain without and with feedback for the circuit values $R_1 = 80\ K\Omega$, $R_2 = 20\ K\Omega$, $R_o = R_D = 10\ K\Omega$ and $g_m = 4000\ \mu S$

sol.

FET amp stage with voltage series feedback



<u>note that:</u>
In the circuit, part of the signal v_o is obtained using a feedback network of resistors R_1 and R_2. The feedback voltage v_f is connected in series with the source voltage without feedback the amplifier.
Gain is:

$$A_v = \frac{V_o}{V_{in}} = -g_m R_L$$

R_L is parallel obtained by Resistors:

$$R_L = R_o // R_D // (R_1 + R_2)$$

The feedback net. provides a feedback factor of:

$$\beta = -\frac{R_2}{R_2 + R_1}$$

The gain of the negative feedback:

$$A_v = \frac{A}{1 + \beta A} = \frac{-g_m R_L}{1 + (-\frac{R_2}{R_2 + R_1})(-g_m R_L)}$$

If $\beta A \gg 1$, we have

$$A_f \cong \frac{1}{\beta} = -\frac{R_2 + R_1}{R_2}$$

For the example:

$R_L = 10\ K\Omega // 10\ K\Omega // (80\ K\Omega + 20\ K\Omega) \cong 5\ K\Omega$

Since $(R_1 + R_2) = 100\ K\Omega$ so neglect $(R_1 + R_2)$

$R_o = R_D = 10\ K\Omega$

Without feedback: $A_v = -g_m R_L$

$= -(4000 \times 10^{-6})(5\ K) = -20$

With feedback factor:

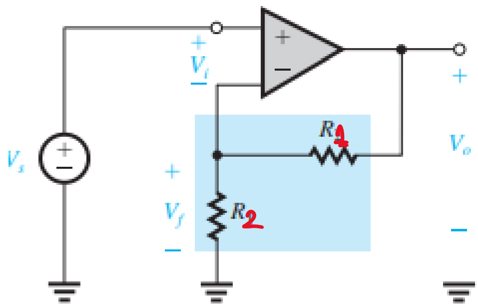
$$\beta = -\frac{R_2}{R_2 + R_1} = -\frac{20}{20 + 80} = -0.2$$

The gain with feedback is:

$$A_f = \frac{A}{1 + \beta A} = -\frac{-20}{1 + (-0.2)(-20)} = -4$$

Example 2:

Calculate the amplifier gain of the circuit in fig (2) for the op-amp gain $A = 100000$ and resistances: $R_1 = 1.8\ K\Omega, \mathrm{R}_2 = 200\ \Omega$



voltage series feedback in op-amp connection

Note: the gain of the op-amp A without feedback, is reduced by β factor.

$$\beta = \frac{R_2}{R_2 + R_1}$$

For this example:

$$\beta = \frac{200}{200 + 1.8K} = 0.1$$

$$A_f = \frac{A}{1 + \beta A} = -\frac{100000}{1 + (0.1)(100000\ )} = 9.999$$

Since $\beta A \gg 1$

$$\therefore A_f \cong \frac{1}{\beta} = \frac{1}{0.1} = 10$$

Gain Stability with feedback:

In addition to the β factor setting a precise gain value. It is also interested to show how stable the Feedback amplifier is compared 5 an amplifier without feedback Differentiating the equation of the gain with feedback A_f.

$$|A_f| = \left|\frac{A}{1+\beta A}\right| \quad \rightarrow \quad (1)$$

$$\left|\frac{\partial A_f}{\partial A}\right| = \left|\frac{(1+\beta A) - A\beta}{(1+\beta A)^2}\right| = \left|\frac{1}{(1+\beta A)^2}\right|$$

$$|\partial A_f| = \left|\frac{\partial A}{(1+\beta A)^2}\right| \quad \rightarrow \quad (2)$$

Divide by eqn. (1)

$$\left|\frac{\partial A_f}{A_f}\right| = \left|\frac{\partial A}{(1+\beta A)^2}\right| \left|\frac{1+\beta A}{A}\right| = \left|\frac{\partial A}{A(1+\beta A)}\right|$$

$$= \left|\frac{1}{(1+\beta A)}\right| \left|\frac{\partial A}{A}\right|$$

For $\beta A \gg 1$

$$\left|\frac{\partial \boldsymbol{A_f}}{\boldsymbol{A_f}}\right| \cong \left|\frac{\mathbf{1}}{(\boldsymbol{\beta A})}\right| \left|\frac{\partial \boldsymbol{A}}{\boldsymbol{A}}\right|$$

This shows that the magnitude of the relative change in gain $\left|\frac{\partial A_f}{A_f}\right|$ is reduced by the factor $|\beta A|$ compared to that without feedback $\left|\frac{\partial A}{A}\right|$

i.e. the gain change of amplifier ***with feedback is very small*** *with respect 5 the change of the gain of amp. Without feedback.*

Example 3:

If an amplifier with gain of -1000 and feedback of $\beta = 0.1$ has a gain change of 20% due to temperature, calculate the change of The feedback amplifier.

sol.

Using equation as an approximation since

$\beta = 0.1, A = -1000 \rightarrow A\beta = -100$

$$\left|\frac{\partial A_f}{A_f}\right| \cong \left|\frac{1}{(\beta A)}\right| \left|\frac{\partial A}{A}\right|$$

Gain change

$$= \left|\frac{1}{(0.1)(-1000)}(20\%)\right|$$

$$= \left|\frac{20\%}{-100}\right| = 0.2\%$$

The improvement is **100** times. This, while the amplifier gain changes from $|A| = |1000|$ by 20%, the with feedback changes from $|A_f| = 10$ by the 0.2%

Since $|A_f| = \left|\frac{A}{1+\beta A}\right|$ or $|A_f| \cong \left|\frac{1}{\beta}\right| = \frac{1}{0.1} = 10$

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$$\frac{20\%}{0.2\%} = 100$$

Feedback amplifier

Phase and Frequency considerations

we have considered the operation of a feedback amplifier which the feedback signal was opposite to the input signal - negative feedback. In any practical circuit this condition occurs only for some midfrequency range & operation. we know that an amplifier gain will change with frequency, dropping off at high frequencies from the midfrequency value. In addition, the phase shift of an amplifier will also change with frequency.

If, as the frequency increases, the phase shift changes then some of the feedback signed will add to the input signal, it is possible for the amplifier to break into oscillations due to the positive feedback. If the amplifier oscillates at some low or high frequency, it is **no longer useful on an amplifier** proper feedback amplifier design require, that the circuit be **stable at all frequencies** not merely(مُجَرّد) those in the range of intent.
otherwise, a transient disturbance could cause a seemingly **unstable amplifier** to suddenly start oscillating.

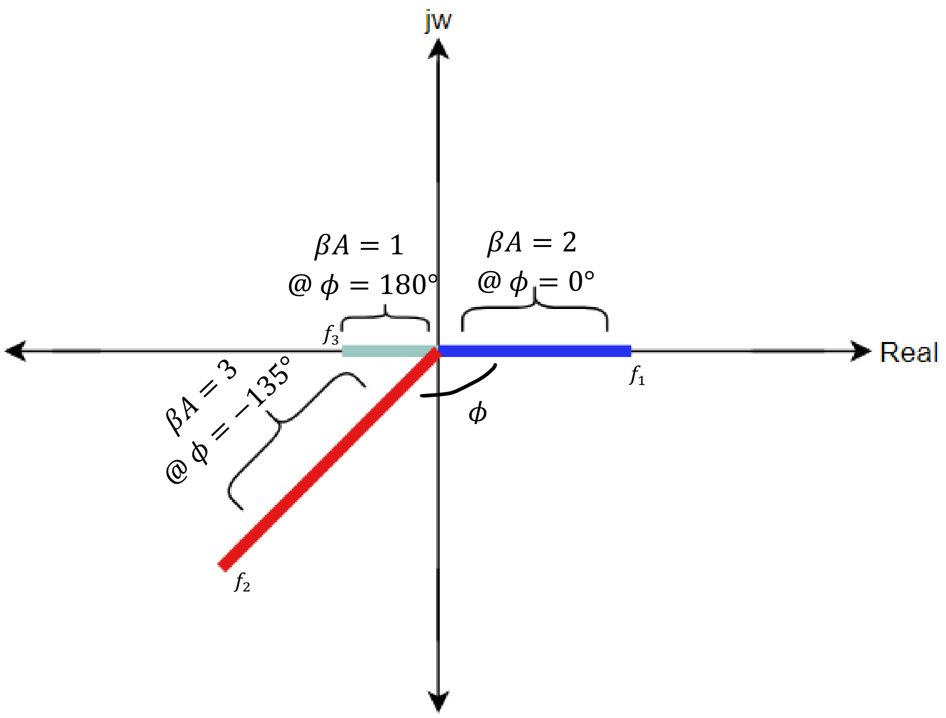
Nyquist criterion

In judging the stability of a feedback amplifier, as a function of frequency, the βA product and the phase Shift between input and output are the determining factors. One of the most popular techniques used to investigate Stability is the Nyquist method.
A Nyquist diagram is used to plot **gain** and **phase shift** as a function of **frequency** in a **complex plane**.

- the Nyquist plot, in effect, combines the two Bode plots of **gain** versus frequency and **phase shift** versus frequency on **"single plot"**

- A Nyquist plot is used to **quickly** show whether an amplifier is stable for all frequencies and how stable the amplifier is relative to some gain or phase shift criteria.

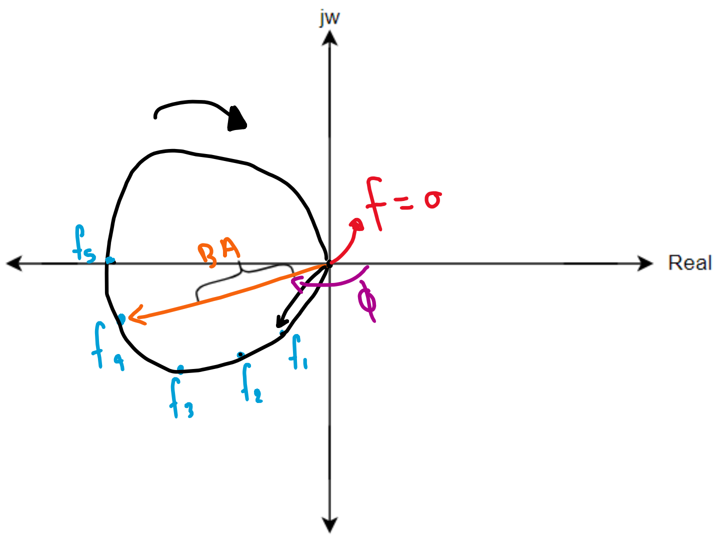
As a start, consider the complex plane shown in fig below. A few points of various gain βA values are shown at a Few different phase shift angles. By using the positive red axis as reference $(0°)$.
a magnitude of $\beta A = 2$ is shown at a phase shift of $0°$ at point 1. Additionally, a magnitude of $\beta A = 3$ at a phase shift of $-135°$ is shown at point 2 and a magnitude of $\beta A = 1$ at $180°$ phase shift is shown at point 3. Thus, points on this plot can represent both gain magnitude of BA and phase shift.



Complex plane
showing typical gain – phase pivots

If the points representing gain and phase shift for an amplifier circuit are plotted at increasing frequency, then The Nyquist plot is obtained as shown by the plot in by below.

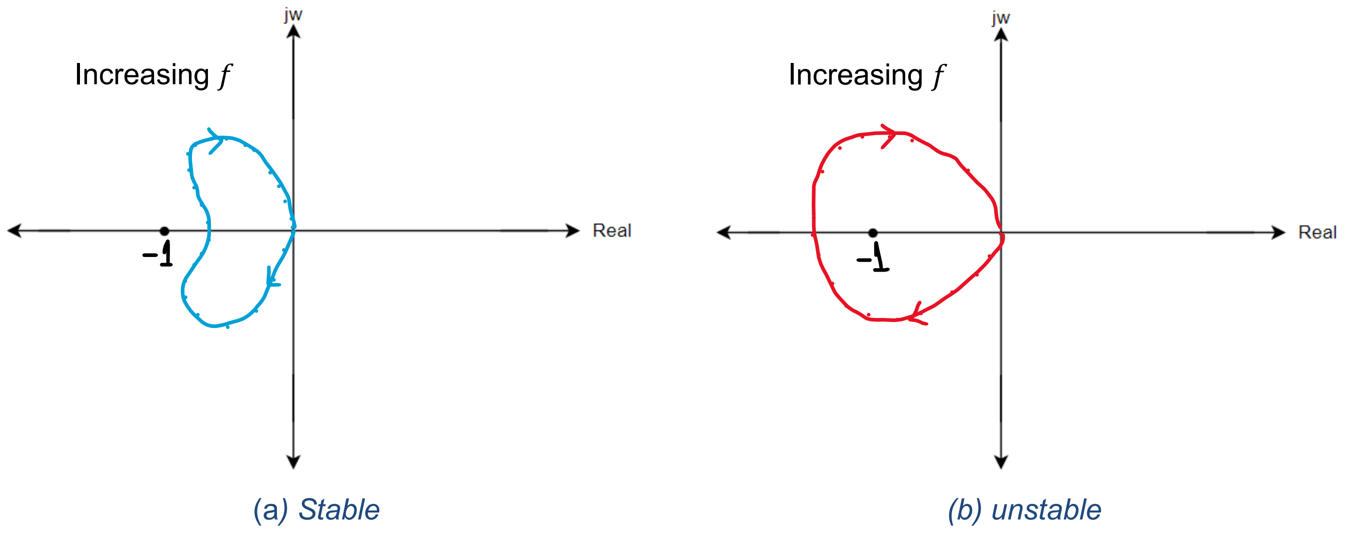
- At the origin the gain is zero of a frequency of 0 (for RC- type coupled).
- At increasing frequency, points F_1, F_2 and F_3 and the phase shift increased as did the magnitude of βA.
- At a representative frequency F_4 in the value of βA is the vector length from the origin to point (4) and the phase shift is the angle of ϕ.
- At a frequency F_5 for the phase shift is $180°$.
- At higher frequencies the gain is shown to decrease back to 0.



Nyquist Plot

The Nyquist conditions for stability can be stated as follows:
"The amplifier is **unstable** if: the Nyquist curve platter **encloses** (**encircles**) the -1 point, and it is stable otherwise".

As example of the Nyquist criteria is demonstrated by the curves in fig below. the Nyquist plot in fig (a) is stable since it does not encircle the -1 point, while that shown in fig (b) is unstable since the curve does encircle the -1 point. Keep in mind that encircling the -1 point means that at a phase shift of $180°$ the loop gain βA is greater than 1. Therefore, the feedback signal is in phase with the input and large enough to result in a large input signed than that applies, with the result that oscillation occurs.



Nyquist plots showing stability conditions
(a) stable (b) unstable

Gain and Phase Margins (BODE plot)

From the Nyquist criterion we know that a feedback amplifier is stable if the loop gain βA is less than unity $(0\ dB)$, when its phase angle is $180°$. we can additionally determine some margins of stability to indicate how close to instability the amplifier is, that is, if the gain βA is less than unity but say 0.95 in value, this would be as relatively stable as another amplifier having, say $\beta A = 0.7$ (both measured at $180°$). of course, amplifiers with loop gains 0.95 and 0.7 are both stable, but one is closer to instability, If the loop gain increased, than the other, we can define the following terms:

Gain Margin (GM): is defined as the negative of the value βA in decibles at the frequency at which the phase angle is $180°$. Thus, $0\ dB$ equal to a value of $\beta A = 1$ is on the border of stability, and any negative decibel value is stable.
The GM may be evaluated in decibels from the curve of fig below.

Phase Margin (PM): is defined as the angle of $180°$ minus the magnitude of the angle at which the value βA is unity $(0\ dB)$.
The PM may also be evaludies directly from the curve of the fig below.