LEC 4

The Stability of Linear feedback systems

Routh- Hurwitz stability criterion

Case 1:

No element in the first column is zero.

Example <u>The characteristic equation of a second order System is.</u>

q(s)= a2 S^2+ a2 S+ a0

the array is written as

$$
\text{even}\left\{
\begin{array}{c|cc}
S^2 & a_2 & a_2 \\
S^1 & a_1 & 0 \\
\hline
S^0 & b_1 & 0
\end{array}\right.
$$

$$b_{n-1} = b_{2-1}$$

Where:

$$b_1 = \frac{a_1 a_0 - (0) a_2}{} = \frac{1}{a_1}(a_1 a_0 - (0) a_2)$$

$$= \frac{-1}{a_1}(a_2(0) - a_0 a_1)$$

$$= \frac{-1}{a_1}\begin{vmatrix} a_2 & a_0 \\ a_1 & 0 \end{vmatrix}$$

$$= \frac{-1}{a_1}(0 - a_2 a_1) = \frac{-1}{a_1}(-a_0 a_1)$$

Therefore, the required for a stable second order system is simply that all the coefficients be positive.

for The Third order to be stable, it is necessary and sufficient That the coefficients be positive and $a_2\ a_1 \geq a_0\ a_3$

The condition other $a_2\, a_1 = a_0\, a_3$ results in a board line stability case, and one pair of roots lies on the majority axis in the s plane.

Numerical example for the case when the characteristic equation results in no zero elements in the first consider a polynomial.

$$q(s) = (s-1+j\sqrt{7})(s-1-j\sqrt{7})(s-3)$$

$$\text{i.e. } q(s) = s^3 + s^2 + 2s + 24 \quad \text{——} \quad (7)$$

The polynomial satisfies that all the coefficients exist and are positive . Therefore, utilizing the Route. Hurwitz array, we have (eq1)

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 1 & 24 \\ s^1 & b_1 & 0 \\ s^0 & c_1 & 0 \end{array}$$

$$b_1 = \frac{1\times 2 - 24\times 1}{1} = -22$$

$$c_1 = \frac{b_1(24) - 1\times 0}{b_1} = \frac{24\, b_1}{b_1} = 24$$

Then the array is

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 1 & 24 \\ s^1 & -22 & 0 \\ s^0 & 24 & 0 \end{array}$$

Since two changes in sign appears in the first column, we find that two roots of q(s) lie in the right hand of **S** plane .

Case 2

Zero in the first column while some other elements of the raw containing a zero in the first column are non-zero.

If only one element in the array in zero, it may be replaced with a small positive number & which is allowed & approach zero after completing the array .

Example, consider the flowing's characteristic equation.

$$q(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$$

The Routh-Hurwitz array is then

odd	s^5	1	2	11
even	s^4	2	4	10
	s^3	$\cancel{0}$ ϵ	6	0
	s^2	c_1	10	0
	s^1	d_1	0	0
	s^0	$\frac{10 d_1}{d_1}$	0	0

$b_1 = \frac{4-4}{2} = 0 \rightarrow \epsilon$

$\frac{44-20}{4} = \frac{24}{4} = 6$

$c_1 = \frac{4\epsilon - 12}{\epsilon}$ and $d_1 = \frac{6c_1 - 10\epsilon}{c_1} = 6 - \frac{10\epsilon}{c_1}$

$d_1 = 6$

$c_1 = \frac{-12}{\epsilon}$ → large negative number since $\epsilon \rightarrow 0$

Then the array is

s^5	1	2	11
s^4	2	4	10
s^3	ϵ	6	0
s^2	$\frac{-12}{\epsilon}$	10	0
s^1	6	0	0
s^0	10	0	0

There are two sign changes due to the large negative number in the first column, **Cs=-12/E** Therefore, the system is unstable, and two roots lies in the night half of the s plane.

Example: **(case 2)**

Garder the characteristic equation $q(s) = s^4 + s^3 + s^2 + s + K$

It is required to determine the gun K which results in borderline stability using post -Hurwitz stability criterion-

sol

The Routh-Hurwitz array is then.

s^4	1	1	K
s^3	1	1	0
s^2	ϵ	K	0
s^1	$\frac{\epsilon - K}{\epsilon}$	0	0
s^0	K	0	0

$$c_1 = \frac{\epsilon - K}{\epsilon} = \frac{\epsilon}{\epsilon} - \frac{K}{\epsilon} = \frac{-K}{\epsilon}$$

$$d_1 = \frac{K(\epsilon - K)}{\epsilon} = \frac{K\epsilon - K^2}{\epsilon} = K - \frac{K^2}{\epsilon} = K$$

i.e. The array is

s^4	1	1	K
s^3	1	1	0
s^2	ϵ	K	0
s^1	$\frac{-K}{\epsilon}$	0	0
s^0	K	0	0

Therefore , any value of K greater Than zero lie positive values The system is unstable . Also, since the last term in the first Column is paid to k, a negative value of k will result in an unstable system. Therefore, the system is unstable for all values of gari k.

Case 3

Forms is the first column, and the other elements of the row containing the zero in the first column are also zero,(we discuss the case when the row consists of a single element which is zero)

case 3 occurs when all the elements in one you are zeros or when the rows consist of a single element which is zero.

Example.

consider the third - order system with a

characteristic equation

$$q(s) = s^3 + 2s^2 + 4s + K$$

where K is an adjustable loop gain Find The value of K for a stable system.

Sol

The Routh array is.

The Routh array is

odd	s^3	1	4
even	s^2	2	K
	s^1	b_1	0
	s^0	c_1	0

$$b_1 = \frac{8-K}{2}, \quad c_1 = \frac{b_1 K - 0}{b_1} = K$$

Then The array

Then The array is

s^3	1	4
s^2	2	K
s^1	$\frac{8-K}{2}$	0
s^0	K	0

therefore, for a stable system, we require That **0 ≤ K ≤ 8**

when K=8, we have two roots on the **jω** axis and a borderline stability case.

Example

<u>In The previous example , find the factors of the characteristic equation when K=8.</u>

Sol

The array when K=8 is

s^3	1	4
s^2	2	K
s^1	0	0
s^0	K	0

when K=8 we obtain row of zeros (case 3) The auxiliary equation U(S), is the equation of the row preceding the row of zeros .The equation of the row preceding the row of zeros is, in this case, obtained from the s^2= row. We recall that this row contains the coefficients of the even powers of S and therefore in this case we have.

$$U(S) = 2s^2 + Ks^0 = 2s^2 + K$$
$$= 2s^2 + 8 = 2(s+4) = 2(s+j2)(s-j2)$$

To show that the auxilian eqn

, U(S) is a factor of the chs equation.

, we divide q(s) by U(s) to obtain.

$$\begin{array}{r|l} & \frac{1}{2}s + 1 \\ \hline 2s^2 + 8 & s^3 + 2s^2 + 4s + 8 \\ & s^3 \quad\quad + 4s \\ \hline & 2s^3 \quad\quad + 8 \\ & 2s^3 \quad\quad + 8 \\ \hline & \quad\quad\quad 0 \end{array}$$

- Therefore, when Ka8, the factor of the chs eqn are.

$$\text{i.e } (s^3 + 2s^2 + 4s + 8) = 2\left(\tfrac{1}{2}s + 1\right)(s+j2)(s-j2)$$
$$= (s+2)(s+2j)(s-2j)$$

The q(S) roots are. $(s+2),\ (s+j2),\ (s-j2)$

Then we have two roots on the jω-axis und deborder stability case and one root on the right of the S- plane.