Trigonometrične formule

Adicijske formule

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha \cdot \operatorname{tg}\beta}$$

Primer:

Izračunaj vrednost $\sin 15^\circ$.

$$\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ = = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Formule za dvojni kot

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$$

Primer:

Izračunaj vrednost $\cos 120^\circ$.

$$\cos 120^\circ = \cos(2 \cdot 60^\circ) = \cos^2(60^\circ) - \sin^2(60^\circ) = \left[\frac{1}{2}\right]^2 - \left[\frac{\sqrt{3}}{2}\right]^2 = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

Formule za polovični kot

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$
$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$
$$\operatorname{tg}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

Primer:

Z uporabo formule za polovični kot, preveri, da $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

$$\cos 30^\circ = \cos\frac{60^\circ}{2} = +\sqrt{\frac{1+\cos 60^\circ}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

Parametrične formule

$$t = \operatorname{tg}\frac{\alpha}{2}, \quad \alpha \neq \pi + 2k\pi,\ k \in Z \quad \Rightarrow \quad \sin\alpha = \frac{2t}{1+t^2}, \quad \cos\alpha = \frac{1-t^2}{1+t^2}, \quad \operatorname{tg}\alpha = \frac{2t}{1-t^2}$$

Primer:

Z uporabo parametrične formule, preveri, da $\cos 60^\circ = \frac{1}{2}$.

$$\cos 60^\circ = \frac{1-t^2}{1+t^2} \overset{(*)}{=} \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2}$$

$$(*)\quad t = \operatorname{tg}\frac{60^\circ}{2} = \frac{1}{\sqrt{3}}$$

Formule prostafereze (»cento rose molto meno belle«)

$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\sin p - \sin q = 2\cos\frac{p+q}{2}\sin\frac{p-q}{2}$$
$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

Primer:

Izračunaj vrednost $\cos 150^\circ - \sin 60^\circ$.

$$\cos 150^\circ - \sin 60^\circ = \cos 150^\circ - \sin(90^\circ - 30^\circ)$$
$$= \cos 150^\circ - \cos 30^\circ = -2\sin\frac{150^\circ + 30^\circ}{2} \cdot \sin\frac{150^\circ - 30^\circ}{2}$$
$$= -2\sin 90^\circ \cdot \sin 60^\circ = -2 \cdot 1 \cdot \frac{\sqrt{3}}{2} = -\sqrt{3}$$

Wernerjeve formule

$$\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

$$\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$

$$\sin\alpha\sin\beta=-\frac{1}{2}\left[\cos(\alpha+\beta)-\cos(\alpha-\beta)\right]$$

Primer:

Dokaži enakost: $\sin 2x\cdot\sin 3x+\cos 4x\cdot\cos x=\cos 2x\cdot\cos x$

$$\underline{\sin 2x\cdot\sin 3x}+\underline{\underline{\cos 4x\cdot\cos x}}=-\frac{1}{2}\left[\cos(2x+3x)-\cos(2x-3x)\right]+\frac{1}{2}\left[\cos(4x+x)+\cos(4x-x)\right]$$

$$=\frac{1}{2}\left[-\cos 5x+\cos x+\cos 5x+\cos 3x\right]=\frac{1}{2}(\cos x+\cos 3x)=\frac{1}{2}\left(2\cos\frac{x+3x}{2}\cdot\cos\frac{x-3x}{2}\right)=\cos 2x\cdot\cos x$$

TRIGONOMETRIČNE ENAČBE

V trigonometričnih enačbah nastopa neznanka kot argument kotnih funkcij. Rešiti trigonometrično enačbo pomeni določiti vrednost vseh kotov, ki enačbi zadoščajo.

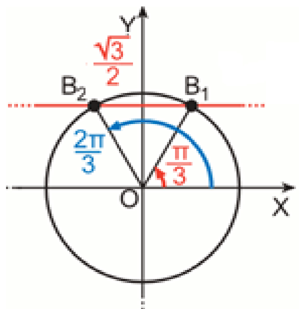
OSNOVNE (ELEMENTARNE) TRIGONOMETRIČNE ENAČBE

1. Osnovne trigonometrične enačbe »**v sinusu**«

Primer:

$$\sin x=\frac{\sqrt{3}}{2}$$

$$\Rightarrow\begin{array}{l} x=\frac{\pi}{3},\ \frac{\pi}{3}+2\pi,\ \frac{\pi}{3}+2\cdot 2\pi,\ \frac{\pi}{3}-2\pi,\ \frac{\pi}{3}-2\cdot 2\pi\ldots \\ x=\frac{2}{3}\pi,\ \frac{2}{3}\pi+2\pi,\ \frac{2}{3}\pi+2\cdot 2\pi,\ \frac{2}{3}\pi-2\pi,\ \frac{2}{3}\pi-2\cdot 2\pi\ldots \end{array}$$



V splošnem:

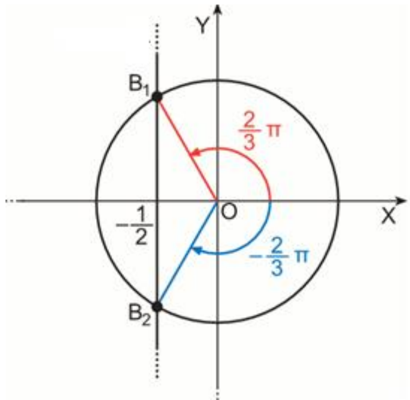
$$\begin{cases}\sin x=a \\ -1\le a\le 1\end{cases}\quad\Rightarrow\quad\begin{array}{l} x=\arcsin a+k\cdot 2\pi,\ k\in\mathbb{Z} \\ x=(\pi-\arcsin a)+k\cdot 2\pi,\ k\in\mathbb{Z}\end{array}$$

2. Osnovne trigonometrične enačbe »**v kosinusu**«

Primer:

$$\cos x=-\frac{1}{2}$$

$$\Rightarrow\begin{array}{l} x=\frac{2}{3}\pi,\ \frac{2}{3}\pi+2\pi,\ \frac{2}{3}\pi-2\pi,\ldots \\ x=-\frac{2}{3}\pi,\ -\frac{2}{3}\pi+2\pi,\ -\frac{2}{3}\pi-2\pi,\ldots \end{array}$$



V splošnem:

$$\begin{cases}\cos x=a \\ -1\le a\le 1\end{cases}\quad\Rightarrow\quad x=\pm\arccos a+k\cdot 2\pi,\ k\in\mathbb{Z}$$

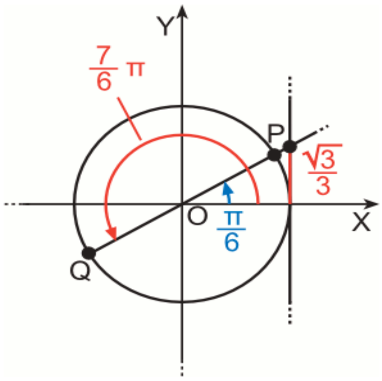
3. Osnovne trigonometrične enačbe »**v tangensu**«

Primer:

$$\operatorname{tg} x=\frac{\sqrt{3}}{3} \Rightarrow x=\frac{\pi}{6},\ \frac{\pi}{6}+\pi,\ \frac{\pi}{6}+2\pi,\ \frac{\pi}{6}-\pi,\ \frac{\pi}{6}-2\pi\ldots$$

V splošnem:

$$\operatorname{tg} x=a \Rightarrow x=\operatorname{arctg} a+k\cdot\pi,\ k\in\mathbb{Z}$$



Vaji:

1. $\frac{11+2\cos 4x}{2}=2\cos 4x+5 \Rightarrow\ldots\Rightarrow \cos 4x=\frac{1}{2} \Rightarrow 4x=\pm\frac{\pi}{3}+2k\pi \Rightarrow x=\pm\frac{\pi}{12}+\frac{k\pi}{2}$

2. $\operatorname{tg}\left(2x-15^\circ\right)=8 \Rightarrow 2x-15^\circ=\operatorname{arctg} 8+k\cdot 180^\circ \Rightarrow x=\frac{\operatorname{arctg} 8+15^\circ}{2}+k\cdot 90^\circ\approx 49^\circ+k\cdot 90^\circ$

POSEBNE ELEMENTARNE ENAČBE

- $\boxed{\sin\alpha=\sin\beta}$

  Dva kota imata isti sinus, čče sta (do periode natančno) enaka ali suplementarna.

  Sledi: $\alpha=\beta+2k\pi, k\in Z \ \vee\ \alpha+\beta=\pi+2k\pi, k\in Z$

  Primer:

  $$\sin\left(x+\frac{\pi}{3}\right)=\sin\left(\frac{\pi}{4}-2x\right)$$

  $$x+\frac{\pi}{3}=\frac{\pi}{4}-2x+2k\pi \quad\vee\quad x+\frac{\pi}{3}+\frac{\pi}{4}-2x=\pi+2k\pi$$

  $$3x=-\frac{\pi}{12}+2k\pi \quad\vee\quad -x=\pi-\frac{7\pi}{12}+2k\pi$$

  $$x=-\frac{\pi}{36}+\frac{2}{3}k\pi, k\in Z \quad\vee\quad x=-\frac{5\pi}{12}+2k\pi, k\in Z$$

- $\boxed{\cos\alpha=\cos\beta}$

  Dva kota imata isti kosinus, čče sta (do periode natančno) enaka ali nasprotna.

  Sledi: $\alpha=\beta+2k\pi, k\in Z \ \vee\ \alpha=-\beta+2k\pi, k\in Z$

- $\boxed{\operatorname{tg}\alpha=\operatorname{tg}\beta}$

  Dva kota imata isti tangens, čče sta (do periode natančno) enaka.

  Sledi: $\alpha=\beta+2k\pi, k\in Z$

Vaje:

1. $\sin\alpha=-\sin\beta$

   Sinus je <u>liha</u> funkcija, zato zgornjo enačbo lahko napišemo kot sledi:

   $\sin\alpha=\sin(-\beta)$.

   Torej: $\alpha=-\beta+2k\pi, k\in Z \ \vee\ \alpha+(-\beta)=\pi+2k\pi, k\in Z\ldots$

2. $\cos\alpha = -\cos\beta$

Kosinus je <u>soda</u> funkcija, zato zgornji »trik« ni možen. Ker imata dva suplementarna kota nasproten kosinus ($\cos(\pi-\beta) = -\cos\beta$), lahko napišemo:

$\cos\alpha = \cos(\pi-\beta)$.

Torej: $\alpha = (\pi-\beta) + 2k\pi, k \in Z \quad \vee \quad \alpha = -(\pi-\beta) + 2k\pi, k \in Z$...

3. $\sin\alpha = \cos\beta$

Ker je kosinus nekega kota enak sinusu komplementarnega kota ($\cos\beta = \sin\left(\frac{\pi}{2}-\beta\right)$) in obratno, sinus nekega kota enak kosinusu komplementarnega kota ($\sin\alpha = \cos\left(\frac{\pi}{2}-\alpha\right)$), lahko gornjo enačbo napišemo na dva načina:

- ✓ $\sin\alpha = \sin\left(\frac{\pi}{2}-\beta\right)$...
- ✓ $\cos\left(\frac{\pi}{2}-\alpha\right) = \cos\beta$...

RAZCEPNE TRIGONOMETRIČNE ENAČBE

<u>Primeri</u>:

1. $2\sin x\cos x - 2\cos x - \sin x + 1 = 0$

$2\cos x(\sin x - 1) - (\sin x - 1) = 0$

$(\sin x - 1)(2\cos x - 1) = 0$

$\sin x - 1 = 0 \quad \Rightarrow \quad \sin x = 1 \quad \Rightarrow \quad x = \frac{\pi}{2} + 2k\pi, k \in Z$

$2\cos x - 1 = 0 \quad \Rightarrow \quad \cos x = \frac{1}{2} \quad \Rightarrow \quad x = \pm\frac{\pi}{3} + 2k\pi, k \in Z$

2. $6\sin^2 x + \frac{2}{\cos x} = 3\operatorname{tg} x + \frac{2\sin 2x}{\cos x} \qquad DO: \ \cos x \neq 0$ oz. $x \neq \frac{\pi}{2} + k\pi, k \in Z$

$6\sin^2 x + \frac{2}{\cos x} = 3\frac{\sin x}{\cos x} + \frac{2\cdot 2\sin x\cos x}{\cos x} \quad /\cdot\cos x$

$6\sin^2 x\cos x + 2 - 3\sin x - 4\sin x\cos x = 0$

$2\sin x\cos x(3\sin x - 2) - (3\sin x - 2) = 0$

$(3\sin x - 2)(2\sin x\cos x - 1) = 0$

$3\sin x - 2 = 0 \quad \Rightarrow \quad \sin x = \frac{2}{3} \quad \Rightarrow \quad x = \arcsin\frac{2}{3} + 2k\pi, k \in Z, \quad x = \pi - \arcsin\frac{2}{3} + 2k\pi, k \in Z$

$2\sin x\cos x - 1 = 0 \quad \Rightarrow \quad \sin 2x = 1 \quad \Rightarrow \quad 2x = \frac{\pi}{2} + 2k\pi, k \in Z \quad \Rightarrow \quad x = \frac{\pi}{4} + k\pi, k \in Z$

Vse rešitve so sprejemljive.

3. $\sin^4 x - \cos^4 x = 0$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = 0 \quad \Rightarrow \quad -(-\sin^2 x + \cos^2 x)\cdot 1 = 0 \quad \Rightarrow \quad -\cos 2x = 0 \quad \Rightarrow \quad \cos 2x = 0$

$2x = \frac{\pi}{2} + k\pi, k \in Z \quad \Rightarrow \quad x = \frac{\pi}{4} + \frac{k\pi}{2}, k \in Z$

4. $\sin 2x \cdot \sin 3x + \cos 4x \cdot \cos x = 0$ ...

$\cos 2x \cdot \cos x = 0$ (glej stran 2)

$\cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2} + k\pi \Rightarrow x = \frac{\pi}{4} + k\frac{\pi}{2}, k \in Z$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi, k \in Z$

ENAČBE, KI JIH REŠUJEMO Z UVEDBO POMOŽNE SPREMENLJIVKE

Primera:

1. $2\cos^2 x - 3\cos x + 1 = 0$ $\boxed{\cos x = t}$

   $2t^2 - 3t + 1 = 0 \Rightarrow t_1 = 1, t_2 = \frac{1}{2}$

   $\cos x = 1 \Rightarrow x = 2k\pi, k \in Z$

   $\cos x = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{3} + 2k\pi, k \in Z$

2. $\operatorname{ctg} x - 6\operatorname{tg} x = 1$ $\boxed{\operatorname{tg} x = t}$ $DO: x \neq 0 + k\pi, k \in Z, \quad x \neq \frac{\pi}{2} + k\pi, k \in Z$

   $\frac{1}{t} - 6t = 1 \Rightarrow 6t^2 + t - 1 = 0 \Rightarrow t_1 = -\frac{1}{2}, \; t_2 = \frac{1}{3}$

   $\operatorname{tg} x = -\frac{1}{2} \Rightarrow x = \operatorname{arctg}\left(-\frac{1}{2}\right) + k\pi, \; k \in \mathrm{Z}$

   $\operatorname{tg} x = \frac{1}{3} \Rightarrow x = \operatorname{arctg}\left(\frac{1}{3}\right) + k\pi, \; k \in \mathrm{Z}$

   Vse rešitve so sprejemljive.

LINEARNE ENAČBE V SINUSU IN KOSINUSU

Imajo obliko $\boxed{a\sin x + b\cos x + c = 0}$. V primeru, da je $c = 0$, jih imenujemo homogene[1] linearne enačbe.

- $c = 0 \Rightarrow a\sin x + b\cos x = 0, \; a, \; b \neq 0$.

  Enačbo rešimo tako, da jo delimo s $\cos x$. Pri tem predpostavljamo, da je $\cos x \neq 0$. Preverimo, če so rešitve enačbe $\cos x = 0$ oz. $x = \frac{\pi}{2} + k\pi, k \in Z$ rešitve dane enačbe:

  $a\sin\left(\frac{\pi}{2} + k\pi\right) + b\left(\frac{\pi}{2} + k\pi\right) \stackrel{?}{=} 0$

  $a \cdot (\pm 1) + b \cdot 0 \stackrel{?}{=} 0 \Rightarrow$ ne, zato je enačba $a\operatorname{tg} x + b = 0$ ekvivalentna dani enačbi.

  Primer:

  $\sqrt{3}\sin x + \cos x = 0 \; /:\cos x(\neq 0)$

  $\operatorname{tg} x = -\frac{1}{\sqrt{3}}\left(= \frac{-1/2}{\sqrt{3}/2}\right) \Rightarrow x = -\frac{\pi}{6} + k\pi, \; k \in \mathbb{Z}$

[1] Polinom je *homogen*, če so vsi njegovi členi iste stopnje.

- $c \neq 0 \;\Rightarrow\; a\sin x + b\cos x + c = 0,\; a,\; b \neq 0$.

  Enačbo rešimo z uporabo parametričnih obrazcev: $\sin\alpha = \frac{2t}{1+t^2}$, $\cos\alpha = \frac{1-t^2}{1+t^2}$, kjer je $t = \operatorname{tg}\frac{\alpha}{2}$.

  Pri tem predpostavljamo, da je $\alpha \neq \pi + 2k\pi, k \in Z$, saj parametrični obrazci veljajo samo pod tem pogojem. Na koncu še preverimo, če so vrednosti $\pi + 2k\pi, k \in Z$ rešitve dane enačbe.

  Primer:

  $\sqrt{3}\sin x + \cos x = 1$

  $\sqrt{3}\cdot\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1 \quad \ldots$

  $t^2 - \sqrt{3}t = 0$

  $t_1 = 0 \;\Rightarrow\; \operatorname{tg}\frac{x}{2} = 0 \;\Rightarrow\; \frac{x}{2} = 0 + k\pi,\; k \in \mathbb{Z} \;\Rightarrow\; x = 2k\pi,\; k \in \mathbb{Z}$

  $t_2 = \sqrt{3} \;\Rightarrow\; \operatorname{tg}\frac{x}{2} = \sqrt{3} \;\Rightarrow\; \frac{x}{2} = \frac{\pi}{3} + k\pi,\; k \in \mathbb{Z} \;\Rightarrow\; x = \frac{2}{3}\pi + 2k\pi,\; k \in \mathbb{Z}$

  Preverimo, če $x = \pi + 2k\pi, k \in Z$ so rešitve začetne enačbe:

  $\sqrt{3}\sin\pi + \cos\pi \overset{?}{=} 1$

  $\sqrt{3}\cdot 0 + (-1) \overset{?}{=} 1$ ne, zato vrednosti $x = \pi + 2k\pi, k \in Z$ niso rešitve dane enačbe.

KVADRATNE NE ENAČBE V SINUSU IN KOSINUSU

Imajo obliko $\boxed{a\sin^2 x + b\sin x\cos x + c\cos^2 x + d = 0}$.

V primeru, da je $d = 0$, jih imenujemo homogene kvadratne enačbe.

- $d = 0 \;\Rightarrow\; a\sin^2 x + b\sin x\cos x + c\cos^2 x = 0,\; a \neq 0$.

  Enačbo rešimo tako, da jo delimo s $\cos^2 x$. Pri tem predpostavljamo, da je $\cos x \neq 0$. Preverimo, če so rešitve enačbe $\cos x = 0$ oz. $x = \frac{\pi}{2} + k\pi, k \in Z$ rešitve dane enačbe:

  $a\sin^2\left(\frac{\pi}{2} + k\pi\right) + b\sin\left(\frac{\pi}{2} + k\pi\right)\cos\left(\frac{\pi}{2} + k\pi\right) + c\cos^2\left(\frac{\pi}{2} + k\pi\right) \overset{?}{=} 0$

  $a\cdot 1 + b\cdot(\pm 1)\cdot 0 + c\cdot 0 \overset{?}{=} 0 \;\Rightarrow\;$ ne, zato je enačba $a\operatorname{tg}^2 x + b\operatorname{tg} x + c = 0$ ekvivalentna dani enačbi.

  Primer:

  $5\sin^2 x - 2\sin x\cos x - 3\cos^2 x = 0 \quad /:\cos^2 x(\neq 0)$

  $5\operatorname{tg}^2 x - 2\operatorname{tg} x - 3 = 0$

  $\boxed{\operatorname{tg} x = t} \;\Rightarrow\; 5t^2 - 2t - 3 = 0$

  $t_1 = 1 \;\Rightarrow\; \operatorname{tg} x = 1 \;\Rightarrow\; x = \frac{\pi}{4} + k\pi, k \in Z$

  $t_2 = -\frac{3}{5} \;\Rightarrow\; \operatorname{tg} x = -\frac{3}{5} \;\Rightarrow\; x = \operatorname{arctg}\left(-\frac{3}{5}\right) + k\pi, k \in Z$

- $d \neq 0 \quad \Rightarrow \quad a\sin^2 x + b\sin x\cos x + c\cos^2 x + d = 0$.

  Enačbo rešimo „s trikom“: $d = d\cdot 1 = d\cdot\left(\sin^2 x + \cos^2 x\right)$ in nato po zgoraj opisanem postopku. Na koncu še preverimo, če so rešitve enačbe $\cos x = 0$ oz. $x = \frac{\pi}{2} + k\pi,\ k \in Z$ rešitve dane enačbe.

Primer:

$$6\sin^2 x + \sin 2x = 4$$

$$6\sin^2 x + 2\sin x\cos x = 4\left(\sin^2 x + \cos^2 x\right)$$

$$2\sin^2 x + 2\sin x\cos x - 4\cos^2 x = 0 \quad /: 2\cos^2 x(\neq 0)^{*}$$

$$\operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0 \qquad \boxed{\operatorname{tg} x = t}$$

$$t^2 + t - 2 = 0$$

$$t_1 = 1 \quad \Rightarrow \quad \operatorname{tg} x = 1 \quad \Rightarrow \quad x = \frac{\pi}{4} + k\pi,\ k \in Z$$

$$t_2 = -2 \quad \Rightarrow \quad \operatorname{tg} x = -2 \quad \Rightarrow \quad x = \operatorname{arctg}(-2) + k\pi,\ k \in Z$$

(*) Preverimo, če so $x = \frac{\pi}{2} + k\pi,\ k \in Z$ rešitve začetne enačbe:

$$6\sin^2\cdot\left(\frac{\pi}{2} + k\pi\right) + \sin\left[2\left(\frac{\pi}{2} + k\pi\right)\right] \stackrel{?}{=} 4$$

$6\cdot 1 + 0 \stackrel{?}{=} 4$ ne, zato vrednosti $x = \frac{\pi}{2} + k\pi,\ k \in Z$ niso rešitve dane enačbe.

SISTEMI TRIGONOMETRIČNIH ENAČB

$$\begin{cases} 4\cos^2 x + 3\cos^2 y = 4 \\ 2\cos x + 5\cos y = 6 \end{cases} \qquad \boxed{\begin{cases} \cos x = u \\ \cos y = v \end{cases}}$$

$$\begin{cases} 4u^2 + 3v^2 = 4 \\ 2u + 5v = 6 \end{cases} \Rightarrow \ldots \Rightarrow \begin{cases} 7v^2 - 15v + 8 = 0 \\ u = \dfrac{6-5v}{2} \end{cases} \Rightarrow \ldots \Rightarrow \begin{cases} u_1 = \dfrac{1}{2} \\ v_1 = 1 \end{cases} \vee \begin{cases} u_2 = \dfrac{1}{7} \\ v_2 = \dfrac{8}{7} \end{cases}$$

$$\begin{cases} \cos x = \dfrac{1}{2} \\ \cos y = 1 \end{cases} \Rightarrow \begin{cases} x = \pm\dfrac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \\ y = 2h\pi,\ h \in \mathbb{Z} \end{cases} \qquad \vee$$

$$\begin{cases} \cos x = \dfrac{1}{7} \\ \cos y = \dfrac{8}{7} \end{cases} \Rightarrow \text{nemogoč}$$

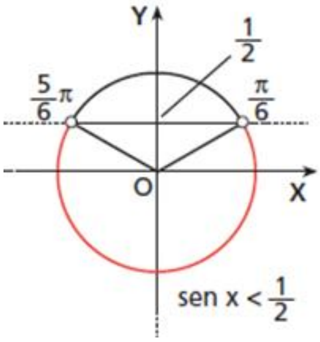
TRIGONOMETRIČNE NEENAČBE

OSNOVNE (ELEMENTARNE) TRIGONOMETRIČNE NEENAČBE

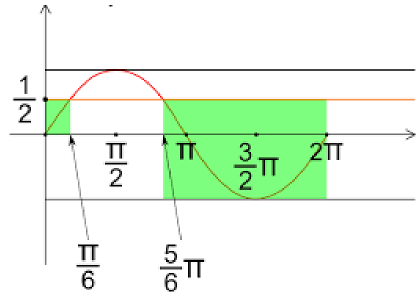
1. Osnovne trigonometrične neenačbe »**v sinusu**«

Primer: $\sin x < \frac{1}{2}$

1. način: s trigonometrično krožnico



2. način: z grafom funkcije sinus (npr v intervalu $[0, 2\pi]$)

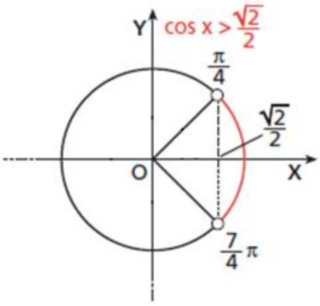


$$-\frac{7}{6}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$$

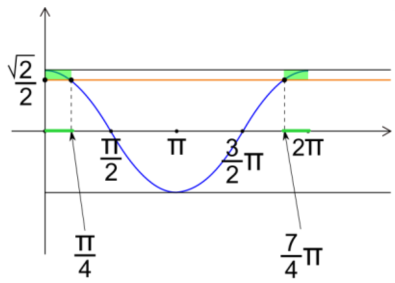
2. Osnovne trigonometrične neenačbe »**v kosinusu**«

Primer: $\cos x > \frac{\sqrt{2}}{2}$

1. način: s trigonometrično krožnico



2. način: z grafom funkcije kosinus (npr v intervalu $[0, 2\pi]$)

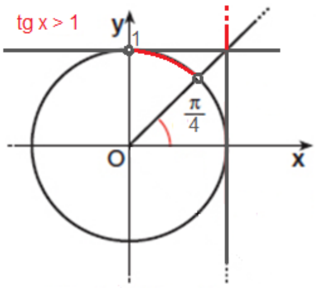


$$-\frac{\pi}{4} + 2k\pi < x < \frac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$$

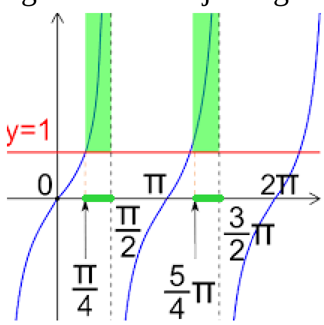
3. Osnovne trigonometrične neenačbe »**v tangensu**«

Primer: $\operatorname{tg} x > 1$

1. način: s trigonometrično krožnico



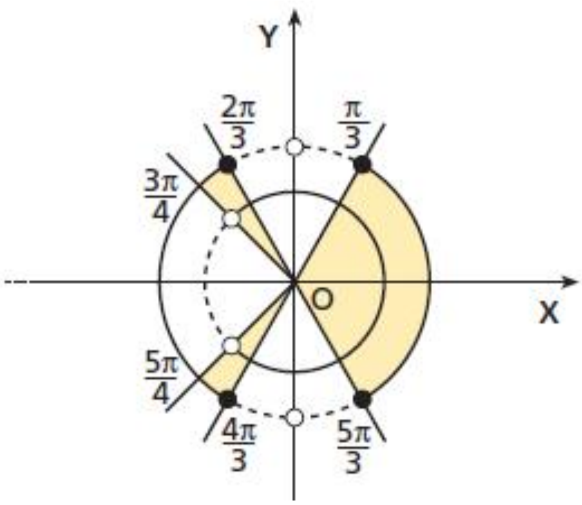
2. način: z grafom funkcije tangens



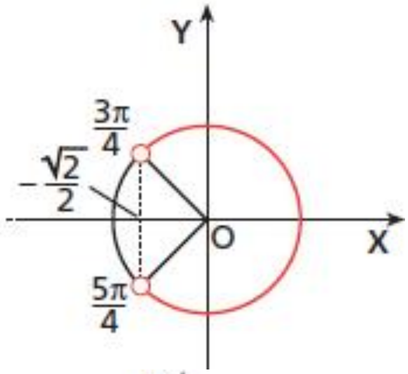
$$\frac{\pi}{4} + k\pi < x < \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$$

SISTEMI TRIGONOMETRIČNIH NEENAČB

$$\begin{cases} \cos x > -\dfrac{\sqrt{2}}{2} \\ |\text{tg}\, x| \le \sqrt{3} \end{cases}$$

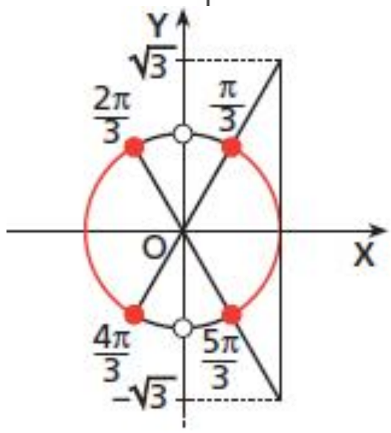


$$\cos x > -\frac{\sqrt{2}}{2}$$



$$|\text{tg}\, x| \le \sqrt{3}$$

$$-\sqrt{3} \le \text{tg}\, x \le \sqrt{3}$$



$$-\frac{\pi}{3} + 2k\pi \le x \le \frac{\pi}{3} + 2k\pi \;\vee$$

$$\frac{2}{3}\pi + 2k\pi \le x < \frac{3}{4}\pi + 2k\pi \;\vee$$

$$\frac{5}{4}\pi + 2k\pi < x \le \frac{4}{3}\pi + 2k\pi,\; k \in \mathbb{Z}$$

RAZCEPNE IN ULOMLJENE TRIGONOMETRIČNE NEENAČBE

Primeri:

1. $4\sin x \cos x - 2\cos x - 2\sin x + 1 \ge 0$

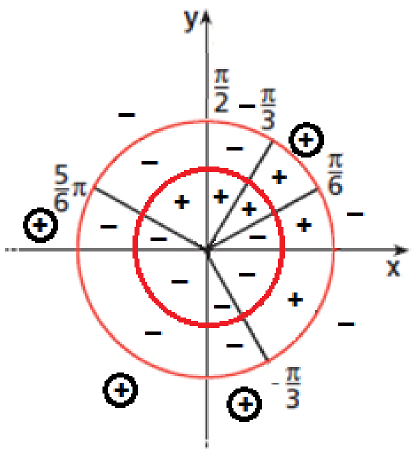
$2\cos x(2\sin x - 1) - (2\sin x - 1) \ge 0$

$(2\sin x - 1)(2\cos x - 1) \ge 0$

$2\sin x - 1 \overset{?}{>} 0 \;\Rightarrow\; \sin x > \frac{1}{2} \;\Rightarrow\; \frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi,\, k \in Z$

$2\cos x - 1 \overset{?}{>} 0 \;\Rightarrow\; \cos x > \frac{1}{2} \;\Rightarrow\; -\frac{\pi}{3} + 2k\pi < x < \frac{\pi}{3} + 2k\pi,\, k \in Z$

$\frac{\pi}{6} + 2k\pi \le x \le \frac{\pi}{3} + 2k\pi \;\vee\; \frac{5}{6}\pi + 2k\pi \le x \le \frac{5}{3}\pi + 2k\pi,\, k \in Z$



2. $\dfrac{2\sin x - 1}{2\cos x - 1} \ge 0$

…

$\frac{\pi}{6} + 2k\pi \le x < \frac{\pi}{3} + 2k\pi \;\vee\; \frac{5}{6}\pi + 2k\pi \le x < \frac{5}{3}\pi + 2k\pi,\, k \in Z$

NEENAČBE, KI JIH REŠUJEMO Z UVEDBO POMOŽNE SPREMENLJIVKE

$$\cos 2x - 3\sin x + 4\sin^2 x > 0$$
$$\cos^2 x - \sin^2 x - 3\sin x + 4\sin^2 x > 0$$
$$1 - \sin^2 x + 3\sin^2 x - 3\sin x > 0$$
$$2\sin^2 x - 3\sin x + 1 > 0 \qquad \boxed{\sin x = t}$$
$$2t^2 - 3t + 1 > 0 \;\Rightarrow\; \mathscr{P}, \bigcup, t_1 = 1, t_2 = \frac{1}{2} \;\Rightarrow\; t < \frac{1}{2} \vee t > 1$$
$$\sin x < \frac{1}{2} \vee \sin x > 1 \;\Rightarrow\; -\frac{7}{6}\pi + 2k\pi < x < \frac{\pi}{6} + 2k\pi, k \in Z$$

LINEARNE NEENAČBE V SINUSU IN KOSINUSU: rešimo jih grafično.

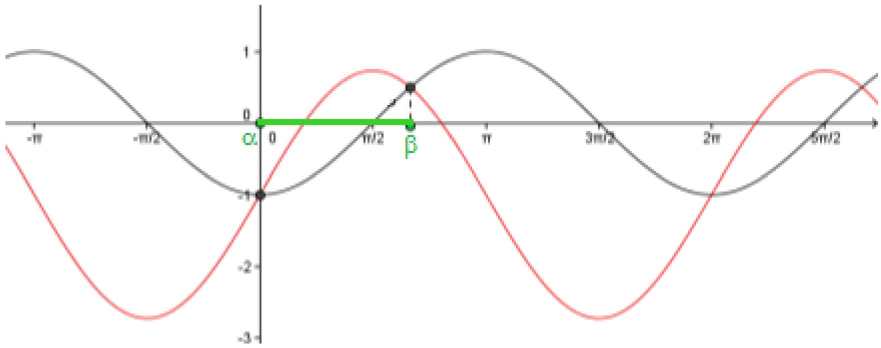
Primer:

$\sqrt{3}\sin x + \cos x \geq 1$

$\sqrt{3}\sin x - 1 \geq -\cos x$

$\alpha + 2k\pi \leq x \leq \beta + 2k\pi,\; k \in \mathbb{Z}$

"Mejnike" izračunamo tako, da rešimo asocirano trigonometrično enačbo:



$$\sqrt{3}\sin x + \cos x = 1$$
$$\sqrt{3} \cdot \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1 \quad \ldots$$
$$t^2 - \sqrt{3}t = 0$$
$$t_1 = 0 \;\Rightarrow\; \mathrm{tg}\frac{x}{2} = 0 \;\Rightarrow\; \frac{x}{2} = 0 + k\pi,\; k \in \mathbb{Z} \;\Rightarrow\; x = 2k\pi,\; k \in \mathbb{Z} \;\overset{k=0}{\Rightarrow}\; \alpha = 0$$
$$t_2 = \sqrt{3} \;\Rightarrow\; \mathrm{tg}\frac{x}{2} = \sqrt{3} \;\Rightarrow\; \frac{x}{2} = \frac{\pi}{3} + k\pi,\; k \in \mathbb{Z} \;\Rightarrow\; x = \frac{2}{3}\pi + 2k\pi,\; k \in \mathbb{Z} \;\overset{k=0}{\Rightarrow}\; \beta = \frac{2}{3}\pi$$

Rešitev neenačbe: $0 + 2k\pi \leq x \leq \frac{2}{3}\pi + 2k\pi,\; k \in \mathbb{Z}$