Simulating the stock market and comparing trading strategies

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In this report, we simulate the stock market using geometric Brownian motion. We furthermore introduce different trading strategies and compare their performance to each other. For this, we introduce agents (investors) that follow a certain strategy. Their profits are then compared to each other after a certain amount of time. We investigate the influence of certain parameters that characterise the stock on the respective performances. Furthermore, the risk that these investors are facing is discussed.

I. INTRODUCTION

The stock market's dynamics have long intrigued economists, mathematicians, and investors alike, offering a complex interplay of factors that drive financial markets. A prominent method is the application of Geometric Brownian Motion (see e.g. [1] and [2]). Geometric Brownian Motion (GBM) models the evolution of stock prices upon introducing the parameters drift and volatility. This approach is supported by studies like the one conducted in [3], which utilized geometric Brownian motion to forecast stock market scenarios, particularly during the volatile period of the COVID-19 pandemic. Additionally, the exploration of various trading strategies in simulated market environments is crucial for understanding market behaviour and investment performance. The work in [4] provides an insightful comparison of different trading strategies against a simple buy-and-hold approach.

This project aims to develop a comprehensive simulation of the stock market using GBM. The ultimate goal is to compare and identify which strategies offer the best balance between risk and return, particularly over the long term. For this, we will introduce agents investing via a certain strategy and compare their respective profits to each other. Furthermore, we will investigate the risk of the investments made (for the considered stock parameters) using the Value at Risk (VaR) method. This simulation will not only contribute to academic understanding but also offer practical insights for investors and traders in navigating the complexities of the stock market.

II. GEOMETRIC BROWNIAN MOTION

The concept of Geometric Brownian Motion (GBM) (an introduction into GBM is given e.g. in [1]) is related to Brownian motion. However, in GBM, it is the logarithm of the randomly varying quantity which performs a Brownian motion. Its most common application is the modeling of stock prices. It is described by a stochastic differential equation as follows:

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

where σ is the *volatility* and μ is the *drift*.

This equation is solved by the following equation for S_t:

$$S_t = S_0 \cdot \exp\big((\mu - \frac{\sigma^2}{2})t + \sigma W_t\big) \tag{2}$$

The simulated stock prices then look something like the stock shown in figure 1. Drift and volatility can be varied

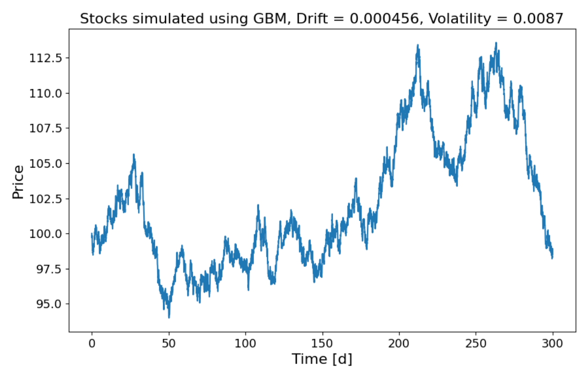

FIG. 1. Simulation of a stock using GBM with drift and volatility given by: $\sigma = 8.7 \times 10^{-3}, \mu = 4.56 \times 10^{-4}$

to alter the shape of the plot in figure 1. The drift determines the general trend of the stock. The higher the drift parameter, the more will the stock be dominated by a general upwardstrend.

The volatility indicates how rapidly the stock varies over a certain period of time. Thus, investors are interested in stocks with high drifts - high overall growth - and depending on the strategy they follow they are interested in stocks with higher or lower volatility. In this report, we compare the performance of different investors to each other that follow different trading strategies. The performances will be compared for stocks of different drifts and volatilities. In order to find realistic parameters for the volatility σ and the drift, we compared our simulations to historic data of the S&P500 index, consisting of the 500 biggest companies in the United States [5]. For the timeframe between 01/12/2022 and 01/12/2023, we calculate volatility and drift for this past stock data of the S&P500. The resulting values are:

$$\sigma = 8.7 \times 10^{-3} \qquad \mu = 4.56 \times 10^{-4} \tag{3}$$

The process of obtaining the drift and volatility from a stock [6] is described shortly in the following.
We calculate the log return, which is the natural logarithm of a stock's price change, for each trading day as follows:

$$R_L = \log(\frac{P_{t+1}}{P_t}) \tag{4}$$

where R_L is the log return and P_t is the stock price on day t. The drift μ represents the average daily logarithmic return while the volatility σ represents the standard deviation of the logarithmic returns. We can thus obtain the drift and the volatility as follows:

$$\mu = \frac{1}{n}\sum_{t=1}^{n} R_L \tag{5}$$

$$\sigma = \sqrt{\frac{\sum_{t=1}^{n}(R_L-\mu)^2}{n}} \tag{6}$$

In the analysis part of this project, we investigate how well certain trading strategies perform on certain stocks with varying values of volatility and drift. If not mentioned otherwise, all the stocks shown in this report are created with the values in equation 3. Furthermore, for all of the stocks, we chose a timestep of `dt = 1/24` if not stated otherwise. This means that the stock is updated 2 times per hour (assuming trading hours from 08:00 am to 08:00 pm). The initial price for each stock in this project is 100 price units.

III. TRADING STRATEGIES

In total, eight different trading strategies will be taken into account. In this section, they shall be presented shortly. For the different strategies, the parameters were chosen based mostly on intuition but also on what turned out to work experimentally.

Buy and Hold

The simplest strategy is the buy and hold strategy. The investor buys a stock as the first possible option he has. And sells it at the last possible option. These options are limited by the timeframe that we set.

Moving Average Strategy

This strategy utilizes the moving average value [7]. The moving average value $A_n[i]$ for day i over n days corresponds to the average over the closing prices of the last n days - i.e. the days $i-1$, $1-2$, ..., $i-20$. This value can act as an indicator if the stock is in an upward or downward trend. This value is used most commonly in two ways.

1. The first Moving Average approach

One of them is to buy when the average is rising. If the moving average has been rising for the last n days, we buy. If it has been falling for the last n days, we sell. This approach will in the following be referred to as the Moving Average strategy.

We chose the parameter `n = 20`.

2. The crossover moving average approach

Another strategy is the Crossover Strategy. Using the average value over two different numbers of days t_{long} and t_{short} (different number of days - one significantly longer than the other) for the calculation of the moving average, we make decisions to buy and sell based on their behaviour relative to each other. If the short time average crosses the long time average and surpasses it, then we buy. Once the two averages cross again, this time the long-time average being the one surpassing, we sell. We show a plot explaining this strategy in figure 2 (with parameters given below). This stock runs over 100 days for the sake of displayment. In this figure, it

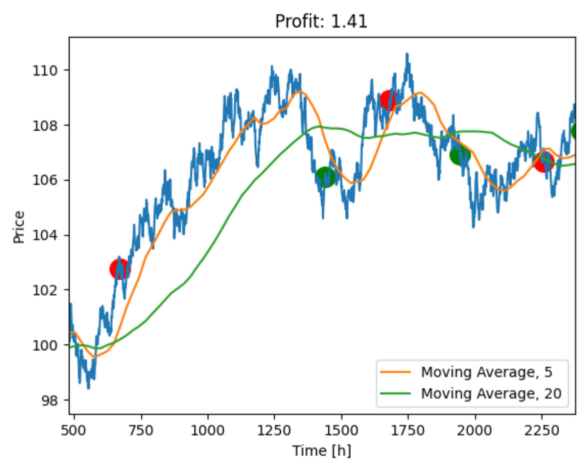

FIG. 2. Illustration of the cross-over trading strategy for a stock running over 100 days. The red circle indicates buying, green circle indicates selling.

is clear that the agent buys (indicated by a red circle), when the short term crosses over the long term average, and sells (indicated by a green circle), once the long term crosses over the short term average.
One technical thing we have to keep in mind when working with the moving average is that the agent can only make buying and selling decisions after n days, so after the first average value is observed. We will keep this in mind and only allow players using other strategies to buy and sell after this period of days has passed. In reality, however, we can look as far in the past of a stock

as we wish.

We chose the parameters `n = 20`, `tLong = 20` and `tShort = 5`.

Mean Reversion Strategy

The mean reversion strategy utilizes the so-called Z-score [8]. The Z score is calculated as follows. The mean value M over the last n days is observed. After this, the deviation D is calculated for each timestep in the stock:

$$D[i] = \text{stock}[i] - M \tag{7}$$

After this, the Z score is calculated using the standard deviation σ:

$$Z = \frac{D}{\sigma} \tag{8}$$

Essentially, this value captures the relation of a given value to the average value. The value of the Z-score is used by investors as an indicator of when to buy or sell. If the Z score is small (smaller than some value `-zLim`), the stock is likely to be undervalued. Thus, buying the stock is a good idea. If the Z score is high (higher than some value `zLim`), the stock is likely to be overvalued. This is an indicator to sell.

We chose the parameters `n = 10`, `zLim = 1.5`.

Scalping

The scalping trading strategy [9] focuses on short term movements of the stock market. A trading strategy like this is called a *short term trading strategy*. The goal is to gain money through making small profits many times. A stock should in the best case have a high volatility for this trading strategy. Technically, there are two variables defined, `timeA` and `timeB`. If the stock has been falling for `timeA`, we sell the stock. If it has been rising for `timeB`, we buy the stock.

We chose the parameters `timeA = 10` and `timeB = 10`

Range Trading

The range trading technique [10] is another *short term trading strategy*. If there is some range present (if we recognize a range in the last few timesteps in the stock) - meaning that the prices do not exceed a certain maximal value (the upper limit of the range) and do not drop lower than a certain minimal value (the lower limit of the range), the trader buys when the current price of the stock is close to the lower limit of the range. She sells, once the stock gets close (a certain measure has to be defined to reason about what closeness means exactly - in our case, we use an offset) to the upper limit.
One other parameter that has to be defined is our limit to how big a range can be. This parameter we call `rangeLimit`. One more parameter is the number of days that this range has to be existent in order for us to consider it a range - this parameter is `numberDays`

Breakout Trading

One other strategy that is using range to reason about buy and sell decisions is the Breakout strategy. For some range, if a price surpasses the upper limit of this range, we buy the stock for this price. Once the stock starts falling, we sell.

The parameters for both the Range Trading as well as the Breakout strategy used are: `offset = 1, rangeLimit = 10, numberDays = 5`

Random Strategy

We include a random strategy. A player buys a (limited) random number of stocks at random times and sells at other random times. This strategy is not considered a serious competition in this collection. The reason we include this strategy is for it to act as a gauge for the other strategies.

The random factor we chose is to buy and sell at every timestep with the random probability of `probability = 1%`. At two timesteps per hour (`dt = 1/24`), this corresponds to buying on average around one stock in four days.

IV. COMPARING TRADING STRATEGIES

For the trading strategies, we introduce agents each playing one certain strategy: there will be one agent buying and holding the stock, one agent following the Crossover Strategy, one agent buying and selling randomly, and so on with the respective parameters discussed above.
For this section, agents are allowed to buy one share of the stock and only hold one share at the same time. This means, if the agent was to buy a share, he first has to sell the current one before he is allowed to buy the next share. The agent following the Buy and Hold strategy then only holds one share for the entirety of the trading period.
The performance is averaged over 100 stocks. This means we generate 100 stocks with the parameters in equation 3 and let each agent invest in all 100 of these stocks. After this, we average over the profits that they made in each investment and these are the profits that are being

discussed. What we get as a result of our analysis is the bar graph depicted in figure 3. In this plot the x-labels mean the trading strategies from left to right: BH: Buy and Hold, MA: Moving Average, CO: Crossover, MR: Mean Reversion, RT: Range Trading, BO: Break Out, SC: Scalping and RD: Random. In figure 3, we see that

FIG. 3. Comparison of trading strategies for one stock with $\sigma = 8.7 \times 10^{-3}$ and $\mu = 4.56 \times 10^{-4}$. The number of holds at the same time is limited to one.

the Buy and Hold strategy clearly performs best - the corresponding agent gets an average profit of almost 3.5 price units, while the others - while making a profit - make significantly less profit. The agent following the Scalping strategy even loses money.

V. INCREASING NUMBER OF HOLDS

One factor one can influence the performance of specific agents with is the ability to hold multiple shares of a stock at the same time. As explained above, agents were only allowed to hold one share of the stock at the same time so far. This number can be varied. However, at each timestep, the agent still decides to buy **one** share of the stock. She is now allowed to buy a given number of shares before she has to sell shares to keep buying shares. From here on, we will refer to the number of allowed shares of a stock that we are allowed to hold at the same time to the number of holds at the same time.
The number of stocks that have been averaged over is increased from 100 to 500 stocks. Though this takes significantly more time, we notice that the results are far from constant when increasing the number of holds and maintaining the number of stocks we average over at 100. This behaviour does make sense. Agents have, upon having the possibility to hold more stocks at the same time, more chances to exploit the stock's price fluctuations - thus the randomness becomes more prominent. For averaging over 500 stocks, the behaviour does get significantly more, however not completely stable. This will restrict the information value of our results. We obtain, for `numberOfHold = 5` and `numberOfHold = 10` the plots in figure 4 and figure 5. For `numberOfHolds = 5`, we see

FIG. 4. Comparison of trading strategies, where 5 holds at the same time are allowed. Standard values for volatility and drift (eq. 3)

FIG. 5. Comparison of trading strategies, where 10 holds at the same time are allowed. Standard values for volatility and drift (eq. 3)

that the performance of all agents - besides of course the performance of the agent following the Buy and Hold strategy - improves. This is due to the fact that multiple holds at the same time give the agents the opportunity to scale their potential profit by a certain number (not exactly, but almost, five!). We see furthermore that the difference between `numberOfHolds = 5` and `numberOfHolds = 10` is not as big - the difference between figure 4 and 5 is rather small compared to the difference between figure 3 and 4. This shows that agents are not particularly interested in holding significantly more than five shares of a stock at the same time.
Thus, if multiple holds at the same time are allowed, the Mean Reversion and the Breakout strategy seem to be (keeping in mind the limitations introduced by our model) well performing trading strategies.
Of course, there is no guarantee that the parameters of

stock in the real world will be close to the ones we modeled the stock with (equation 3). This analysis is for a stock with very specific parameters.

VI. VARIATION OF THE PARAMETERS

After finishing the analysis of player's performances on one stock, we now vary the parameters (drift μ and volatility σ) of this stock. We looked at historical data of stocks to obtain appropriate parameters for the volatility and the drift. We made use of the yahoo finance [11] collection of past data of stocks. We investigate the performance of agents following the strategies discussed above and vary volatility and drift in the following ranges:

- Volatility: $\sigma \in [4 \times 10^{-3}, 1.3 \times 10^{-2}]$
- Drift: $\mu \in [1 \times 10^{-4}, 8 \times 10^{-4}]$

The differences in the trading strategies are depicted in figures 6, 7, 8 and 9. We have chosen to display the figures for the parameter `numberOfHolds = 1`, because this is the most stable and simple one.

FIG. 6. Comparison of trading strategies for stocks with high drift value ($\mu = 8 \times 10^{-4}$) and unchanged volatility ($\sigma = 8.7 \times 10^{-3}$)

Let's investigate these plots one by one. The plot with increased drift value, figure 6, shows the plot in figure 3 scaled. This makes sense since a high drift corresponds to a stock that gains - on average - more. The relative magnitudes between the trading strategies also changed. This indicates that, if a stock is performing well, it makes increasingly more sense to use strategies other than Buy and Hold.

The plot for a low drift value, figure 7, shows that in this case all agents lose money. The Buy and Hold agent loses the least amount of money. Also, in this plot, we see that the scalping strategy seems to be the worst idea to choose for all parameters and the number of allowed holds - so far.

The plot with high volatility, figure 8, shows strongly varying performances for the different agents. This

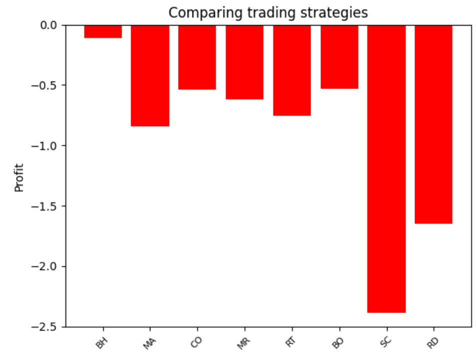

FIG. 7. Comparison of trading strategies for stocks with low drift value ($\mu = 1 \times 10^{-4}$) and unchanged volatility ($\sigma = 8.7 \times 10^{-3}$)

FIG. 8. Comparison of trading strategies for stocks with unchanged drift value ($\mu = 4.56 \times 10^{-4}$) and high volatility ($\sigma = 1.3 \times 10^{-2}$)

makes sense as well, as volatility controls how much the stock varies over time. The results for this particular set of parameters were thus especially unstable (when averaging over - still - 500 stocks).

Lastly, the low volatility stock, figure 9. We see that there is a lot of similarity to the simple case, figure 3, however all strategies (besides the Buy and Hold one) have improved by one price unit. This behaviour can be explained as well. The less the stock varies, the less risky is the investment and thus, the less likely it is for agents to lose money. But of course, the volatility does not affect the total return that a stock has over time.

VII. VARYING THE TIMESTEP

In this section, we want to briefly discuss the effect that the values of the timestep have on the performance of certain trading strategies. The timestep, defined in our case as the number of values in the stock per day, is set to `dt = 1/24`, corresponding to 24 values per day.

FIG. 9. Comparison of trading strategies for stocks with unchanged drift value ($\mu = 4.56 \times 10^{-4}$) and low volatility ($\sigma = 4 \times 10^{-3}$)

We want to investigate, for the *short term* trading strategies, if there are any differences in the performance if we alter the timestep to be smaller. If we choose to update the stock every minute for example, corresponding to a timestep of `dt = 1/(24*60)`, we give the short term trader many more possibilities to sell and to buy. The long term trader who is focused on the moving average over the last 20 days, for example, might not be interested in these new details that he can get out of his stock. The short term trader might very well do. In the following, we will restrict the number of days of trading to 50, since this is enough to investigate the change in performance of the respective trading strategies. Furthermore, to create an environment as realistic as possible for short term traders, we set the allowed number of holds to 10 and the volatility of the stock to high ($\sigma = 1.3 \times 10^{-2}$) and left the drift at the standard value. The resulting comparison is shown in figure 10. As we can see in this figure, the

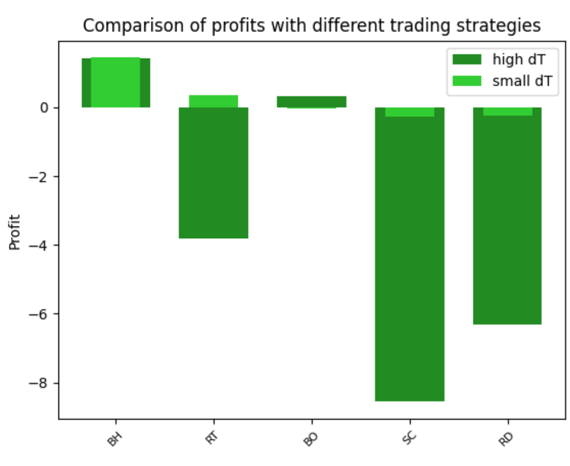

FIG. 10. Influence of the timestep on the performance of short term trading strategies

agents following the short range trading strategies clearly perform better for a smaller timestep `dt = 1/(24*60)`. This exposes one of the (many) limitations of our model. Since the volatility σ is high for this plot, the results are not stable. However, for all runs that were executed, the short term trading strategies performed significantly better on stocks where a smaller timestep was used.

VIII. VALUE AT RISK

In the previous sections, the focus lies on how to maximise profit. However, almost all investments come with risks and uncertainties. In this section, we analyse the gains and losses of different types of stocks from a risk perspective to help investors make sound decisions based on their circumstances.

Risk in investment is the uncertainty about the actual return on investment smaller than the expected return on investment in the future. In the financial industry, risk measurement can be carried out in various ways. One of them is using the Value at Risk (VaR) [12] method. VaR is widely accepted nowadays and considered the standard method of measuring risk. It can be defined as the **estimated maximum loss** that will be obtained during a certain time period under normal market conditions at a certain confidence level.

The three most common methods for calculating VaR are the normal distribution, the historical simulation method, and the exponential weighted moving average (EWMA). The advantage of using the normal distribution method is that it is simple and easy to calculate. The historical simulation method assumes that past gains and losses can continue to reflect the distribution of gains and losses for the next period. The first two methods assume that all past returns have equal weight. The Exponentially Weighted Moving Average (EWMA) method, on the other hand, assigns different weights to returns in an exponentially decreasing manner.

Since we use the given value of drift and volatility, we will use the first method in this section, assuming that asset gains and losses follow a normal distribution, and simulate them using the Monte Carlo method. The Monte Carlo method takes repeated measurements of random variables from a normal distribution to determine the probability of each output and then assigns a confidence interval output.

In the simulation, an agent with 10^6 price units will use the simple Buy and Hold strategy and hold the stock for **ten** trading days. The simulation will show how the volatility and drift values will influence the gains and losses. Figure 11 shows the results of the standard values (given in equation 3) after 10^5 simulations. The biggest loss at confidence level 95% is around 40 189 price units as can be seen in the plot. This is about 4%, which is a very reasonable amount for ten days of trading.

As a next step, the values of volatility and drift are varied and the respective other value is kept constant to obtain four sets of simulation results. The result of this compar-

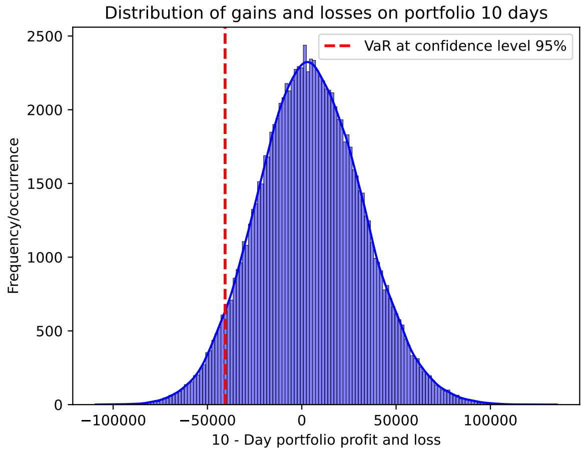

FIG. 11. Standard VaR method; distribution of the profit for 10 days of trading using the volatility and drift given in equation 3.

ison is shown in figure 12. When volatility is high, stock

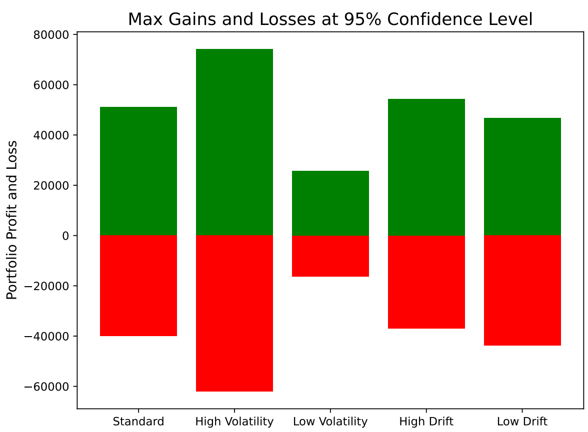

FIG. 12. Comparison of maximal gains and losses for the different stocks from section VI.

prices exhibit greater fluctuations over time, leading to unpredictable price movements. This scenario increases the potential for higher gains at a given price point but simultaneously elevates the risk of substantial losses. In such contexts, a simple Buy and Hold strategy might prove more effective. Conversely, when the volatility is lower, the associated investment risk markedly decreases. This is coherent with our results in section VI. The lower the volatility, the better do the strategies (besides Buy and Hold) perform.
The drift value correlates with the expected return on investment. In scenarios of standard volatility, where higher drift rates are associated with greater gains and fewer losses. This aligns with the simulation results. This is compatible with the results in section VI as well. Drift is directly correlated with total profit, as can be seen in figures 6 and 7.
Practically, risk can be mitigated by diversifying investments across a portfolio comprising different types of stocks, each assigned varying weights, and by selecting an appropriate trading strategy for simulation and evaluation. This, however, lies outside of the scope of this project.

IX. CONCLUSION

We do obtain a number of realistic results using the methods introduced in this report. It has been discussed that the Buy and Hold strategy performs best when the number of held shares of a stock at the same time is restricted to one. If we increase the number of allowed holds, other trading strategies outperform the Buy and Hold strategy. However, only for a positive drift and a volatility that is not too high - so only for very specific parameters. For a low drift for example, it has been shown that the Buy and Hold strategy is by far the best choice. The profit of the trading strategies seems to be a scaled version of the Buy and Hold strategy. If the drift is high and the Buy and Hold agent profits, other agents make even more profit. If it loses or only makes a small amount of profit, other agents lose even more. We see that the risk involved in following these trading strategies is way higher.
We have discussed that the timestep we chose ($dt = 1/24$) is too small to get reasonable results for the short term trading strategies. This exposes one of the limitations of our model.
Also, the fact that even when considering agents being able to hold multiple stocks at the same time, they can - in our model - never buy more than one stock at the same timestep. Depending on the situation, it would make sense on some occasions to buy more stocks, and on other occasions to buy less. For example, if the range is very stable over a long time, this would be an indication for an agent using the Range trading strategy to buy more than one stock at the same timestep.
Lastly, GBM itself is limited in modeling the stocks in the first place. We do have to keep in mind that we are evaluating the performance of the trading strategies using simulations, **not** real-life stocks.

X. FURTHER IDEAS

We want to end this report by introducing two ideas on how to further approach the project.
One could introduce multiple agents that invest in one stock and thus model the influence that agents have on the stock market itself. If the number of agents that buy a certain stock is high enough, the evolution of this stock is affected.

One could furthermore introduce the idea of a portfolio for each agent, in which the agent holds several stocks. Based on the respective trading strategies, agents will distribute their money differently amongst the different constituents of the portfolio.

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