Task 2: Trapezoid Rule for Integration

Concept: The trapezoid rule approximates definite integrals by dividing the area under a curve into trapezoids and summing their areas.

How to Use:

1. **Choose interval $[a, b]$ and number of subintervals n**
2. **Calculate width: $h = \frac{b-a}{n}$**
3. **Find evaluation points: $x_i = a + ih$ for $i = 0, 1, 2, \ldots, n$**
4. **Apply the composite formula:**

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)]$$

Example: Approximate $\int_0^1 e^{x^2}\, dx$ with $n = 4$

- **$h = 0.25$, points: 0, 0.25, 0.5, 0.75, 1**
- **Function values: $f(0) = 1, f(0.25) = 1.0677, f(0.5) = 1.2840, f(0.75) = 1.6644, f(1) = 2.7183$**
- **Result: $\frac{0.25}{2}[1 + 2(1.0677 + 1.2840 + 1.6644) + 2.7183] = 1.4903$**
- **(Actual value: 1.4627, error ≈ 1.9%)**

Task 3: Matrix Normalization and AI Applications

Concept: Matrix normalization scales data to improve computational stability and model performance.

Common Methods:

- **Min-Max Normalization: $X' = \frac{X - \min(X)}{\max(X) - \min(X)}$ (scales to [0,1])**
- **Z-Score Normalization: $X' = \frac{X-\mu}{\sigma}$ (mean=0, std=1)**

AI Applications:

1. **Feature Scaling: Essential for distance-based algorithms (k-NN, SVM) to prevent feature dominance**
    - **Example: Normalizing pixel values [0-255] to [0-1] in image classification**
2. **Batch Normalization: Critical in deep learning networks**
    - **Formula: $\hat{x} = \frac{x-\mu_B}{\sqrt{\sigma_B^2+\epsilon}}$**
    - **Example: CNNs for image recognition, stabilizes training between layers**
    - **Benefits: faster convergence, higher learning rates, reduced vanishing gradients**
3. **Neural Network Training: Improves gradient flow and reduces internal covariate shift**
    - **Example: RNNs for sequence processing, normalizes hidden states**

Resources:

https://nm.mathforcollege.com/NumericalMethodsTextbookUnabridged/chapter-07.02-trapezoidal-rule-of-integration.html

https://testbook.com/maths/trapezoidal-rule

https://patrickwalls.github.io/mathematicalpython/integration/trapezoid-rule/