Matlab Analytical (set 2)

r=0

r=2

a) Enclosed Volume

$\phi = 45° = \frac{\pi}{4}$ $\quad V = \int dv$

$\phi = 90° = \frac{\pi}{2}$ $\quad = \int_{\pi/4}^{\pi/2}\int_{\pi/4}^{\pi/2}\int_{0}^{2} r^2 \sin\theta \, dr \, d\theta \, d\phi$

$\theta = 45° = \frac{\pi}{4}$

$\theta = 90° = \frac{\pi}{2}$ $\quad = \int_{\pi/4}^{\pi/2}\int_{\pi/4}^{\pi/2} \sin\theta \left.\frac{r^3}{3}\right|_0^2 d\theta d\phi$

$$= \int_{\pi/4}^{\pi/2}\int_{\pi/4}^{\pi/2} \sin\theta \left(\frac{8}{3}\right) d\theta d\phi$$

$$= \int_{\pi/4}^{\pi/2} \frac{8}{3} \int_{\pi/4}^{\pi/2} \sin\theta \, d\theta \, d\phi$$

$$= \int_{\pi/4}^{\pi/2} \frac{8}{3} \left. -\cos\theta \right|_{\pi/4}^{\pi/2} d\phi$$

$$= \int_{\pi/4}^{\pi/2} \frac{8}{3} \left(-\cos\left(\frac{\pi}{2}\right) - \left(-\cos\left(\frac{\pi}{4}\right)\right)\right)$$

$$= \int_{\pi/4}^{\pi/2} \frac{8}{3} \left(0 - \left(-\frac{\sqrt{2}}{2}\right)\right) d\phi$$

$$= \int_{\pi/4}^{\pi/2} \frac{8\sqrt{2}}{6} d\phi$$

$$= \frac{8\sqrt{2}}{6}\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$= \frac{8\sqrt{2}}{6}\left(\frac{\pi}{4}\right)$$

$$= \frac{8\sqrt{2}\pi}{24}$$

$$V = \frac{\sqrt{2}\pi}{3} \approx 1.48$$

b) Area of the closed surface S

$$S_{Total} = S_{r_1} + S_{r_2} + S_{\theta_1} + S_{\theta_2} + S_{\phi_1} + S_{\phi_2}$$

$$S_{r_1} = \int_{\pi/4}^{\pi/2}\int_{\pi/4}^{\pi/2} r^2 \sin\theta \, d\theta \, d\phi \quad \Big| r=0$$

$$= 0 \cdot \int_{\pi/4}^{\pi/2}\int_{\pi/4}^{\pi/2} \sin\theta \, d\theta \, d\phi$$

$$S_{r_1} = 0$$

$$S_{r_2} = \int_{\pi/4}^{\pi/2}\int_{\pi/4}^{\pi/2} r^2 \sin\theta \, d\theta \, d\phi \quad \Big| r=2$$

$$= 4\int_{\pi/4}^{\pi/2}\int_{\pi/4}^{\pi/2} \sin\theta \, d\theta \, d\phi$$

$$= 4\int_{\pi/4}^{\pi/2} -\cos\theta \Big]_{\pi/4}^{\pi/2} d\phi$$

$$= 4\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$= 4\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\pi}{4}\right)$$

$$S_{r_2} = \frac{\sqrt{2}\pi}{2}$$

$$S_{\phi_1} = \int_{\pi/4}^{\pi/2}\int_{0}^{2} r \, d\theta \, dr \quad \phi = \frac{\pi}{2}$$

$$= \int_{\pi/4}^{\pi/2} r\Big]_0^2 d\theta$$

$$= 2 \cdot \theta \Big]_{\pi/4}^{\pi/2}$$

$$= 2 \cdot \left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$= 2 \cdot \frac{\pi}{4}$$

$$S_{\phi_1} = \frac{\pi}{2}$$

$$S_{\theta_1} = \int_{\pi/4}^{\pi/2}\int_{0}^{2} r\sin\theta \, d\phi \, dr \quad \Big| \theta = \pi/2$$

$$= \sin\left(\frac{\pi}{2}\right) \int_{\pi/4}^{\pi/2}\int_{0}^{2} r \, d\theta \, dr$$

$$= 1 \cdot \frac{\pi}{2}\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$= 2 \cdot \frac{\pi}{4}$$

$$S_{\theta_1} = \frac{\pi}{2}$$

$$S_{\phi_2} = \int_{\pi/4}^{\pi/2}\int_{0}^{2} r \, d\theta \, dr \quad \phi = \frac{\pi}{4}$$

$$S_{\phi_2} = S_{\phi_1} = \frac{\pi}{2}$$

$$S_{\theta_2} = \int_{\pi/4}^{\pi/2}\int_{0}^{2} r\sin\theta \, d\phi \, dr \quad \Big| \theta = \frac{\pi}{4}$$

$$= \sin\left(\frac{\pi}{4}\right) \int_{\pi/4}^{\pi/2} \frac{r^2}{2}\Big|_0^2 d\phi$$

$$= \frac{\sqrt{2}}{2} \cdot 2 \cdot \phi \Big]_{\pi/4}^{\pi/2}$$

$$= \sqrt{2} \cdot \left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$= \sqrt{2} \cdot \frac{\pi}{4}$$

$$S_{\theta_2} = \frac{\sqrt{2}\pi}{4}$$

$$\longrightarrow S_{Total} = S_{r_1} + S_{r_2} + S_{\theta_1} + S_{\theta_2} + S_{\phi_1} + S_{\phi_2}$$

$$= 0 + \frac{\sqrt{2}\pi}{2} + \frac{\pi}{2} + \frac{\sqrt{2}\pi}{4} + \frac{\pi}{2} + \frac{\pi}{2}$$

$$= \frac{\sqrt{2}\pi}{2} + \frac{\sqrt{2}\pi}{4} + \frac{3\pi}{2}$$

$$S_{Total} = \frac{3\pi(2+\sqrt{2})}{4} \approx 8.045$$

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