LEC 4

The Stability of Linear feedback systems

Routh- Hurwitz stability criterion

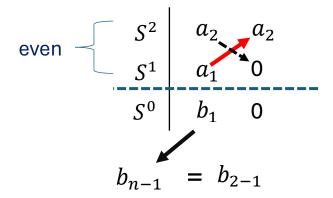
Case 1:

No element in the first column is zero.

Example The characteristic equation of a second order System is.

$$q(s)=a2 S^2+a2 S+a0$$

the array is written as



Where:

$$b_1 = \frac{q_1 q_0 - (0)a_2}{q_1} = \frac{1}{q_1} \left(\frac{q_1 q_0 - 10(a_2)}{a_0 q_1} \right)$$

$$= \frac{1}{q_1} \left(\frac{q_2 q_0}{a_0 q_1} - \frac{1}{q_1} \left(-\frac{q_0 q_1}{a_1} \right) - \frac{1}{q_1} \left(-\frac{q_0 q_1}{a_1} \right)$$

$$= \frac{1}{q_1} \left(\frac{q_1 q_0 - 10(a_2)}{a_1 q_1} - \frac{1}{q_1} \left(-\frac{q_0 q_1}{a_1} \right) - \frac{1}{q_1} \left(-\frac{q_0 q_1}{a_1} \right)$$

Therefore, the required for a stable second order system is simply that all the coefficients be positive.

for The Third order to be stable, it is necessary and sufficient That the coefficients be positive and a_2 $a_1 \ge a_0$ a_3

The condition other a_2 a_1 = a_0 a_3 results in a board line stability case, and one pair of roots lies on the majority axis in the s plane.

<u>Numerical example</u> for the case when the characteristic equation results in no zero elements in the first consider a polynomial.

The polynomial satisfies that all the coefficients exist and are positive . Therefore, utilizing the Route. Hurwitz array, we have (eq1)

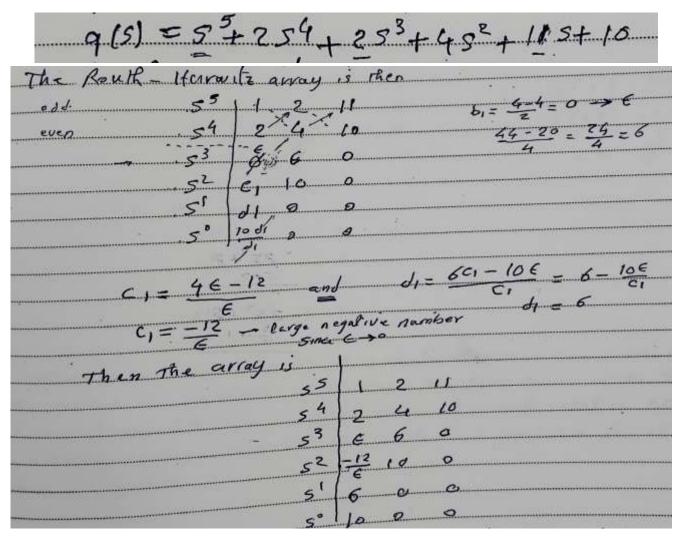
Since two changes in sign appears in the first column, we find that two roots of q(s) lie in the right hand of $\bf S$ plane.

Case 2

Zero in the first column while some other elements of the raw containing a zero in the first column are non-zero.

If only one element in the array in zero, it may be replaced with a small positive number & positive number & amp; which is allowed & proach zero after completing the array.

Example, consider the flowing's characteristic equation.



There are two sign changes due to the large negative number in the first column, **Cs=-12/E** Therefore, the system is unstable, and two roots lies in the night half of the s plane.

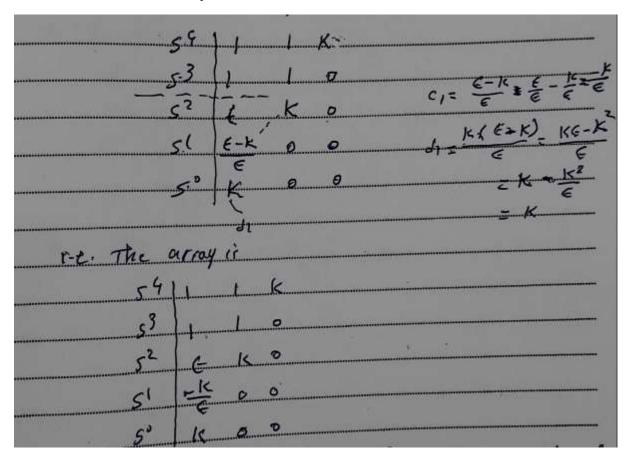
Example: (case 2)

Garder the characteristic equation

It is required to determine the gun K which results in borderline stability using post -Hurwitz stability criterion-

sol

The Routh-Hurwitz array is then.



Therefore, any value of K greater Than zero lie positive values The system is unstable. Also, since the last term in the first Column is paid to k, a negative value of k will result in an unstable system. Therefore, the system is unstable for all values of gari k.

Case 3

Forms is the first column, and the other elements of the row containing the zero in the first column are also zero, (we discuss the case when the row consists of a single element which is zero)

case 3 occurs when all the elements in one you are zeros or when the rows consist of a single element which is zero.

Example.

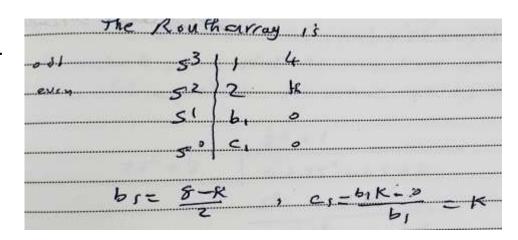
consider the third - order system with a

characteristic equation

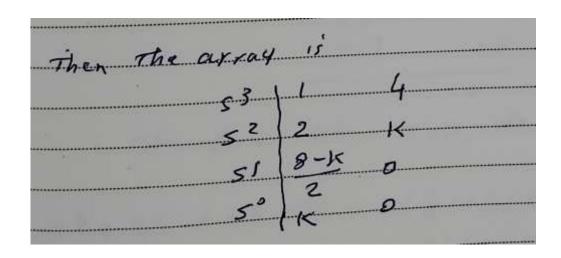
where K is an adjustable loop gain Find The value of K for a stable system.

Sol

The Routh array is.



Then The array



therefore, for a stable system, we require That $0 \le K \le 8$ when K=8, we have two roots on the $j\omega$ axis and a borderline stability case.

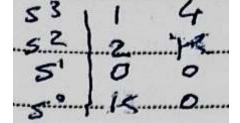
Example

In The previous example, find the factors of the characteristic equation when

K=8.

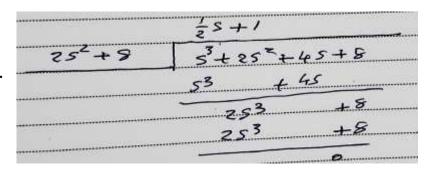
Sol

The array when K=8 is

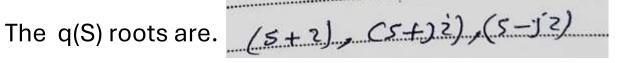


when K=8 we obtain row of zeros (case 3) The auxiliary equation U(S), is the equation of the row preceding the row of zeros . The equation of the row preceding the row of zeros is, in this case, obtained from the s^2= row. We recall that this row contains the coefficients of the even powers of S and therefore in this case we have.

To show that the auxilian eqn , U(S) is a factor of the chs equation. , we divide q(s) by U(s) to obtain.



Therefore, when Ka8, the factor of the chs eqn are.



Then we have two roots on the $j\omega$ -axis und deborder stability case and one root on the right of the S- plane.

