Computer Vision, Assignment 3

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2 The Fundamental Matrix

Exercise 1

Given that we gave the calibrated 2-camera system $P1 = [I \mid \mathbf{0}]$ and $P2 = [KR \mid Kt]$ we can calculate the fundamental matrix F from the following equation:

$$F = [P2 \cdot \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}]_x \cdot P2 \cdot P1^{-1} \tag{1}$$

where $[A]_x$ denotes the cross product matrix of a 3D vector. The fundamental matrix is $F = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{bmatrix}$.

$$e_1 = F[x, 1]^{-1} = F \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} \tag{2}$$

Correct points: $(2, 0)$ and $(2, 1)$.

Exercise 2

To compute the epipoles, we have to compute the camera centers in world coordinates. For $P1$, $C_1 = [0, 0, 0, 1]^T$. For the second camera $P2$ we set:

$$P2 = [p1\ p2\ p3\ p4] \rightarrow M = [p1\ p2\ p3] \tag{3}$$

After checking that the M matrix is non-singular, we compute C_2 as such:

$$C_2 = [-M^{-1} \cdot p4 \quad 1]^T = [-1\ \ -1\ 0\ 1]^T \tag{4}$$

Thus, the epipoles are:

$$e_1 = P1\ C_2 = [-1\ \ -1\ 0] \tag{5}$$
$$e_2 = P2\ C_1 = [2\ 2\ 0] \tag{6}$$

The computation of the fundamental matrix is exactly as the previews exercise, which gives $F = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{bmatrix}$

We confirm $e_2^T F = 0$ and $F e_1 = 0$

(OPTIONAL)
Since our camera is has the form of $P2 = [A\ t]$, the according to (3) we have

$$A = M;\ p4 = t \tag{7}$$

By making use of the fact that A is invertible, we know that we don't have a camera at infinity, and consequently we have the following camera centers:

$$C_1 = [0\ 0\ 0\ 1]^T \tag{8}$$
$$C_2 = [-A^{-1}t]^T \tag{9}$$

Thus the epipoles will be:

$$e_1 = P1 * C_2 = [I\ 0] * [-A^{-1}\ t\ 0]^T \tag{10}$$
$$e_2 = P2 * C_1 = [A\ t] * [0\ 0\ 0\ 1]^T \tag{11}$$

Our fundamental matrix is:

$$F = [t]_x A \tag{12}$$

So we have:

$$e_2^T\ F = [0\ 0\ 0]\ F = 0 \tag{13}$$

$$F\ e_1 = [t]_x A\ [I\ 0] * [-A^{-1}\ t]^T = [t]_x\ A\ (-A^{-1}t)) \Leftrightarrow \tag{14}$$
$$F\ e_1 = -[t]_x\ t \tag{15}$$

So we need to show that $[t]_x\ t = 0$ Supposing $t = [t_1\ t_2\ t_2]^t$. Then we have :

$$[t]_x\ t = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} [t_1\ t_2\ t_2]^t = [0\ 0\ 0]^T \tag{16}$$

which finished the proof.
The fundamental matrix F must be of rank 2. The rank of a matrix shows the maximum of independent rows or columns. By having a determinant of zero, we reduce the independencies of the matrix so that there are 7 degrees of freedom.

Exercise 3

After solving the following system:

$$\widetilde{x_2}^T \widetilde{F} \widetilde{x_1} = x_2^T F x_1 \tag{17}$$

we have $F = N_2^T \widetilde{F} N_1$

Computer Exercise 1

The exercise's results/plots are produced by running the `ce_1.m` file, changing only the normalization according to the first or the second task. Bare in mind that if the points and the lines are plotted in the first figure, then the files must be ran section by section.
The mean distance for the normalized case $E_{norm} = 0.3612$ and for the non normalized case is $E_{un} = 0.4878$, thus we have a small numerical improvement by normalizing the points.

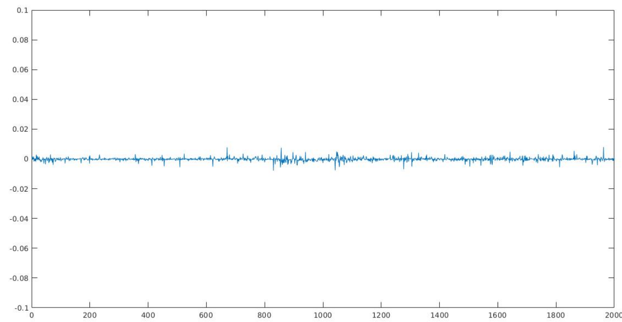

Figure 1: Deviation of epipolar constraint for all the points

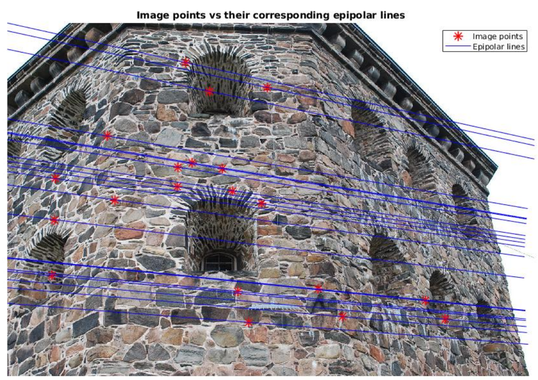

Figure 2: Image with 20 randomly sampled points and their corresponding epipolar lines

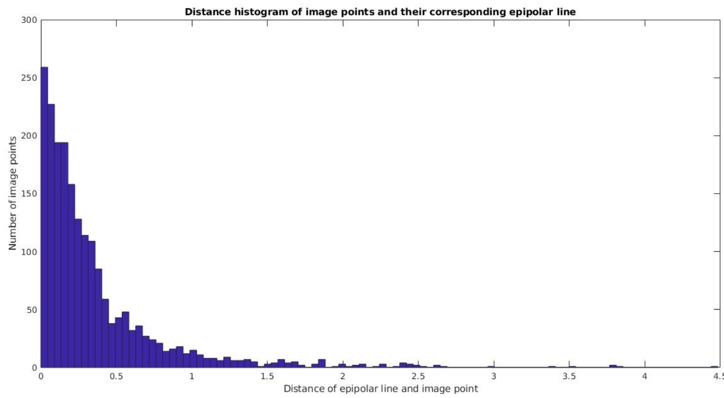

Figure 3: Distance histogram of epipolar lines and image points (left skewed)

Exercise 4

For both points we add a fourth dimension at 1, so that they are homogeneous. To find the epipolar line e_2, since we have the fundamental matrix, we can calculate it by finding the nullspace of the transpose of F, that is:

$$e_2 = null(F^T) = [0.7071 \; 0 \; -07071]^T \tag{18}$$

We calulate the $P2$ matrix as it is defined and we get $P2 = \begin{bmatrix} 0.7071 & 0 & 0 & 0.7071 \\ 0 & -1.4142 & -1.4142 & 0 \\ 0.7071 & 0 & 0 & -0.7071 \end{bmatrix}$

Using the second camera matrix we project the 2 points by $x^i_{pr2} = P2 * x^i$. Thus we have $x^1_{pr2} = [1.4142 \; -7.0711 \; 0]$ and $x^1_{pr2} = [2.8284 \; -4.2426 \; 1.4142]$, for both of which the constraint holds true. We find the camera center of $P2$ as in exercise 2. We see that $det(M_2) = 0$, which means that the center is at infinity with $C_2 = [null(M_2) \; 0]^T = [0 \; -0.7071 \; 0.7071 \; 0]^T$

Computer Exercise 2

The projected points and the image points are shown below in the first plot, and the 3d reconstruction at the second plot. The non normalized camera matrices are :

$$P1 = [I \mid 0] \text{ and } P_2 = \begin{bmatrix} -0.0016 & 0.0057 & 0.2163 & 0.9763 \\ 0.0070 & -0.0257 & -0.9763 & 0.2163 \\ 0.0000 & 0.0000 & -0.0273 & 0.0001 \end{bmatrix}$$

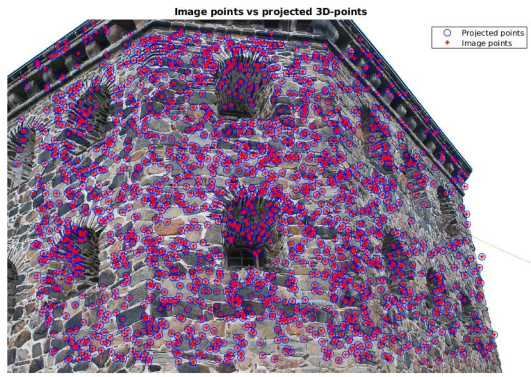

Figure 4: Projected vs image points

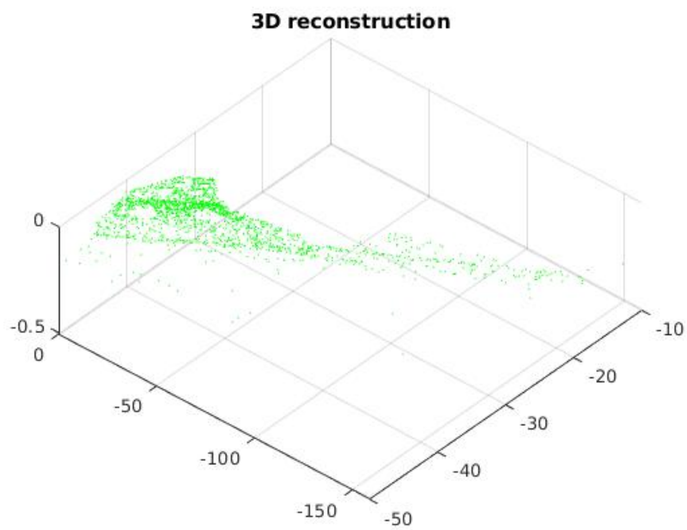

Figure 5: 3D-Reconstruction

3 Essential Matrix

Exercise 5 (OPTIONAL)

$$[t]_x^T[t]_x = (USV^T)^T USV^T = VS^TU^TUSV^T = VS^TSV^T \Leftrightarrow (V \ is \ orthonormal) \tag{19}$$
$$[t]_x^T[t]_x = VS^2V^T \tag{20}$$

Thus S diagonalizes $[t]_x^T[t]_x$.
w can be an eigenvector. So the following must hold true:

$$[t]_x^T[t]_x w = \lambda w \tag{21}$$

So we must show that $[t]_x^T[t]_x = -t \times (t \times w)$
Computer Exercise 3
The necessary plots for this exercise are show below. the Essential matrix is:

$$E = \begin{bmatrix} -8.8885 & -1005.8067 & 377.07825 \\ 1252.5231 & 78.3677 & -2448.1743 \\ -472.78884 & 2550.1917 & 1 \end{bmatrix}$$

and the mean distance in $\overline{d} = 2.0838$

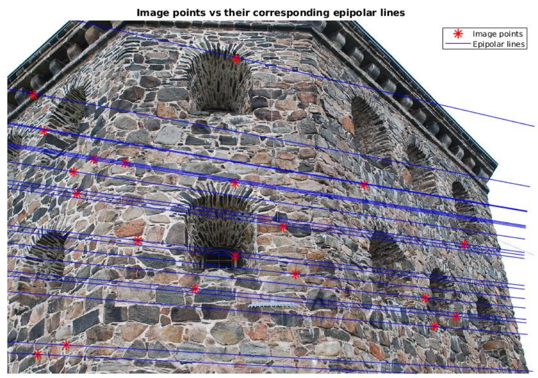

Figure 6: Image with 20 randomly sampled points and their corresponding epipolar lines

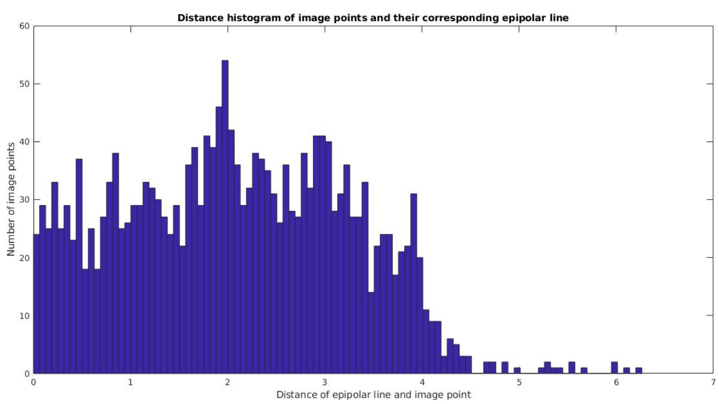

Figure 7: Distance histogram of epipolar lines and image points (left skewed)

Exercise 6 (Solved in class)

Computer Exercise 4

The plots for this exercise are produced by `ce_4.m` and are presented below. There is the same issue as Computer Exercise 1, so maybe the file should be ran in sections. After plotting the cameras and the points in 3D, the best solution was chosen (camera 2) and was hardcoded for the comparison between the projection and the image points.

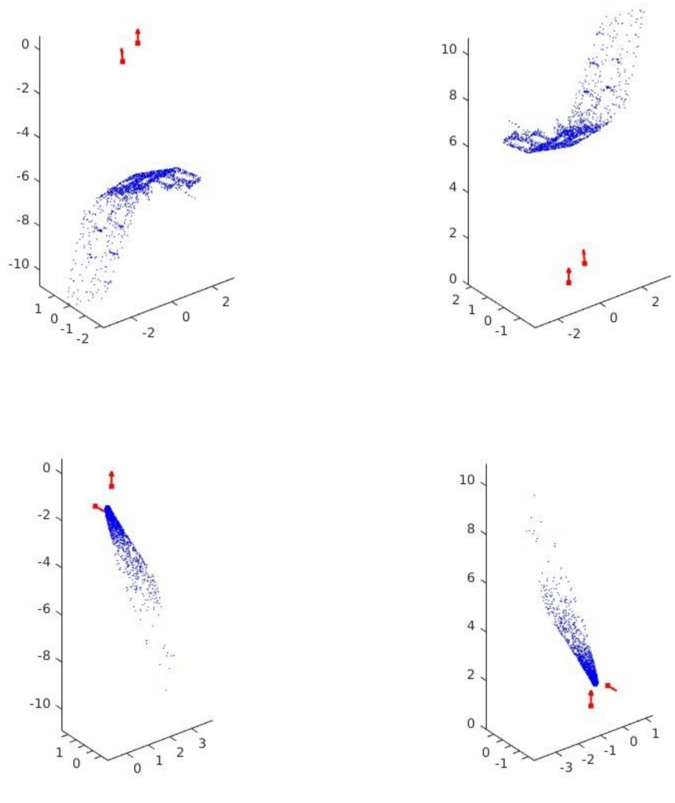

Figure 8: Points in 3D with plotted cameras. The best one is the second (top right))

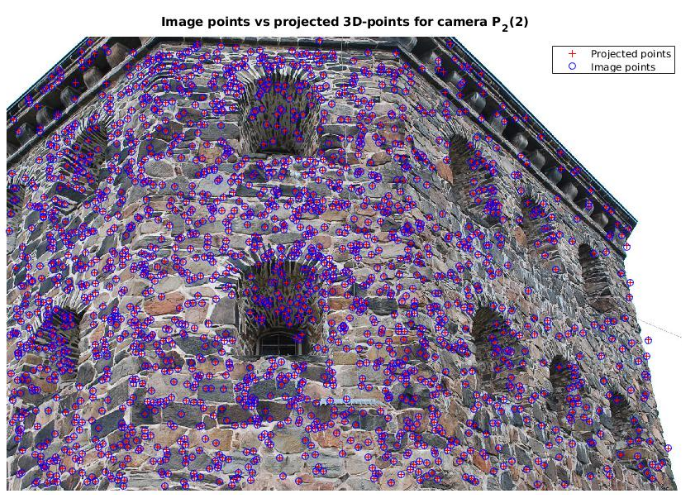

Figure 9: Projected vs image points