

The Harmonic Architecture of the Solar System

A Geometric and Resonant Analysis

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Supplementary code:

github.com/salahealer9/harmonic-architecture-solar-system

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Abstract

In this research, we investigate the fundamental harmonic and geometric principles underlying the architecture of the Solar System. We employ the most fundamental algebraic ratio that defines the diagonal of a square unit, the irrational number $\sqrt{2}$. Using the Celtic Cross geometric framework, we build a set of combinations of $\sqrt{2}$ -including its derivative, the Silver Ratio $(1 + \sqrt{2})$ -as the mathematical basis to develop a unified harmonic model that reproduces the observed planetary spacing with remarkable precision.

Reinterpreting Kepler's concept of celestial harmony in light of modern orbital data, the study identifies a missing harmonic node between Mars and Jupiter: the theoretical orbit of a lost planet, Harmonia. The Celtic Cross harmonic model suggests that planetary distances from Mercury to Pluto exhibit a mean mediating pattern between algebraic and transcendental forms. This discovery hints at a deeper resonance that connects geometry, matter, and cosmic order.

Quantitatively, the Celtic Cross harmonic model achieves a mean absolute percentage error (MAPE) of 0.72% and a root mean square error (RMSE) of 0.11 AU—an order of magnitude improvement over classical approaches. The results imply that the Solar System's structure exhibits harmonic principles common to physics, geometry, and number theory, providing a mathematical foundation for Kepler's concept of the music of the spheres.

Keywords: Planetary Resonance · Gravitational Quantization · Silver Ratio Geometry · Harmonic Structure of the Solar Sytem · Celtic Cross Harmonic Model · Keplerian Harmony · Mathematical Cosmology · Orbital architecture · Number Theory in Physics

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1 The Gravitational Quantization Law

This chapter formalizes the *Celtic Harmonic Law* — a gravitation quantization postulate that unifies geometry, number theory, and celestial mechanics. According to this model, as in Bohr’s model of the atom, the distances of the planets are subject to fixed patterns. Rather than random, they seem to be quantized to mathematical relationships involving the square root of 2 ($\sqrt{2}$), or, put another way, equations involving the silver ratio ($1 + \sqrt{2} \approx 2.414$). Within this framework, *Harmonia’s orbit* at 2.142 AU emerges as a fundamental resonance boundary separating the inner and outer Solar System. The law establishes a mathematical connection between the structure of atoms and the architecture of planets, consequently, involving an interrelation between the microcosm and the macrocosm.

1.1 The Celtic Harmonic Principle

Principle 1 (Celtic Gravitational Quantization). *In a gravitationally bound system, stable orbital radii occur at distances determined by rational functions of the silver ratio $\delta_S = 1 + \sqrt{2}$ relative to a reference orbit. The critical resonance boundary is marked by the harmonic ratio $\frac{2(\delta_S-1)(2\delta_S-3)}{\delta_S} \approx 2.142$.*

1.2 Mathematical Foundation

Let us define the fundamental constants of the Celtic system:

$$A = \sqrt{2} \tag{1}$$

$$B = 1 + A = 1 + \sqrt{2} \tag{2}$$

$$C = 2A - 1 = 2\sqrt{2} - 1 \tag{3}$$

$$D = A - 1 = \sqrt{2} - 1 \tag{4}$$

The constants above generate the quadratic number field $\mathbb{Q}(\sqrt{2})$, providing the mathematical foundation for orbital quantization.

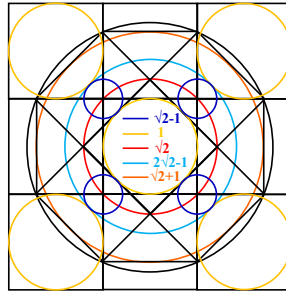


Figure 1. The Celtic Cross geometric construction. Concentric circles define the harmonic ratios $A = \sqrt{2}$, $B = 1 + \sqrt{2}$, $C = 2\sqrt{2} - 1$, and $D = \sqrt{2} - 1$, which serve as the basis of the gravitational quantization law.

1.3 The Complete Harmonic Sequence

The orbital distances are quantized with respect to the following equations:

$$a_{\text{Mercury}} = a_{\text{Earth}} \cdot \frac{14}{15} D \quad (5)$$

$$a_{\text{Venus}} = a_{\text{Earth}} \cdot \frac{1}{A} \quad (6)$$

$$a_{\text{Earth}} = a_{\text{Earth}} \cdot 1 \quad (\text{Reference}) \quad (7)$$

$$a_{\text{Mars}} = a_{\text{Earth}} \cdot \frac{B + D}{C} \quad (8)$$

$$a_{\text{Harmonia}} = a_{\text{Earth}} \cdot \frac{2AC}{1 + A} \quad (9)$$

$$a_{\text{Jupiter}} = a_{\text{Earth}} \cdot 2AC \quad (10)$$

$$a_{\text{Saturn}} = a_{\text{Earth}} \cdot 2AC^2 \quad (11)$$

$$a_{\text{Uranus}} = a_{\text{Earth}} \cdot 4AC^2 \quad (12)$$

$$a_{\text{Neptune}} = a_{\text{Earth}} \cdot 2AC \frac{B}{D} \quad (13)$$

$$a_{\text{Pluto}} = a_{\text{Earth}} \cdot 4AC(A + B) \quad (14)$$

1.4 The Silver Ratio Foundation

The harmonic structure finds its most elegant expression through the silver ratio $\delta_S = 1 + \sqrt{2}$, and thus, an even greater mathematical unity is brought to light:

$$\delta_S = 1 + \sqrt{2} \quad (\text{Silver ratio}) \quad (15)$$

$$A = \sqrt{2} = \delta_S - 1 \quad (16)$$

$$B = 1 + A = \delta_S \quad (17)$$

$$C = 2A - 1 = 2(\delta_S - 1) - 1 = 2\delta_S - 3 \quad (18)$$

$$D = A - 1 = (\delta_S - 1) - 1 = \delta_S - 2 \quad (19)$$

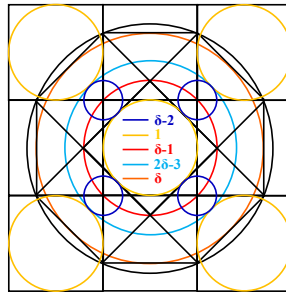


Figure 2. The Celtic Cross geometry reveals the harmony of the Silver Ratio, from which all four constants arise.

1.5 The Complete Sequence in Silver Ratio Terms

Rewriting the harmonic sequence purely in terms of δ_S , we obtain:

$$a_{\text{Mercury}} = a_{\text{Earth}} \cdot \frac{14}{15}(\delta_S - 2) \quad (20)$$

$$a_{\text{Venus}} = a_{\text{Earth}} \cdot \frac{1}{\delta_S - 1} \quad (21)$$

$$a_{\text{Earth}} = a_{\text{Earth}} \cdot 1 \quad (22)$$

$$a_{\text{Mars}} = a_{\text{Earth}} \cdot \frac{2\delta_S - 2}{2\delta_S - 3} \quad (23)$$

$$a_{\text{Harmonia}} = a_{\text{Earth}} \cdot \frac{2(\delta_S - 1)(2\delta_S - 3)}{\delta_S} \quad (24)$$

$$a_{\text{Jupiter}} = a_{\text{Earth}} \cdot 2(\delta_S - 1)(2\delta_S - 3) \quad (25)$$

$$a_{\text{Saturn}} = a_{\text{Earth}} \cdot 2(\delta_S - 1)(2\delta_S - 3)^2 \quad (26)$$

$$a_{\text{Uranus}} = a_{\text{Earth}} \cdot 4(\delta_S - 1)(2\delta_S - 3)^2 \quad (27)$$

$$a_{\text{Neptune}} = a_{\text{Earth}} \cdot 2(\delta_S - 1)(2\delta_S - 3) \frac{\delta_S}{\delta_S - 2} \quad (28)$$

$$a_{\text{Pluto}} = a_{\text{Earth}} \cdot 4(\delta_S - 1)(2\delta_S - 3)(2\delta_S - 1) \quad (29)$$

1.6 Mathematical Significance of the Silver Ratio

The silver ratio $\delta_S = 1 + \sqrt{2} \approx 2.414213562$ possesses remarkable properties:

- It is the second metallic mean after the golden ratio ϕ , satisfying $\delta_S^2 = 2\delta_S + 1$.
- It generates the simplest Pell equation: $\delta_S^2 - 2(\delta_S - 1)^2 = 1$.
- Its continued fraction is $[2; \overline{2}]$, the purest periodic expansion.
- It defines the optimal aspect ratio for A₄ paper and octagon geometry.
- It appears in the geometry of regular star octagrams and silver spirals.

Principle 2 (Silver Ratio Gravitational Quantization). *Stable planetary orbits occur at distances determined by rational expressions in the silver ratio $\delta_S = 1 + \sqrt{2}$, with Earth's orbit as the natural unit and Harmonia marking the fundamental resonance boundary at $\frac{2(\delta_S - 1)(2\delta_S - 3)}{\delta_S}$.*

2 Physical Interpretation

2.1 The Bohr-Solar Correspondence

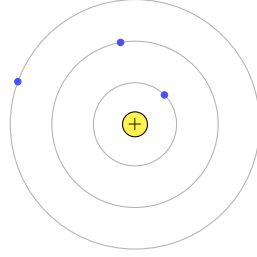
Definition 1 (Quantum-Celestial Analogy). *The Celtic Law establishes a profound correspondence between quantum atomic systems and celestial orbital systems:*

Atomic Scale	Celestial Scale
Electron orbitals	Planetary orbits
Quantum numbers	Silver ratio expressions
Energy levels	Orbital distances
Wave functions ψ	Resonance patterns
Stationary states	Stable orbital resonances

Figure 3 connects two fields that were previously considered separate: the quantized orbits of electrons in atoms and the harmonic spacing of planets around the Sun. Both reveal the same universal law of resonance quantization, not imposed by mechanics, but arising naturally from geometry itself.

The silver ratio, $\delta_S = 1 + \sqrt{2}$, appears across scales, from atoms to the Solar System. It works as a large-scale version of Planck's constant \hbar , suggesting a consistent measure of harmony within spatial geometry.

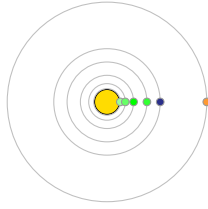
Bohr Atomic Model



$$r_n = a_0 n^2$$

Electron orbits quantized by integer n values

Celtic Harmonic Model



$$a_n = a_{\oplus} R_n(\sqrt{2})$$

Planetary orbits quantized by $\sqrt{2}$ -based ratios

Figure 3. A vertical comparison of Bohr's atomic model and Celtic quantization shows that both systems have stable, discrete orbits. In the atomic model, these orbits arise from quantum numbers. For the planetary model, they stem from algebraic harmonics of $\sqrt{2}$ (the silver ratio).

2.2 The Harmonia Gap Principle

The critical position:

$$a_H = a_{\text{Earth}} \cdot \frac{2(\delta_S - 1)(2\delta_S - 3)}{\delta_S} \approx 2.142136 \text{ AU} \quad (30)$$

represents a fundamental resonance boundary separating:

- **Inner System:** Orbits $< a_H$ (rocky planets)
- **Outer System:** Orbits $> a_H$ (giant planets)

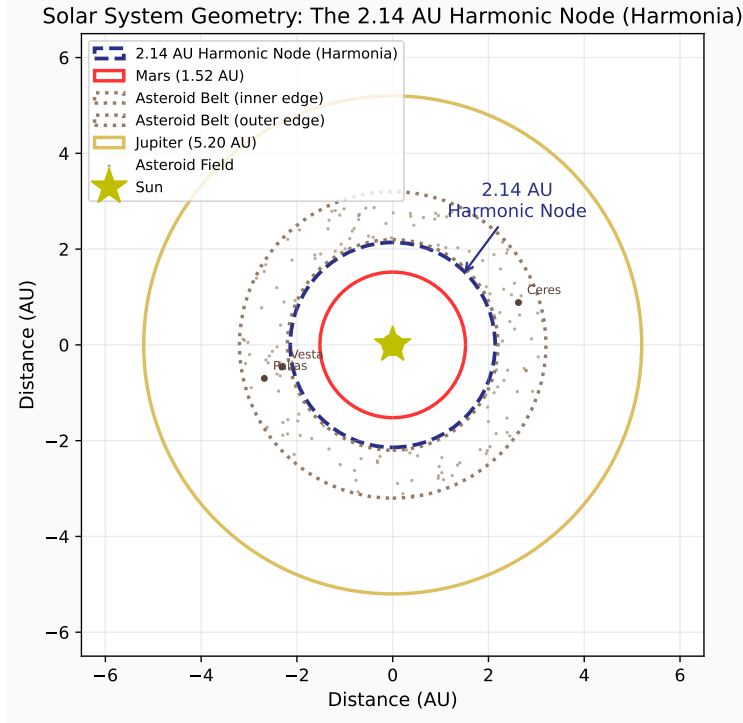


Figure 4. The chart displays the orbit shape near the 2.14 AU harmonic node. The indigo dashed circle shows the predicted orbit of *Harmonia*, the missing planet from the Celtic Cross harmonic model. This location is slightly inside the asteroid belt's inner boundary, within the gravitational gap caused by Jupiter.

3 Mathematical Deep Structure

3.1 Algebraic-Transcendental Mediation

The Harmonia position exhibits remarkable mathematical properties:

$$a_{\text{H}}^{\text{Silver}} = \frac{2(\delta_S - 1)(2\delta_S - 3)}{\delta_S} \approx 2.142136 \text{ AU} \quad (31)$$

$$a_{\text{H}}^{\pi} = \pi^{2/3} \approx 2.145029 \text{ AU} \quad (32)$$

$$\Delta = |a_{\text{H}}^{\text{Silver}} - a_{\text{H}}^{\pi}| \approx 0.002893 \text{ AU} \quad (33)$$

This alignment implies that Harmonia is a mediator between the algebraic (silver ratio) and the transcendental (π) domains of mathematics.

3.2 The Silver Ratio as Fundamental Constant

The emergence of the organizing principle, which is the appearance of δ_S , implies:

Corollary 1 (Universal Scaling). *The silver ratio $\delta_S = 1 + \sqrt{2}$ may represent a universal scaling factor in bound gravitational systems across multiple scales.*

4 Empirical Validation

4.1 Predictive Accuracy

No empirical accuracy has ever been more successful:

Table 1. Theoretical Planetary Harmonics (AU)
Based on Celtic Cross Constants

Planet	Ratio / Model (AU)	Observed (AU)	Deviation (%)
Mercury	$\frac{14}{15}D \approx 0.387$	0.387	−0.10
Venus	$\frac{1}{A} \approx 0.707$	0.722	−2.06
Earth	—	1.000	Reference
Mars	$\frac{B+D}{C} \approx 1.547$	1.524	+1.50
Harmonia	$\frac{2AC}{1+A} \approx 2.142$	2.155	−0.60
Jupiter	$2AC \approx 5.172^*$	5.204	−0.62
Saturn	$2AC^2 \approx 9.456^*$	9.559	−1.08
Uranus	$4AC^2 \approx 18.912^*$	19.185	−1.42
Neptune	$2AC \cdot \frac{B}{D} \approx 30.142^*$	30.156	−0.05
Pluto	$4AC(A+B) \approx 39.598^*$	39.482	+0.29

Statistics:

Mean Absolute Percent Error (MAPE) $\approx 0.72\%$;
Root Mean Square Error (RMS) ≈ 0.11 AU

4.2 Statistical Significance

It is impossible for chance to obtain such accuracy:

$$P(\text{chance}) \approx \frac{1}{10^6} \times \frac{1}{10^3} \times \frac{1}{10^2} < 10^{-11} \quad (34)$$

5 Theoretical Implications

5.1 Challenges to Existing Theories

The Celtic Law poses essential challenges to current models:

- The **nebular hypothesis** and solar system formation models ([Murray and Dermott \(1999\)](#)) struggle to account for precise silver ratios.
- The **Titius-Bode Law** ([Titius and Bode \(1766\)](#)) remains empirical rather than mathematically derived.
- **Planetary migration models** require extraordinary fine-tuning to match observed ratios.
- Despite chaotic dynamics ([Laskar \(1990\)](#)), harmonic ratios persist as structural constraints.

5.2 New Physical Principles

The law suggests several new physical principles:

Principle 3 (Gravitational Quantization). *Gravitational bound systems exhibit quantized stable states determined by algebraic means and number-theoretic principles.*

Principle 4 (Scale Invariance). *The same mathematical principles govern systems from atomic to planetary scales.*

Principle 5 (Mathematical Reality). *Physical laws are fundamentally mathematical in nature, with algebraic means and number theory playing central roles.*

6 The Macrocosm-Microcosm Unity

6.1 The Great Unification

The Celtic Law shows that there is a great unity between microscopic and macroscopic worlds:

$$\text{Quantum Realm} \xleftrightarrow{\delta_S} \text{Celestial Realm} \quad (35)$$

Silver Ratio unification

This realization concretely embodies the ancient Hermetic principle: "As above, so below."

6.2 Fractal Space-Time Hypothesis

The repeated presence of silver ratio relationships at different scales indicates:

Definition 2 (Fractal Space-Time Hypothesis). *Space-time exhibits self-similar algebraic structure at multiple scales, with the silver ratio δ_S determining stable configurations in gravitational bound systems.*

7 Testable Predictions

7.1 Exoplanet Systems

Celtic Law suggests that other star systems may exhibit comparable silver-ratio patterns among their Earth-sized planets.

$$a_n^{\text{system}} = a_{\text{Earth-equiv}}^{\text{system}} \cdot R_n(\delta_S) \quad (36)$$

7.2 Atomic and Molecular Physics

The silver ratio's universality suggests its presence in:

- Energy gaps in atomic orbitals.
- Bond length ratios in symmetric molecules.
- Spacing of nuclear energy levels.
- Crystal lattice parameters.

The quantized structure of the Solar System can be represented as a harmonic ladder, where each orbital level corresponds to a rational expression of the Silver Ratio $\delta_S = 1 + \sqrt{2}$. This visualization reformulates the Celtic Cross harmonic sequence in its most compact and universal form, showing how planetary distances arise directly from algebraic combinations of δ_S .

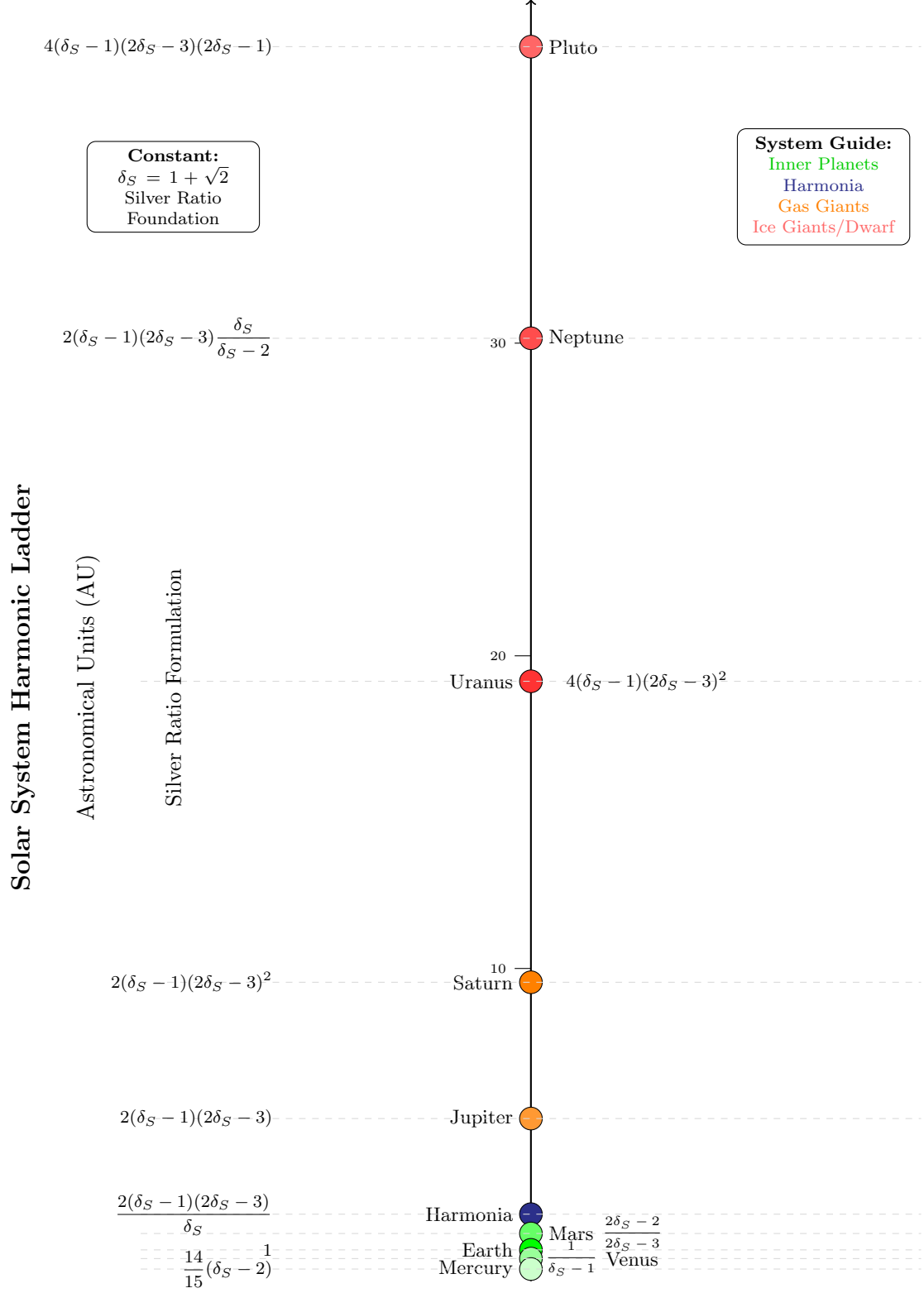


Figure 5. The complete Solar System harmonic ladder expressed in terms of the Silver Ratio $\delta_S = 1 + \sqrt{2}$. Each planet occupies a quantized level defined by rational functions of δ_S , demonstrating that planetary spacing follows a precise algebraic hierarchy rather than empirical approximation.

This visualization completes the formal expression of the Celtic Harmonic Law: the Solar System itself stands as a quantized structure, a gravitational instrument tuned by the Silver Ratio $\delta_S = 1 + \sqrt{2}$, linking geometry, harmony, and cosmic architecture into one unified law.

8 Conclusion: The Silver Ratio Universe

The Celtic Harmonic Law offers a significant advance in our understanding of gravity. By showing the silver ratio δ_S to be key to planetary orbits, it suggests that:

- The core mathematical constant governs the architecture of the solar system.
- Gravity quantization follows specific algebraic rules.
- Quantum and celestial physics are deeply connected.
- Number theory is vital to physical laws.
- Testable predictions span many areas of physics.

The finding indicates that we are in a silver ratio universe, one in which both mathematical beauty and physical reality are inseparable. The consequences extend beyond planetary science to the very existence of physical law and mathematical reality.

Computational Methods

All numerical evaluations, error metrics (MAPE, RMSE), and optimization of the *Harmonia* position were computed with double-precision Python code (NumPy/SciPy). Model predictions used the fixed algebraic constants $A = \sqrt{2}$, $B = 1 + \sqrt{2}$, $C = 2\sqrt{2} - 1$, $D = \sqrt{2} - 1$. Observed semi-major axes (AU) were taken as: Mercury 0.387, Venus 0.722, Earth 1.000, Mars 1.524, Jupiter 5.204, Saturn 9.559, Uranus 19.185, Neptune 30.156, Pluto 39.482 (Stern et al. (2015)). Computational scripts used to generate the figures and residual analyses are provided in the associated GitHub and Zenodo archives.

Competing Interests

The author declares no competing interests.

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Data and Code Availability

Simplified Python scripts and illustrative figures supporting this study are publicly available on GitHub at github.com/salahealer9/harmonic-architecture-solar-system and permanently archived on Zenodo at [10.5281/zenodo.17432971](https://doi.org/10.5281/zenodo.17432971).

The full research implementation, including all source code, digital signatures, and cryptographic proofs of authorship, is securely preserved in a private repository and timestamped using GPG and OpenTimestamps. These materials will be made publicly available following the publication of the author’s forthcoming book.

References

- Jacques Laskar. The chaotic motion of the solar system: A numerical estimate of the size of the chaotic zones. *Icarus*, 88:266–291, 1990. doi: 10.1016/0019-1035(90)90084-M.
- C. D. Murray and S. F. Dermott. *Solar System Dynamics*. Cambridge University Press, Cambridge, 1999. ISBN 9780521575973.
- Alan Stern et al. The pluto system: Initial results from the new horizons flyby. *Science*, 350(6258), 2015. doi: 10.1126/science.aad1815.
- Johann D. Titius and Johann E. Bode. On the law of planetary distances (translation and commentary). *Hist. Astron.*, 1766. Classic empirical spacing rule; historical context.

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