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(1) Consider,

* Rotation θ about z axis.

$$R_z(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* Rotation ϕ about y axis

$$R_y(\phi) = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

* Rotation ψ about x-axis.

$$R_x(\psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix}$$

(2) (a) Give $p(\hat{x}, \hat{y})$ in the image normalized Co-ordinate System.

Generally $\hat{x} = \dfrac{X}{Z}$ $\hat{y} = \dfrac{Y}{Z}$

for all values of Z 3D-line will be

$$(Z\hat{x},\ Z\hat{y},\ Z)$$

2 (b) Given a line l (a, b, c) then we represent as $a\hat{x} + b\hat{y} + c = 0$ in Normalized Co-ordinate System.

$\forall$ all values of Z and some constant d.

$$X = Z\hat{x}, \quad Y = Z\hat{y} \quad \text{---} (1)$$

the plane eqn in general form is $ax + by + cZ + d = 0$, --- (2)

from (1) & (2)

$$aZ\hat{x} + bZ\hat{y} + cZ + d = 0$$

as $Z = 1$ $\quad a\hat{x} + b\hat{y} + c + d = 0$

3 (a) Given $f = 50mm$

image surface 1000×1000 Pixels.

Pixel length $0.05mm$ in each dimension.

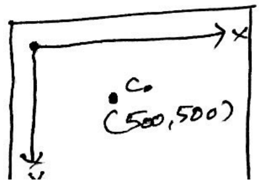
$\frac{1}{k} = 0.05 \qquad \frac{1}{l} = 0.05$

$k = 20$ pixels mm^{-1} $\qquad l = 20$ pixels mm^{-1}

$\alpha = kf = 1000$

$\beta = kf = 1000$

for $c_0 (x_0, y_0)$ assume positive x axis and positive y-axis and Given $90°$ in clock wise direction.



$x_0 = 500$

$y_0 = 500$

$\theta = -90°$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & x_0 \\ 0 & -\beta/\sin\theta & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 1000 & -1000\cot(-90^\circ) & 500 \\ 0 & \frac{+1000}{\sin(-90^\circ)} & 500 \\ 0 & 0 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} 1000 & 0 & 500 \\ 0 & \frac{+1000}{(-1)} & 500 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1000 & 0 & 500 \\ 0 & -1000 & 500 \\ 0 & 0 & 1 \end{bmatrix}$$

3(b) Given.

$O_c = (5,4,2)$ in world co-ordinate system.

rotated 30° about x-axis in clockwise direction.

$\theta = -30^\circ$

$t = (5,4,2)$

$$R_x(-30^\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos -30^\circ & \sin -30^\circ & 0 \\ 0 & -\sin -30^\circ & \cos -30^\circ & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \end{bmatrix}$$

Now,

$$M = \begin{bmatrix} \alpha r_1^T - \alpha\cot\theta\, r_2^T + x_0 r_3^T & \alpha t_1 - \alpha\cot\theta\, t_2 + x_0 t_3 \\ \beta/\sin\theta\, r_2^T + y_0 r_3^T & \beta/\sin\theta\, t_2 + y_0 t_3 \\ r_3^T & t_3 \end{bmatrix}$$

$$\alpha r_1^T - \alpha \cot\theta\, r_2^T + x_0 r_3^T$$

$$= 1000 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T - 1000\ (0) \begin{bmatrix} 0 \\ \sqrt{3}/2 \\ -1/2 \end{bmatrix} + 500 \begin{bmatrix} 0 \\ 1/2 \\ \sqrt{3}/2 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 \\ 500 \times \sin 30 \\ 500 \times \cos 30 \end{bmatrix}^T$$

$$\beta / \sin\theta \quad r_2^T + y_0 r_3^T$$

$$= -1000 \begin{bmatrix} 0 \\ \sqrt{3}/2 \\ -1/2 \end{bmatrix}^T + 500 \begin{bmatrix} 0 \\ 1/2 \\ \sqrt{3}/2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1000 \cos 30 + 500 \sin 30^\circ \\ 1000 \sin 30^\circ + 500 \cos 30^\circ \end{bmatrix}^T$$

$$\alpha_1 \cancel{t_1 + x_0 t_3} = \overset{10}{\cancel{(1000)(5) + (500)(2)}} = \cancel{6000}$$

$$\cancel{-\beta t_2 + y_0 t_3 = -(1000)(4) + (500)(2)} = \cancel{-3000}$$

$$\cancel{t_3} = 2$$

Converting $(5, 4, 2)$ into pixel Co-ordinates.

$0.05\,mm = 1$ Pixel.

$5000\ mm = ?$

$$= \frac{5000}{0.05} = 1 \times 10^5$$

Similarly.

$\cancel{4} \longrightarrow 8\times10^4$ pixels.

$2 \rightarrow 4\times10^4$ pixels.

now,

$\alpha t_1 - \alpha\cot\theta\, t_2 + x_0 t_3$

$= 10^8 + 2000\times10^4$

$= 10^8 + 2\times10^7$

$= 12\times10^7$

$\beta/\sin\theta + y_0 t_3$

$= -8\times10^7 + 2\times10^7$

$= -6\times10^7$.

$t_3 = 4\times10^4$

$$M = \begin{bmatrix} 1000 & 250 & 250\sqrt{3} & 12\times10^7 \\ 0 & 250-500\sqrt{3} & 500+250\sqrt{3} & -6\times10^7 \\ 0 & 0.5 & \frac{\sqrt{3}}{2} & 4\times10^4 \end{bmatrix}$$

③ⓑ
② Let the 3D-line be.

$$P = P_0 + tD.$$

but when $t \to \infty$

$1/t \to 0$

$$= \frac{1}{t} P_0 + D$$

$$= D.$$

$$= \begin{bmatrix} D_x \\ D_y \\ D_z \\ 0 \end{bmatrix}$$

As the lines are vertical in world and parallel to Y-axis $= \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$$P = \frac{1}{Z} MP$$

$$x = \frac{m_1 . P}{m_3 . P}$$

$$Y = \frac{m_2 . P}{m_3 P}.$$

$$= M P$$

$$= \begin{bmatrix} 250 \\ 250 - 500\sqrt{3} \\ 1/2 \end{bmatrix}$$

$$x = \frac{X}{Z}$$

$$= \frac{250}{1/2} = \cancel{(250 \times 2)}(250\times 2) = 500$$

$$y = \frac{Y}{Z} = \frac{250 - 500\sqrt{3}}{1/2} = \cancel{125 + 500\sqrt{3}}.$$

$$500 - 1000\sqrt{3}.$$

$$= \begin{bmatrix} 500 \\ 500 - 1000\sqrt{3} \\ 1 \end{bmatrix}$$

$$v' = (500,\ 500 - 1000\sqrt{3})$$

3 b

3. set of $\parallel^{el}$ lines in horizontal plane.

$$\text{the } D = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$P = MP.$$

$$= \begin{bmatrix} 1000 + 250\sqrt{3}. \\ 500 + 250\sqrt{3}. \\ \sqrt{3}/2 \end{bmatrix}$$

$$P = \frac{P}{Z} = \begin{bmatrix} \frac{2000}{\sqrt{3}} + 500 \\ \frac{1000}{\sqrt{3}} + 500 \\ 1 \end{bmatrix}$$

$$v' = \left(\frac{2000}{\sqrt{3}} + 500,\ \frac{1000}{\sqrt{3}} + 500\right)$$

(3) (b)

(4) we have two points.

$v_1' = (500,\ 500 - 1000\sqrt{3})$, $v_2' = \left(\frac{2000}{\sqrt{3}} + 500,\ \frac{1000}{\sqrt{3}} + 500\right)$

now, line eqn = $y = mx\,c$.

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= \cancel{\frac{2000}{\sqrt{3}}} \quad \frac{\frac{1000}{\sqrt{3}} + \cancel{500} - \cancel{500} + 1000\sqrt{3}}{\frac{2000}{\sqrt{3}} + \cancel{500} - \cancel{500}}$$

$$m = \frac{4000}{2000} = 2$$

$$y = 2x + c.$$

now to get c value substitute v_2'.

$$\frac{1000}{\sqrt{3}} + 500 = \frac{4000}{\sqrt{3}} + 1000 + c.$$

$$c = -\frac{3000}{\sqrt{3}} - 500$$

$$c = -1000\sqrt{3} - 500$$

$$\boxed{y = 2x - 1000\sqrt{3} - 500}$$

Now S.T vanishing points of horizontal lines lie on line.

Now, consider horizontal lines in a direction

$$D = \begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

Now,

$$p = MP$$

$$= \begin{bmatrix} 4000 + 1000\sqrt{3} \\ 0 \\ 2000 + 1000\sqrt{3} \\ 2\sqrt{3} \end{bmatrix}$$

$$x = \frac{2000}{\sqrt{3}} + 500$$

$$y = \frac{1000}{\sqrt{3}} + 500.$$

Now line Eqn. $y = 2x - 1000\sqrt{3} - 500.$

$$y = \frac{4000}{\sqrt{3}} + 1000 - 1000\sqrt{3} - 500.$$

$$= \frac{1000}{\sqrt{3}} + 500$$

$$= y$$

$\therefore$ hence Proved.

Scanned with CamScanner