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Improved Radius Selection in Sphere Decoder for MIMO System

R. P. Yadav¹, Garima Mathur², Sonjuhi Mrinalee³ and Honey Priya Garg⁴

Abstract - The MLD (Maximum-likelihood decoding) of a particular random code over an AWGN (additive white Gaussian noise) channel requires an exhaustive search over all the possible code-word, and so the computational complexity of the optimal decoding scheme is exponential in the length of the code-word. A new type of the detection technique called the sphere decoding algorithm is proposed to lower the computational complexity. The principle of the sphere decoding algorithm is to search the closest lattice point to the received signal within a sphere radius, where every codeword is represented by a lattice point in a lattice field.

Index Terms - Fincke - Pohst (FP), Maximum Likelihood Decoder (MLD), Multiple Input Multiple Output (MIMO), Partial Euclidean Distance (PED), Sphere Decoder (SD), Schnorr-Euchner (SE).

NOMENCLATURE

In the paper, vectors are regarded as column vectors. These are written in lowercase bold characters. For matrices, uppercase bold characters are used. $\|\cdot\|$, \cdot^T , \cdot^H and $E\{\cdot\}$ denote the Euclidean norm, transpose, complex conjugate transpose and expected value, respectively.

I. INTRODUCTION

MIMO is the promising technology for next generation wireless communication system, as it obtains high throughput and diversity gain on rich scattering environment. As ML detection is optimal bit error rate detector for MIMO system which is infeasible due to the complexity of ML when a large number of antennas used together with high order modulation scheme. Therefore, SD is the lattice decoding algorithm, which is introduced for MIMO system to reduce the complexity of ML detection. It is a very tempting approach to reduce the implementation complexity of ML detector, significantly. This paper proposes Sphere Decoder that efficiently reduces complexity. We have two methods for SD. These are respectively – FP-method and SE method. FP-enumeration scheme directly detects the constellation points. Fincke-pohst was the first to give this sphere decoder strategy in the year 1984. In 1993 Viterbo and Biglieri first used the SD in communications for soft decoding of the Golaycode. The work was to find the closest lattice point for a single antenna fading channels Schnorr- Euchner tried to improve Fincke and Pohst’s decoding algorithm. It involves searching the lattice points

hyper-sphere in the increasing order of distance. Agrellet. al. inside the developed the Closest Lattice Point Search Algorithm, to generalize the Schnorr-Euchner’s algorithm for decoding MIMO system. Chan and Lee invented hybrid version for ordering of Schnorr-Euchner and the Fincke-Pohst’s sphere decoding algorithm. Kannan introduced an alternate version of ordering the Fincke- Pohst’s based SD decoding algorithm. The Kannan’s algorithm recursively searches all the lattice points inside a rectangular parallelepiped centered at the query point with edges along the Gram-Schmidt vectors of a proper basis of the lattice. It was later extended to the sphere decoding algorithm of MIMO systems by Hassibi and Vikalo, to overcome the demerits of linear and non-linear decoders. We have discussed in section 2 sphere decoder in details. Section 3 and 4 will discuss about types of sphere decoder. The points covered are its working and how they vary according to their processing time. In the last section, we propose our work on radius search strategy.

II. SPHERE DECODER

In communication systems where the signal is transmitted using a digital modulation like QPSK or QAM, the signal space diagram forms a regular grid. On certain conditions such a grid can be described with the help of lattice theory. In doing so, the maximum-likelihood detection problem is equivalent to the task of finding the closest point in a lattice. This problem was overcome by SD algorithm, where the main challenge was to determine the search radius d of the sphere. This is due to the fact that, if the chosen radius d is too large we obtain too many leaf nodes within the sphere and the search remain exponentially in size, whereas, if d is too small we obtain no leaf node inside the sphere and the search must be restarted by increasing the radius after a huge computational loss. The sphere decoding can be considered as a depth-first search approach with tree pruning process. In SD algorithm, the most important issue is the strategy based on which signals “hypotheses” are tested per level. The SD algorithm for SM MIMO systems has two types of searching strategies, the Fincke - Pohst (FP) and the Schnorr-Euchner (SE).

III. FINCKE - POHST STRATEGY

The Fincke - Pohst (FP) strategy is considered to be the original sphere decoding algorithm [1]. This strategy was first used in digital communications theory by Viterbo and Biglieri, which it was applied to find the closest point for a single antenna fading channels. This method considers all hypotheses in natural order, and the search is starting with the first one as shown in Figure 1. If a point is found, the radius is updated (reduced) and so forth. An important and critical aspect of the FP strategy is that a search radius must be initialized appropriately. However,

¹MNIT, Jaipur, Rajasthan, India

²JEC-Kukas, Jaipur, Rajasthan, India

E-mail: ¹rp_yadav@yahoo.com, ²garimamathurjec@gmail.com,

³sonjuhimrinalee@yahoo.com and ⁴honeygarg693@gmail.com

if the sphere radius d is too large, many lattice points will have to be computed and a large number of points may also be cancelled out. If it is too small, no points will be found and the algorithm must then be restarted with a larger searching radius. Both of these factors negatively influence the overall computation time, and thus it is well-known that one of the main drawbacks of the FP strategy is the sensitivity of its performance to the choice of initial search radius d . A recommend choice is the distance to the Babai point (ZF), which is the first returned point in the search set. Then, it could be assured that at least one lattice point will be found inside the sphere.

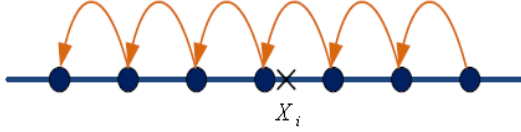


Figure 1: Fincke - Pohst Strategy

IV. SCHNORR – EUCHNER STRATEGY

Schnorr-Euchner (S-E) introduced an algorithm which does not require an initial radius estimate [2]. It added a small but significant refinement to the FP approach. The FP strategy searches the admissible nodes without any ordering, whereas in SE strategy, the admissible nodes of each level are spanned in a zigzag order starting with the closest middle point as depicted in Figure 2.

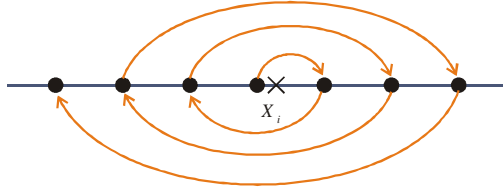


Figure 2: Schnorr-Euchner Strategy

SE strategy considers symbol close to the Babai point (ZF) solution and if a point is found, then the radius is updated (reduced) and so on. It was also concluded that the SE enumeration is more efficient than FP and has lower computational complexity by reordering the constellation searching at each level. The tree is explored depth-first and child nodes at level i are prioritized in the increasing order of PEDs. Furthermore, the initial search radius d is set to infinity, and is updated with the PED of each new candidate solution. The S-E enumeration finds eligible solutions faster, and is the foundation for most of SD extensions. However, the S-E algorithm requires the real-valued decomposition of the channel matrix and the receive vector, which doubles the tree depth [3, 4].

In this paper, the tree with complex valued constellations is considered, i.e. without performing real valued decomposition. In order to avoid the problem of the choice of initial radius d , the ZF estimate is used to set the radius as shown in the following equation

$$x^{ZF} = Q(\tilde{x}) \quad (1)$$

Where $Q(\cdot)$ performs minimum distance quantization of each element of \tilde{x} to the constellation points in use. Using the quantized points the initial radius is obtained as follows:

$$d = \sum_{l=1}^{N_t} \left| \sum_{l=1}^{N_t} r_{l,l} (\tilde{x}_l - x_l^{ZF}) \right|^2 \quad (2)$$

Since the computation of \tilde{x} is part of the SD, the additional complexity is limited to the complexity of Equations (1) and (2) which is negligible. Moreover, further complexity reduction can be achieved by employing the radius update strategy by setting the distance of every new candidate as a new search radius [5].

V. RADIUS SEARCH

It is already mentioned, the MLD is not feasible for larger number of transmit antennas and/or higher modulation schemes. A feasible option is the application of SDs, whose computational complexity is independent of the total number of possible transmit vectors (Ω).

SDs achieves similar performance to MLD with reasonable computational complexity. This is due to the fact that SD examine only the vector candidates which fall within a hyper sphere (N_r dimensional region) of a given radius d centered at the receive vector, instead of examining the entire possible transmit vectors as shown in Figure 3. As it can be clearly seen from this figure, the closest point inside the sphere will also be the closest point for the whole set of points.

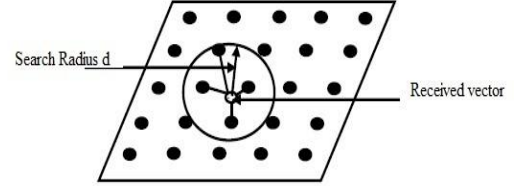


Figure 3: A 2- Dimensional Geometric Representation of Sphere Decoding

From Figure 3, we can see that the number of vector candidates within the sphere is significantly smaller and this makes the SD computationally efficient. The sphere decoding performs the ML search by imposing a constraint on the exhaustive search of Equation 1 as:

$$X_{ml} = \text{argmin} \|y - Hx_j\|^2 \leq d^2 \quad (3)$$

Where d is the radius of the sphere and equation 3 is the sphere constraint equation.

Only imposing the sphere constraint using Equation 3 does not lead to complexity reductions as the challenge has merely been shifted from finding the closest point to identifying points that lie inside the sphere. Hence, the complexity is only reduced if the sphere constraint can be checked other than exhaustively searching through all possible candidate vectors [6,7]. Therefore, Equation 3 needs further processing as follows

$$\begin{aligned} \|y - Hx_j\|^2 &= (y - Hx_j)^H (y - Hx_j) \\ &= (\tilde{x} - x_j)^H H^H H (\tilde{x} - x_j) \end{aligned} \quad (4)$$

where $\tilde{x} = (H^H H)^{-1} H^H y$ is the unconstrained solution. Moreover, after QR decomposition of the channel matrix H we obtain:

$$X_{ml} = \operatorname{argmin} (\tilde{x} - x_j)^H R^H R (\tilde{x} - x_j) \quad (5)$$

$$X_{ml} \leq d^2 \quad (6)$$

where Q is the orthogonal matrix and R is the upper triangular matrix which is obtained by QR decomposition of H . The upper triangular nature of R enables the sphere decoder to decide whether the point is inside the sphere or not, before computing the total sphere constraint equation [8, 9]. Besides, due to this structure of matrix R , Equation 6 can be transformed from matrix manipulation to summation expression as follows

$$\sum_{l=i}^{N_t} \left\| \sum_{l=i}^{N_t} r_{i,l} (\tilde{x}_l - x_l^m) \right\|^2 \leq d^2 \quad (7)$$

where $r_{i,l}$ is $(i,l)^{\text{th}}$ element of R , \tilde{x}_l is the l^{th} element of \tilde{x} and x_l^m is the m^{th} element of the M point constellation used at the l^{th} layer. With Equation 6, the exhaustive search has turned out to be an iterative tree traversal, where the leaves at the bottom level correspond to all possible symbol vectors x and the M possible values of the entry x_{N_t} define its top level (root level) as illustrated in Figure 4 for a single root. This enables us to uniquely describe the node at level i ($i = 1, 2, \dots, N_t$) by the partial vectors $x^{(i)} = [x_i, x_{i+1}, \dots, x_{N_t}]$ as described in the figure.

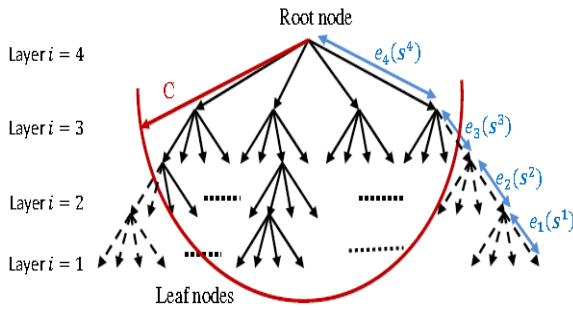


Figure: 4 Search tree diagram of generalized sphere decoding algorithm

We define branch cost function (increments) associated with nodes in the i^{th} layer as:

$$e_i(x^{(i)}) = \left| \sum_{l=i}^{N_t} r_{i,l} (\tilde{x}_l - x_l^m) \right|^2 \\ = \left| \sum_{l=i}^{N_t} r_{i,l} \tilde{x}_l - \sum_{l=i+1}^{N_t} r_{i,l} x_l^m - r_{i,i} x_i^m \right|^2 \quad (8)$$

where $x^{(i)}$ is the partial vector of x with $x_1 = 0$, $x_{i-1} = 0$. From Equation 7, it can be deduced that the computation of the squared Euclidean distance is a recursive process where intermediate computations are reused [10,11].

We can further decompose Equation 7 to separate the part influenced by x_i^m as follows:

$$E_i(x^{(i)}) = \left| b_{i+1} x^{(i+1)} - r_{i,i} x_i^m \right|^2 \\ b_{i+1} x^{(i+1)} = r_{i,i} \tilde{x}_i + \sum_{l=i+1}^{N_t} r_{i,l} (\tilde{x}_l - x_l^m) \quad (9)$$

It is obvious from equation 9 that $b_{i+1} x^{(i+1)}$ is common to all the children of the node under consideration [12]. So, the branch cost function can be obtained with minimum effort by pre-computing $b_{i+1} x^{(i+1)}$ and simply evaluating equation 9 for all $x_i^m \in M$. The distance of the partial vector is obtained as:

$$T_i(x^{(i)}) = T_{i+1}(x^{(i+1)}) + e_i(x^{(i)}) \quad (10)$$

where $T_i(x^{(i)})$ is called partial Euclidean distance (PED) which is non-decreasing function and for $i = N_t$, $T_{i+1}(x^{(i+1)})$ initialized to zero. Thus, the decoding process can be regarded as descending down a tree in which each node has M branches [13, 14]. Since branch increments $e_i(x^{(i)})$ are non-negative, it follows immediately that the PED of a node ignore the sphere constraint, given by:

$$T_i(x^{(i)}) \leq d^2 \quad (11)$$

The PED's of all its descendents will also discarded the sphere constraint. Consequently, the tree can be pruned below this node. This approach effectively reduces the number of vector symbols (i.e. leaf of the tree) to be checked. When the tree traversal is finished the leaf with the lowest $T_1(x)$ is chosen as corresponding to the transmitted vector, depending on the tree traversal.

VI. SIMULATION RESULT

In this section, we have discussed simulation result of the proposed schemes in terms of the bit-error rate by using MATLAB Programmed. Here, we have compared the result of different decoders for 4 X 4 MIMO systems and denote them by different colors. We find that the BER performance of our proposed Sphere Decoder is better than that of the conventional

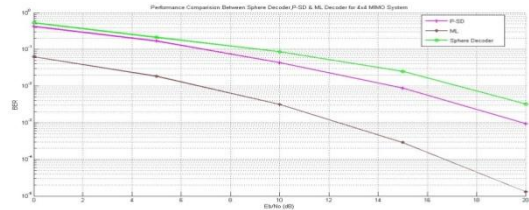


Figure 5: Comparison between Different Decoders

VII. CONCLUSION

The generalized sphere decoder was modified to overcome an impairment normally overlooked in the general equation for calculating the radius of the search sphere. In our proposed sphere decoder, larger number of overall iterations causes the decoder to be nearer to the optimal ML solution. It has been viewed here that proposed sphere decoder has performed better in terms of complexity and the performance as compared to the generalized sphere decoder and performs near optimal to the ML decoder.

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