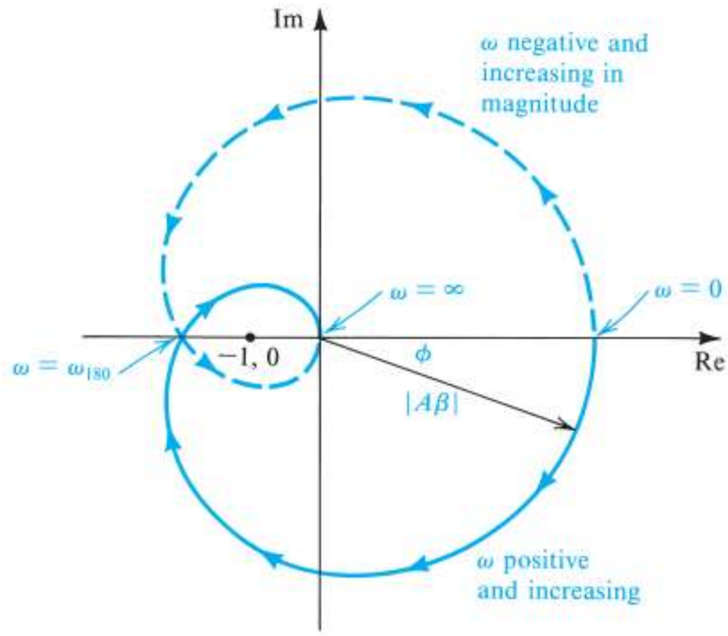
Control

Part 4

Notes on Nyquist Plot
+
BODE Plot Gain and Phase Rules

Notes on Nyquist plot (for amplifiers with feedback)

It is a formalized approach for testing the stability It is simply a polar plot of the loop gain with frequency (ω) used as a parameter.



Nyquist Plot of an unstable amplifier

Note that the radial distance is $|A\beta|$ and the angle is the phase angle ϕ. The solid fine plot for positive frequencies since the loop gain - and for that matter any gain function of a physical network - has a magnitude that is an **even** function of frequency and a phase that is an **odd** function of frequency, the $A\beta$ plot for negative frequencies, shown in fig above as a broken line, can be drawn as a mirror image through the Re axis.

The Nyquist plot intersects the negative red axis at the frequency Wiso. Thus, if this intersection occurs to the left of the point $(-\mathbf{1}, \mathbf{0})$, we know that the magnitude of the boy gain at this frequency is greater than unity and the amplifier will be **unstable**. On The other hand, if the intersection occurs to the right of the point $(-\mathbf{1}, \mathbf{0})$ the amplifier will be **stable**. It follows that if the Nyquist plot **encircles** the point (-1, 0) then the amplifier will be **unstable**. It should be noted that this statement is a simplified version of the Nyquist criterion.

Example 1:

Consider a feedback amplifier for which the open loop transfer function $A(S)$ is given by:

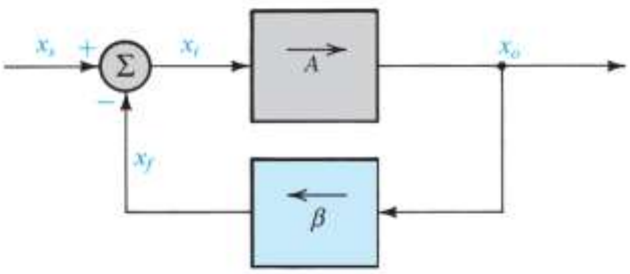
$$A(s) = \left(\frac{10}{1 + \left(\frac{s}{10^4}\right)}\right)^3$$

Let the feedback factor β be a constant independent of frequency. Find the frequency ω, at which the phase shift- is $180°$. Then, show that the feedback amplifier will be Stable if the Feedback factor β is less than a critical Value β_{cr} and unstable if $\beta \geq \beta_{cr}$ and find the value of β_{cr}

sol.

The loop gain $A\beta$

$$\beta A(s) = 10^3\beta\left(\frac{1}{1 + \left(\frac{s}{10^4}\right)}\right)^3$$



$$|\beta A(s)| = 10^3\beta \left|\left(\frac{1}{1+j10^{-4}\omega}\right)^3\right| \qquad S = j\omega$$

$$\angle\beta A(s) = \angle\frac{\left(10^{\frac{1}{3}}\sqrt[3]{\beta}\right)^3}{(1+j10^{-4}\omega)^3} = \frac{\angle\left(10^{\frac{1}{3}}\sqrt[3]{\beta}\right)^3}{\angle(1+j10^{-4}\omega)^3}$$

$$= \frac{0°}{-3\tan^{-1}(10^{-4}\omega)}$$

the phase of $\beta A(s) = \angle\beta A(s) = 0 - 3\tan^{-1}(10^{-4}\omega)$

for phase of $180°$:

$$3\tan^{-1}(10^{-4}\omega) = 180°$$

$$\tan^{-1}(10^{-4}\omega) = 60°$$

$$10^{-4}\omega = \tan 60 = \sqrt{3}$$

$$\therefore \omega = \frac{\sqrt{3}}{10^{-4}} = \sqrt{3}\times 10^4\ rad/sec$$

To find $\beta_{cr}@|A\beta| = 1$:

$$1 = \beta_{cr}\left(\frac{10}{1+j10^{-4}\omega}\right)^3 \quad @\ \omega = \sqrt{3}\times 10^4$$

$$\therefore \beta_{cr} = 0.008$$

BODE Plot

A simple technique exists for obtaining an approximate plot of the magnitude and phase of a transfer function given its poles and zeros. The resulting diagrams are Called Bude plats (The method was developed by H. Bode)
In summary, to obtain the Bode-plot for the magnitude of a transfer function, the asymptotic plot for each pole and zero is first drawn. The slope of the high-frequency asymptotic of the curve corresponding to a zero is $+20\ dB/decade$, while that for a pole is $-20\ dB/decade$ (which equal to $-6\ dB/octave$). the various plots are then added together, and the overall curve is shifted vertically by an amount determined by the multiplicative constant of the transfer function.

A transfer function in the form:

$$T(s) = a_m \frac{(s - z_1)(s - z_2) \cdots (s - z_m)}{(s - p_1)(s - P_2) \cdots (s - p_n)}$$

a_m is a multiplicative constant.
$z_1, z_2, \ldots$ are the zeros of the transfer function.
$P_1, P_2, \ldots$ are the poles.

A transfer function is completely specified in terms of its poles and zeros together with the value of multiplicative constant. It follows that the magnitude response in decibels of the network can be obtained by summing together terms of the form: $20 \log (\sqrt{a^2 + \omega^2})$, and the phase response can be obtained by summing together terms of the form: $\tan^{-1}(\frac{\omega}{a})$. In both cases the terms corresponding to poles are summed with negative signs.

For convenience we can extract the constant a and write the typical magnitude term in the form:

$$20 \log \left(\sqrt{1 + \frac{\omega^2}{a^2}} \right)$$

On a plot of decibels versus log frequency the term a gives rise to the curve and straight - line asymptotes.

اللي فات دوت كان شرح لما نقابل Pole أو Zero بيكون تأثيره ايه على ال Bode plot

والجدول دوت بيلخص الموضوع

	Pole	Zero
Magnitude	-20 dB/decade Actual Mag @ pole: -3 dB	+20 dB/decade Actual Mag @ zero: +3 dB
Phase	-90° Actual Phase @ pole: -45°	LHP zero: +90° Actual Phase @ zero: +45°
		RHP zero: مش علينا -90° Actual Phase @ zero: -45°

جزء ال RHP Zero الدكتور مضربش بيه مثال ولا شرحه فمش لازم تعرفها

دلوقتي هنشرح مثال ونطبق عليه الكلام اللي فات دوت بالتفصيل الممل

Example 2:

An amplifier has the voltage transfer function of:

$$T(s) = \frac{10s}{\left(1 + \frac{s}{10^2}\right)\left(1 + \frac{s}{10^5}\right)}$$

(a) Find the poles and zeros and sketch the magnitude of the gain versus frequency. Find approximate values for the gain at $\omega = 10\,, 10^3$, and $10^6\ rad/sec$.

(b) Find the Bode plot for the phase of the given transfer function.

sol.

(a) The transfer-function factors in a convenient form: $(1+\frac{s}{a})$

The given transfer function has zeros at $s_{z_1}=0$ and at $s_{z_2}=\infty$. The poles are as follows: one at $s_{p_1}=-10^2$ rad/s and one at $s_{p_2}=10^5\ rad/s$

لو عاوز تعرف جبنا ال poles ازاى فببساطة:

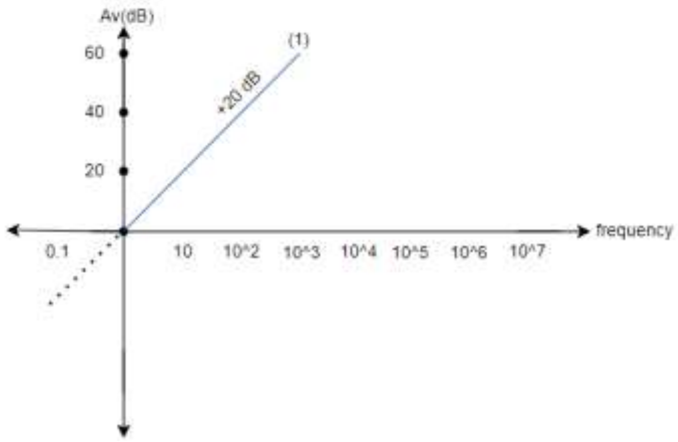
$$1+\frac{s}{10^2}=0\ \rightarrow\ \frac{s}{10^2}=-1\rightarrow s=-10^2\ \rightarrow\ \omega=|s|=10^2$$

وال Pole التاني بنفس الطريقة وكذلك ال zero وموضوع ال ∞ مش قصتنا دلوقتي 😁

هنشرح دلوقتي تأثير كل pole and zero في رسمة ال magnitude

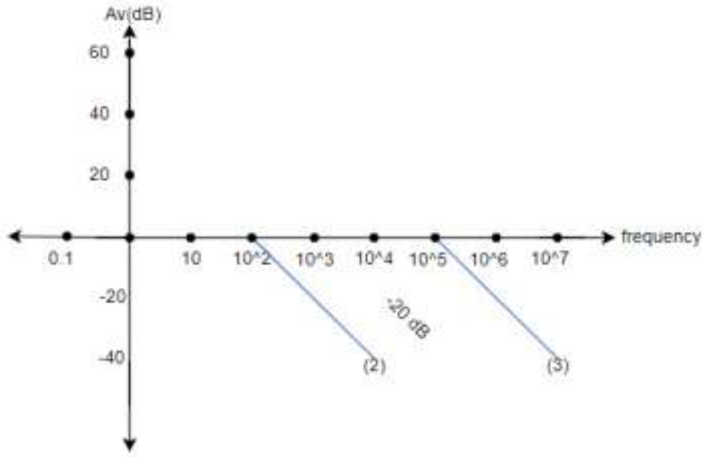
1) curve 1, which is a straight line with $+20\ dB/decade$ Stop, corresponds to the storm (that is, the year at seo) in the numerator.

كل ال zero هيعمله انه يزود الميل بمقدار $+20\ dB/decade$



2) The pole at $s=-10^2$ results in curve 2, which consists of two asymptotes intersecting at $\omega=10^2$.
3) Similarly, the pole of $s=-10^5$ is represented by curve 3, where the intersection of the asympitotes is at $\omega=10^5$.

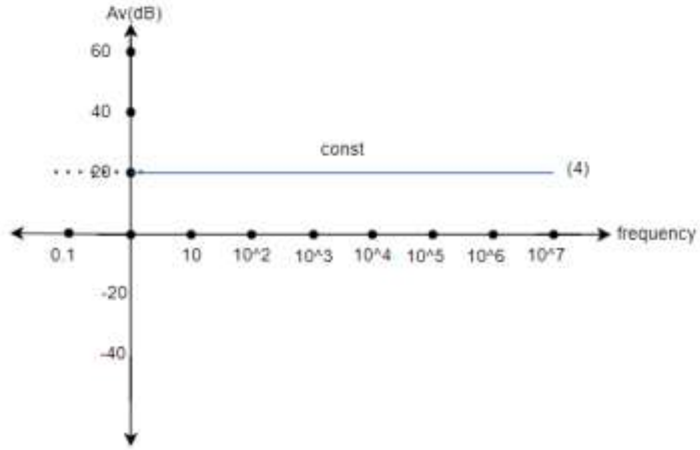
ال pole هيعملي العكس هيقلل الميل بمقدار 20- dB/decade زى الرسمة كده



4) Finally, curve 4 represents the multiplicative constant of value 10.

جت ازاى ؟ فيه ثابت في السؤال اللي هو 10 هنحوله بس لل dB فيكون كالآتي:

$$A_v = 20\log a = 20\log 10 = 20\ dB$$



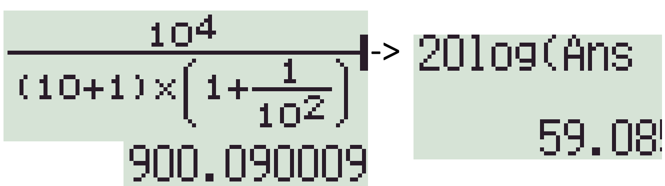
5) Adding the four curves results in the asymptotic Bode plot of the amplifier gain (curve 5). Note that since the two poles are widely separated, the gain will be very close to $60\ dB$ (or $A_v = 10^3$ since $20log\ 10^3 = 60\ dB$) over the frequency range (mid band gain) from 10^2 to $10^5\ rad/sec$.

هنا بقى هنجمع الكلام اللي فات كله في الرسمة كلها مع بعض

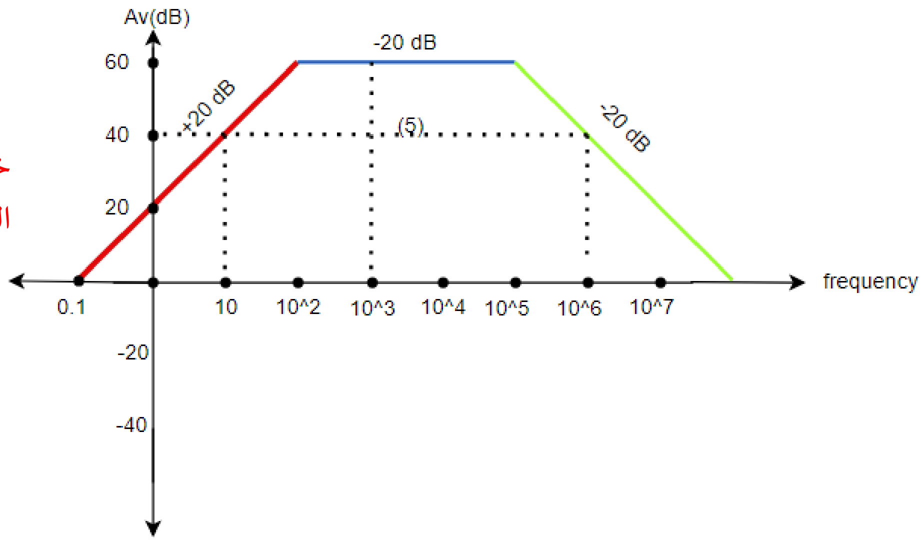
طيب هو جاب ال $A_v = 10^3 = 60\ dB$ منين ؟

هنعوض في ال $T(s)$ اللي هي ال gain بأى تردد في الفترة من 10^2 الي 10^5 مكان ال s

عوضت هنا ب 10^3

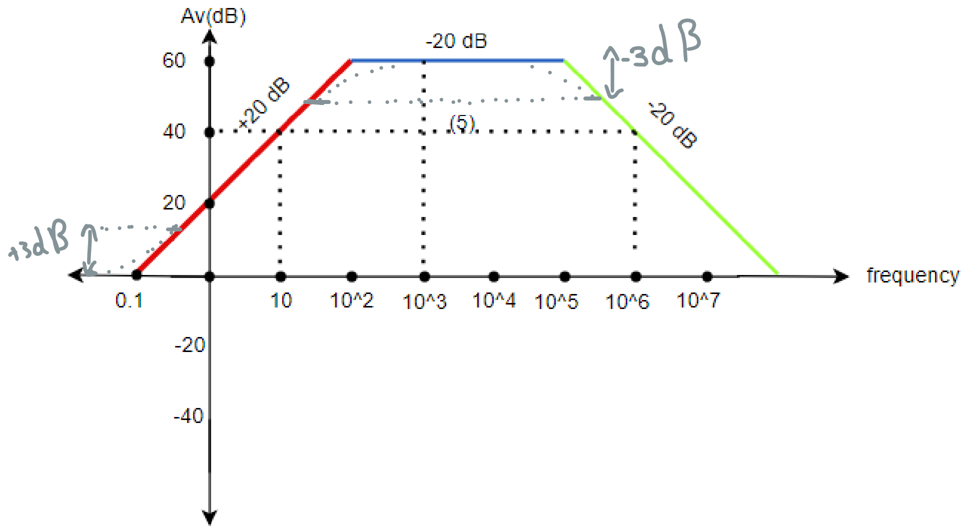


وكده ديت التجميعة النهائية اللي بنسميها **asymptotic or approximate plot**



6) At the two corner frequencies (10^2 and 10^5 rad/sec) the gain will be approx. $3\ dB$ below the maximum of $60\ dB$

عند كل Pole بينزل $-3\ dB$ وعند كل zero بنطلع $+3\ dB$
وديت الرسمة النهائية



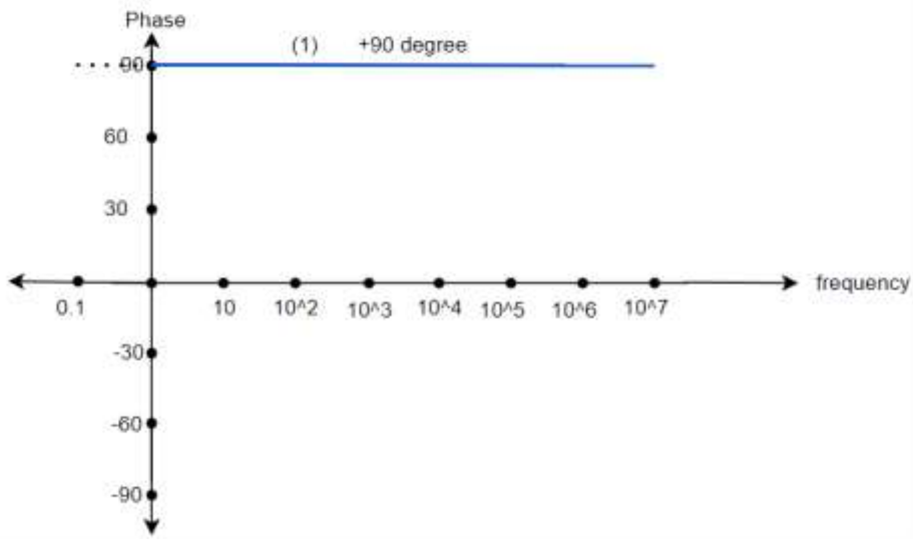
<u>#Note:</u> The 3 specific frequencies the value of the gain that obtained from Bode plot are:

ω	Approximate gain
10	$40\ dB$
10^3	$60\ dB$
10^6	$40\ dB$

(b) The Bode diagram for the phase of the transfer function is platted as follows:

1) The zero at $s = 0$ gives rise to a constant $+90°$ phase function represented by curve 1 in the following figure.

كل ال zero هيعمله انه يزود ال Phase ب $+90°$



2) The pole at $\omega = -10^2$ gives rise to the phase function:

$$\phi_1 = -\tan^{-1}\frac{\omega}{10^2}$$

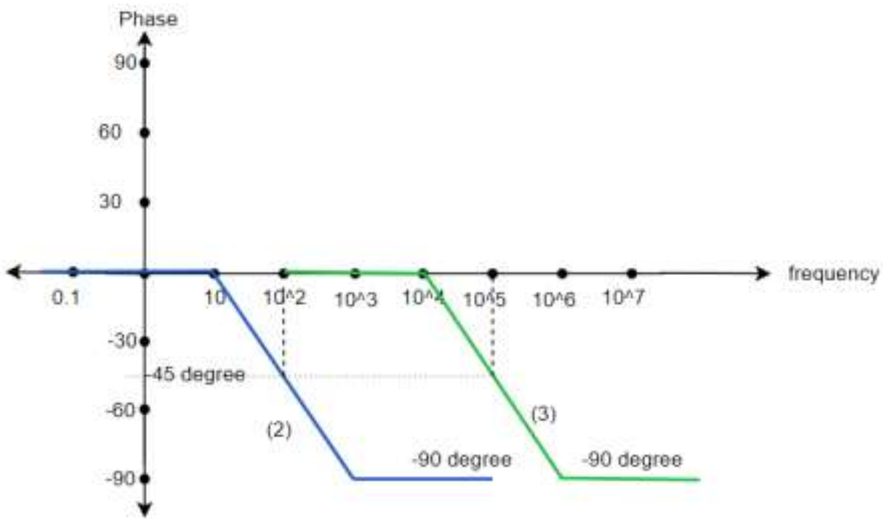
(the leading minus sign is since this singularity is a pole)
The asymptotic plot for this function is given by curve 2 in fig below.

3) The pole at $s = -10^5$ gives rise to the phase function:

$$\phi_2 = -\tan^{-1}\frac{\omega}{10^5}$$

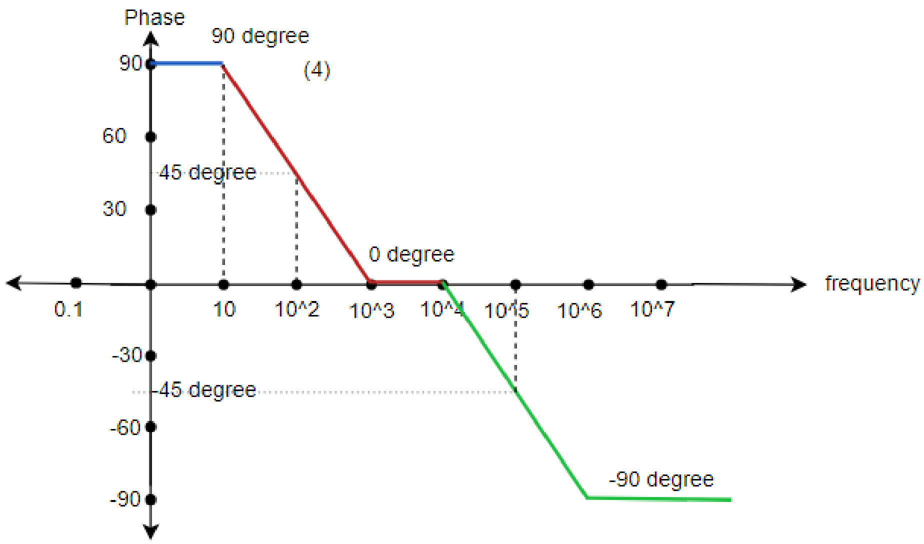
whose asymptotic plot is given by are curve 3.

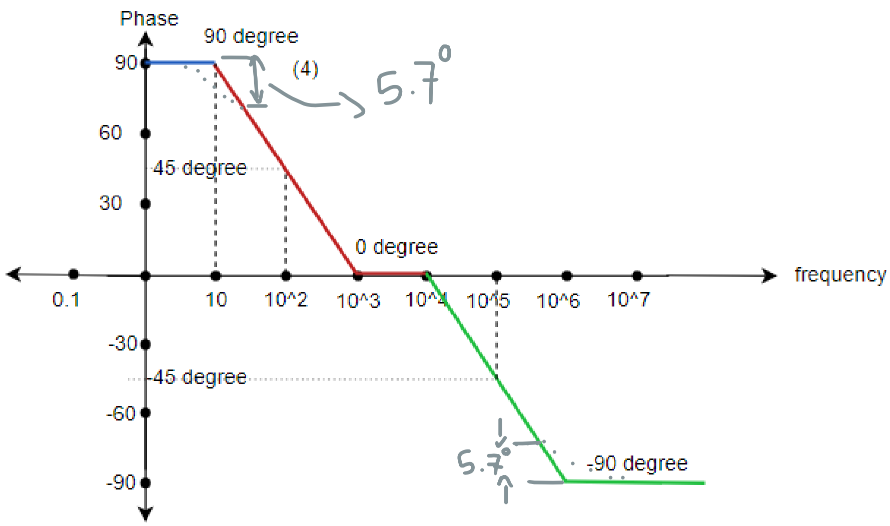
ال pole هيعملي العكس هيقلل ال Phase ب $-90°$ بحيث انه لما يوصل عند ال pole يكون قل ب $-45°$ وبعد كده يقل ال $-45°$ الباقية



4) the overall Phase response (curve 4) is obtained by direct summation of the three plots.

هنجمع اللي فات مع بعض فنوصل للشكل دوت





#Note: From the Bode plot for the phase of the given transfer function we see that:

- at $100\ rad/sec$, the amplifier phase **leads** by $45°$.
- at $10^5\ rad/sec$, the phase **lags** by 45°.

#Remark: For constructing Bode plots. it is convenient to express the transfer function factors in the form $(1+\frac{s}{a})$.

أنا عارف اني مزود حاجات كتير من عندي في المحاضرة ديت بس صدقني..