***Home Work 1***

***S. M. Salah Uddin Kadir ID: 1800503***

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***[1] Find the solution to the recurrence relation 𝑎­n = 2𝑎n-1 + 5𝑎n-2 − 6an-3 for n >= 3, with the initial conditions 𝑎0 = 1; 𝑎1= 2; and 𝑎2 = 3.***

***Solution***:

The characteristic equation of this recurrence relation is,

r3 – 2r2 – 5r +6 = 0

Or, (r-1) (r+2) (r-3) = 0 // Using rational root test

Hence, the characteristic roots are,

r1 = 1, r2 =-2, and r3 = 3

So, the solution is,

an = A.(1)n + B.(-2)n + C.(3)n ,where A, B, and C are constants.

Therefore,

a0 = A + B + C = 1

a1 = A – 2B + 3C = 2 and

a2 = A + 4B + 9C = 3

Solving these equations, we get,

A = 5/6;

B = - (2/15);

C = 3/10;

So, the final solution is,

an = 5/6 – (2/15).(-2)n + (1/10)3(n+1)

***[2] Let’s modify the above recurrence relation to non-homogenous.***

***𝑎­n = 2𝑎n-1 + 5𝑎n-2 − 6an-3 + 3n***

***Solve the recurrence relation with boundary conditions of your own. Choice the initial values (and clearly state them at the beginning of the solution) so that you can find the constants for the solution easier.***

***Solution***:

This is a non-linear homogeneous equation where its associated homogeneous equation,

an = 2a­n-1 + 5an-2 -6an-3 ,and

f(n)= 3n

Let, the initial conditions are a0 = 1, a1 = 47/10, and a2 = 96/5;

Now, the characteristic equation of its associated homogeneous relation is,

r3 – 2r2 – 5r +6 = 0

So, (r-1) (r+2) (r-3) = 0

So, the characteristic roots are,

r1 = 1, r2 =-2, and r3 = 3

Hence,

ah = A.(1)n + B.(-2)n + C.(3)n , where A, B, and C are constants

Since, f(n) = 3n and r3 = 3

So, at = Dn3n , where D is constant

After putting the solution in the recurrence relation,

We get,

Dn3n = 2D(n-1)3(n-1) + 5D(n-2)3(n-2) – 6D(n-3)3(n-3) + 3n

Or, Dn33 = 2D(n-1)32 + 5D(n-2).3 – 6D(n-3) + 33

Or, 27Dn = 18Dn – 18D + 15Dn – 30D - 6Dn + 18D + 27

Or, D = 27/30

Or, D = 9/10

So,

at = (9/10)n3n = (n/10)3(n+2)

Hence, the solution for the recurrence relation can be written as,

an = A + B.(-2)n + C.(3)n + (n/10)3(n+2)

Using initial conditions,

a0 = A + B + C = 1

a1 = A -2B + 3C + 27/10 = 47/10

Or, a1 = A – 2B + 3C = 2

a2 = A + 4B + 9C + 81/5 = 96/5

Or, a2 = A + 4B + 9C = 3

Solving these equations, we get,

A = 5/6;

B = - (2/15);

C = 3/10;

So, the final solution is,

an = 5/6 – (2/15).(-2)­n+(1/10).3(n + 1) + (n/10).3(n+2)

***[3] For the maximum sum problem, I gave you 4 algorithms with different complexity. This question is dealing with the one with a linear solution. (a) Rewrite the algorithm in a recursive way. (b) Write a recurrence relation to compute the time complexity of the recursive algorithm. (c) Solve the recurrence relation.***

***Solution:***

(a)

MaxSum(X, index=0, max\_here=0, max\_sum=0)

{

If(index == arr\_size)

return max\_sum;

max\_here = max(0, max\_here + X[index]);

max\_sum = max(max\_sum, max\_here);

return MaxSum(X, index+1, max\_here, max\_sum);

}

(b)

T(n) = 1 + T(n-1) ,where n>1, T(1) = 1

(c)

We have,

T(n) = 1 + T(n-1)

= 1 + 1 + T(n-2)

= 2 + 1 + T(n-3)

…………

= k + T(n-k)

………….

= n – 1 + T(1)

= n - 1 + 1

= n

So, T(n) = Θ(n)

***[4] Solve the following recurrence relation,***

***g(n) = 2g() – g() + 3***

***for all 2i where i >= 2, g(1) = 1, and g(2) = 4.***

***Solution***:

Let, n = 2k ,and g(n) = ak= g(2k)

Now, we can write the recurrence as,

ak = 2ak-1 – ak-2 + 3, for k >= 2

a1 = 1 ,and

a2 = 4

Using operator E, we may write the equation,

(E-1)2<ak> = <3>

The annihilator for the right-hand side is (E-1), so we obtain

(E-1)3<ak> = <0>

Thus the characteristic root is 1 with a multiplicity of 3, so

ak= c1 + c2k + c3k2

We have,

a2 – 2a1 +a0 = 3

so, a0 = 1

Solving for the constants, we obtain

c1 = 1

c2 = -

c3 =

So,

ak = 1 – (3/2)k + (3/2)k2

The solution is,

g(n) = 1 – (3/2)lgn + (3/2)(lgn)2

***[5] Use the Principle of Induction to prove the following,***

***=***

***for x ≠ 1 and integers n ≥ 0.***

***Solution:***

Basis of Induction, n=0

*=*

x0 == 1

Inductive Hypothesis, n = k

*=*

Inductive Step, n = k + 1

*=*

Right side of the inductive step,

Left side of the inductive step,

=

= +

=

=

Which is identical to the right side.