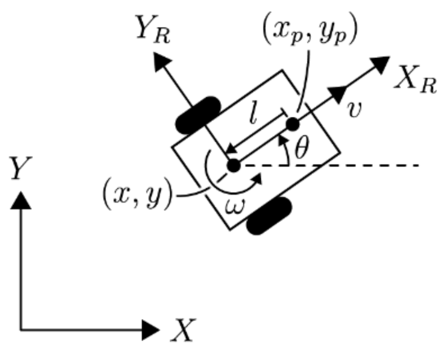
Suppose you are tasked with devising a robotic system that can paint patterns on athletic fields and roadways. The chosen platform is a differentially driven robot with a paint sprayer mounted a distance of l from the center of rotation along the X_R axis as shown below in Figure 1:



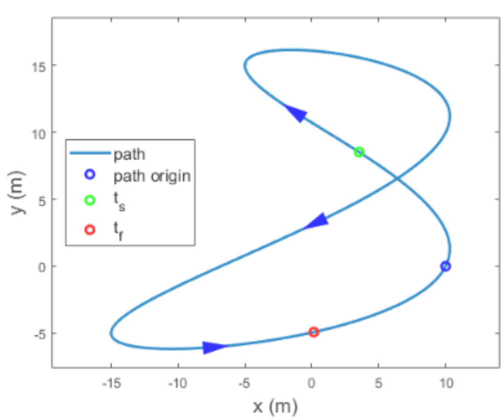
Considering v and ω as the control inputs to the system, the nonlinear state space equations are given by:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad y = \begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} x + l\cos(\theta) \\ y + l\sin(\theta) \end{bmatrix}$$

Where (x, y, θ) is the position of the robot and (x_p, y_p) is the position of the paint sprayer.

Controller Design

Given the problem statement from the previous unit reproduced above, implement a control law to drive the robot such that the location of the paint sprayer follows the trajectory given below:



described by the following set of parametric equations:

$$x(t) = 10\cos\left(\frac{\pi t}{5}\right) + 5\sin\left(\frac{\pi t}{10}\right)$$

$$y(t) = 10\sin\left(\frac{\pi t}{10}\right) - 5\cos\left(\frac{\pi t}{10}\right) + 5$$

The painting mechanism is programmed to begin spraying at $t_s = 2.5\text{s}$ and stop at $t_f = 18\text{s}$. During that period, the position of the sprayer, (x_p, y_p), must remain within 0.15m of the desired path.

Use input-output linearization and an appropriate control law to achive the desired performance. The end result is a function:

```
function x_dot = controller(t, state_vec)
    % your code here
end
```

that will be passed to ode45 to simulated the system:

```
[t_out, robot_path] = ode45(@controller, [0 18], [0 0 0]');
```

with the following initial conditions: $[x\ \ y\ \ \theta]^T = [0\ \ 0\ \ 0]^T$, $t_0 = 0$, $t_f = 18$, and $l = 0.1\mathrm{m}$.

The submission will be sent by replying to this email in the same thread. You need to submit: -

1- The source code of your solution.
2- A pdf report of one page of your result.
3- Your CV.

The deadline for the task is 26th of September 2022 at 11:59 PM.