COSC 6320
Spring 2018
Homework 1

All work submitted should be done by the student alone. If you may use any reference material (books and journal articles), you must provide proper citation. You are not allowed to copy the solutions from any source. Collaboration between students is not allowed. On problems selected from the textbook, use the definitions from the book. Definitions from different books may be slightly different.

For some of the problems, you may want to review "finding rational roots" from high school algebra. Keep in mind, not all roots are rational, but if you can find one, you can reduce the polynomial by one degree which is much easier.

[1] (Homogenous Linear Recurrence Relation, 20 points) Find the solution to the recurrence relation

$$a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$$

for n >= 3, with the initial conditions a_0 = 1; a_1= 2; and a_2 = 3.

[2] (Non-homogenous Linear Recurrence Relation, 20 points) Let's modify the above recurrence relation to non-homogenous.

$$a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3} + 3^n.$$

Solve the recurrence relation with boundary conditions of your own. Choice the initial values (and clearly state them at the beginning of the solution) so that you can find the constants for the solution easier.

[3] (Recurrence Relation, 20 points) For the maximum sum problem, I gave you 4 algorithms with different complexity. This question is dealing with the one with a linear solution. (a) Rewrite the algorithm in a recursive way. (b) Write a recurrence relation to compute the time complexity of the recursive algorithm. (c) Solve the recurrence relation.

[4] (Recurrence with Transformation, 20 points) Solve the following recurrence relation,

$$g(n) = 2g\left(\frac{n}{2}\right) - g\left(\frac{n}{4}\right) + 3,$$

for all 2^i where *i* >= 2, *g(1) = 1*, and *g(2) = 4*.

[5] (Proof by Induction, 20 points) Use the Principle of Induction to prove the following:

$$\sum_{i=0}^{n} x^i = \frac{1 - x^{n+1}}{1 - x},$$

for $x \neq 1$ and integers $n \geq 0$.