In [31]:
```
#          ##########################################################################################
#          #                                      IN THE NAME OF ALLAH                              #
#          #                                CABLE FOOT SUSPENSION BRIDGE 01                         #
#          #----------------------------------------------------------------------------------------#
#          #                  THIS PROGRAM WRITTEN BY SALAR DELAVAR GHASHGHAEI (QASHQAI)            #
#          #                           EMAIL: salar.d.ghashghaei@gmail.com                          #
#          ##########################################################################################
```

Suspension Foot Bridge:

A Suspension Foot Bridge is a type of bridge that uses the features of a suspension bridge but is specifically designed for pedestrian use. These bridges are typically used to cross natural obstacles like valleys, rivers, or difficult terrain, whereas regular footbridges are simpler in design and construction.

Features of a Suspension Foot Bridge:

1. Suspension Structure:
   - In a suspension footbridge, steel or wire cables are suspended to support the bridge deck. These cables are anchored to tall towers on each side of the bridge, transferring the load from the deck to the towers.
   - Unlike regular footbridges, which are usually made of simple beams (either concrete or steel), a suspension footbridge's primary structure consists of cables and towers.
2. Design Purpose:
   - These bridges are typically designed for pedestrian use, but compared to regular footbridges, they can span greater distances and are ideal for crossing deep valleys, rivers, or other natural barriers.
   - Suspension footbridges are often built in natural parks, hiking trails, or areas that require crossing significant natural obstacles.
3. Load Characteristics:
   - Suspension footbridges generally carry less load than full-scale suspension bridges (designed for vehicles or trains), as they are only built to support pedestrians. However, they still rely on suspended cables to carry the load.
   - The load on these bridges is primarily pedestrian traffic, but in some cases, they might also support bicycles or animals.
4. Advantages:
   - Longer Spans: Suspension footbridges can cover longer distances than regular pedestrian bridges without requiring additional supports in the middle of the span.
   - Crossing Natural Obstacles: These bridges are ideal for crossing rivers, valleys, and rugged terrain where building traditional bridges is difficult.
   - Aesthetic and Integration with Nature: Suspension footbridges often have a beautiful design that blends well with the natural surroundings, especially in tourist or recreational areas.
5. Construction and Installation:
   - Building a suspension footbridge requires specialized equipment for installing cables and towers. These bridges are commonly used in areas where the terrain is difficult, or access to traditional infrastructure is limited.
   - Installing cables and towers in such challenging locations requires precise engineering and safety considerations.

Examples: Suspension Foot Bridges in Mountainous Areas: Suspension footbridges are commonly found in mountainous regions or nature reserves where they allow pedestrians to cross valleys and rivers. These bridges provide access for both locals and tourists to difficult-to-reach areas.

2. Suspension Footbridges in Parks: In some natural parks or recreational areas, suspension footbridges are used to cross rivers or valleys. These bridges serve both a practical purpose and act as tourist attractions.

Differences from Regular Suspension Bridges:

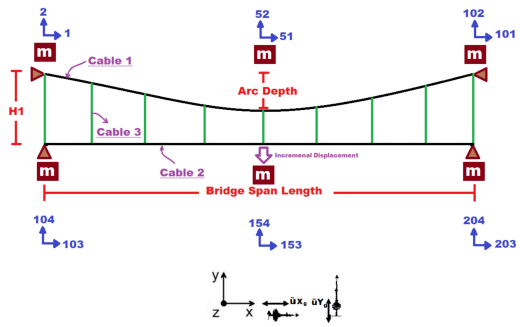
- Lower Load Capacity: While regular suspension bridges are designed to carry vehicles, trains, or heavy loads, and can bear significant stress, suspension footbridges are designed only for pedestrian use and therefore carry much lighter loads.
- Smaller Dimensions: Suspension footbridges are generally smaller in scale and have shorter spans than full-size suspension bridges. They are primarily intended for pedestrians and are rarely used for vehicle traffic.

Conclusion: A Suspension Foot Bridge is a type of suspension bridge specifically designed for pedestrian use. Due to the use of suspended cables, these bridges can span longer distances and are very useful in areas where crossing valleys, rivers, or difficult terrain is necessary.

In [32]:
```
# Load the image
def PLOT_IMAGE(image):
    import matplotlib.pyplot as plt
    import matplotlib.image as mpimg
    image = mpimg.imread(image_path)

    # Display the image
    plt.figure(figsize=(15, 8))
    plt.imshow(image)
    plt.axis('off')  # Hide axes
    plt.show()

image_path = 'OPENSEES_CABLE_SUSPENSION_BRIDGE_01.png'
PLOT_IMAGE(image_path)
```



In [33]:
```
# WIKIPEDIA: Simple suspension bridge
'https://en.wikipedia.org/wiki/Simple_suspension_bridge'
# IMAGE:
'https://www.waratahbridgeconstructions.com.au/pedestrian-bridges-north-east-victoria.html'
'https://www.moodie.com.au/?product=cable-bridges'
```

```
# PAPER: Study on Suspension Bridge: Analysis and Construction criteria
'https://www.ijsdr.org/papers/IJSDR1704077.pdf'
```

Out[33]:
```
'https://www.ijsdr.org/papers/IJSDR1704077.pdf'
```

In [34]:
```
#import the os module
import os
import time
import numpy as np
import openseespy.opensees as op
import matplotlib.pyplot as plt
```

In [35]:
```
#to create a directory at specified path with name "Data"
os.mkdir('C:\\OPENSEESPY_SALAR')
```

```
---------------------------------------------------------------------------
FileExistsError                           Traceback (most recent call last)
Cell In[35], line 2
      1 #to create a directory at specified path with name "Data"
----> 2 os.mkdir('C:\\OPENSEESPY_SALAR')

FileExistsError: [WinError 183] Cannot create a file when that file already exists: 'C:\\OPENSEESPY_SALAR'
```

In [36]:
```
FOLDER_NAME = 'OPENSEES_CABLE_SUSPENSION_BRIDGE'
dir = f"C:\\OPENSEESPY_SALAR\\{FOLDER_NAME}\\"
if not os.path.exists(dir):
    os.makedirs(dir)
```

In [37]:
```
# OUTPUT DATA ADDRESS:
SALAR_DIR = f'C://OPENSEESPY_SALAR//{FOLDER_NAME}//';
```

In [38]:
```
## DELETE ALL FILES IN DIRECTORY
def DELETE_FOLDER_CONTANTS(folder_path):
    import os
    for filename in os.listdir(folder_path):
        file_path = os.path.join(folder_path, filename)
        try:
            if os.path.isfile(file_path) or os.path.islink(file_path):
                os.unlink(file_path)
        except Exception as e:
            print(f"Failed to delete {file_path}. Reason: {e}")
    print("Deletion done")

FOLDER_PATH = f'C:\\OPENSEESPY_SALAR\\{FOLDER_NAME}'  # Specify the folder path
#DELETE_FOLDER_CONTANTS(FOLDER_PATH)
```

In [39]:
```
def CURRENT_TIME():
    import time
    t = time.localtime()
    current_time = time.strftime("%H:%M:%S", t)
    print(f"Current time (HH:MM:SS): {current_time}\n\n")

# ---------------------
def plot_shapes(initial_coords, displacements):
    import numpy as np
    import matplotlib.pyplot as plt
    """
    Plot the initial and deformed shapes of the arch.

    Parameters:
    - initial_coords: Initial coordinates of the nodes.
    - displacements: List of displacements for each node.
    """
    # Plot initial and deformed shape
    plt.figure(figsize=(12, 8))
    plt.plot(initial_coords[:num_nodes, 0], initial_coords[:num_nodes, 1], 'bo-', label='Initial Cable Shape')
    plt.plot(deformed_coords[:num_nodes, 0], deformed_coords[:num_nodes, 1], 'ro-', label='Deformed Cable Shape')
    plt.plot(initial_coords[num_nodes:, 0], initial_coords[num_nodes:, 1], 'go-', label='Initial Deck Shape')
    plt.plot(deformed_coords[num_nodes:, 0], deformed_coords[num_nodes:, 1], 'yo-', label='Deformed Deck Shape')

    # Plot connecting elements between the cable and the beam
    for i in range(num_nodes):
        plt.plot([initial_coords[i, 0], initial_coords[num_nodes + i, 0]],
                 [initial_coords[i, 1], initial_coords[num_nodes + i, 1]], 'c--', label='Initial Connecting Elements' if i == 0 else "")
        plt.plot([deformed_coords[i, 0], deformed_coords[num_nodes + i, 0]],
                 [deformed_coords[i, 1], deformed_coords[num_nodes + i, 1]], 'm--', label='Deformed Connecting Elements' if i == 0 else "")

    # Mark simply supported nodes with red triangles
    support_nodes = [0, num_nodes - 1, num_nodes, 2*num_nodes - 1]
    plt.plot(initial_coords[support_nodes, 0], initial_coords[support_nodes, 1], 'r^', markersize=20, label='Simply Supported Nodes')

    plt.xlabel('X [mm]')
    plt.ylabel('Y [mm]')
    plt.legend()
    plt.title('Pushover Analysis: Cable and Deck Deformed Shapes')
    plt.show()

# ---------------------

def plot_reactions(disp_x, reaction_x, disp_y, reaction_y):
    import matplotlib.pyplot as plt
    """
    Plot the base reaction versus displacement.

    Parameters:
    - disp_x: List of x displacements of the middle node.
    - reaction_x: List of base reactions in x.
    - disp_y: List of y displacements of the middle node.
    - reaction_y: List of base reactions in y.
    """
    plt.figure(figsize=(10, 6))
    plt.plot(disp_x, reaction_x, 'b-')
    plt.xlabel('Displacement X [mm]')
    plt.ylabel('Base Reaction X [N]')
    plt.title('Base Reaction X vs. Displacement X')
    plt.show()

    plt.figure(figsize=(10, 6))
    plt.plot(disp_y, reaction_y, 'r-')
    plt.xlabel('Displacement Y [mm]')
    plt.ylabel('Base Reaction Y [N]')
```

```
    plt.title('Base Reaction Y vs. Displacement Y')
    plt.show()

# --------------------
"""
When OK equals -1, it generally indicates that the command or operation was not executed
because it was already in progress or had already been completed. This can happen if you
try to run a command that is already running or has been completed in a previous step.

When OK equals -2, it typically indicates that the command or operation was not executed
because it was not recognized or not implemented. This could mean that the command
is either misspelled, not available in the current version of OpenSees, or not applicable to the current context.

When OK equals -3, it typically means that the command or operation failed.
This could be due to various reasons, such as incorrect input parameters,
syntax errors, or issues with the model setup.
"""
def ANALYSIS(OK, INCREMENT, TOLERANCE, MAX_ITERAIONS):
    import openseespy.opensees as op
    test = {1:'NormDispIncr', 2: 'RelativeEnergyIncr', 4: 'RelativeNormUnbalance',5: 'RelativeNormDispIncr', 6: 'NormUnbalance'}
    algorithm = {1:'KrylovNewton', 2: 'SecantNewton' , 4: 'RaphsonNewton',5: 'PeriodicNewton', 6: 'BFGS', 7: 'Broyden', 8: 'NewtonLineSearch'}

    for i in test:
        for j in algorithm:
            if OK != 0:
                if j < 4:
                    op.algorithm(algorithm[j], '-initial')

                else:
                    op.algorithm(algorithm[j])

                op.test(test[i], TOLERANCE, MAX_ITERAIONS)
                OK = op.analyze(INCREMENT)
                print(test[i], algorithm[j], OK)
                if OK == 0:
                    break
            else:
                continue
```

In [ ]:

```
#          ##########################################################################################
#          #                                          IN THE NAME OF ALLAH                          #
#          #                         PUSHOVER ANALYSIS OF CABLE SUSPENSION BRIDGE 01                #
#          #----------------------------------------------------------------------------------------#
#          #                  THIS PROGRAM WRITTEN BY SALAR DELAVAR GHASHGHAEI (QASHQAI)            #
#          #                            EMAIL: salar.d.ghashghaei@gmail.com                         #
#          ##########################################################################################
```

In [40]:

```
# --------------------
#   PUSHOVER ANALYSIS
# --------------------
def PUSHOVER_ANALYSIS(LINEAR, L, H1, arc_depth, E_cable, Cable_Dia_01, Cable_Dia_02, Cable_Dia_03, num_nodes, MAX_DISP, disp_incr, MAX_ITERATIONS, TOLERANCE):
    import openseespy.opensees as ops
    import numpy as np
    import matplotlib.pyplot as plt

    A_cable_01 = (np.pi * Cable_Dia_01 **2) / 4  # [mm^2] Horizontal Longitudinal Cable Area Bottom
    A_cable_02 = (np.pi * Cable_Dia_02 **2) / 4  # [mm^2] Horizontal Longitudinal Cable Area Top
    A_cable_03 = (np.pi * Cable_Dia_03 **2) / 4  # [mm^2] Vertical Longitudinal Cable Area Top

    # Define model builder
    ops.wipe()
    ops.model('basic', '-ndm', 2, '-ndf', 2)
    dx = L / (num_nodes - 1)

    # Create nodes
    for i in range(num_nodes):
        x = i * dx
        y = H1 - arc_depth * np.sin(np.pi * x / L)
        ops.node(i + 1, x, y)
        ops.node(num_nodes + i + 1, i * dx, 0.0)

    # Define boundary conditions (fixed at both ends)
    ops.fix(1, 1, 1)              # TOP CABLE
    ops.fix(num_nodes, 1, 1)      # TOP CABLE
    ops.fix(num_nodes + 1, 1, 1)  # BOTTOM CABLE - DECK
    ops.fix(2 * num_nodes, 1, 1)  # BOTTOM CABLE - DECK

    # Define material properties
    if LINEAR == True:
        ops.uniaxialMaterial('Elastic', 1, E_cable)
        #                                 TENSION    COMPRESSION
        #ops.uniaxialMaterial('Elastic', 1, E_cable, 0, 0.5 * E_cable)
    if LINEAR == False:
        Fy_cable = 355   # [N/mm^2] Yield strength of the cable
        b0 = 0.01
        ops.uniaxialMaterial('Steel01', 1, Fy_cable, E_cable, b0)

    # Define truss elements
    for i in range(num_nodes - 1):
        ops.element('corotTruss', i + 1, i + 1, i + 2, A_cable_02, 1)  # Cable element
        ops.element('corotTruss', num_nodes + i + 1, num_nodes + i + 1, num_nodes + i + 2, A_cable_01, 1)  # Deck element

    # Connect each node of the cable with the corresponding node of the deck using truss elements
    for i in range(num_nodes):
        ops.element('corotTruss', 2 * num_nodes + i + 1, i + 1, num_nodes + i + 1, A_cable_03, 1)

    #mid_node = num_nodes // 2 + 1
    mid_node = int(num_nodes + 0.5 * num_nodes)
    # Define load pattern with displacement at the middle span
    ops.timeSeries('Linear', 1)
    ops.pattern('Plain', 1, 1)
    ops.load(mid_node, 0.0, -1)  # Apply a horizontal load at node
    disp_incr = 0.01    # [mm] Incremental Displacement
    n_steps = int(np.abs(MAX_DISP / disp_incr))  # Analysis Steps

    # Define analysis parameters
    ops.system('BandGeneral')
    ops.numberer('Plain')
    ops.constraints('Plain')
    ops.integrator('DisplacementControl', mid_node, 2, -disp_incr)
    ops.test('EnergyIncr', TOLERANCE, MAX_ITERATIONS)
    ops.algorithm('ModifiedNewton')
```

```
    ops.analysis('Static')

    # OUTPUT DATA
    ops.recorder('Node', '-file', f"{SALAR_DIR}DTH_PUSH.txt",'-time', '-node', mid_node, '-dof', 1,2, 'disp')# Displacement Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_PUSH_01.txt",'-time', '-node', 1, '-dof', 1,2, 'reaction')# Base Shear Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_PUSH_02.txt",'-time', '-node', num_nodes, '-dof', 1,2, 'reaction')# Base Shear Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_PUSH_03.txt",'-time', '-node', num_nodes + 1, '-dof', 1,2, 'reaction')# Base Shear Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_PUSH_04.txt",'-time', '-node', 2*num_nodes, '-dof', 1,2, 'reaction')# Base Shear Time History

    DISPLACEMENTS = []
    DISP_X , DISP_Y = [], []
    BASE_REACTION_X, BASE_REACTION_Y = [], []

    # Perform the analysis with increments
    for i in range(n_steps):
        OK = ops.analyze(1)
        ANALYSIS(OK, 1, TOLERANCE, MAX_ITERATIONS)
        DISPLACEMENTS.append([ops.nodeDisp(j + 1) for j in range(num_nodes * 2)])
        DISP_X.append(ops.nodeDisp(mid_node, 1))
        DISP_Y.append(ops.nodeDisp(mid_node, 2))
        x1 = ops.nodeResponse(1, 1, 6)
        x2 = ops.nodeResponse(num_nodes, 1, 6)
        x3 = ops.nodeResponse(num_nodes + 1, 1, 6)
        x4 = ops.nodeResponse(2*num_nodes, 1, 6)

        y1 = ops.nodeResponse(1, 2, 6)
        y2 = ops.nodeResponse(num_nodes, 2, 6)
        y3 = ops.nodeResponse(num_nodes + 1, 2, 6)
        y4 = ops.nodeResponse(2*num_nodes, 2, 6)
        BASE_REACTION_X.append(x1+x2+x3+x4) # CABLE REACION-X
        BASE_REACTION_Y.append(y1+y2+y3+y4) # CABLE REACION-Y
        #print(f'STEP: {i+1}')

    # Get initial and final node coordinates for plotting
    initial_coords = np.array([ops.nodeCoord(i + 1) for i in range(num_nodes * 2)])
    deformed_coords = initial_coords + np.array(DISPLACEMENTS[-1])

    # Output all unconstrained node displacements
    for i in range(num_nodes * 2):
        disp = ops.nodeDisp(i + 1)
        print(f"Node {i + 1}: X Displacement = {disp[0]}, Y Displacement = {disp[1]}")

    return initial_coords, deformed_coords, DISPLACEMENTS, DISP_X, DISP_Y, BASE_REACTION_X, BASE_REACTION_Y
```

In [41]:

```
# --------------------
#   PUSHOVER ANALYSIS
# --------------------
# Parameters for the analysis
L = 50000.0        # [mm] Bridge span lentgh
H1 = 3500.0        # [mm] Height of Top Cable
arc_depth = 1000.0 # [mm]
E_cable = 210e5    # [N/mm^2] Modulus of elasticity Cable
Cable_Dia_01 = 25  # [mm] Horizontal Longitudinal Cable Diameter Bottom
Cable_Dia_02 = 18  # [mm] Horizontal Longitudinal Cable Diameter Top
Cable_Dia_03 = 10  # [mm] Vertical Longitudinal Cable Diameter Top
num_nodes = 51     # Cable Arc Number of nodes
MAX_DISP = -500.0  # [mm] Maximum Displacment
disp_incr = -0.01  # [mm] Incremental Displacement

MAX_ITERATIONS = 50000  # Maximum number of iterations
TOLERANCE = 1.0e-10     # Tolerance for convergence

LINEAR = True # False: Cable Nonlinear Materials Properties

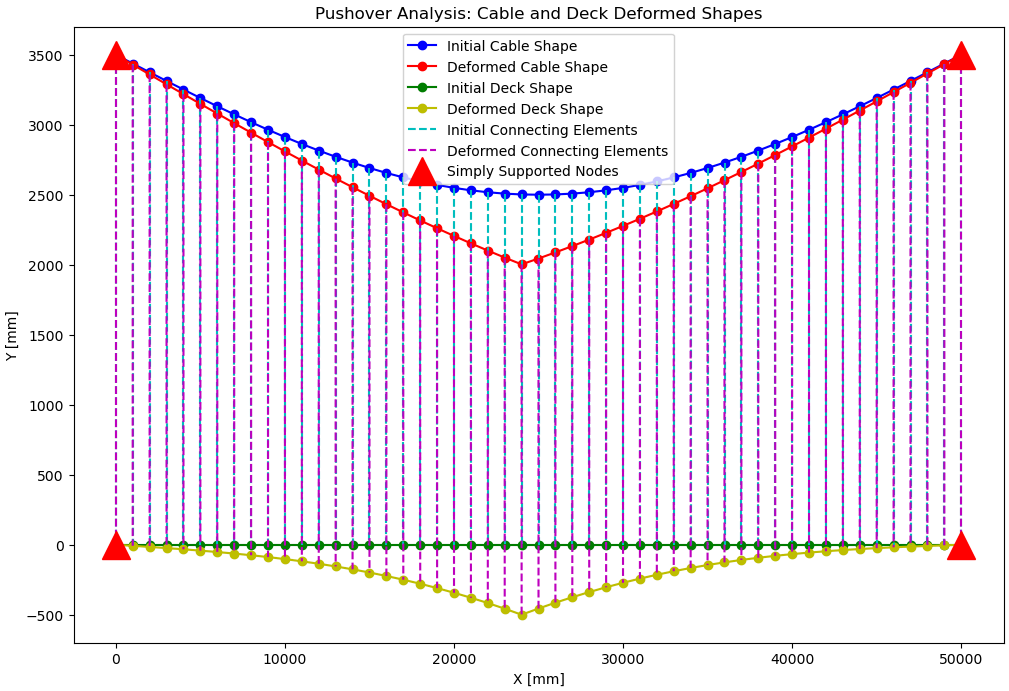
starttime = time.process_time()
# Run the analysis
results = PUSHOVER_ANALYSIS(LINEAR, L, H1, arc_depth, E_cable, Cable_Dia_01, Cable_Dia_02, Cable_Dia_03, num_nodes, MAX_DISP, disp_incr, MAX_ITERATIONS, TOLERANCE)
initial_coords, deformed_coords, DISPLACEMENTS, DISP_X, DISP_Y, BASE_REACTION_X, BASE_REACTION_Y = results
totaltime = time.process_time() - starttime
print(f'\nTotal time (s): {totaltime:.4f} \n\n')
```

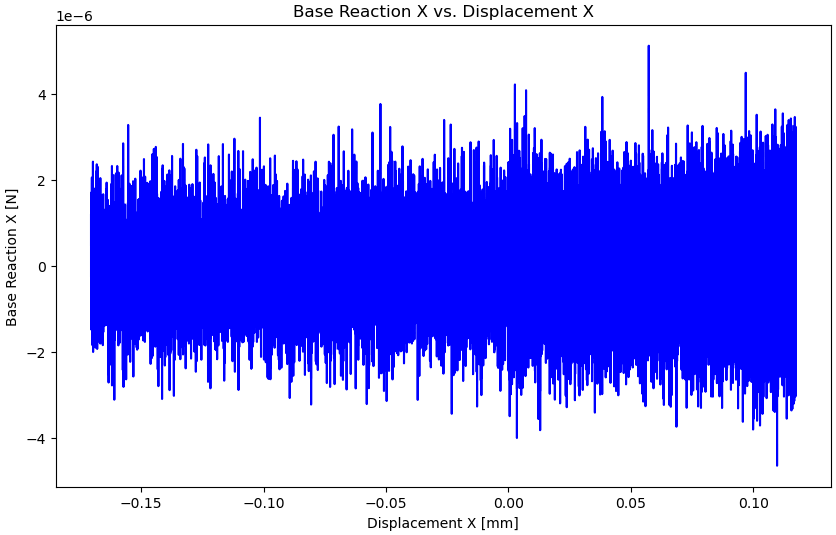
```
WARNING: CTestEnergyIncr::test() - failed to converge
after: 50000 iterations
 current EnergyIncr: -nan(ind) (max: 1e-10)     Norm deltaX: nan, Norm deltaR: nan
ModifiedNewton::solveCurrentStep() -the ConvergenceTest object failed in test()
StaticAnalysis::analyze() - the Algorithm failed at step: 0 with domain at load factor -nan(ind)
OpenSees > analyze failed, returned: -3 error flag
```

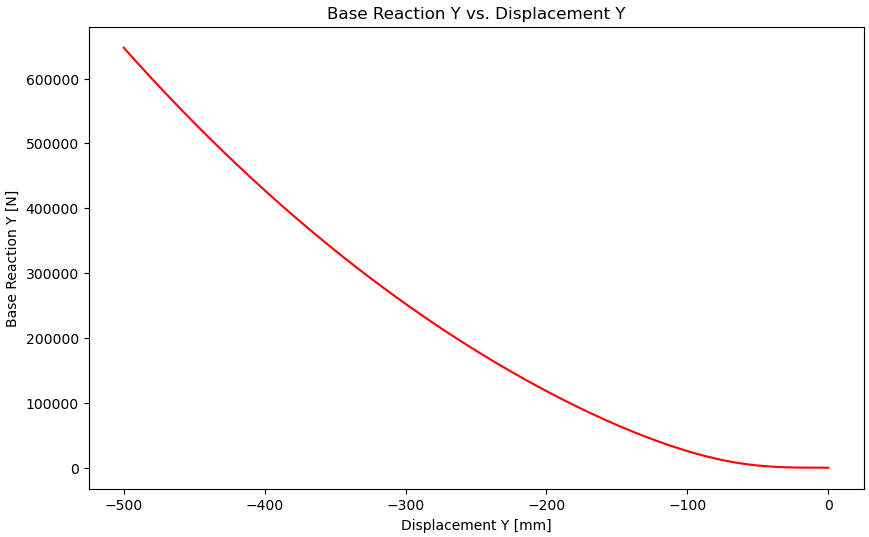
```
NormDispIncr KrylovNewton 0
Node 1: X Displacement = 0.0, Y Displacement = 0.0
Node 2: X Displacement = 0.3199093321564988, Y Displacement = -7.806389908584831
Node 3: X Displacement = 0.631027760133854, Y Displacement = -15.764664966975404
Node 4: X Displacement = 0.9247555455256964, Y Displacement = -24.026106496111915
Node 5: X Displacement = 1.1928702307870171, Y Displacement = -32.74081689215971
Node 6: X Displacement = 1.4277073923384542, Y Displacement = -42.057135691408995
Node 7: X Displacement = 1.622332110862012, Y Displacement = -52.12106165685267
Node 8: X Displacement = 1.770698284729665, Y Displacement = -63.07568290146361
Node 9: X Displacement = 1.8677931008152302, Y Displacement = -75.06061701609693
Node 10: X Displacement = 1.9097642032318738, Y Displacement = -88.21146317507215
Node 11: X Displacement = 1.8940273651560144, Y Displacement = -102.65926820367797
Node 12: X Displacement = 1.8193527683809405, Y Displacement = -118.53000860839629
Node 13: X Displacement = 1.6859283256916666, Y Displacement = -135.94409059115955
Node 14: X Displacement = 1.4953988381728838, Y Displacement = -155.01587009100749
Node 15: X Displacement = 1.250880158343515, Y Displacement = -175.85319491640485
Node 16: X Displacement = 0.956947925340774, Y Displacement = -198.556971044306
Node 17: X Displacement = 0.6196008446537753, Y Displacement = -223.2207551615983
Node 18: X Displacement = 0.2461988961689911, Y Displacement = -249.93037550380964
Node 19: X Displacement = -0.15462273573297428, Y Displacement = -278.7635829973928
Node 20: X Displacement = -0.5730628141799169, Y Displacement = -309.78973462838667
Node 21: X Displacement = -0.9982847005899349, Y Displacement = -343.0695108359222
Node 22: X Displacement = -1.4185665547848574, Y Displacement = -378.6546685562685
Node 23: X Displacement = -1.821468717721201, Y Displacement = -416.58782978268636
Node 24: X Displacement = -2.1939860062174326, Y Displacement = -456.9017142816989
Node 25: X Displacement = -2.511670554771266, Y Displacement = -499.3906115988735
Node 26: X Displacement = -2.547112242136458, Y Displacement = -455.62603297097127
Node 27: X Displacement = -2.657280625809106, Y Displacement = -414.0553153571602
Node 28: X Displacement = -2.8191839800275544, Y Displacement = -374.90342719373695
Node 29: X Displacement = -3.0193758337812246, Y Displacement = -338.1565908826925
Node 30: X Displacement = -3.244595967746345, Y Displacement = -303.79090227054826
Node 31: X Displacement = -3.4819469820560487, Y Displacement = -271.7730186968144
Node 32: X Displacement = -3.7190877861838976, Y Displacement = -242.06029416544825
Node 33: X Displacement = -3.9444152535021972, Y Displacement = -214.60095197313652
Node 34: X Displacement = -4.147231127818948, Y Displacement = -189.3342954123924
Node 35: X Displacement = -4.317891598800236, Y Displacement = -166.19095535900598
Node 36: X Displacement = -4.447937265183557, Y Displacement = -145.0931733086479
Node 37: X Displacement = -4.530201541491634, Y Displacement = -125.95511822781721
Node 38: X Displacement = -4.55889592913094, Y Displacement = -108.68323542832194
Node 39: X Displacement = -4.529670959493224, Y Displacement = -93.17662556083904
Node 40: X Displacement = -4.439652018139383, Y Displacement = -79.32745174773434
Node 41: X Displacement = -4.287449668585613, Y Displacement = -67.02137283210837
Node 42: X Displacement = -4.073144505121741, Y Displacement = -56.13800070193991
Node 43: X Displacement = -3.7982469701719217, Y Displacement = -46.55137964796155
Node 44: X Displacement = -3.4656329670143715, Y Displacement = -38.13048572492054
Node 45: X Displacement = -3.0794564775824953, Y Displacement = -30.739744102436582
Node 46: X Displacement = -2.6450407523323833, Y Displacement = -24.239562409355027
Node 47: X Displacement = -2.1687499699352424, Y Displacement = -18.486878091067872
Node 48: X Displacement = -1.6578435644342842, Y Displacement = -13.335717810695044
Node 49: X Displacement = -1.1203156825277303, Y Displacement = -8.637766931193015
Node 50: X Displacement = -0.5647224603638313, Y Displacement = -4.242947115834292
Node 51: X Displacement = 0.0, Y Displacement = 0.0
Node 52: X Displacement = 0.0, Y Displacement = 0.0
Node 53: X Displacement = 0.24112605465504822, Y Displacement = -7.8072747869822665
Node 54: X Displacement = 0.48105567210462, Y Displacement = -15.766397713668386
Node 55: X Displacement = 0.7185279680729767, Y Displacement = -24.02864811754546
Node 56: X Displacement = 0.9521548079230172, Y Displacement = -32.744127433061436
Node 57: X Displacement = 1.1803604652083914, Y Displacement = -42.06117471138081
Node 58: X Displacement = 1.4013251633564807, Y Displacement = -52.12578815303226
Node 59: X Displacement = 1.6129337000797217, Y Displacement = -63.081054631283514
Node 60: X Displacement = 1.8127302728272177, Y Displacement = -75.06658928193129
Node 61: X Displacement = 1.9978805244068158, Y Displacement = -88.21798725340128
Node 62: X Displacement = 2.1651417115164655, Y Displacement = -102.6662897199856
Node 63: X Displacement = 2.310841766928375, Y Displacement = -118.53746625716987
Node 64: X Displacement = 2.4308678803800152, Y Displacement = -135.95191565820377
Node 65: X Displacement = 2.520665066063016, Y Displacement = -155.02398723317057
Node 66: X Displacement = 2.575245018423436, Y Displacement = -175.86152457499296
Node 67: X Displacement = 2.5892053854758474, Y Displacement = -198.56543370202027
Node 68: X Displacement = 2.5567594127425624, Y Displacement = -223.22927739674802
Node 69: X Displacement = 2.4717757340734416, Y Displacement = -249.9388974592113
Node 70: X Displacement = 2.3278279106821866, Y Displacement = -278.7720664873375
Node 71: X Displacement = 2.1182531493437795, Y Displacement = -309.79817069104934
Node 72: X Displacement = 1.8362194672132621, Y Displacement = -343.07792514772893
Node 73: X Displacement = 1.4748004161407329, Y Displacement = -378.6631228239429
Node 74: X Displacement = 1.027056242454488, Y Displacement = -416.59642107811
Node 75: X Displacement = 0.4860821835797728, Y Displacement = -456.9121116977464
Node 76: X Displacement = -0.17053444703073659, Y Displacement = -499.999999969026
Node 77: X Displacement = -0.8828766685547187, Y Displacement = -455.63735237292235
Node 78: X Displacement = -1.4752667052045902, Y Displacement = -414.06533383497389
Node 79: X Displacement = -1.9699453163437748, Y Displacement = -374.91354199889855
Node 80: X Displacement = -2.37334110328229, Y Displacement = -338.1667019131928
Node 81: X Displacement = -2.6920666666837616, Y Displacement = -303.80094547678414
Node 82: X Displacement = -2.932865023031619, Y Displacement = -271.78294601563357
Node 83: X Displacement = -3.1025195899647424, Y Displacement = -242.07006185542613
Node 84: X Displacement = -3.207768280920555, Y Displacement = -214.61051514818658
Node 85: X Displacement = -3.255223075297706, Y Displacement = -189.34360697575613
Node 86: X Displacement = -3.2512962214099574, Y Displacement = -166.19996759809206
Node 87: X Displacement = -3.2021341003860795, Y Displacement = -145.10184063433705
Node 88: X Displacement = -3.1135596365658453, Y Displacement = -125.96339987017544
Node 89: X Displacement = -2.991023985646228, Y Displacement = -108.6910972796879
Node 90: X Displacement = -2.8395680679613786, Y Displacement = -93.18404073981579
Node 91: X Displacement = -2.6637943431529436, Y Displacement = -79.33439979813383
Node 92: X Displacement = -2.4678490466121494, Y Displacement = -67.02783774405411
Node 93: X Displacement = -2.255414930161023, Y Displacement = -56.14396812890243
Node 94: X Displacement = -2.029714372321433, Y Displacement = -46.55683379065218
Node 95: X Displacement = -1.7935225500832521, Y Displacement = -38.135406366880446
Node 96: X Displacement = -1.5491901971652293, Y Displacement = -30.744104226997216
Node 97: X Displacement = -1.29867531605593, Y Displacement = -24.24332672229692
Node 98: X Displacement = -1.043583065104162, Y Displacement = -18.490002638388983
Node 99: X Displacement = -0.7852129097940771, Y Displacement = -13.338150736543284
Node 100: X Displacement = -0.5246120109768905, Y Displacement = -8.639450284798523
Node 101: X Displacement = -0.2626337237928343, Y Displacement = -4.243819502893362
Node 102: X Displacement = 0.0, Y Displacement = 0.0

Total time (s): 89.4219
```

In [42]:
```
# Plot results
plot_shapes(initial_coords, DISPLACEMENTS)
plot_reactions(DISP_X, BASE_REACTION_X, DISP_Y, BASE_REACTION_Y)
```







In [24]:
```
#          ##########################################################################################
#          #                                    IN THE NAME OF ALLAH                                  #
#          #                     FREE VIBRATION ANALYSIS OF CABLE SUSPENSION BRIDGE 01               #
#          #-----------------------------------------------------------------------------------------#
#          #                THIS PROGRAM WRITTEN BY SALAR DELAVAR GHASHGHAEI (QASHQAI)               #
```

```
#         #                          EMAIL: salar.d.ghashghaei@gmail.com                          #
#         ########################################################################################
```

In [25]:

```
# -------------------------
#   FREE VIBRATION ANALYSIS
# -------------------------
def FREE_VIBRATION_ANALYSIS(damping, damping_ratio, LINEAR, duration, dt, TOTAL_MASS, u0, L, H1, arc_depth, E_cable, Cable_Dia_01, Cable_Dia_02, Cable_Dia_03, num_nodes, MAX_ITERATIONS
    import openseespy.opensees as ops
    import numpy as np
    import matplotlib.pyplot as plt

    A_cable_01 = (np.pi * Cable_Dia_01 **2) / 4  # [mm^2] Horizontal Longitudinal Cable Area Bottom
    A_cable_02 = (np.pi * Cable_Dia_02 **2) / 4  # [mm^2] Horizontal Longitudinal Cable Area Top
    A_cable_03 = (np.pi * Cable_Dia_03 **2) / 4  # [mm^2] Vertical Longitudinal Cable Area Top

    # Define model builder
    ops.wipe()
    ops.model('basic', '-ndm', 2, '-ndf', 2)
    KE = 1;
    dx = L / (num_nodes - 1)
    MASS = TOTAL_MASS / num_nodes
    # Create nodes
    for i in range(num_nodes):
        X = i * dx
        Y1 = H1 - arc_depth * np.sin(np.pi * X / L)
        ops.node(i + 1, X, Y1)
        ops.node(num_nodes + i + 1, i * dx, 0.0)
        # Define mass
        ops.mass(num_nodes + i + 1, MASS, MASS, 0)

    # Define boundary conditions (fixed at both ends)
    ops.fix(1, 1, 1)              # TOP CABLE
    ops.fix(num_nodes, 1, 1)      # TOP CABLE
    ops.fix(num_nodes + 1, 1, 1)  # BOTTOM CABLE
    ops.fix(2 * num_nodes, 1, 1)  # BOTTOM CABLE

    # Define material properties
    if LINEAR == True:
        ops.uniaxialMaterial('Elastic', 1, E_cable)
        #                                 TENSION    COMPRESSION
        #ops.uniaxialMaterial('Elastic', 1, E_cable, 0, 0.5 * E_cable)
    if LINEAR == False:
        Fy_cable = 355   # [N/mm^2] Yield strength of the cable
        b0 = 0.01
        ops.uniaxialMaterial('Steel01', 1, Fy_cable, E_cable, b0)

    # Define truss elements
    for i in range(num_nodes - 1):
        ops.element('corotTruss', i + 1, i + 1, i + 2, A_cable_02, 1)  # Top Cable element
        ops.element('corotTruss', num_nodes + i + 1, num_nodes + i + 1, num_nodes + i + 2, A_cable_01, 1)  # Bottom Cable Element

    # Connect each node of the cable with the corresponding node of the deck using truss elements
    for i in range(num_nodes):
        ops.element('corotTruss', 2 * num_nodes + i + 1, i + 1, num_nodes + i + 1, A_cable_03, 1) # Vertical Cable Element

    #mid_node = num_nodes // 2 + 1
    mid_node = int(num_nodes + 0.5 * num_nodes)
    # Define load pattern with displacement at the middle span
    ops.timeSeries('Linear', 1)
    ops.pattern('Plain', 1, 1)
    ops.load(mid_node, 0.0, -1)  # Apply a horizontal load at node
    ops.constraints('Transformation')
    ops.numberer('RCM')
    ops.system('BandGeneral')
    ops.algorithm('Newton')
    ops.test('EnergyIncr', TOLERANCE, MAX_ITERATIONS)
    ops.integrator('DisplacementControl', mid_node, 2, u0)
    ops.analysis('Static')
    ops.analyze(1)

    ops.setTime(0.0)

    # Wipe analysis and reset time
    ops.wipeAnalysis()
    ops.remove('loadPattern', 1)
    ops.system('UmfPack')

    # Dynamic analysis
    ops.constraints('Transformation')
    ops.numberer('RCM')
    ops.system('UmfPack')
    ops.test('EnergyIncr', TOLERANCE, MAX_ITERATIONS)
    ops.integrator('Newmark', 0.5, 0.25)
    ops.algorithm('Newton')

    if damping == True:
        # Calculate Rayleigh damping factors
        omega1 = np.sqrt(KE / TOTAL_MASS)
        omega2 = 2 * omega1  # Just an assumption for two modes
        a0 = damping_ratio * (2 * omega1 * omega2) / (omega1 + omega2)
        a1 = damping_ratio * 2 / (omega1 + omega2)
        # Apply Rayleigh damping
        ops.rayleigh(a0, a1, 0, 0)

    ops.analysis('Transient')

    # OUTPUT DATA
    ops.recorder('Node', '-file', f"{SALAR_DIR}DTH_DYN.txt",'-time', '-node', mid_node, '-dof', 1,2, 'disp')# Displacement Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}VTH_DYN.txt",'-time', '-node', mid_node, '-dof', 1,2,3, 'vel')  # Velocity Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}ATH_DYN.txt",'-time', '-node', mid_node, '-dof', 1,2,3, 'accel') # Acceleration Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_DYN_01.txt",'-time', '-node', 1, '-dof', 1,2, 'reaction')# Base Shear Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_DYN_02.txt",'-time', '-node', num_nodes, '-dof', 1,2, 'reaction')# Base Shear Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_DYN_03.txt",'-time', '-node', num_nodes + 1, '-dof', 1,2, 'reaction')# Base Shear Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_DYN_04.txt",'-time', '-node', 2*num_nodes, '-dof', 1,2, 'reaction')# Base Shear

    DISPLACEMENTS = []
    DISP_X , DISP_Y, VELOCITY, ACCELERATION = [], [], [], []
    BASE_REACTION_X, BASE_REACTION_Y = [], []

    stable = 0
    current_time = 0.0
     # Perform the analysis with increments
```

```
    while stable == 0 and current_time < duration:
        OK = ops.analyze(1, dt)
        ANALYSIS(OK, 1, TOLERANCE, MAX_ITERATIONS)
        current_time = ops.getTime()
        DISPLACEMENTS.append([ops.nodeDisp(j + 1) for j in range(num_nodes * 2)])
        DISP_X.append(ops.nodeDisp(mid_node, 1))
        DISP_Y.append(ops.nodeDisp(mid_node, 2))
        VELOCITY.append(ops.nodeVel(mid_node, 2))
        ACCELERATION.append(ops.nodeAccel(mid_node, 2))
        x1 = ops.nodeResponse(1, 1, 6)
        x2 = ops.nodeResponse(num_nodes, 1, 6)
        x3 = ops.nodeResponse(num_nodes + 1, 1, 6)
        x4 = ops.nodeResponse(2*num_nodes, 1, 6)
        y1 = ops.nodeResponse(1, 2, 6)
        y2 = ops.nodeResponse(num_nodes, 2, 6)
        y3 = ops.nodeResponse(num_nodes + 1, 2, 6)
        y4 = ops.nodeResponse(2*num_nodes, 2, 6)
        BASE_REACTION_X.append(x1+x2+x3+x4) # CABLE REACION-X
        BASE_REACTION_Y.append(y1+y2+y3+y4) # CABLE REACION-Y
        KE =  BASE_REACTION_Y[-1] / DISP_Y[-1] # Effective Lateral Stiffness
        #print(f'STEP: {i+1}')

    # Get initial and final node coordinates for plotting
    initial_coords = np.array([ops.nodeCoord(i + 1) for i in range(num_nodes * 2)])
    deformed_coords = initial_coords + np.array(DISPLACEMENTS[-1])

    # Output all unconstrained node displacements
    for i in range(num_nodes * 2):
        disp = ops.nodeDisp(i + 1)
        print(f"Node {i + 1}: X Displacement = {disp[0]}, Y Displacement = {disp[1]}")

    return initial_coords, deformed_coords, DISPLACEMENTS, VELOCITY, ACCELERATION, DISP_X, DISP_Y, BASE_REACTION_X, BASE_REACTION_Y
```

In [26]:

```
# Define the plotting function
def plot_time_history(TIME, DISP_X_undamped, DISP_X_damped, DISP_Y_undamped, DISP_Y_damped,
                      VELOCITY_undamped, VELOCITY_damped, ACCELERATION_undamped, ACCELERATION_damped,
                      BASE_REACTION_X_undamped, BASE_REACTION_X_damped, BASE_REACTION_Y_undamped, BASE_REACTION_Y_damped):
    import matplotlib.pyplot as plt
    import numpy as np

    plt.figure(figsize=(14, 26))

    # Displacement X
    plt.subplot(6, 1, 1)
    P1 = np.max(np.abs(np.array(DISP_X_undamped)))
    P2 = np.max(np.abs(np.array(DISP_X_damped)))
    plt.plot(TIME, DISP_X_undamped, color='black', label=f'Undamped: {P1:.4e}')
    plt.plot(TIME, DISP_X_damped, color='red', label=f'Damped: {P2:.4e}')
    plt.xlabel('Time [s]')
    plt.ylabel('Displacement X [mm]')
    plt.legend()

    # Displacement Y
    plt.subplot(6, 1, 2)
    P1 = np.max(np.abs(np.array(DISP_Y_undamped)))
    P2 = np.max(np.abs(np.array(DISP_Y_damped)))
    plt.plot(TIME, DISP_Y_undamped, color='black', label=f'Undamped: {P1:.4e}')
    plt.plot(TIME, DISP_Y_damped, color='red', label=f'Damped: {P2:.4e}')
    plt.xlabel('Time [s]')
    plt.ylabel('Displacement Y [mm]')
    plt.legend()

    # Velocity
    plt.subplot(6, 1, 3)
    P1 = np.max(np.abs(np.array(VELOCITY_damped)))
    P2 = np.max(np.abs(np.array(VELOCITY_undamped)))
    plt.plot(TIME, VELOCITY_undamped, color='black', label=f'Undamped: {P1:.4e}')
    plt.plot(TIME, VELOCITY_damped, color='red', label=f'Damped: {P2:.4e}')
    plt.xlabel('Time [s]')
    plt.ylabel('Velocity [mm/s]')
    plt.legend()

    # Acceleration
    plt.subplot(6, 1, 4)
    P1 = np.max(np.abs(np.array(ACCELERATION_undamped)))
    P2 = np.max(np.abs(np.array(ACCELERATION_damped)))
    plt.plot(TIME, ACCELERATION_undamped, color='black', label=f'Undamped: {P1:.4e}')
    plt.plot(TIME, ACCELERATION_damped, color='red', label=f'Damped: {P2:.4e}')
    plt.xlabel('Time [s]')
    plt.ylabel('Acceleration [mm/s^2]')
    plt.legend()

    # Base Reaction X
    plt.subplot(6, 1, 5)
    P1 = np.max(np.abs(np.array(BASE_REACTION_X_undamped)))
    P2 = np.max(np.abs(np.array(BASE_REACTION_X_damped)))
    plt.plot(TIME, BASE_REACTION_X_undamped, color='black', label=f'Undamped: {P1:.4e}')
    plt.plot(TIME, BASE_REACTION_X_damped, color='red', label=f'Damped: {P2:.4e}')
    plt.xlabel('Time [s]')
    plt.ylabel('Base Reaction X [N]')
    plt.legend()

    # Base Reaction Y
    plt.subplot(6, 1, 6)
    P1 = np.max(np.abs(np.array(BASE_REACTION_Y_undamped)))
    P2 = np.max(np.abs(np.array(BASE_REACTION_Y_damped)))
    plt.plot(TIME, BASE_REACTION_Y_undamped, color='black', label=f'Undamped: {P1:.4e}')
    plt.plot(TIME, BASE_REACTION_Y_damped, color='red', label=f'Damped: {P2:.4e}')
    plt.xlabel('Time [s]')
    plt.ylabel('Base Reaction Y [N]')
    plt.legend()

    plt.suptitle('Time History of Free Vibration Analysis: Damped vs Undamped')
    plt.tight_layout(rect=[0, 0.03, 1, 0.95])
    plt.show()
```

In [27]:

```
# -------------------------
#   FREE VIBRATION ANALYSIS
# -------------------------
# Parameters for the analysis
L = 50000.0         # [mm] Bridge span lentgh
H1 = 3500.0         # [mm] Height of Top Cable
```

```
arc_depth = 1000.0  # [mm]
E_cable = 210e5     # [N/mm^2] Modulus of elasticity Cable
Cable_Dia_01 = 25   # [mm] Horizontal Longitudinal Cable Diameter Bottom
Cable_Dia_02 = 18   # [mm] Horizontal Longitudinal Cable Diameter Top
Cable_Dia_03 = 10   # [mm] Vertical Longitudinal Cable Diameter Top
num_nodes = 51      # Cable Arc Number of nodes

TOTAL_MASS = 500000.0  # [kg] Total Mass of Structure
u0 = 2.0               # [mm] Initial displacement
damping_ratio = 0.05   # Damping ratio
duration = 50.0        # [s] Duration of the analysis in seconds
dt = 0.01              # Time step in seconds

MAX_ITERATIONS = 10000  # Maximum number of iterations
TOLERANCE = 1.0e-14     # Tolerance for convergence

import time
starttime = time.process_time()

# Run the undamped analysis
damping = False
LINEAR = True # False: Cable NonLinear Materials Properties
results_undamped = FREE_VIBRATION_ANALYSIS(damping, damping_ratio, LINEAR, duration, dt, TOTAL_MASS, u0, L, H1, arc_depth, E_cable, Cable_Dia_01, Cable_Dia_02, Cable_Dia_03, num_nodes,
initial_coords, deformed_coords, DISPLACEMENTS_undamped, VELOCITY_undamped, ACCELERATION_undamped, DISP_X_undamped, DISP_Y_undamped, BASE_REACTION_X_undamped, BASE_REACTION_Y_undamped

# Run the damped analysis
damping = True
LINEAR = True # False: Cable NonLinear Materials Properties
results_damped = FREE_VIBRATION_ANALYSIS(damping, damping_ratio, LINEAR, duration, dt, TOTAL_MASS, u0, L, H1, arc_depth, E_cable, Cable_Dia_01, Cable_Dia_02, Cable_Dia_03, num_nodes, N
initial_coords, deformed_coords, DISPLACEMENTS_damped, VELOCITY_damped, ACCELERATION_damped, DISP_X_damped, DISP_Y_damped, BASE_REACTION_X_damped, BASE_REACTION_Y_damped = results_damp

totaltime = time.process_time() - starttime
print(f'\nTotal time (s): {totaltime:.4f} \n\n')
```

Node 1: X Displacement = 0.0, Y Displacement = 0.0
Node 2: X Displacement = -0.0060080285929951325, Y Displacement = -0.09561323463466348
Node 3: X Displacement = -0.011861368101459054, Y Displacement = -0.18913514782977003
Node 4: X Displacement = -0.017409051801990132, Y Displacement = -0.27848266326808296
Node 5: X Displacement = -0.022507495314215 94, Y Displacement = -0.3615891846792667
Node 6: X Displacement = -0.027024029350335637, Y Displacement = -0.43641275510935623
Node 7: X Displacement = -0.030840245147584534, Y Displacement = -0.5009441186472807
Node 8: X Displacement = -0.03385509486112691, Y Displacement = -0.5532146526426195
Node 9: X Displacement = -0.03598769314098611, Y Displacement = -0.5913041389681727
Node 10: X Displacement = -0.0371797709387947, Y Displacement = -0.613348343546968
Node 11: X Displacement = -0.03739773819661379, Y Displacement = -0.6175463741152317
Node 12: X Displacement = -0.0366343183848854 35, Y Displacement = -0.602167787123886
Node 13: X Displacement = -0.03490972476677465, Y Displacement = -0.5655594156872907
Node 14: X Displacement = -0.03227235567483926, Y Displacement = -0.5061518916333355
Node 15: X Displacement = -0.028798993862640412, Y Displacement = -0.4224658359676727
Node 16: X Displacement = -0.02459450302209153, Y Displacement = -0.31311769341571466
Node 17: X Displacement = -0.019791022705050173, Y Displacement = -0.17682518815795872
Node 18: X Displacement = -0.014546671033091976, Y Displacement = -0.012412379425652426
Node 19: X Displacement = -0.009043772574362046, Y Displacement = 0.18118570276721332
Node 20: X Displacement = -0.0034866365155758693, Y Displacement = 0.40491885979061 46
Node 21: X Displacement = 0.0019010824034236624, Y Displacement = 0.6596179807495808
Node 22: X Displacement = 0.006879400867173785, Y Displacement = 0.9459917674420825
Node 23: X Displacement = 0.011195355064763816, Y Displacement = 1.2646238465932016
Node 24: X Displacement = 0.014586019139360326, Y Displacement = 1.6159057115413942
Node 25: X Displacement = 0.016685525845849043, Y Displacement = 1.982360267064577
Node 26: X Displacement = 0.015817346337994476, Y Displacement = 1.5827670787783283
Node 27: X Displacement = 0.016501769131742814, Y Displacement = 1.1986093641051365
Node 28: X Displacement = 0.018515361022481722, Y Displacement = 0.8476257660667 09
Node 29: X Displacement = 0.021589531410905898, Y Displacement = 0.529684265642504
Node 30: X Displacement = 0.025457782500837776, Y Displacement = 0.24445761755015744
Node 31: X Displacement = 0.029859624654618 11, Y Displacement = -0.008510601089632145
Node 32: X Displacement = 0.034544424805533 63, Y Displacement = -0.22980409667014448
Node 33: X Displacement = 0.03927502792603657, Y Displacement = -0.4201315579629968
Node 34: X Displacement = 0.04383109525805321, Y Displacement = -0.5803238628298513
Node 35: X Displacement = 0.048012108263183036, Y Displacement = -0.7113308035047032
Node 36: X Displacement = 0.051639992787406504, Y Displacement = -0.8142173431104059
Node 37: X Displacement = 0.05456132417414063, Y Displacement = -0.8901594179074599
Node 38: X Displacement = 0.056649080917386734, Y Displacement = -0.9404393015632028
Node 39: X Displacement = 0.057803921801709776, Y Displacement = -0.9664405494805213
Node 40: X Displacement = 0.057954969224880 85, Y Displacement = -0.9696425429304212
Node 41: X Displacement = 0.057060089404370 34, Y Displacement = -0.9516146543227563
Node 42: X Displacement = 0.055105668321802935, Y Displacement = -0.9140100565182431
Node 43: X Displacement = 0.052105890410439865, Y Displacement = -0.8585592005267121
Node 44: X Displacement = 0.048101535026716 09, Y Displacement = -0.7870629872968078
Node 45: X Displacement = 0.043158313529581986, Y Displacement = -0.7013856605462575
Node 46: X Displacement = 0.037364777201634 67, Y Displacement = -0.6034474487364657
Node 47: X Displacement = 0.030829833163543596, Y Displacement = -0.49521698529723207
Node 48: X Displacement = 0.023679911749706 25, Y Displacement = -0.378703537129877 03
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Node 30: X Displacement = 0.02545782554747551 6, Y Displacement = 0.24445697561946236
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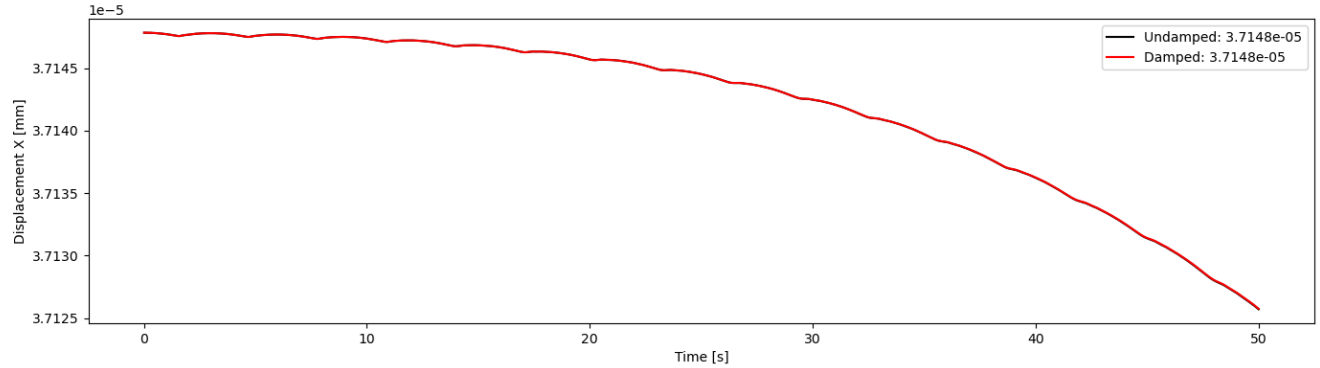
Node 102: X Displacement = 0.0, Y Displacement = 0.0

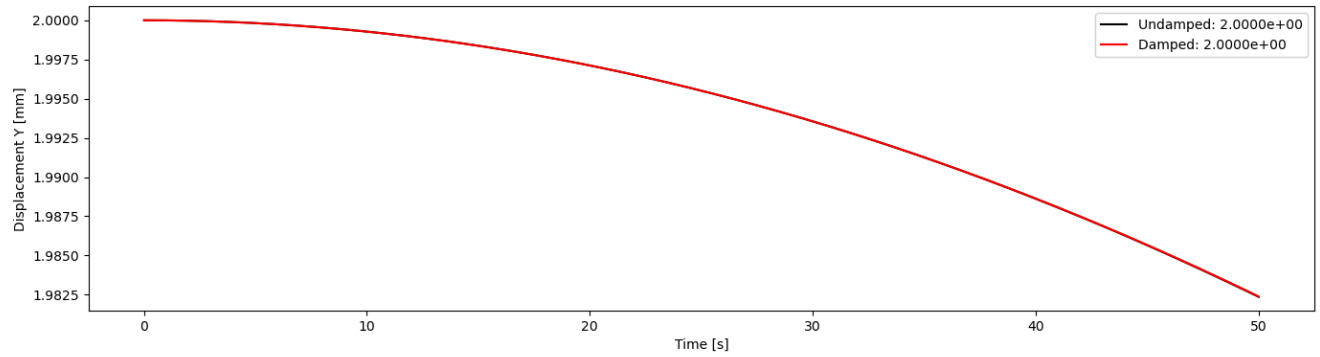
Total time (s): 7.9844
```

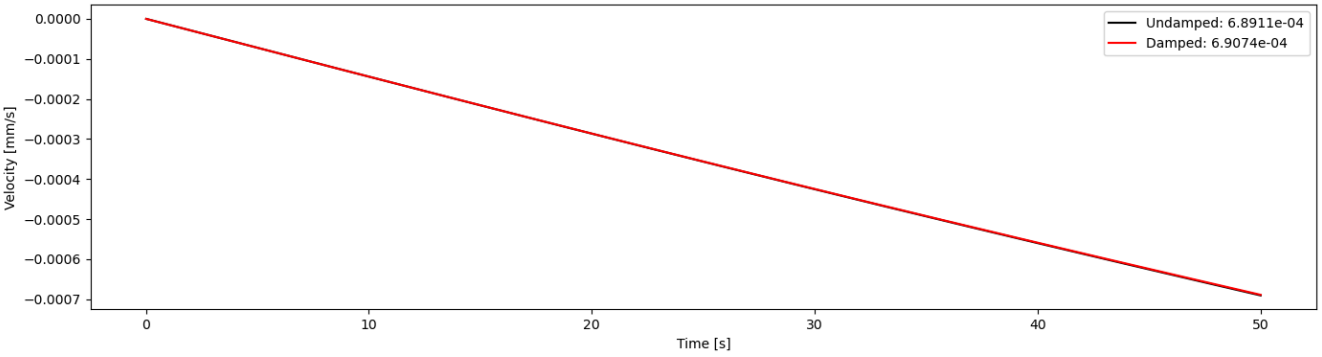
In [28]:
```
# Plotting the time history
TIME = np.arange(dt, duration+2*dt, dt)

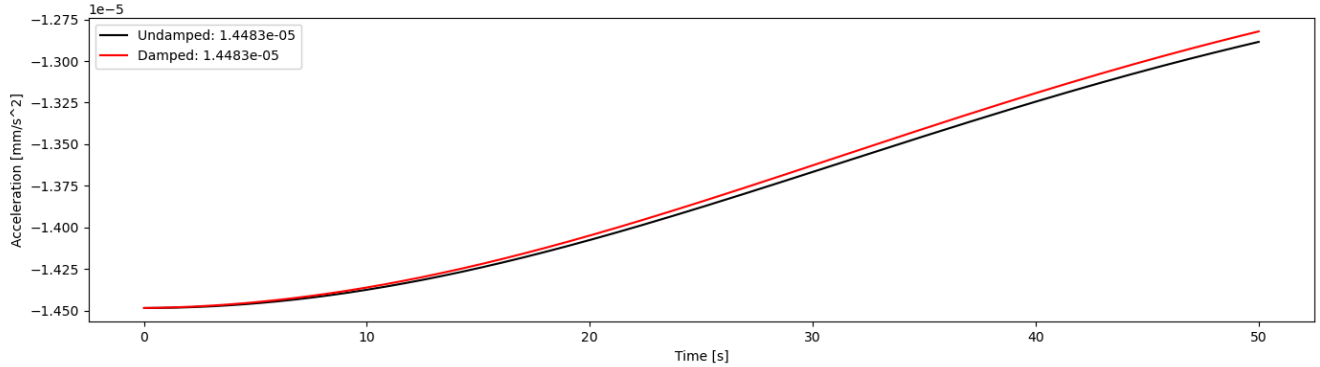
plot_time_history(TIME, DISP_X_undamped, DISP_X_damped, DISP_Y_undamped, DISP_Y_damped,
                  VELOCITY_undamped, VELOCITY_damped, ACCELERATION_undamped, ACCELERATION_damped,
                  BASE_REACTION_X_undamped, BASE_REACTION_X_damped, BASE_REACTION_Y_undamped, BASE_REACTION_Y_damped)
```

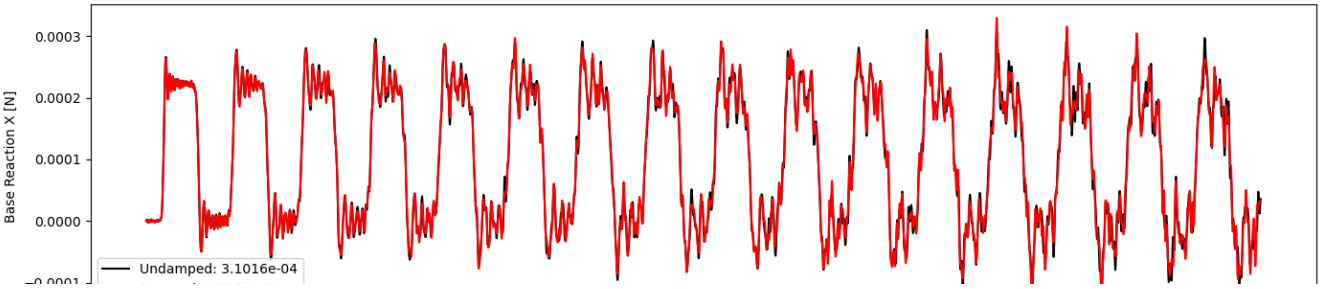
Time History of Free Vibration Analysis: Damped vs Undamped

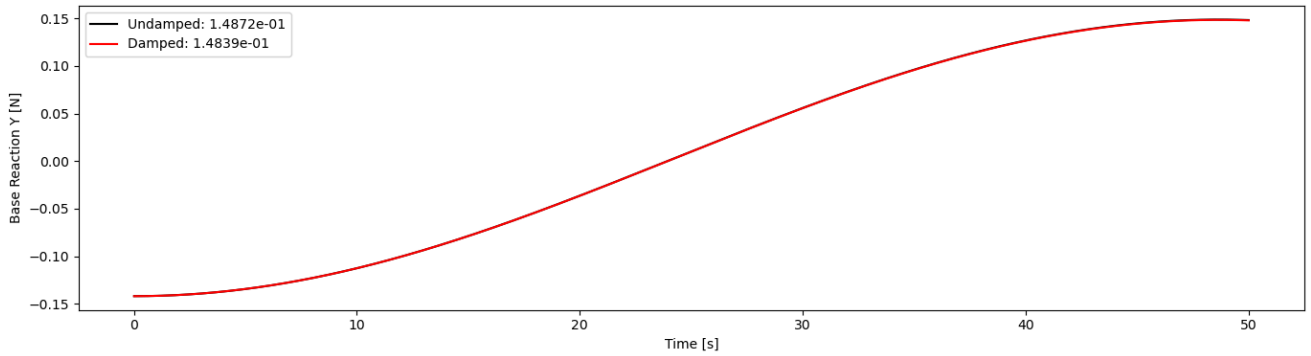




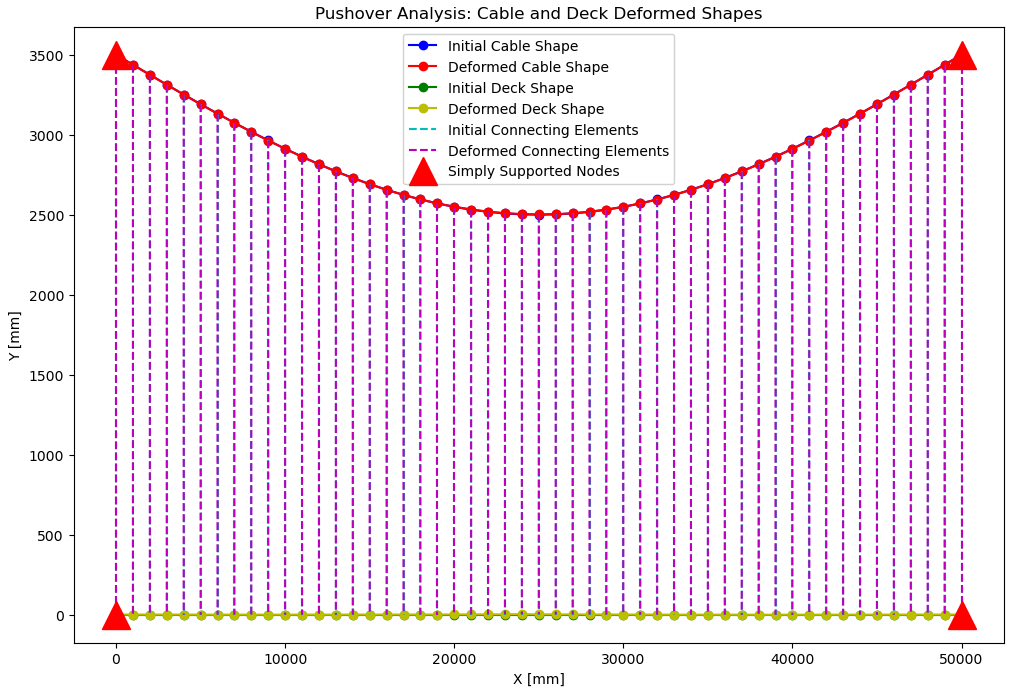


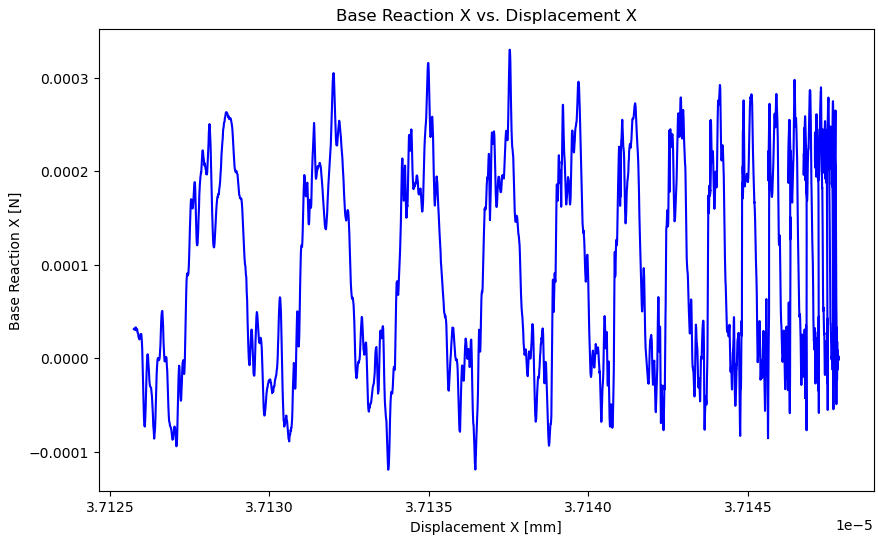


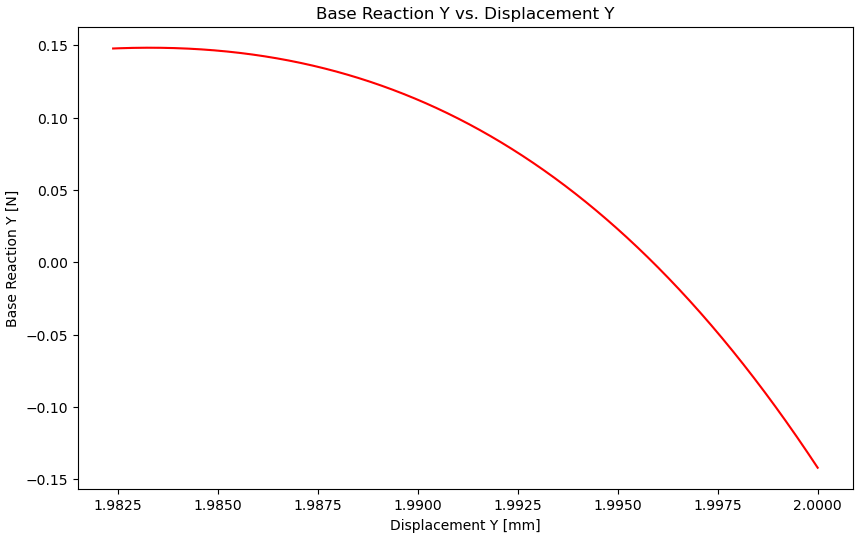




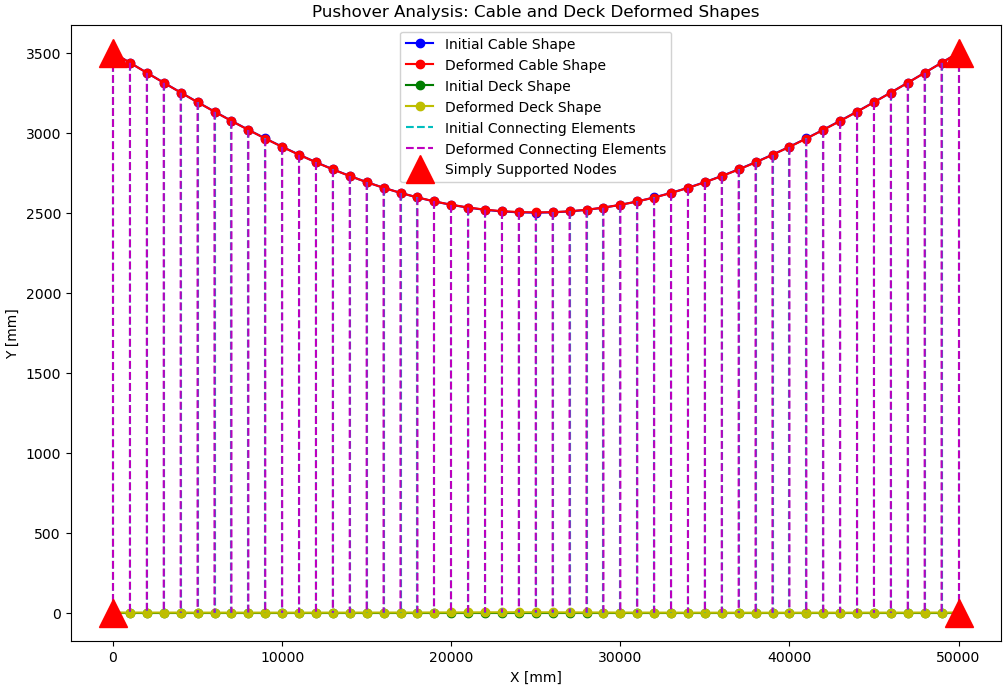
In [29]:
```
# Plot damped results
plot_shapes(initial_coords, DISPLACEMENTS_damped)
plot_reactions(DISP_X_damped, BASE_REACTION_X_damped, DISP_Y_damped, BASE_REACTION_Y_damped)
```

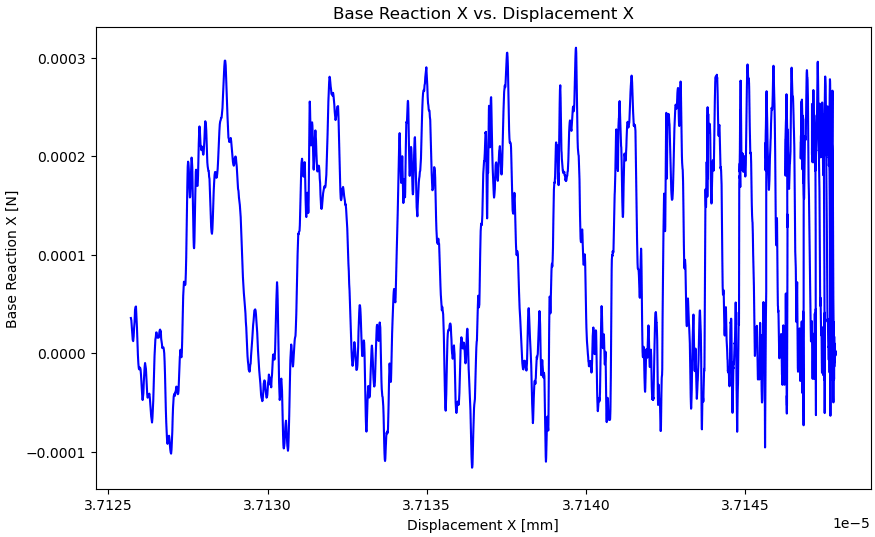


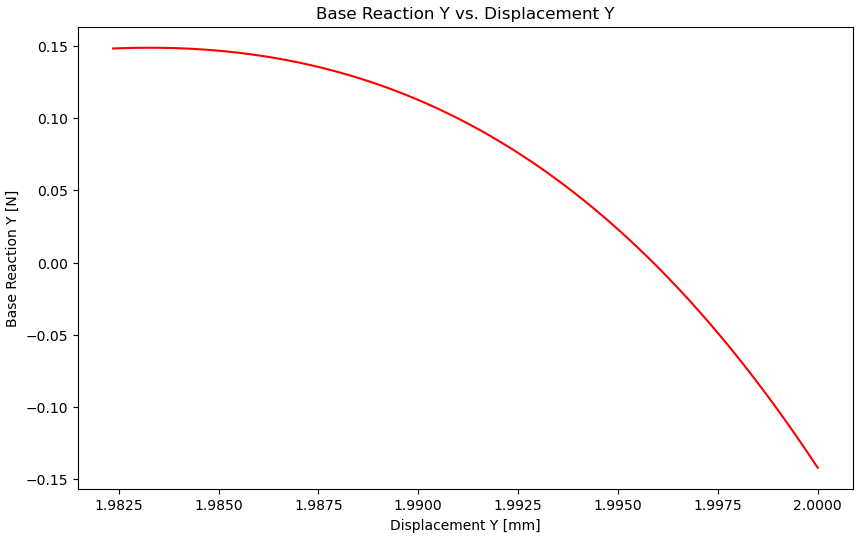




In [30]:
```
# Plot undamped results
plot_shapes(initial_coords, DISPLACEMENTS_undamped)
plot_reactions(DISP_X_undamped, BASE_REACTION_X_undamped, DISP_Y_undamped, BASE_REACTION_Y_undamped)
```







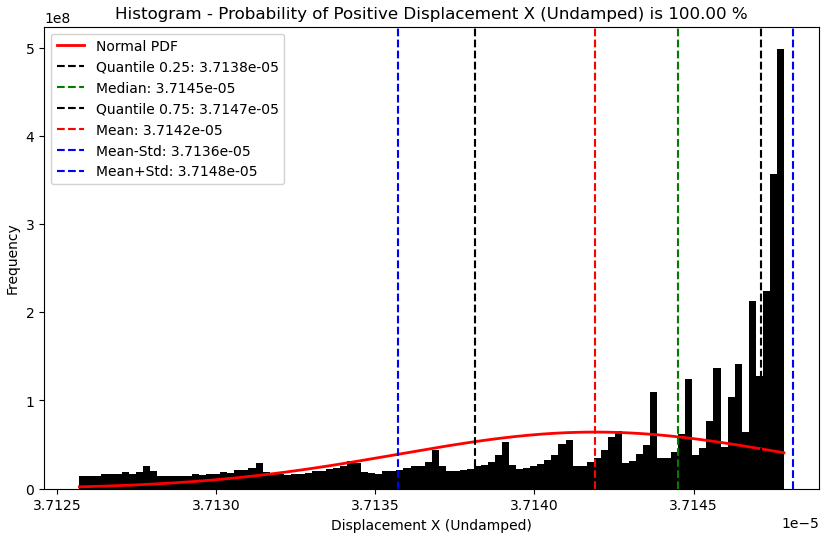
In [23]:

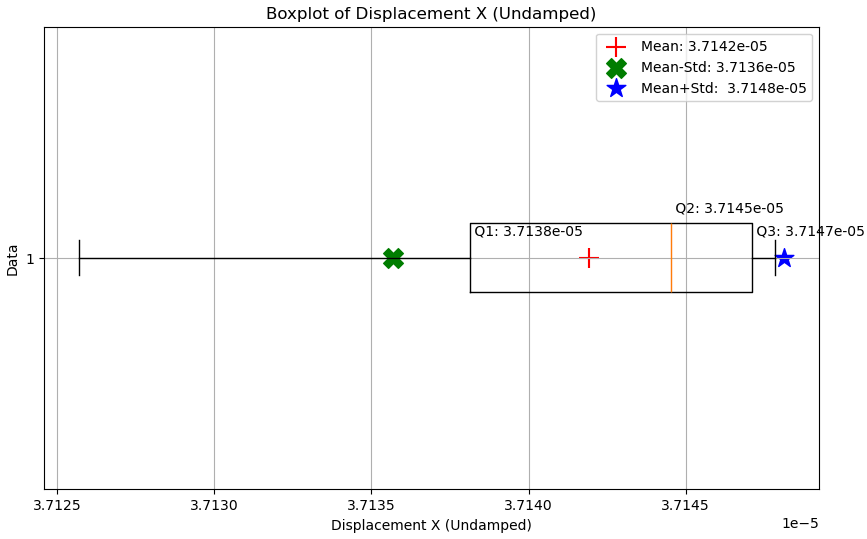
```python
def HISROGRAM_BOXPLOT(X, HISTO_COLOR, LABEL):
    import numpy as np
    import matplotlib.pyplot as plt
    X = np.array(X)
    print("-------------------------")
    from scipy.stats import skew, kurtosis
    MINIMUM = np.min(X)
    MAXIMUM = np.max(X)
    #MODE = max(set(X), key=list(X).count)
    MEDIAN = np.quantile(X, .50)#q2
    MEAN = np.mean(X)
    STD = np.std(X)
    q1 = np.quantile(X, .25)
    q3 = np.quantile(X, .75)
    SKEW = skew(X)
    KURT = kurtosis(X)
    #SKEW = (MEAN - MODE) / STD
    #KURT = (np.mean((X - MEAN)**4) / STD**4)
    # Estimate confidence intervals of the output variable
    lower_bound = np.quantile(X, .05)
    upper_bound = np.quantile(X, .95)
    print("Box-Chart Datas: ")
    print(f'Minimum: {MINIMUM:.4e}')
    print(f'First quartile: {q1:.4e}')
    #print(f'Mode: {MODE:.4e}')
    print(f'Median: {MEDIAN:.4e}')
    print(f'Mean: {MEAN:.4e}')
    print(f'Std: {STD:.4e}')
    print(f'Third quartile: {q3:.4e}')
    print(f'Maximum: {MAXIMUM :.4e}')
    print(f'Skewness: {skew(X) :.4e}')
    print(f'kurtosis: {kurtosis(X) :.4e}')
    print(f"90% Confidence Interval: ({lower_bound:.4e}, {upper_bound:.4e})")
    print("-------------------------")

    plt.figure(figsize=(10,6))
    # Plot histogram of data
    count, bins, ignored = plt.hist(X, bins=100, color=HISTO_COLOR, density=True, align='mid')#, edgecolor="black"

    # Plot lognormal PDF
    x = np.linspace(min(bins), max(bins), 10000)
    pdf = (np.exp(-(x - MEAN)**2 / (2 * STD**2)) / (STD * np.sqrt(2 * np.pi)))
    plt.plot(x, pdf, linewidth=2, color='r', label="Normal PDF")

    # Plot vertical lines for risk measures
    plt.axvline(q1, color="black", linestyle="--", label=f"Quantile 0.25: {q1:.4e}")
    plt.axvline(MEDIAN, color="green", linestyle="--", label=f"Median: {MEDIAN:.4e}")
    plt.axvline(q3, color="black", linestyle="--", label=f"Quantile 0.75: {q3:.4e}")
    #plt.axvline(MODE, color="purple", linestyle="--", label=f"Mode: {MODE:.4e}")
    plt.axvline(MEAN, color="red", linestyle="--", label=f"Mean: {MEAN:.4e}")
    plt.axvline(MEAN-STD, color="blue", linestyle="--", label=f"Mean-Std: {MEAN-STD:.4e}")
    plt.axvline(MEAN+STD, color="blue", linestyle="--", label=f"Mean+Std: {MEAN+STD:.4e}")
    plt.xlabel(LABEL)
    plt.ylabel("Frequency")
    prob = np.sum(X > 0) / len(X)
    plt.title(f"Histogram - Probability of Positive {LABEL} is {100*prob:.2f} %")
    plt.legend()
    #plt.grid()
    plt.show()

    #Plot boxplot with outliers
    plt.figure(figsize=(10,6))
    plt.boxplot(X, vert=0)
    # Write the quartile data on the chart
    plt.text(q1, 1.05, f" Q1: {q1:.4e}")
    plt.text(MEDIAN, 1.1, f" Q2: {MEDIAN:.4e}")
    plt.text(q3, 1.05, f" Q3: {q3:.4e}")
    #plt.text(MODE, 1.15, f" Mode: {MODE:.4e}")

    #plt.text(MEAN, 0.9, f" Mean: {MEAN:.4e}")
    #plt.text(MEAN-STD, 0.9, f" Mean-Std: {MEAN-STD:.4e}")
    #plt.text(MEAN+STD, 0.9, f" Mean+Std: {MEAN+STD:.4e}")
    plt.scatter(MEAN, 1, color="red", marker="+", s=200, label=f"Mean: {MEAN:.4e}")
    plt.scatter(MEAN-STD, 1, color="green", marker="X", s=200, label=f"Mean-Std: {MEAN-STD:.4e}")
    plt.scatter(MEAN+STD, 1, color="blue", marker="*", s=200, label=f"Mean+Std:  {MEAN+STD:.4e}")
    plt.xlabel(LABEL)
    plt.ylabel("Data")
    plt.title(f"Boxplot of {LABEL}")
    plt.legend()
```

```
    plt.grid()
    plt.show()
# ------------------------------------------------
def HISTOGRAM_BOXPLOT_PLOTLY( DATA, XLABEL='X', TITLE='A', COLOR='cyan'):
    # Plotting histogram and boxplot
    import plotly.express as px
    fig = px.histogram(x=DATA, marginal="box", color_discrete_sequence=[COLOR])
    fig.update_layout(title=TITLE, xaxis_title=XLABEL, yaxis_title="Frequency")
    fig.show()
```

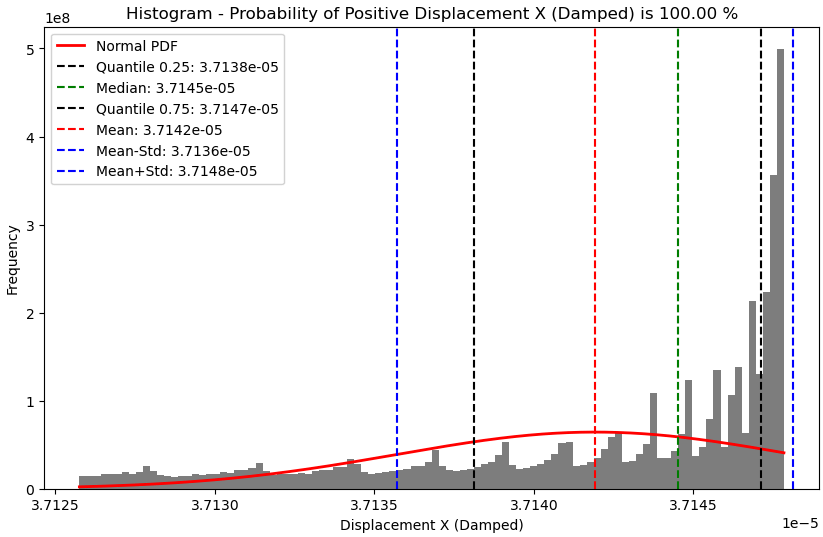
In [32]:
```
HISROGRAM_BOXPLOT(DISP_X_undamped, HISTO_COLOR='black', LABEL='Displacement X (Undamped)')
HISROGRAM_BOXPLOT(DISP_X_damped, HISTO_COLOR='grey', LABEL='Displacement X (Damped)')
#HISTOGRAM_BOXPLOT_PLOTLY(DISP_X_undamped, XLABEL='Displacement X (Undamped)', TITLE='Displacement X (Undamped)', COLOR='black')
#HISTOGRAM_BOXPLOT_PLOTLY(DISP_X_damped, XLABEL='Displacement X (Damped)', TITLE='Displacement X (Damped)', COLOR='grey')
```

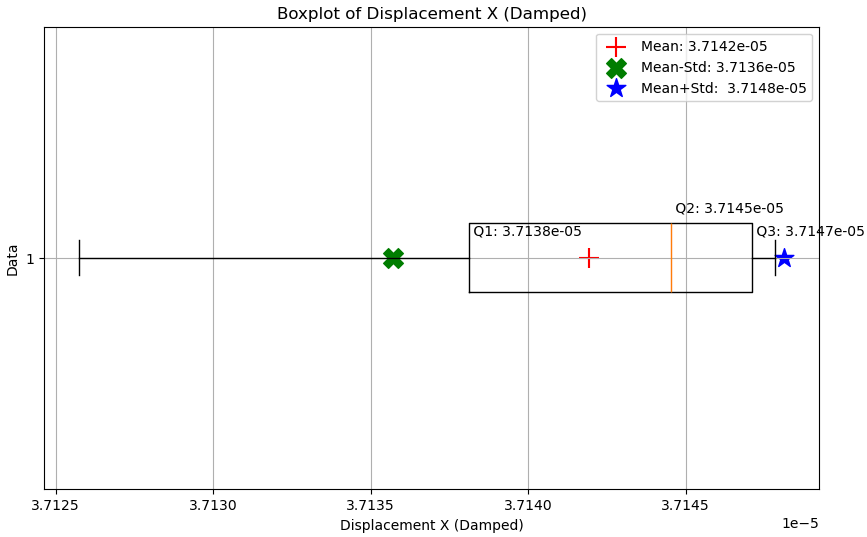
```
-------------------------
Box-Chart Datas:
Minimum: 3.7126e-05
First quartile: 3.7138e-05
Median: 3.7145e-05
Mean: 3.7142e-05
Std: 6.2137e-09
Third quartile: 3.7147e-05
Maximum: 3.7148e-05
Skewness: -1.0058e+00
kurtosis: -1.8075e-01
90% Confidence Interval: (3.7129e-05, 3.7148e-05)
-------------------------
```





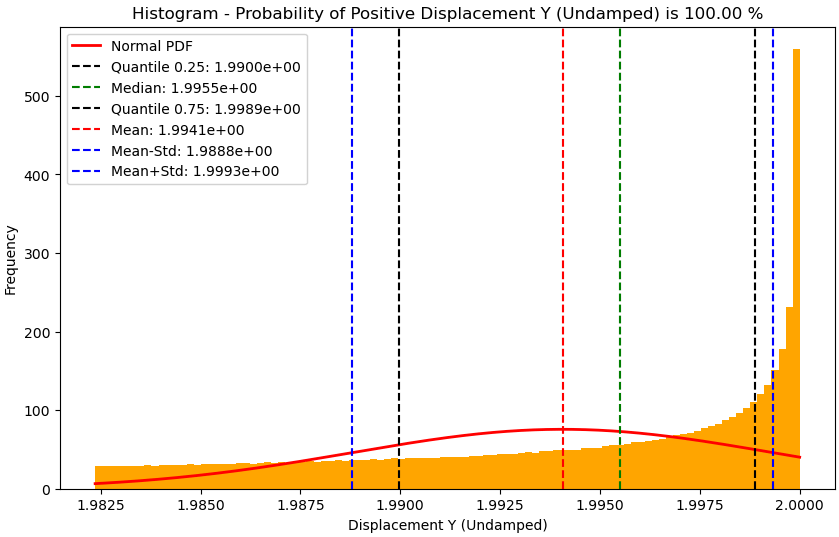

```
-------------------------
Box-Chart Datas:
Minimum: 3.7126e-05
First quartile: 3.7138e-05
Median: 3.7145e-05
Mean: 3.7142e-05
Std: 6.2016e-09
Third quartile: 3.7147e-05
Maximum: 3.7148e-05
Skewness: -1.0050e+00
kurtosis: -1.8262e-01
90% Confidence Interval: (3.7129e-05, 3.7148e-05)
-------------------------
```

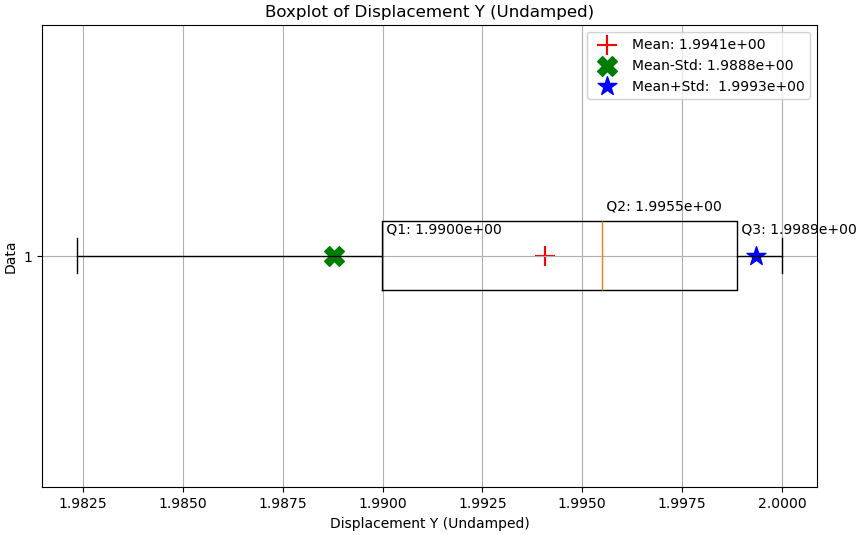




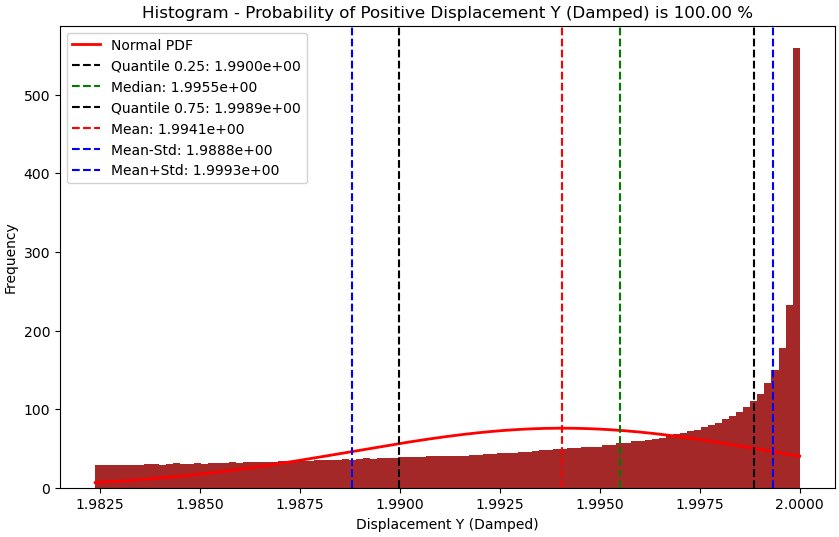
In [33]:
```
HISROGRAM_BOXPLOT(DISP_Y_undamped, HISTO_COLOR='orange', LABEL='Displacement Y (Undamped)')
HISROGRAM_BOXPLOT(DISP_Y_damped, HISTO_COLOR='brown', LABEL='Displacement Y (Damped)')
#HISTOGRAM_BOXPLOT_PLOTLY(DISP_Y_undamped, XLABEL='Displacement Y (Undamped)', TITLE='Displacement Y (Undamped)', COLOR='orange')
#HISTOGRAM_BOXPLOT_PLOTLY(DISP_Y_damped, XLABEL='Displacement Y (Damped)', TITLE='Displacement Y (Damped)', COLOR='brown')
```

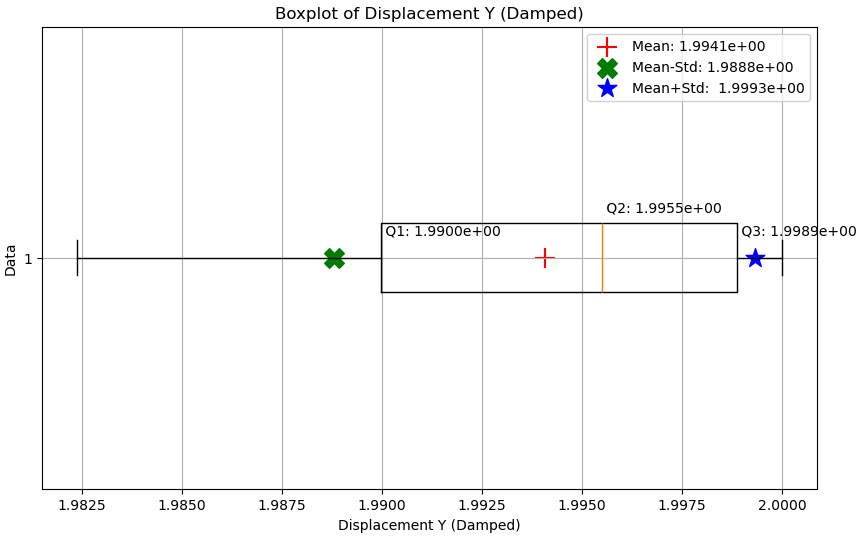
```
-------------------------
Box-Chart Datas:
Minimum: 1.9824e+00
First quartile: 1.9900e+00
Median: 1.9955e+00
Mean: 1.9941e+00
Std: 5.2758e-03
Third quartile: 1.9989e+00
Maximum: 2.0000e+00
Skewness: -6.2305e-01
kurtosis: -8.8164e-01
90% Confidence Interval: (1.9840e+00, 2.0000e+00)
-------------------------
```





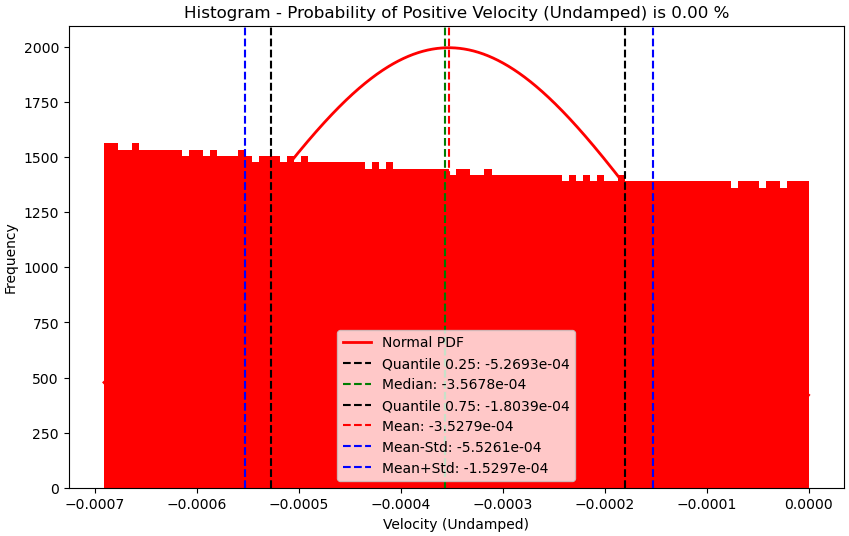

```
-------------------------
Box-Chart Datas:
Minimum: 1.9824e+00
First quartile: 1.9900e+00
Median: 1.9955e+00
Mean: 1.9941e+00
Std: 5.2681e-03
Third quartile: 1.9989e+00
Maximum: 2.0000e+00
Skewness: -6.2241e-01
kurtosis: -8.8257e-01
90% Confidence Interval: (1.9841e+00, 2.0000e+00)
-------------------------
```

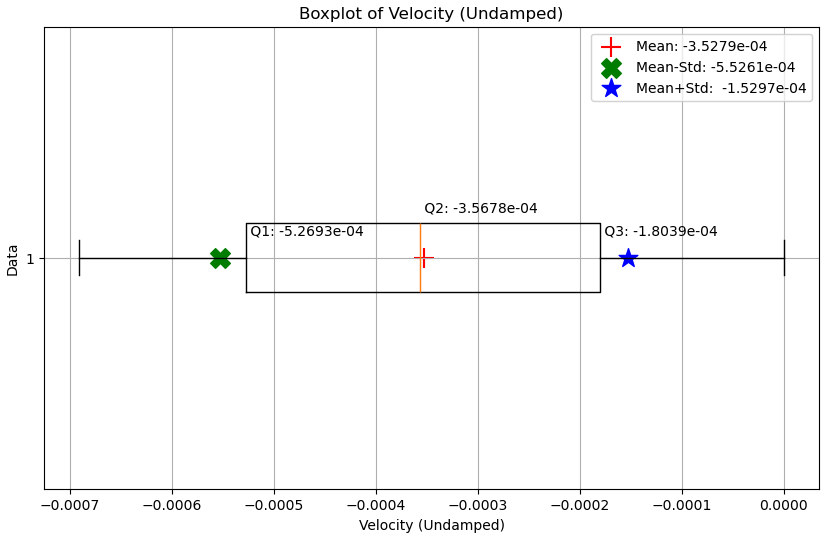




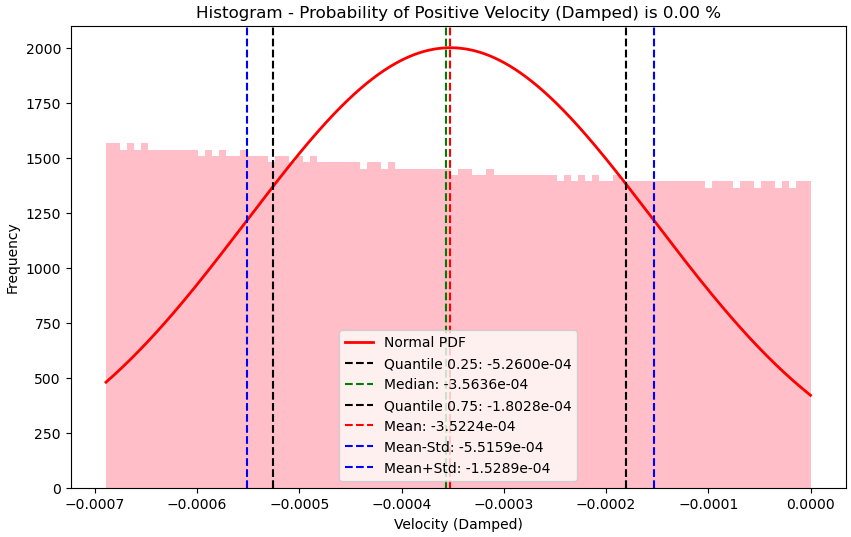
In [34]:
```
HISROGRAM_BOXPLOT(VELOCITY_undamped, HISTO_COLOR='red', LABEL='Velocity (Undamped)')
HISROGRAM_BOXPLOT(VELOCITY_damped, HISTO_COLOR='pink', LABEL='Velocity (Damped)')
#HISTOGRAM_BOXPLOT_PLOTLY(VELOCITY_undamped, XLABEL='Velocity (Undamped)', TITLE='Velocity (Undamped)', COLOR='red')
#HISTOGRAM_BOXPLOT_PLOTLY(VELOCITY_damped, XLABEL='Velocity (Damped)', TITLE='Velocity (Damped)', COLOR='pink')
```

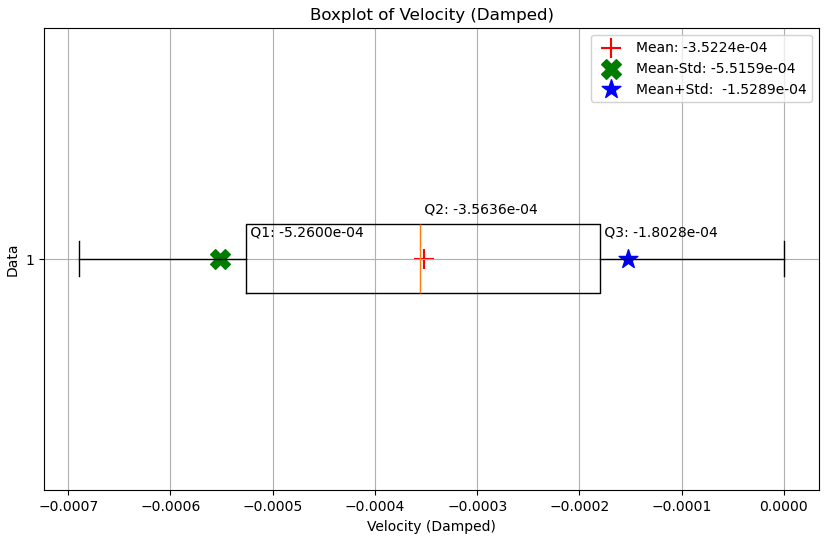
```
-------------------------
Box-Chart Datas:
Minimum: -6.9074e-04
First quartile: -5.2693e-04
Median: -3.5678e-04
Mean: -3.5279e-04
Std: 1.9982e-04
Third quartile: -1.8039e-04
Maximum: -7.2413e-08
Skewness: 4.6243e-02
kurtosis: -1.2022e+00
90% Confidence Interval: (-6.5842e-04, -3.6273e-05)
-------------------------
```






```
-------------------------
Box-Chart Datas:
Minimum: -6.8911e-04
First quartile: -5.2600e-04
Median: -3.5636e-04
Mean: -3.5224e-04
Std: 1.9935e-04
Third quartile: -1.8028e-04
Maximum: -7.2413e-08
Skewness: 4.7925e-02
kurtosis: -1.2020e+00
90% Confidence Interval: (-6.5695e-04, -3.6269e-05)
-------------------------
```

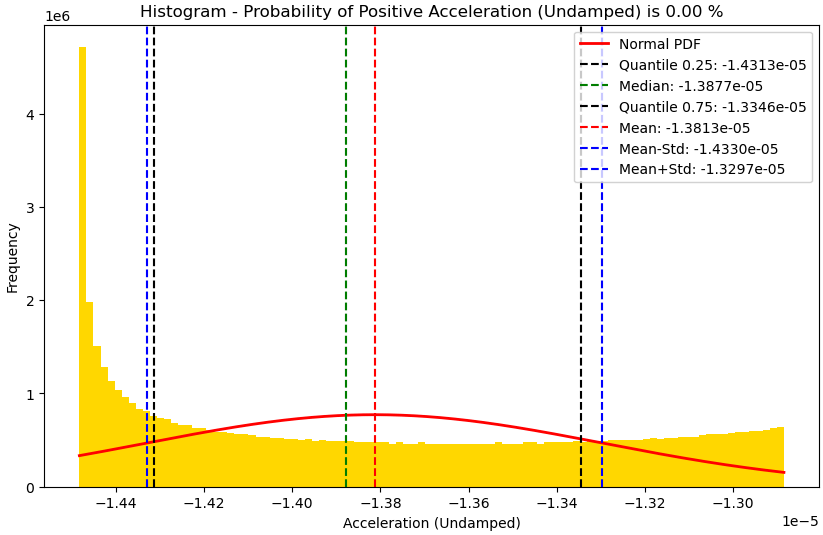


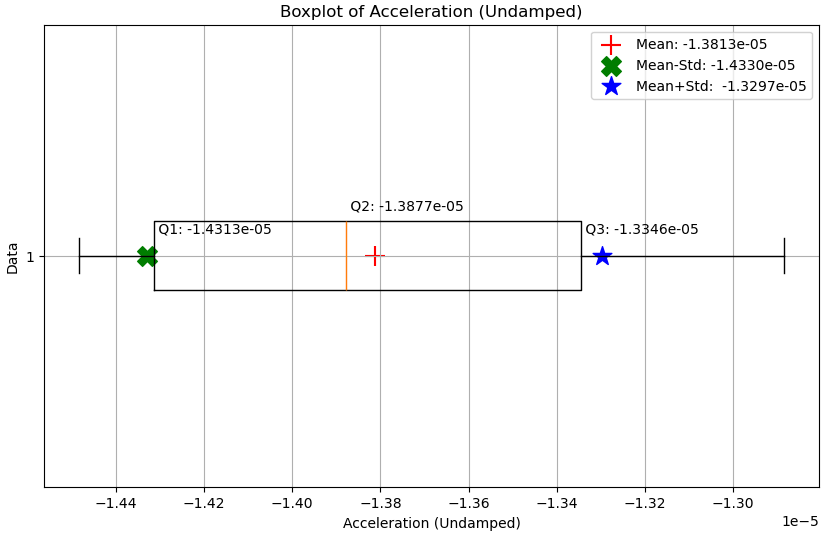


In [35]:
```
HISROGRAM_BOXPLOT(ACCELERATION_undamped, HISTO_COLOR='gold', LABEL='Acceleration (Undamped)')
HISROGRAM_BOXPLOT(ACCELERATION_damped, HISTO_COLOR='brown', LABEL='Acceleration (Damped)')
```

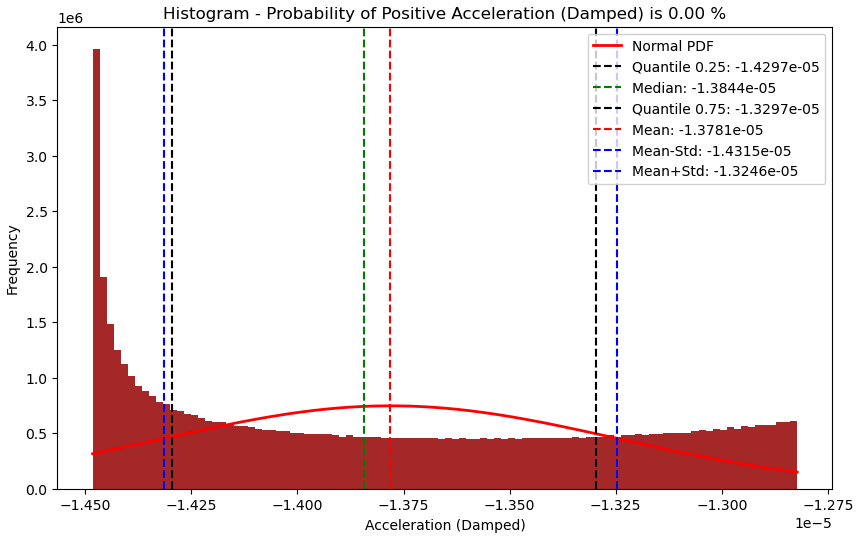
```
#HISTOGRAM_BOXPLOT_PLOTLY(ACCELERATION_undamped, XLABEL='Acceleration (Undamped)', TITLE='Acceleration (Undamped)', COLOR='gold')
#HISTOGRAM_BOXPLOT_PLOTLY(ACCELERATION_damped, XLABEL='Acceleration (Damped)', TITLE='Acceleration (Damped)', COLOR='brown')
```

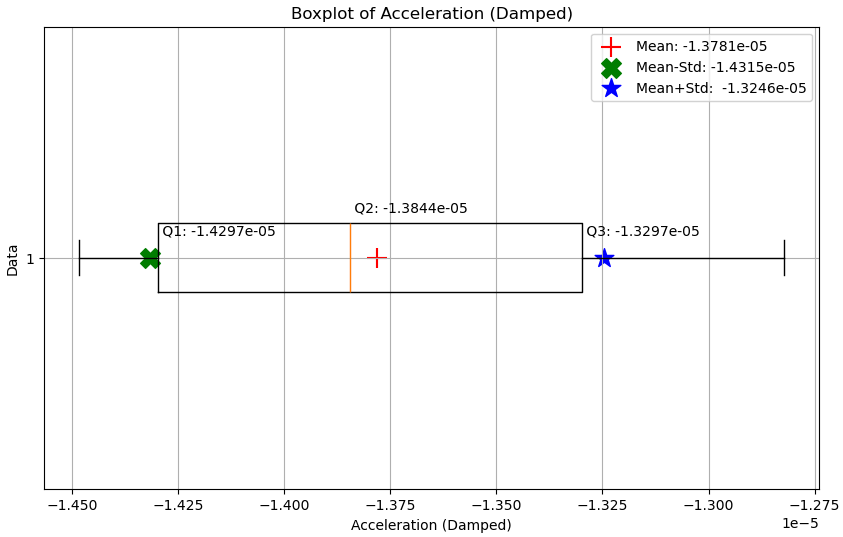
```
-------------------------
Box-Chart Datas:
Minimum: -1.4483e-05
First quartile: -1.4313e-05
Median: -1.3877e-05
Mean: -1.3813e-05
Std: 5.1642e-07
Third quartile: -1.3346e-05
Maximum: -1.2885e-05
Skewness: 2.5677e-01
kurtosis: -1.3241e+00
90% Confidence Interval: (-1.4476e-05, -1.2967e-05)
-------------------------
```





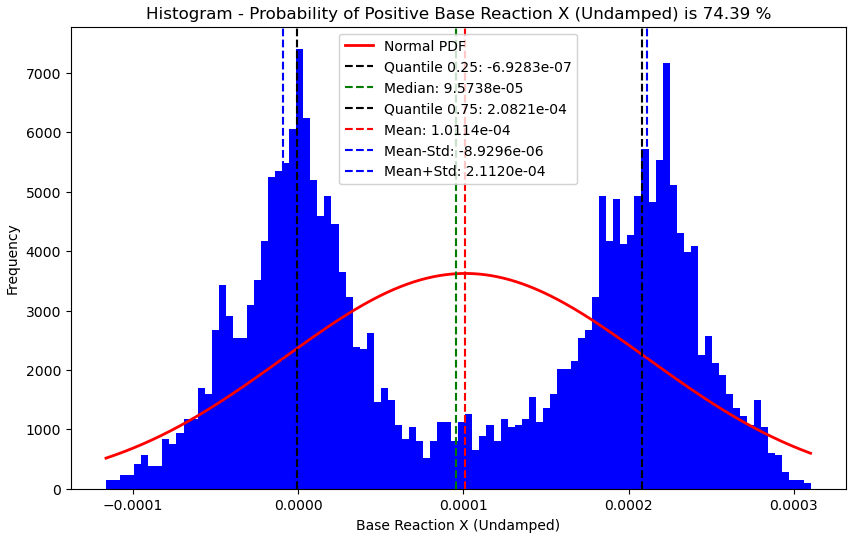

```
-------------------------
Box-Chart Datas:
Minimum: -1.4483e-05
First quartile: -1.4297e-05
Median: -1.3844e-05
Mean: -1.3781e-05
Std: 5.3434e-07
Third quartile: -1.3297e-05
Maximum: -1.2822e-05
Skewness: 2.4734e-01
kurtosis: -1.3235e+00
90% Confidence Interval: (-1.4472e-05, -1.2907e-05)
-------------------------
```

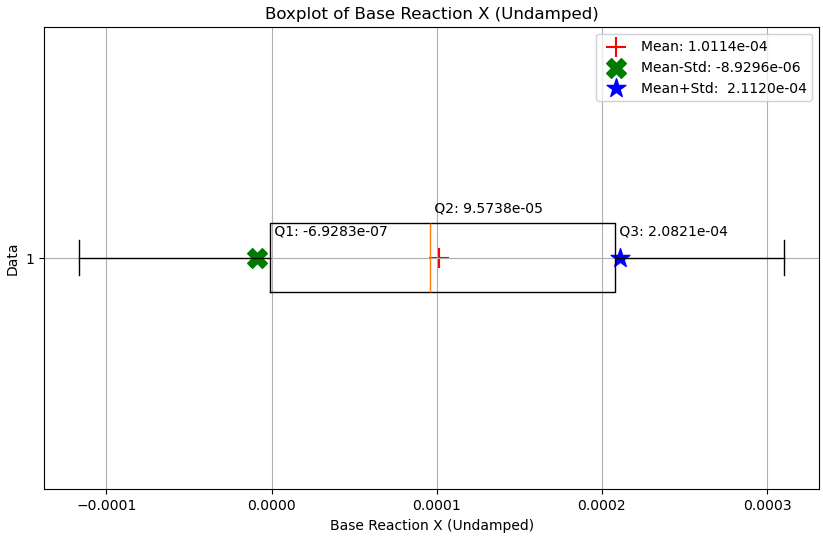




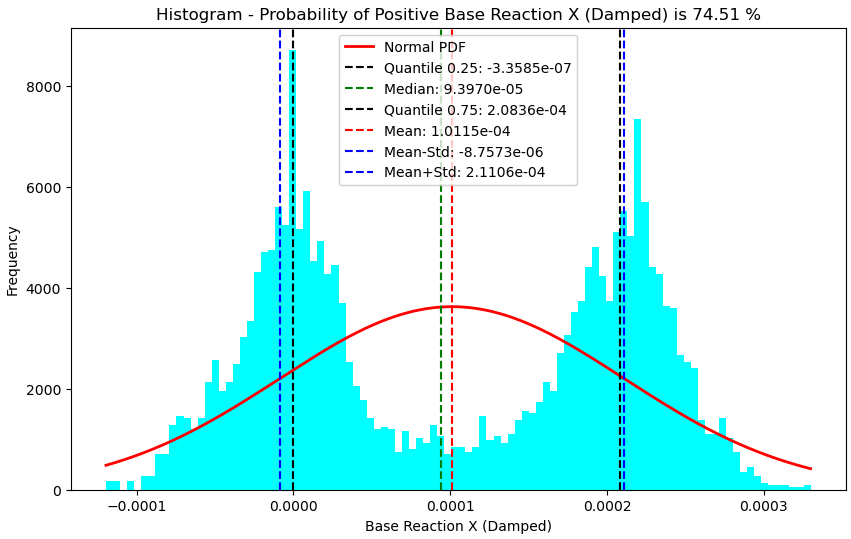
In [36]:
```
HISROGRAM_BOXPLOT(BASE_REACTION_X_undamped, HISTO_COLOR='blue', LABEL='Base Reaction X (Undamped)')
HISROGRAM_BOXPLOT(BASE_REACTION_X_damped, HISTO_COLOR='cyan', LABEL='Base Reaction X (Damped)')
#HISTOGRAM_BOXPLOT_PLOTLY(BASE_REACTION_X_undamped, XLABEL='Base Reaction X (Undamped)', TITLE='Base Reaction X (Undamped)', COLOR='blue')
#HISTOGRAM_BOXPLOT_PLOTLY(BASE_REACTION_X_damped, XLABEL='Base Reaction X (Damped)', TITLE='Base Reaction X (Damped)', COLOR='cyan')
```

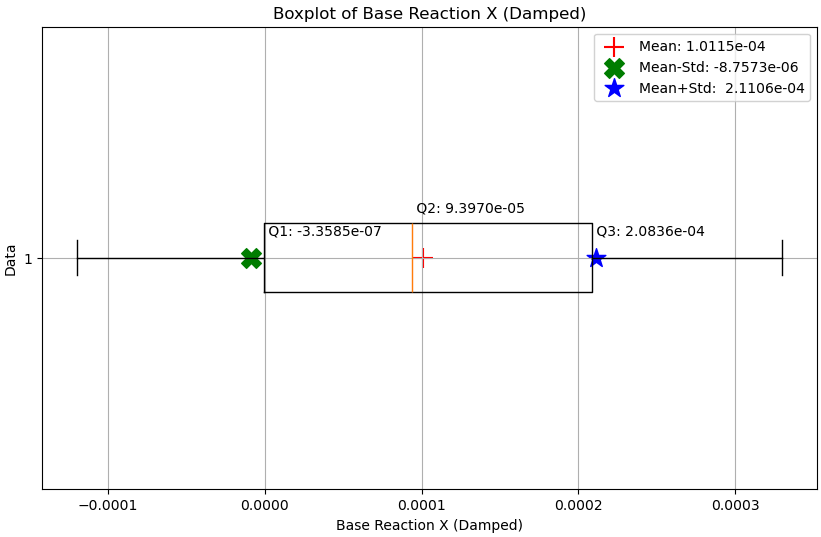
```
-------------------------
Box-Chart Datas:
Minimum: -1.1630e-04
First quartile: -6.9283e-07
Median: 9.5738e-05
Mean: 1.0114e-04
Std: 1.1007e-04
Third quartile: 2.0821e-04
Maximum: 3.1016e-04
Skewness: 1.9720e-02
kurtosis: -1.5392e+00
90% Confidence Interval: (-5.0007e-05, 2.5391e-04)
-------------------------
```





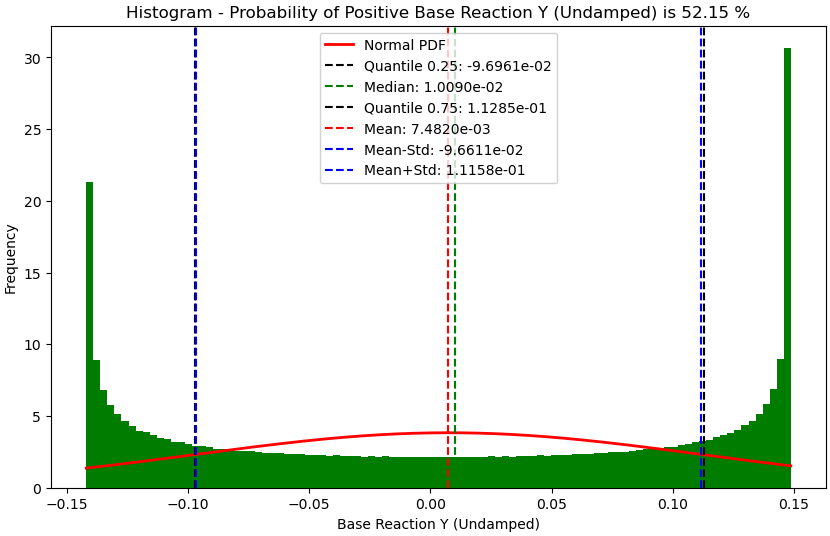

```
-------------------------
Box-Chart Datas:
Minimum: -1.1947e-04
First quartile: -3.3585e-07
Median: 9.3970e-05
Mean: 1.0115e-04
Std: 1.0991e-04
Third quartile: 2.0836e-04
Maximum: 3.2975e-04
Skewness: 1.8111e-02
kurtosis: -1.5364e+00
90% Confidence Interval: (-5.2773e-05, 2.5308e-04)
-------------------------
```

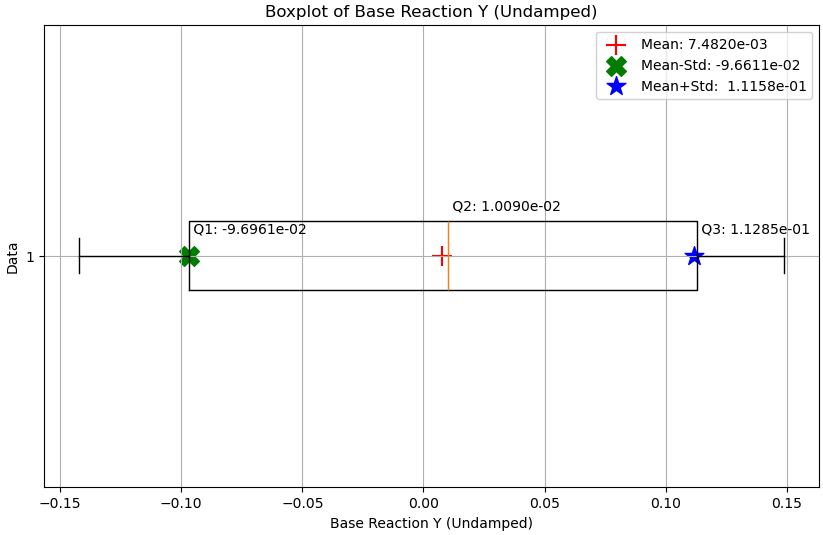




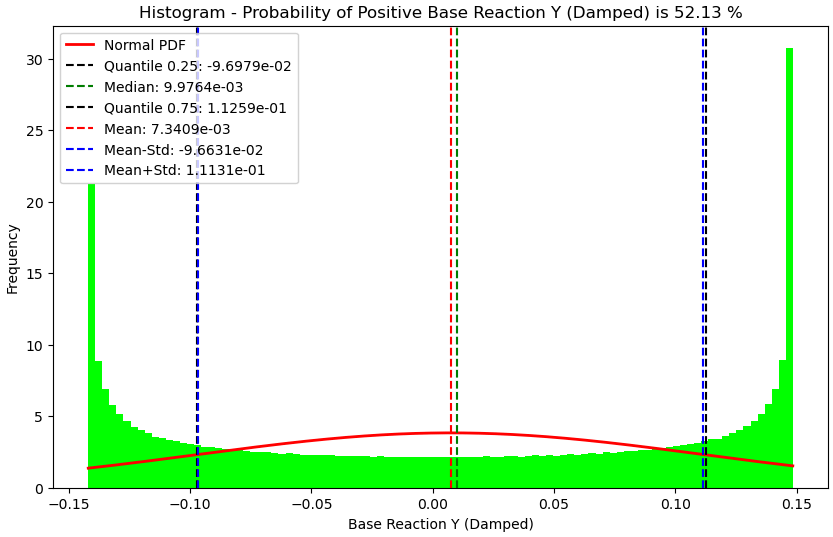
In [37]:
```
HISROGRAM_BOXPLOT(BASE_REACTION_Y_undamped, HISTO_COLOR='green', LABEL='Base Reaction Y (Undamped)')
HISROGRAM_BOXPLOT(BASE_REACTION_Y_damped, HISTO_COLOR='lime', LABEL='Base Reaction Y (Damped)')
#HISTOGRAM_BOXPLOT_PLOTLY(BASE_REACTION_Y_undamped, XLABEL='Base Reaction Y (Undamped)', TITLE='Base Reaction Y (Undamped)', COLOR='green')
#HISTOGRAM_BOXPLOT_PLOTLY(BASE_REACTION_Y_damped, XLABEL='Base Reaction Y (Damped)', TITLE='Base Reaction Y (Damped)', COLOR='lime')
```

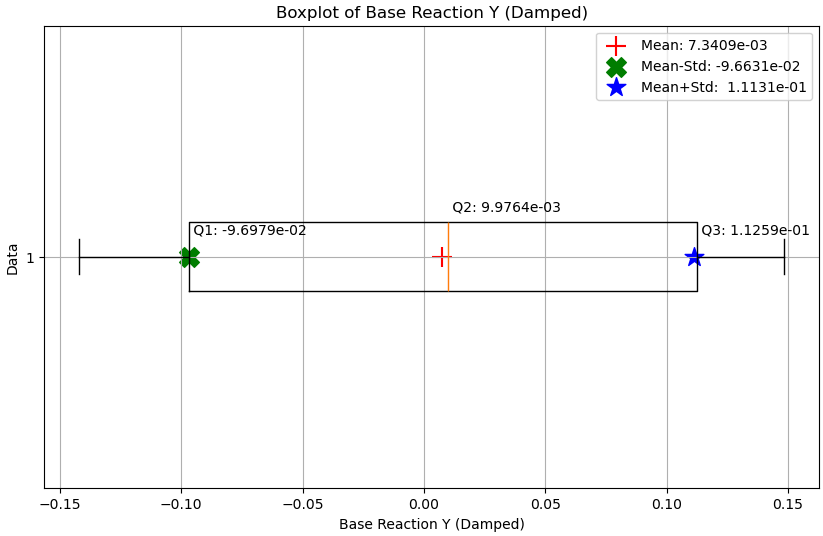
```
-------------------------
Box-Chart Datas:
Minimum: -1.4199e-01
First quartile: -9.6961e-02
Median: 1.0090e-02
Mean: 7.4820e-03
Std: 1.0409e-01
Third quartile: 1.1285e-01
Maximum: 1.4872e-01
Skewness: -4.2545e-02
kurtosis: -1.5124e+00
90% Confidence Interval: (-1.4008e-01, 1.4825e-01)
-------------------------
```






```
-------------------------
Box-Chart Datas:
Minimum: -1.4199e-01
First quartile: -9.6979e-02
Median: 9.9764e-03
Mean: 7.3409e-03
Std: 1.0397e-01
Third quartile: 1.1259e-01
Maximum: 1.4839e-01
Skewness: -4.2987e-02
kurtosis: -1.5123e+00
90% Confidence Interval: (-1.4008e-01, 1.4792e-01)
-------------------------
```





In [15]:
```
#          ##########################################################################################
#          #                                    IN THE NAME OF ALLAH                                  #
```

```
#         #                          DYNAMIC ANALYSIS OF CABLE SUSPENSION BRIDGE 01                 #
#         #-------------------------------------------------------------------------------------#
#         #                  THIS PROGRAM WRITTEN BY SALAR DELAVAR GHASHGHAEI (QASHQAI)             #
#         #                              EMAIL: salar.d.ghashghaei@gmail.com                    #
#         ##########################################################################################
```

In [47]:

```
# -------------------
#   DYNAMIC ANALYSIS
# -------------------
def DYNAMIC_ANALYSIS(damping, damping_ratio, LINEAR, periodTF, duration, dt, TOTAL_MASS, L, H1, arc_depth, E_cable, Cable_Dia_01, Cable_Dia_02, Cable_Dia_03, num_nodes, MAX_ITERATIONS,
    import openseespy.opensees as ops
    import numpy as np
    import matplotlib.pyplot as plt

    #GMfact = 9810              # standard acceleration of gravity or standard acceleration
    GMfact = 1;
    iv0 = 0.0                  # [mm/s] Initial velocity applied to the node
    st_iv0 = 0.0               # [s] Initial velocity applied starting time
    KE = 1000;                 # [N/mm] Lateral column Effective Stiffness
    A_cable_01 = (np.pi * Cable_Dia_01 **2) / 4  # [mm^2] Horizontal Longitudinal Cable Area Bottom
    A_cable_02 = (np.pi * Cable_Dia_02 **2) / 4  # [mm^2] Horizontal Longitudinal Cable Area Top
    A_cable_03 = (np.pi * Cable_Dia_03 **2) / 4  # [mm^2] Vertical Longitudinal Cable Area Top

    # Define model builder
    ops.wipe()
    ops.model('basic', '-ndm', 2, '-ndf', 2)
    dx = L / (num_nodes - 1)
    MASS = TOTAL_MASS / num_nodes
    # Create nodes
    for i in range(num_nodes):
        X = i * dx
        Y1 = H1 - arc_depth * np.sin(np.pi * X / L)
        ops.node(i + 1, X, Y1)
        ops.node(num_nodes + i + 1, i * dx, 0.0)
        # Define mass
        ops.mass(num_nodes + i + 1, MASS, MASS, 0)

    # Define boundary conditions (fixed at both ends)
    ops.fix(1, 1, 1)             # TOP CABLE
    ops.fix(num_nodes, 1, 1)     # TOP CABLE
    ops.fix(num_nodes + 1, 1, 1)  # BOTTOM CABLE
    ops.fix(2 * num_nodes, 1, 1)  # BOTTOM CABLE

    # Define material properties
    if LINEAR == True:
        ops.uniaxialMaterial('Elastic', 1, E_cable)
        #                                TENSION    COMPRESSION
        #ops.uniaxialMaterial('Elastic', 1, E_cable, 0, 0.5 * E_cable)
    if LINEAR == False:
        Fy_cable = 355   # [N/mm^2] Yield strength of the cable
        b0 = 0.01
        ops.uniaxialMaterial('Steel01', 1, Fy_cable, E_cable, b0)

    # Define truss elements
    for i in range(num_nodes - 1):
        ops.element('corotTruss', i + 1, i + 1, i + 2, A_cable_02, 1)  # Top Cable element
        ops.element('corotTruss', num_nodes + i + 1, num_nodes + i + 1, num_nodes + i + 2, A_cable_01, 1)  # Bottom Cable Element

    # Connect each node of the cable with the corresponding node of the deck using truss elements
    for i in range(num_nodes):
        ops.element('corotTruss', 2 * num_nodes + i + 1, i + 1, num_nodes + i + 1, A_cable_03, 1) # Vertical Cable Element

    #mid_node = num_nodes // 2 + 1
    mid_node = int(num_nodes + 0.5 * num_nodes)

    # Dynamic analysis setup
    ops.constraints('Transformation')
    ops.numberer('RCM')
    ops.system('UmfPack')
    ops.test('EnergyIncr', TOLERANCE, MAX_ITERATIONS)
    #ops.integrator('CentralDifference')
    ops.integrator('HHT', 0.9)
    #ops.integrator('Newmark', 0.5, 0.25)
    ops.algorithm('ModifiedNewton')

    # Define analysis type
    ops.analysis('Transient')

    # Define time series for input motion (Acceleration time history)
    ops.timeSeries('Path', 1, '-dt', 0.01, '-filePath', 'OPENSEES_SPRING_SEISMIC_01.txt', '-factor', GMfact, '-startTime', st_iv0) # SEISMIC-X
    ops.timeSeries('Path', 2, '-dt', 0.01, '-filePath', 'OPENSEES_SPRING_SEISMIC_02.txt', '-factor', GMfact) # SEISMIC-Y

    # Define load patterns
    # pattern UniformExcitation $patternTag $dof -accel $tsTag <-vel0 $vel0> <-fact $cFact>
    ops.pattern('UniformExcitation', 1, 1, '-accel', 1, '-vel0', iv0, '-fact', 1.0) # SEISMIC-X
    ops.pattern('UniformExcitation', 2, 2, '-accel', 2)                             # SEISMIC-Y

    # Perform eigenvalue analysis to determine modal periods
    if periodTF == True:
        eigenvalues01 = ops.eigen('-genBandArpack', 1) # eigenvalue mode 1
        #eigenvalues01 = ops.eigen('-fullGenLapack', 1) # eigenvalue mode 1
        #eigenvalues01 = ops.eigen('-symmBandLapack', 1) # eigenvalue mode 1
        #Omega = np.power(eigenvalues01, 0.5)
        #Omega = np.power(max(min(eigenvalues01), min(eigenvalues02), min(eigenvalues03)), 0.5)
        Omega = np.power(min(eigenvalues01), 0.5)
        modal_period = 2 * np.pi / np.sqrt(Omega)  # [Second]
        frequency = 1 / modal_period               # [Hertz]

    if periodTF == False:
        modal_period = 0.0            # [Second]
        frequency = 0.0               # [Hertz]

    if damping == True:
        # Calculate Rayleigh damping factors
        omega1 = np.sqrt(KE / TOTAL_MASS)
        omega2 = 2 * omega1  # Just an assumption for two modes
        a0 = damping_ratio * (2 * omega1 * omega2) / (omega1 + omega2)
        a1 = damping_ratio * 2 / (omega1 + omega2)
        # Apply Rayleigh damping
        ops.rayleigh(a0, a1, 0, 0)

    # OUTPUT DATA
```

```
    ops.recorder('Node', '-file', f"{SALAR_DIR}DTH_DYN.txt",'-time', '-node', mid_node, '-dof', 1,2, 'disp')# Displacement Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}VTH_DYN.txt",'-time', '-node', mid_node, '-dof', 1,2,3, 'vel')  # Velocity Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}ATH_DYN.txt",'-time', '-node', mid_node, '-dof', 1,2,3, 'accel') # Acceleration Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_DYN_01.txt",'-time', '-node', 1, '-dof', 1,2, 'reaction')# Base Shear Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_DYN_02.txt",'-time', '-node', num_nodes, '-dof', 1,2, 'reaction')# Base Shear Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_DYN_03.txt",'-time', '-node', num_nodes + 1, '-dof', 1,2, 'reaction')# Base Shear Time History
    ops.recorder('Node', '-file', f"{SALAR_DIR}BTH_DYN_04.txt",'-time', '-node', 2*num_nodes, '-dof', 1,2, 'reaction')# Base Shear

    DISPLACEMENTS = []
    DISP_X , DISP_Y, VELOCITY, ACCELERATION = [], [], [], []
    BASE_REACTION_X, BASE_REACTION_Y = [], []

    stable = 0
    current_time = 0.0
     # Perform the analysis with increments
    while stable == 0 and current_time < duration:
        OK = ops.analyze(1, dt)
        ANALYSIS(OK, 1, TOLERANCE, MAX_ITERATIONS)
        current_time = ops.getTime()
        DISPLACEMENTS.append([ops.nodeDisp(j + 1) for j in range(num_nodes * 2)])
        DISP_X.append(ops.nodeDisp(mid_node, 1))
        DISP_Y.append(ops.nodeDisp(mid_node, 2))
        VELOCITY.append(ops.nodeVel(mid_node, 2))
        ACCELERATION.append(ops.nodeAccel(mid_node, 2))
        x1 = ops.nodeResponse(1, 1, 6)
        x2 = ops.nodeResponse(num_nodes, 1, 6)
        x3 = ops.nodeResponse(num_nodes + 1, 1, 6)
        x4 = ops.nodeResponse(2*num_nodes, 1, 6)
        y1 = ops.nodeResponse(1, 2, 6)
        y2 = ops.nodeResponse(num_nodes, 2, 6)
        y3 = ops.nodeResponse(num_nodes + 1, 2, 6)
        y4 = ops.nodeResponse(2*num_nodes, 2, 6)
        BASE_REACTION_X.append(x1+x2+x3+x4) # CABLE REACION-X
        BASE_REACTION_Y.append(y1+y2+y3+y4) # CABLE REACION-Y
        KE =  BASE_REACTION_Y[-1] / DISP_Y[-1] # Effective Lateral Stiffness
        #print(f'STEP: {i+1}')

    # Get initial and final node coordinates for plotting
    initial_coords = np.array([ops.nodeCoord(i + 1) for i in range(num_nodes * 2)])
    deformed_coords = initial_coords + np.array(DISPLACEMENTS[-1])

    # Output all unconstrained node displacements
    for i in range(num_nodes * 2):
        disp = ops.nodeDisp(i + 1)
        print(f"Node {i + 1}: X Displacement = {disp[0]}, Y Displacement = {disp[1]}")

    ops.wipe()
    print("Period: ", modal_period)
    print("Frequency: ", frequency)
    if damping == False:
        print(f' Undamping Structure Dynamic Analysis Done.')
    if damping == True:
        print(f' Damping Structure Dynamic Analysis Done.')

    return initial_coords, deformed_coords, DISPLACEMENTS, VELOCITY, ACCELERATION, DISP_X, DISP_Y, BASE_REACTION_X, BASE_REACTION_Y
```

In [48]:

```
def plot_time_history(DISP_X_undamped, DISP_X_damped, DISP_Y_undamped, DISP_Y_damped,
                      VELOCITY_undamped, VELOCITY_damped, ACCELERATION_undamped, ACCELERATION_damped,
                      BASE_REACTION_X_undamped, BASE_REACTION_X_damped, BASE_REACTION_Y_undamped, BASE_REACTION_Y_damped):
    import matplotlib.pyplot as plt
    import numpy as np
    # Assuming you have 'time' array representing the time steps
    time = [i * dt for i in range(len(DISPLACEMENTS_undamped))]
    plt.figure(figsize=(14, 20))

    # Plot Displacements in X direction
    plt.subplot(6, 1, 1)
    P1 = DISP_X_undamped
    P2 = DISP_X_damped
    P11 = np.max(np.abs(P1))
    P22 = np.max(np.abs(P2))
    P3 = P11 / P22
    plt.plot(time, P1, color='black', label=f'Undamped X: {P11:.5e}')
    plt.plot(time, P2, color='red', label=f'Damped X: {P22:.5e}')
    plt.title(f'Displacement Time History (X direction) - Amplification Factor: {P3:.3f}')
    plt.xlabel('Time [s]')
    plt.ylabel('Displacement [m] (X direction)')
    plt.legend()

    # Plot Displacements in Y direction
    plt.subplot(6, 1, 2)
    P1 = DISP_Y_undamped
    P2 = DISP_Y_damped
    P11 = np.max(np.abs(P1))
    P22 = np.max(np.abs(P2))
    P3 = P11 / P22
    plt.plot(time, P1, color='black', label=f'Undamped Y: {P11:.5e}')
    plt.plot(time, P2, color='red', label=f'Damped Y: {P22:.5e}')
    plt.title(f'Displacement Time History (Y direction) - Amplification Factor: {P3:.3f}')
    plt.xlabel('Time [s]')
    plt.ylabel('Displacement [m] (Y direction)')
    plt.legend()

    # Plot Velocities
    plt.subplot(6, 1, 3)
    P1 = VELOCITY_undamped
    P2 = VELOCITY_damped
    P11 = np.max(np.abs(P1))
    P22 = np.max(np.abs(P2))
    P3 = P11 / P22
    plt.plot(time, P1, color='black', label=f'Undamped: {P11:.5e}')
    plt.plot(time, P2, color='red', label=f'Damped: {P22:.5e}')
    plt.title(f'Velocity Time History - Amplification Factor: {P3:.3f}')
    plt.xlabel('Time [s]')
    plt.ylabel('Velocity [m/s] (Y direction)')
    plt.legend()

    # Plot Accelerations
    plt.subplot(6, 1, 4)
    P1 = ACCELERATION_undamped
    P2 = ACCELERATION_damped
    P11 = np.max(np.abs(P1))
```

```
    P22 = np.max(np.abs(P2))
    P3 = P11 / P22
    plt.plot(time, P1, color='black', label=f'Undamped: {P11:.5e}')
    plt.plot(time, P2, color='red', label=f'Damped: {P22:.5e}')
    plt.title(f'Acceleration Time History - Amplification Factor: {P3:.3f}')
    plt.xlabel('Time [s]')
    plt.ylabel('Acceleration [m/s²] (Y direction)')
    plt.legend()

    # Plot Base Reactions in X direction
    plt.subplot(6, 1, 5)
    P1 = BASE_REACTION_X_undamped
    P2 = BASE_REACTION_X_damped
    P11 = np.max(np.abs(P1))
    P22 = np.max(np.abs(P2))
    P3 = P11 / P22
    plt.plot(time, P1, color='black', label=f'Undamped: {P11:.5e}')
    plt.plot(time, P2, color='red', label=f'Damped: {P22:.5e}')
    plt.title(f'Base Reaction Time History (X direction) - Amplification Factor: {P3:.3f}')
    plt.xlabel('Time [s]')
    plt.ylabel('Base Reaction [N] (X direction)')
    plt.legend()

    # Plot Base Reactions in Y direction
    plt.subplot(6, 1, 6)
    P1 = BASE_REACTION_Y_undamped
    P2 = BASE_REACTION_Y_damped
    P11 = np.max(np.abs(P1))
    P22 = np.max(np.abs(P2))
    P3 = P11 / P22
    plt.plot(time, P1, color='black', label=f'Undamped: {P11:.5e}')
    plt.plot(time, P2, color='red', label=f'Damped: {P22:.5e}')
    plt.title(f'Base Reaction Time History (Y direction) - Amplification Factor: {P3:.3f}')
    plt.xlabel('Time [s]')
    plt.ylabel('Base Reaction [N] (Y direction)')
    plt.legend()

    plt.tight_layout()
    plt.show()
```

In [49]:

```
# -------------------
#   DYNAMIC ANALYSIS
# -------------------
# Parameters for the analysis
L = 50000.0         # [mm] Bridge span lentgh
H1 = 3500.0         # [mm] Height of Top Cable
arc_depth = 1000.0  # [mm]
E_cable = 210e5     # [N/mm^2] Modulus of elasticity Cable
Cable_Dia_01 = 25   # [mm] Horizontal Longitudinal Cable Diameter Bottom
Cable_Dia_02 = 18   # [mm] Horizontal Longitudinal Cable Diameter Top
Cable_Dia_03 = 10   # [mm] Vertical Longitudinal Cable Diameter Top
num_nodes = 51      # Cable Arc Number of nodes

TOTAL_MASS = 500000.0  # [kg] Total Mass of Structure
damping_ratio = 0.05   # Damping ratio
duration = 50.0        # [s] Duration of the analysis in seconds
dt = 0.01              # Time step in seconds

MAX_ITERATIONS = 10000  # Maximum number of iterations
TOLERANCE = 1.0e-14     # Tolerance for convergence

import time
starttime = time.process_time()

# Run the undamped analysis
damping = False
LINEAR = True # False: Cable NonLinear Materials Properties
periodTF = False
results_undamped = DYNAMIC_ANALYSIS(damping, damping_ratio, LINEAR, periodTF, duration, dt, TOTAL_MASS, L, H1, arc_depth, E_cable, Cable_Dia_01, Cable_Dia_02, Cable_Dia_03, num_nodes,
initial_coords, deformed_coords, DISPLACEMENTS_undamped, VELOCITY_undamped, ACCELERATION_undamped, DISP_X_undamped, DISP_Y_undamped, BASE_REACTION_X_undamped, BASE_REACTION_Y_undamped

# Run the damped analysis
damping = True
LINEAR = True # False: Cable NonLinear Materials Properties
periodTF = False
results_damped = DYNAMIC_ANALYSIS(damping, damping_ratio, LINEAR, periodTF, duration, dt, TOTAL_MASS, L, H1, arc_depth, E_cable, Cable_Dia_01, Cable_Dia_02, Cable_Dia_03, num_nodes, MA
initial_coords, deformed_coords, DISPLACEMENTS_damped, VELOCITY_damped, ACCELERATION_damped, DISP_X_damped, DISP_Y_damped, BASE_REACTION_X_damped, BASE_REACTION_Y_damped = results_damp

totaltime = time.process_time() - starttime
print(f'\nTotal time (s): {totaltime:.4f} \n\n')
```

Node 1: X Displacement = 0.0, Y Displacement = 0.0
Node 2: X Displacement = -0.0027149136862820965, Y Displacement = -0.0435567618976172 65
Node 3: X Displacement = -0.0025178038686 10918, Y Displacement = -0.04074069392452289
Node 4: X Displacement = -0.0023078010755906183, Y Displacement = -0.03769434475642839
Node 5: X Displacement = -0.0021022577713170247, Y Displacement = -0.0346839438557570 16
Node 6: X Displacement = -0.0019025800731 2154, Y Displacement = -0.03172173693899921
Node 7: X Displacement = -0.0017100589546797569, Y Displacement = -0.028819414637991293
Node 8: X Displacement = -0.0015258725111800653, Y Displacement = -0.025988431017593523
Node 9: X Displacement = -0.0013510673945118 63, Y Displacement = -0.023239958541443787
Node 10: X Displacement = -0.0011865423010942 54, Y Displacement = -0.02058484398233741
Node 11: X Displacement = -0.0010330338083749206, Y Displacement = -0.0180335656565965 9602
Node 12: X Displacement = -0.0008911047567153665, Y Displacement = -0.0155961921360386 56
Node 13: X Displacement = -0.0007611353728797475, Y Displacement = -0.013282342411244052
Node 14: X Displacement = -0.0006433172743087112, Y Displacement = -0.01110114802549537
Node 15: X Displacement = -0.0005376504469161407, Y Displacement = -0.009061216983481717
Node 16: X Displacement = -0.0004439432453345 0236, Y Displacement = -0.007170599788990378
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Node 18: X Displacement = -0.0002907040898931 3273, Y Displacement = -0.0038665331026337168
Node 19: X Displacement = -0.0002298726678750 8908, Y Displacement = -0.002466122873132193
Node 20: X Displacement = -0.0001784224293464 7, Y Displacement = -0.0012410535613773465
Node 21: X Displacement = -0.0001353067108874096, Y Displacement = -0.0001961597891846 7798
Node 22: X Displacement = -9.934741091696255e-05, Y Displacement = 0.0006644348445130573
Node 23: X Displacement = -6.925356940710637e-05, Y Displacement = 0.001337334043009977
Node 24: X Displacement = -4.364172567769229e-05, Y Displacement = 0.0018198822407185639
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Node 27: X Displacement = 2.1058323641873623e-05, Y Displacement = 0.0021101755513325705
Node 28: X Displacement = 4.3642311016199895e-05, Y Displacement = 0.0018198831359133487
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Node 31: X Displacement = 0.0001353072809927246, Y Displacement = -0.0001961577222 4865642
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Node 33: X Displacement = 0.0002298732302743 8874, Y Displacement = -0.0024661204395333134
Node 34: X Displacement = 0.0002907046511523 3667, Y Displacement = -0.0038665306143188774
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Node 44: X Displacement = 0.0015258730362813 9, Y Displacement = -0.025988427538239194
Node 45: X Displacement = 0.0017100594845515 24, Y Displacement = -0.028819411243644642
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Node 77: X Displacement = -0.0007000297267974719, Y Displacement = 0.0022064000771452 22
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Node 92: X Displacement = -0.0004551654312 3403237, Y Displacement = -0.018034018355884772
Node 93: X Displacement = -0.0004553330908 1952005, Y Displacement = -0.020585263610173 88
Node 94: X Displacement = -0.0004542516136 6228113, Y Displacement = -0.023240342321557052
Node 95: X Displacement = -0.0004529843409 5821718, Y Displacement = -0.025988776082765702
Node 96: X Displacement = -0.0004526734416 1857082, Y Displacement = -0.028819718050549024
Node 97: X Displacement = -0.0004528491832 4151104, Y Displacement = -0.03172199580454852
Node 98: X Displacement = -0.0004513263239 4332898, Y Displacement = -0.034684155449197056
Node 99: X Displacement = -0.0004585385253049 0497, Y Displacement = -0.037694506576390875
Node 100: X Displacement = -0.0003514808388 6170784, Y Displacement = -0.040740815515426916
Node 101: X Displacement = -0.0001922553391 9643305, Y Displacement = -0.043559336162014 04
Node 102: X Displacement = 0.0, Y Displacement = 0.0
Period: 0.0
Frequency: 0.0
 Undamping Structure Dynamic Analysis Done.
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Node 2: X Displacement = -0.0025569140376183546, Y Displacement = -0.041022975892697475
Node 3: X Displacement = -0.0023708530619 48677, Y Displacement = -0.03836429918416687
Node 4: X Displacement = -0.0021731277131 50373, Y Displacement = -0.03549640013525909
Node 5: X Displacement = -0.0019796019457837197, Y Displacement = -0.032662364981026315
Node 6: X Displacement = -0.0017915981593881864, Y Displacement = -0.029873700369789844
Node 7: X Displacement = -0.0016103316963372 55, Y Displacement = -0.027141412013271242

```
Node 8: X Displacement = -0.001436911635693057, Y Displacement = -0.02447628293043563
Node 9: X Displacement = -0.0012723233106233984, Y Displacement = -0.02188883103683384
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Node 12: X Displacement = -0.00083923519312216, Y Displacement = -0.014692878389439164
Node 13: X Displacement = -0.0007168559732250078, Y Displacement = -0.012514586681515266
Node 14: X Displacement = -0.0006059161754545716, Y Displacement = -0.010461178636891043
Node 15: X Displacement = -0.0005064157862075425, Y Displacement = -0.008540757957718706
Node 16: X Displacement = -0.0004181743872359723 4, Y Displacement = -0.006760903501870243
Node 17: X Displacement = -0.0003408340007512906 7, Y Displacement = -0.005128639370167705
Node 18: X Displacement = -0.0002738647347316124 3, Y Displacement = -0.003650407186561504 6
Node 19: X Displacement = -0.0002165731401600472 7, Y Displacement = -0.002332040679440304 4
Node 20: X Displacement = -0.0001681131477385179, Y Displacement = -0.001178742659910787
Node 21: X Displacement = -0.0001274994107745303, Y Displacement = -0.000195064511702434 22
Node 22: X Displacement = -9.362284508219386e-05, Y Displacement = 0.000615111749732055
Node 23: X Displacement = -6.52681191747468e-05, Y Displacement = 0.001248588803275402 7
Node 24: X Displacement = -4.1132820226334286e-05, Y Displacement = 0.001702866663427162
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Node 28: X Displacement = 4.1133357535386694e-05, Y Displacement = 0.001702867449942442 3
Node 29: X Displacement = 6.526865297277432e-05, Y Displacement = 0.001248589962738949 8
Node 30: X Displacement = 9.362337416068156e-05, Y Displacement = 0.000615113262124040 3
Node 31: X Displacement = 0.0001274999346836967 5, Y Displacement = -0.000195062696247706 27
Node 32: X Displacement = 0.0001681136673638203 3, Y Displacement = -0.001178740633718306 7
Node 33: X Displacement = 0.0002165736572976020 5, Y Displacement = -0.002332038543135007 7
Node 34: X Displacement = 0.0002738652506268945, Y Displacement = -0.003650405001275991 7
Node 35: X Displacement = 0.0003408345154589 99, Y Displacement = -0.005128637136688779
Node 36: X Displacement = 0.0004181748990895 4955, Y Displacement = -0.006760901178576752
Node 37: X Displacement = 0.0005064162927426626, Y Displacement = -0.008540755489329269
Node 38: X Displacement = 0.0006059166744545991, Y Displacement = -0.010461175976835973
Node 39: X Displacement = 0.0007168564635541571, Y Displacement = -0.012514583814799233
Node 40: X Displacement = 0.0008392356753320268, Y Displacement = -0.014692875345417128
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Node 44: X Displacement = 0.0014369121186329 12, Y Displacement = -0.024476279859852363
Node 45: X Displacement = 0.0016103321842471444, Y Displacement = -0.027141409021319254
Node 46: X Displacement = 0.0017915986527885 85, Y Displacement = -0.029873697464928257
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Node 49: X Displacement = 0.0023708542832691676, Y Displacement = -0.03836430799713927
Node 50: X Displacement = 0.0025570586090367204, Y Displacement = -0.04102527683065716
Node 51: X Displacement = 0.0, Y Displacement = 0.0
Node 52: X Displacement = 0.0, Y Displacement = 0.0
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Node 54: X Displacement = -0.0003312178283286201, Y Displacement = -0.038364404984026323
Node 55: X Displacement = -0.0004315820656338785 6, Y Displacement = -0.035496555777753657
Node 56: X Displacement = -0.0004828403576687509, Y Displacement = -0.032662567976111269
Node 57: X Displacement = -0.0004970398333040806, Y Displacement = -0.029873948084985794
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Node 64: X Displacement = -0.0004741097561593149, Y Displacement = -0.01251507135341186
Node 65: X Displacement = -0.0004396030345747897, Y Displacement = -0.01046168662285111
Node 66: X Displacement = -0.0004089560372055562, Y Displacement = -0.00854128684786756
Node 67: X Displacement = -0.0003930775633635509 4, Y Displacement = -0.006761450989261663
Node 68: X Displacement = -0.0003400627636620061 6, Y Displacement = -0.005129203254861813
Node 69: X Displacement = -0.0003456807285014761 3, Y Displacement = -0.003650985377188666 4
Node 70: X Displacement = -0.0003492886628329365, Y Displacement = -0.0023326311884335 88
Node 71: X Displacement = -0.0003555017384790007 1, Y Displacement = -0.001179343603440454 5
Node 72: X Displacement = -0.0003601556571447209 2, Y Displacement = -0.000195674105671209 96
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Node 79: X Displacement = -0.0003637785448202113 5, Y Displacement = 0.001702241828791504 6
Node 80: X Displacement = -0.0003630866718363739 2, Y Displacement = 0.001247968116379750 4
Node 81: X Displacement = -0.0003623614322330143, Y Displacement = 0.000614496760073589 3
Node 82: X Displacement = -0.0003601202807413303 2, Y Displacement = -0.000195672230411402 1
Node 83: X Displacement = -0.0003554593486858638 8, Y Displacement = -0.001179341504916821
Node 84: X Displacement = -0.0003492329264698922 9, Y Displacement = -0.002332628969990926 8
Node 85: X Displacement = -0.0003435117552344116 9, Y Displacement = -0.003650983100866264
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Node 93: X Displacement = -0.0005191110402934358, Y Displacement = -0.019389663420850 49
Node 94: X Displacement = -0.0005089766993323514, Y Displacement = -0.021889193023734762
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Node 99: X Displacement = -0.0004430097556975048 9, Y Displacement = -0.035496552886740 25
Node 100: X Displacement = -0.0003296702565367218, Y Displacement = -0.038364413331991 71
Node 101: X Displacement = -0.0001803307385195948 7, Y Displacement = -0.041025339625236 56
Node 102: X Displacement = 0.0, Y Displacement = 0.0
Period:  0.0
Frequency:  0.0
 Damping Structure Dynamic Analysis Done.

Total time (s): 10.8594
```
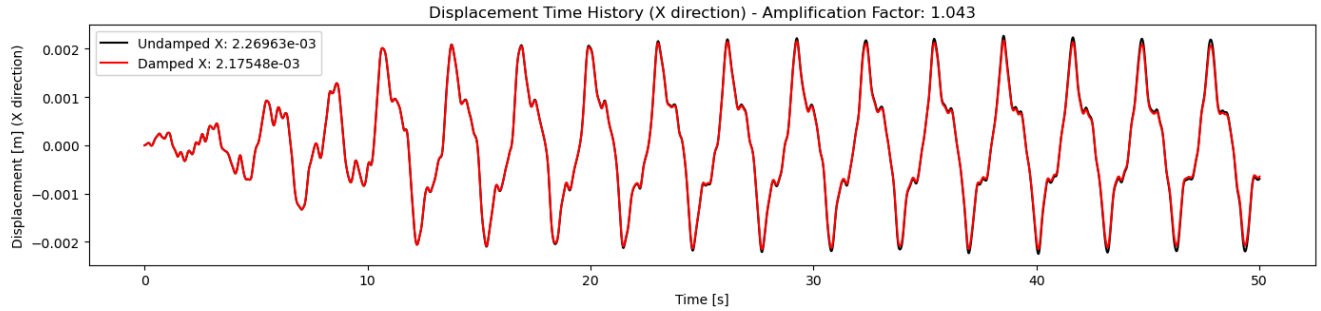
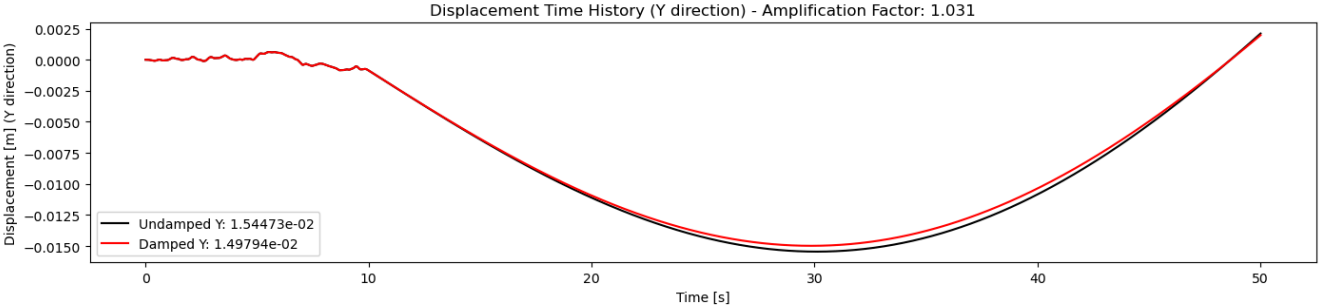
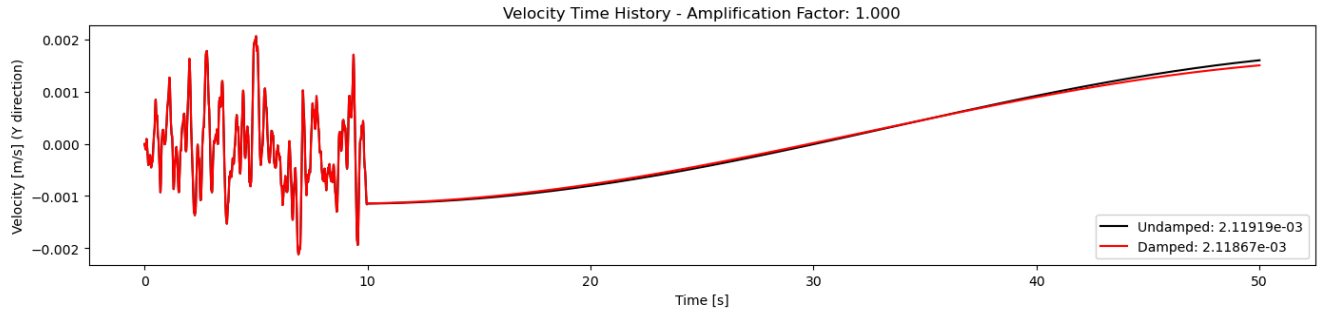
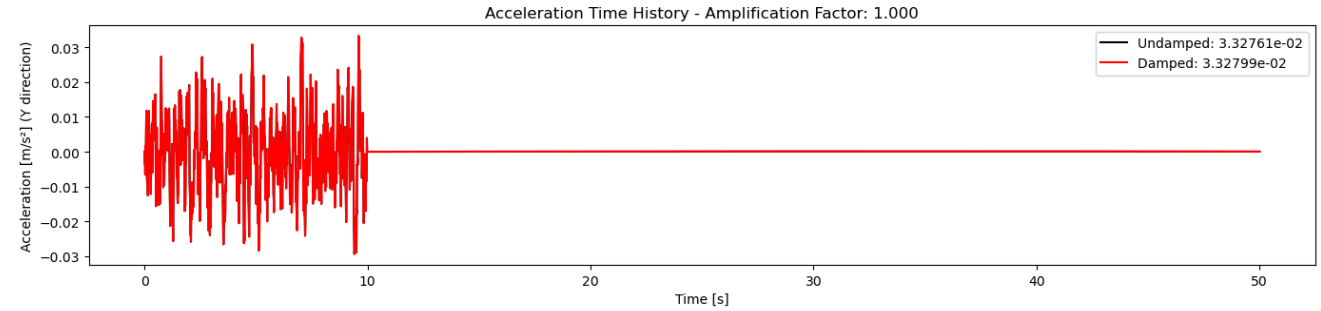
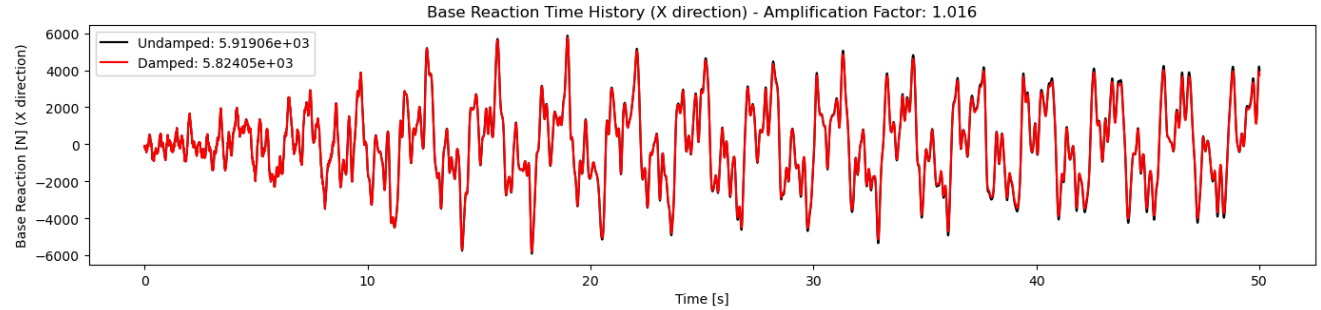

In [50]:
```
plot_time_history(DISP_X_undamped, DISP_X_damped, DISP_Y_undamped, DISP_Y_damped, VELOCITY_undamped, VELOCITY_damped, ACCELERATION_undamped, ACCELERATION_damped, BASE_REACTION_X_undamp
```

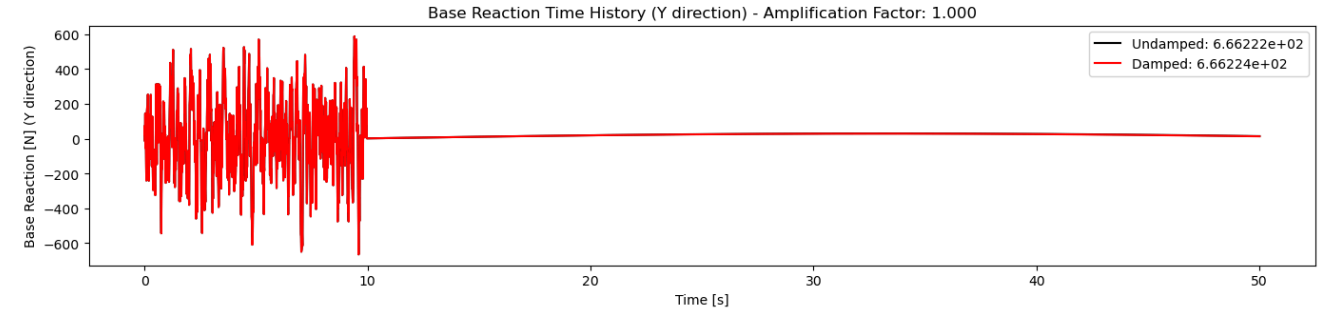

Displacement Time History (X direction) - Amplification Factor: 1.043
Undamped X: 2.26963e-03
Damped X: 2.17548e-03
Displacement [m] (X direction)
Time [s]


Displacement Time History (Y direction) - Amplification Factor: 1.031
Undamped Y: 1.54473e-02
Damped Y: 1.49794e-02
Displacement [m] (Y direction)
Time [s]


Velocity Time History - Amplification Factor: 1.000
Undamped: 2.11919e-03
Damped: 2.11867e-03
Velocity [m/s] (Y direction)
Time [s]

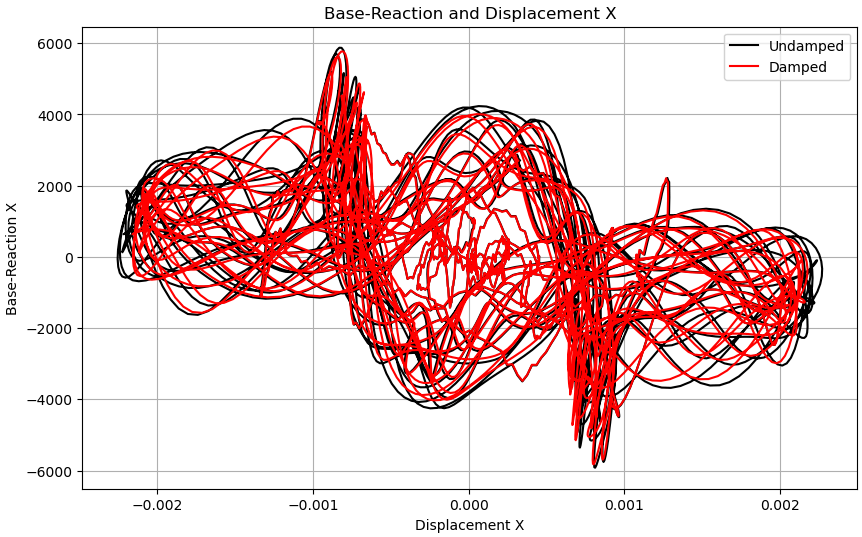

Acceleration Time History - Amplification Factor: 1.000
Undamped: 3.32761e-02
Damped: 3.32799e-02
Acceleration [m/s²] (Y direction)
Time [s]


Base Reaction Time History (X direction) - Amplification Factor: 1.016
Undamped: 5.91906e+03
Damped: 5.82405e+03
Base Reaction [N] (X direction)
Time [s]

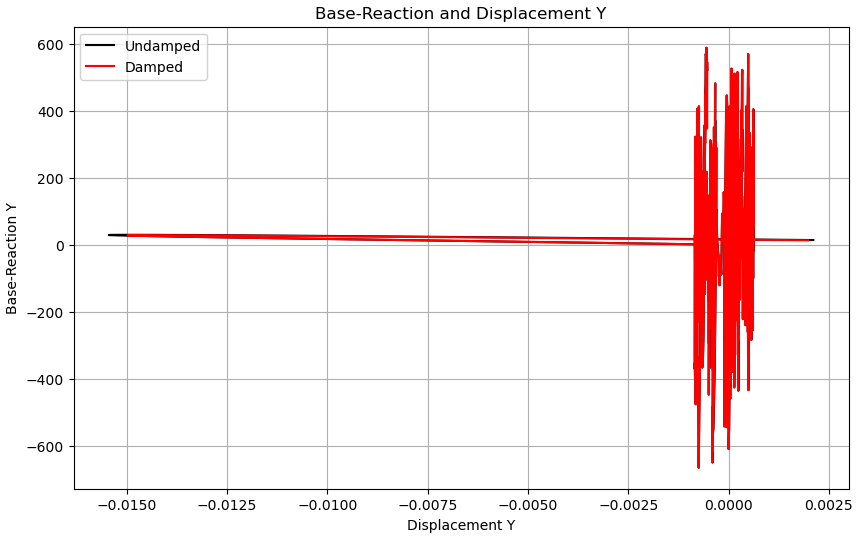

Base Reaction Time History (Y direction) - Amplification Factor: 1.000
Undamped: 6.66222e+02
Damped: 6.66224e+02
Base Reaction [N] (Y direction)
Time [s]

In [51]:
```
def PLOT_2D(X1, Y1, X2, Y2, XLABEL, YLABEL, TITLE):
    plt.figure(figsize=(10, 6))
    plt.plot(X1, Y1, label='Undamped', color='black')
    plt.plot(X2, Y2, label='Damped', color='red')
    plt.xlabel(XLABEL)
    plt.ylabel(YLABEL)
    plt.title(TITLE)
    plt.grid(True)
    #plt.semilogy()
    plt.legend()
    plt.show()
```

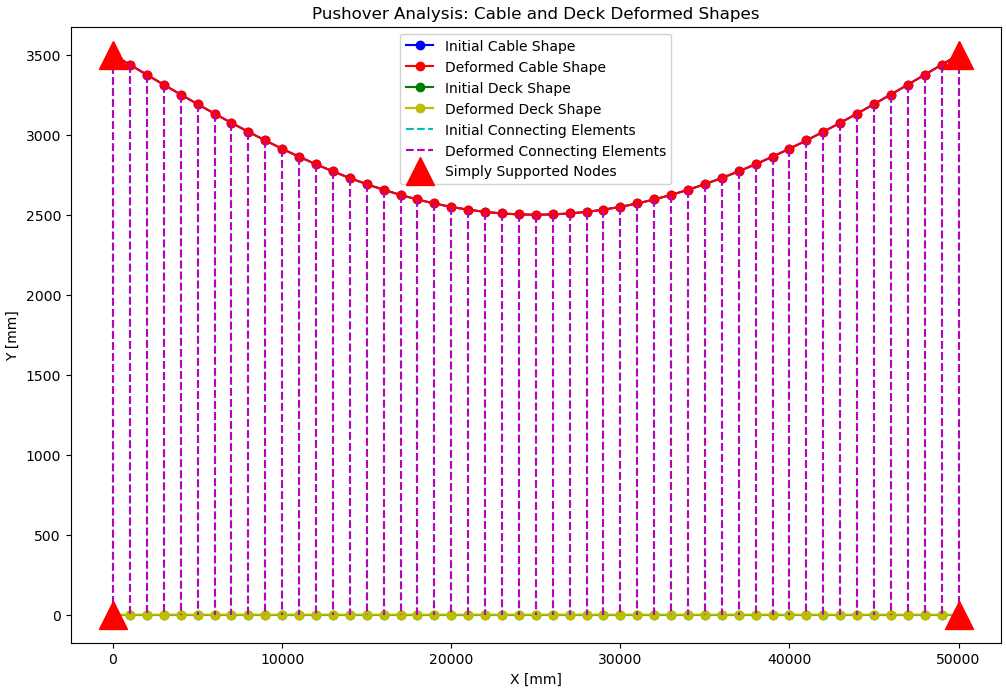
In [53]:
```
### BASE REACTION & DISPALCEMENT IN X:
X1 =  DISP_X_undamped
X2 =  DISP_X_damped
Y1 = BASE_REACTION_X_undamped
Y2 = BASE_REACTION_X_damped
XLABEL = 'Displacement X'
YLABEL = 'Base-Reaction X'
TITLE = 'Base-Reaction and Displacement X'

PLOT_2D(X1, Y1, X2, Y2, XLABEL, YLABEL, TITLE)
```



In [54]:
```
### BASE REACTION & DISPALCEMENT IN Y:
X1 =  DISP_Y_undamped
X2 =  DISP_Y_damped
Y1 = BASE_REACTION_Y_undamped
Y2 = BASE_REACTION_Y_damped
XLABEL = 'Displacement Y'
YLABEL = 'Base-Reaction Y'
TITLE = 'Base-Reaction and Displacement Y'

PLOT_2D(X1, Y1, X2, Y2, XLABEL, YLABEL, TITLE)
```



In [55]:
```
# Plot damped results
plot_shapes(initial_coords, DISPLACEMENTS_damped)
```



Plotting a histogram for the results of dynamic analysis of a structure, such as displacements, accelerations, and support reactions, helps analyze the statistical distribution and trends in the data. The following insights can be drawn from these histograms:

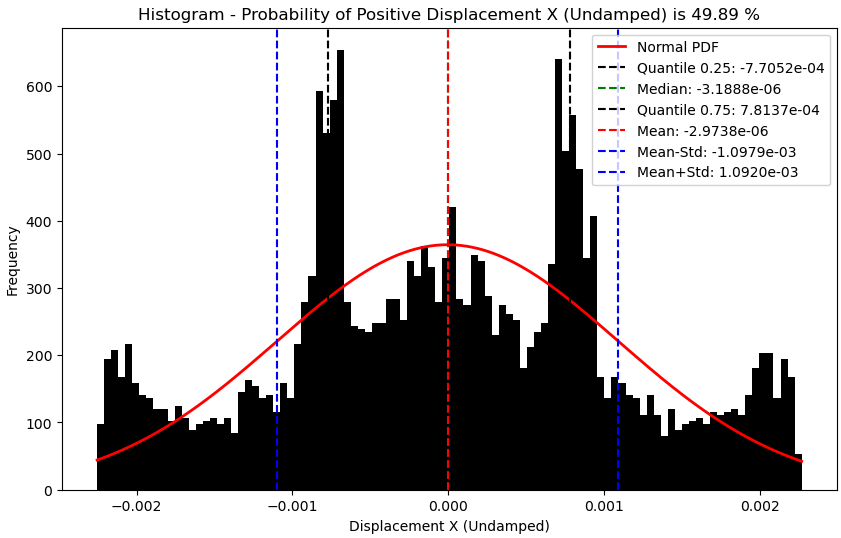
1. Range Identification

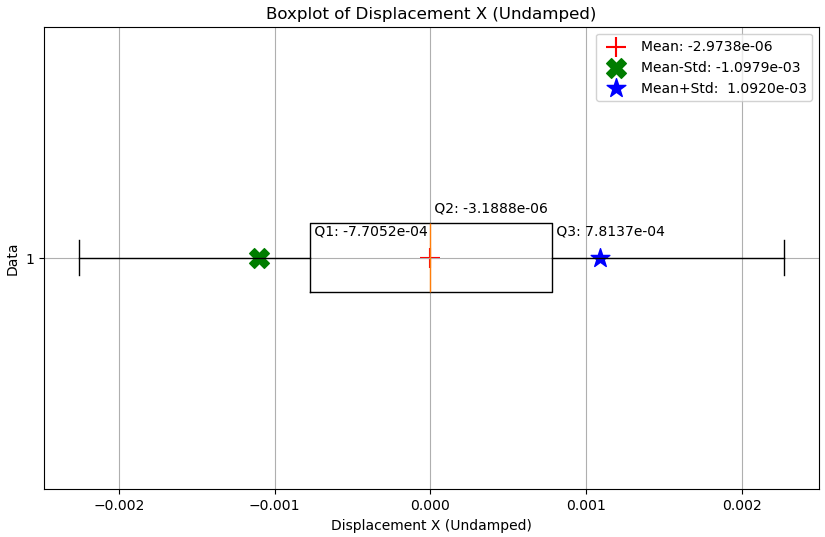
The histogram shows the range within which displacements, accelerations, or support reactions vary. This is crucial for assessing the stability of the structure and determining the allowable design limits. 2. Central Tendency From the histogram, measures like mean, median, or mode can be extracted. This helps identify the most likely values and provides an overview of the dominant behavior of the structure. 3. Data Dispersion The histogram illustrates the spread of the data (e.g., through its width and shape). A wide histogram may indicate nonlinear behavior or unusual effects in the structure. 4. Anomaly Detection Unexpected peaks or asymmetric distributions may indicate abnormal structural behavior (e.g., weaknesses or inelastic behavior). It can also reveal specific phenomena like resonances or amplification effects. 5. Probability of Specific Values The histogram's shape can indicate the probability distribution of the data (e.g., normal, uniform, or Poisson). This is useful for simulations and predicting future structural behavior. 6. Behavior Under Specific Conditions Analyzing histograms of acceleration or displacement can reveal dominant modes of the structure and their impact on overall response. These insights are helpful in designing vibration control systems (e.g., dampers). 7. Impact of External Forces or Loading An unusual histogram shape might suggest the influence of extraordinary dynamic loads (e.g., severe earthquakes or impact forces). Conclusion: Histograms are a simple yet effective tool for analyzing dynamic data. They help engineers and designers:

Understand the overall behavior of a structure. Assess the likelihood of extreme or unexpected responses. Define appropriate design or retrofitting criteria. For deeper analysis, histograms can be combined with other statistical tools, such as standard deviation or spectral analysis.

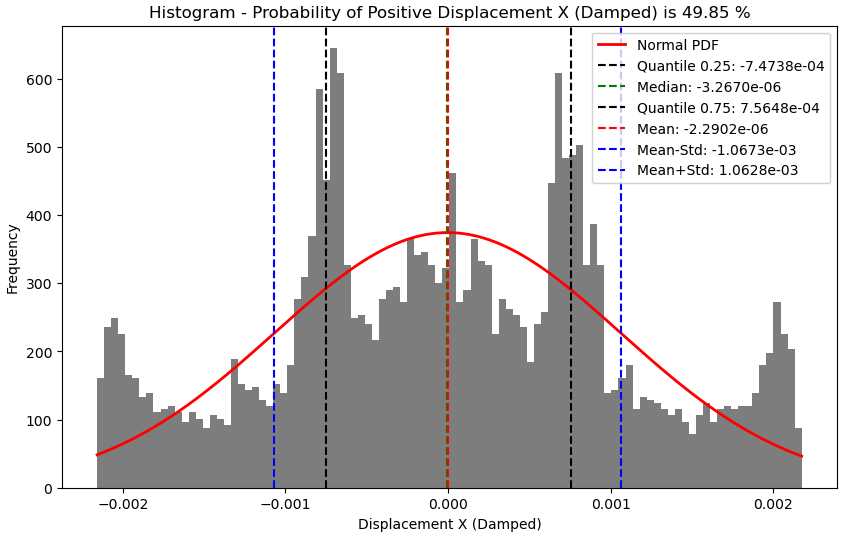
In [56]:
```
HISROGRAM_BOXPLOT(DISP_X_undamped, HISTO_COLOR='black', LABEL='Displacement X (Undamped)')
HISROGRAM_BOXPLOT(DISP_X_damped, HISTO_COLOR='grey', LABEL='Displacement X (Damped)')
#HISTOGRAM_BOXPLOT_PLOTLY(DISP_X_undamped, XLABEL='Displacement X (Undamped)', TITLE='Displacement X (Undamped)', COLOR='black')
#HISTOGRAM_BOXPLOT_PLOTLY(DISP_X_damped, XLABEL='Displacement X (Damped)', TITLE='Displacement X (Damped)', COLOR='grey')
```

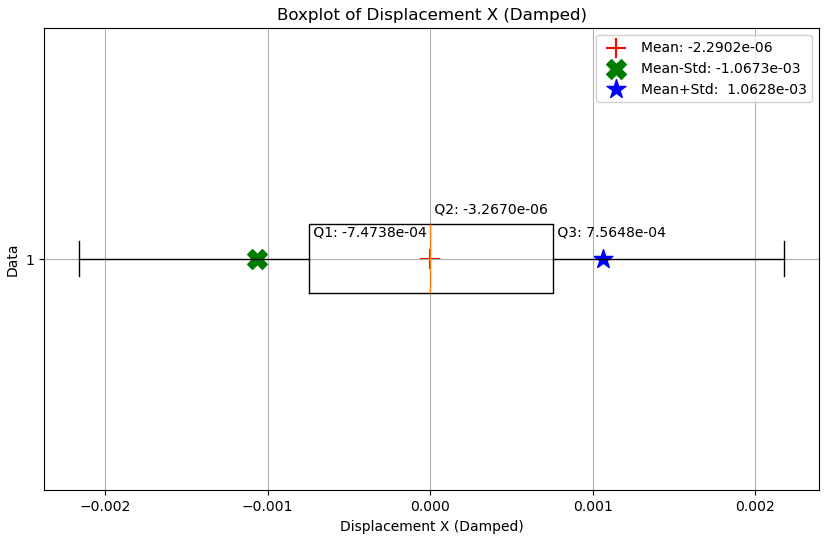
```
-------------------------
Box-Chart Datas:
Minimum: -2.2531e-03
First quartile: -7.7052e-04
Median: -3.1888e-06
Mean: -2.9738e-06
Std: 1.0949e-03
Third quartile: 7.8137e-04
Maximum: 2.2696e-03
Skewness: 4.0488e-03
kurtosis: -6.1986e-01
90% Confidence Interval: (-1.9638e-03, 1.9613e-03)
-------------------------
```






```
-------------------------
Box-Chart Datas:
Minimum: -2.1582e-03
First quartile: -7.4738e-04
Median: -3.2670e-06
Mean: -2.2902e-06
Std: 1.0650e-03
Third quartile: 7.5648e-04
Maximum: 2.1755e-03
Skewness: 6.7665e-03
kurtosis: -6.2866e-01
90% Confidence Interval: (-1.9084e-03, 1.9175e-03)
-------------------------
```

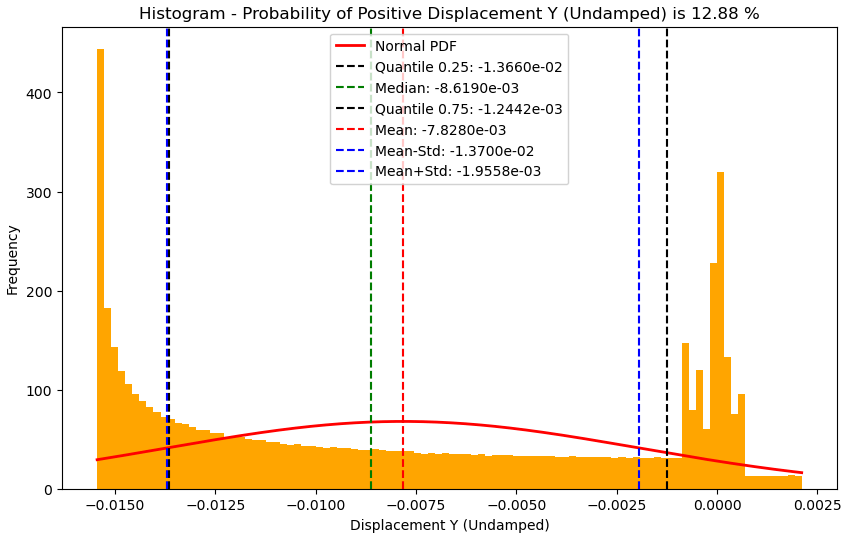


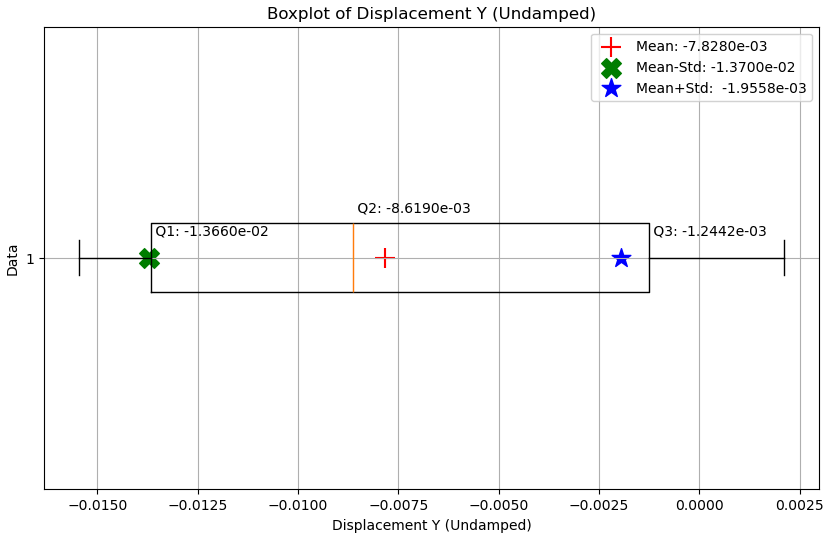


In [57]:
```
HISROGRAM_BOXPLOT(DISP_Y_undamped, HISTO_COLOR='orange', LABEL='Displacement Y (Undamped)')
HISROGRAM_BOXPLOT(DISP_Y_damped, HISTO_COLOR='brown', LABEL='Displacement Y (Damped)')
```

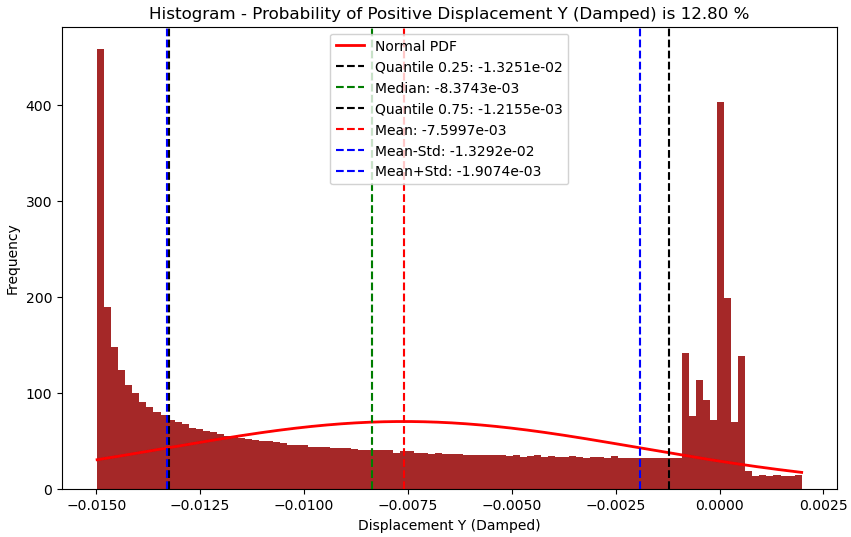
```
#HISTOGRAM_BOXPLOT_PLOTLY(DISP_Y_undamped, XLABEL='Displacement Y (Undamped)', TITLE='Displacement Y (Undamped)', COLOR='orange')
#HISTOGRAM_BOXPLOT_PLOTLY(DISP_Y_damped, XLABEL='Displacement Y (Damped)', TITLE='Displacement Y (Damped)', COLOR='brown')
```

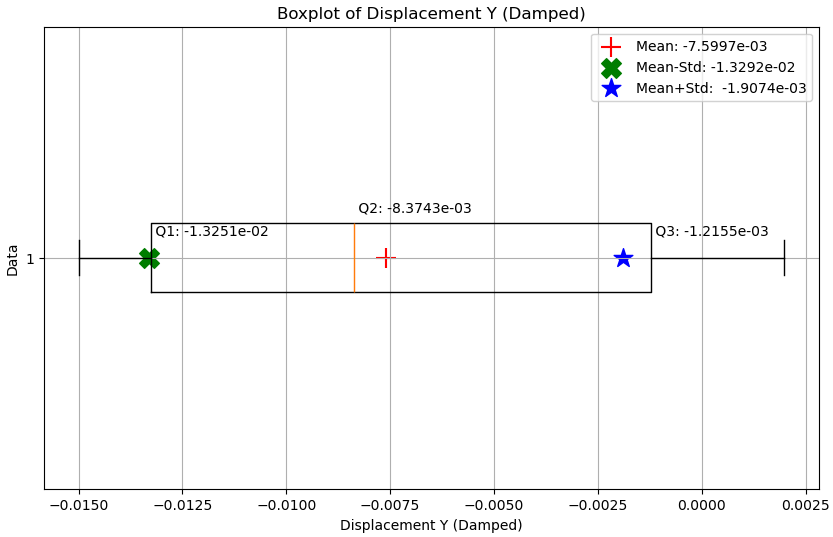
```
-------------------------
Box-Chart Datas:
Minimum: -1.5447e-02
First quartile: -1.3660e-02
Median: -8.6190e-03
Mean: -7.8280e-03
Std: 5.8722e-03
Third quartile: -1.2442e-03
Maximum: 2.1095e-03
Skewness: 1.5596e-01
kurtosis: -1.5264e+00
90% Confidence Interval: (-1.5375e-02, 3.3226e-04)
-------------------------
```





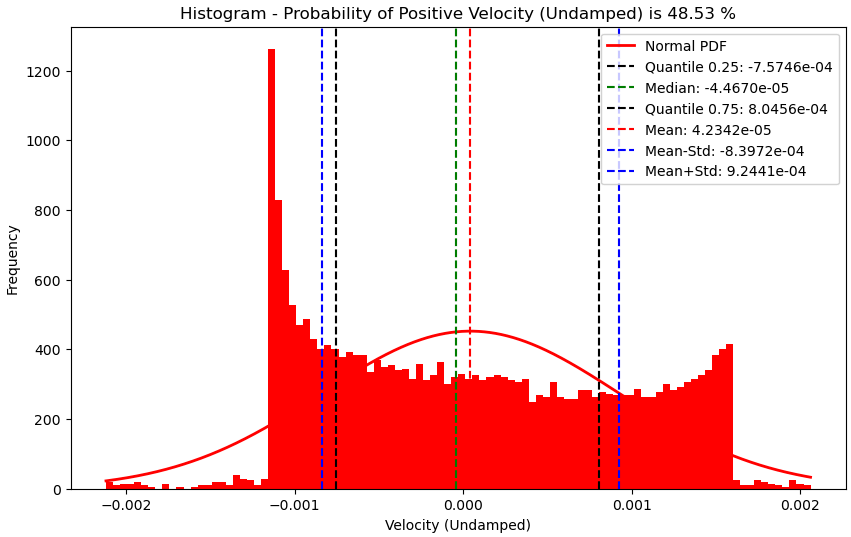

```
-------------------------
Box-Chart Datas:
Minimum: -1.4979e-02
First quartile: -1.3251e-02
Median: -8.3743e-03
Mean: -7.5997e-03
Std: 5.6923e-03
Third quartile: -1.2155e-03
Maximum: 1.9755e-03
Skewness: 1.5760e-01
kurtosis: -1.5264e+00
90% Confidence Interval: (-1.4909e-02, 3.2601e-04)
-------------------------
```

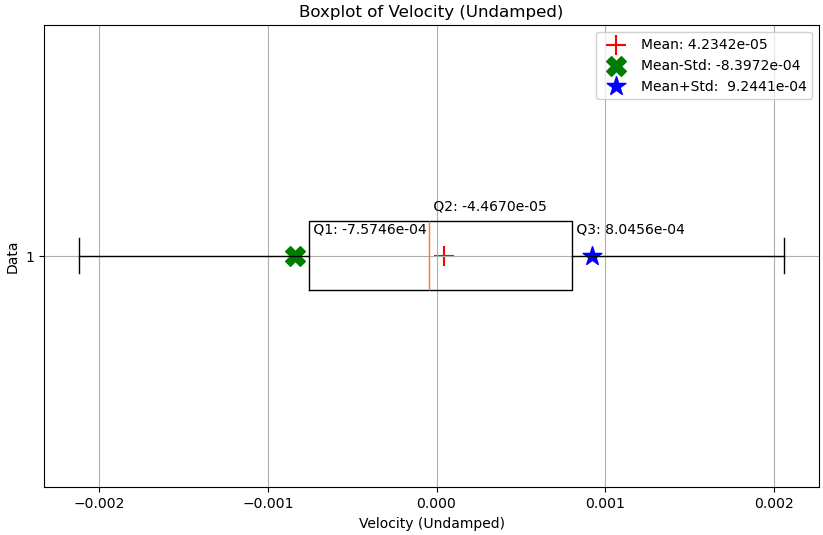




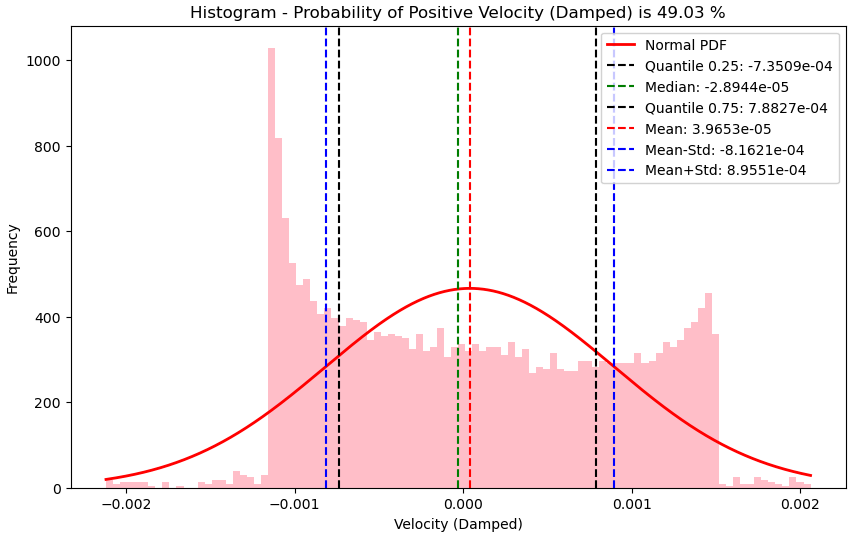
In [58]:
```
HISROGRAM_BOXPLOT(VELOCITY_undamped, HISTO_COLOR='red', LABEL='Velocity (Undamped)')
HISROGRAM_BOXPLOT(VELOCITY_damped, HISTO_COLOR='pink', LABEL='Velocity (Damped)')
#HISTOGRAM_BOXPLOT_PLOTLY(VELOCITY_undamped, XLABEL='Velocity (Undamped)', TITLE='Velocity (Undamped)', COLOR='red')
#HISTOGRAM_BOXPLOT_PLOTLY(VELOCITY_damped, XLABEL='Velocity (Damped)', TITLE='Velocity (Damped)', COLOR='pink')
```

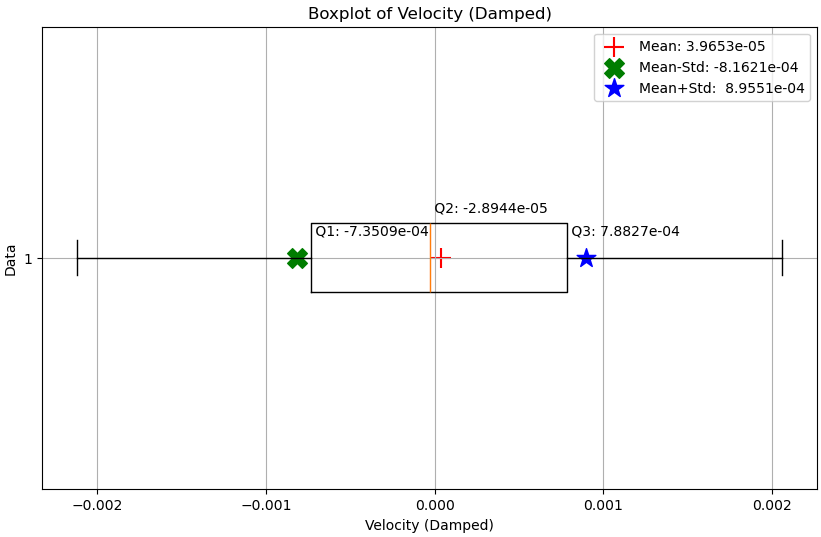
```
-------------------------
Box-Chart Datas:
Minimum: -2.1192e-03
First quartile: -7.5746e-04
Median: -4.4670e-05
Mean: 4.2342e-05
Std: 8.8206e-04
Third quartile: 8.0456e-04
Maximum: 2.0610e-03
Skewness: 2.0954e-01
kurtosis: -1.1562e+00
90% Confidence Interval: (-1.1298e-03, 1.4930e-03)
-------------------------
```





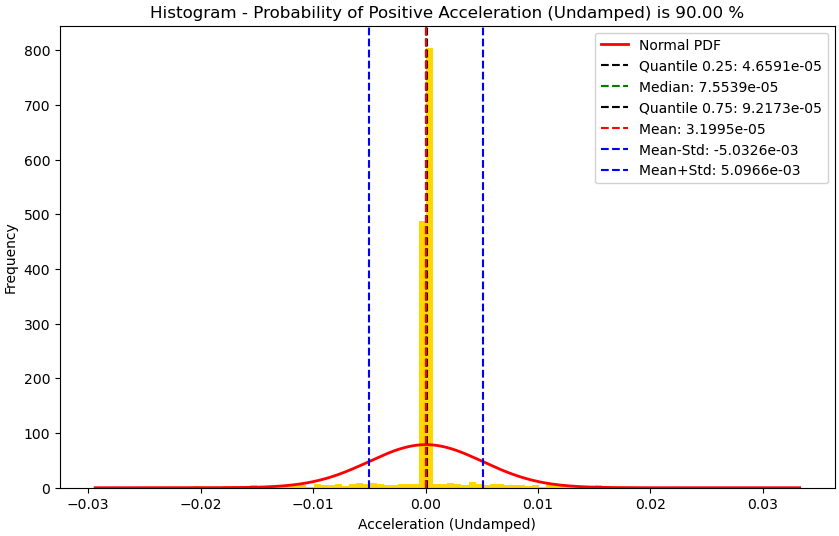

```
-------------------------
Box-Chart Datas:
Minimum: -2.1187e-03
First quartile: -7.3509e-04
Median: -2.8944e-05
Mean: 3.9653e-05
Std: 8.5586e-04
Third quartile: 7.8827e-04
Maximum: 2.0602e-03
Skewness: 1.6260e-01
kurtosis: -1.1619e+00
90% Confidence Interval: (-1.1213e-03, 1.4132e-03)
-------------------------
```

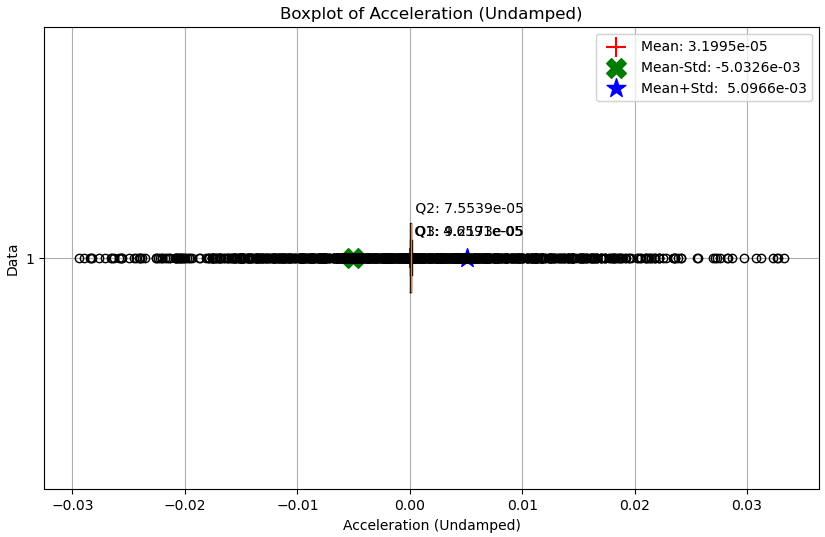




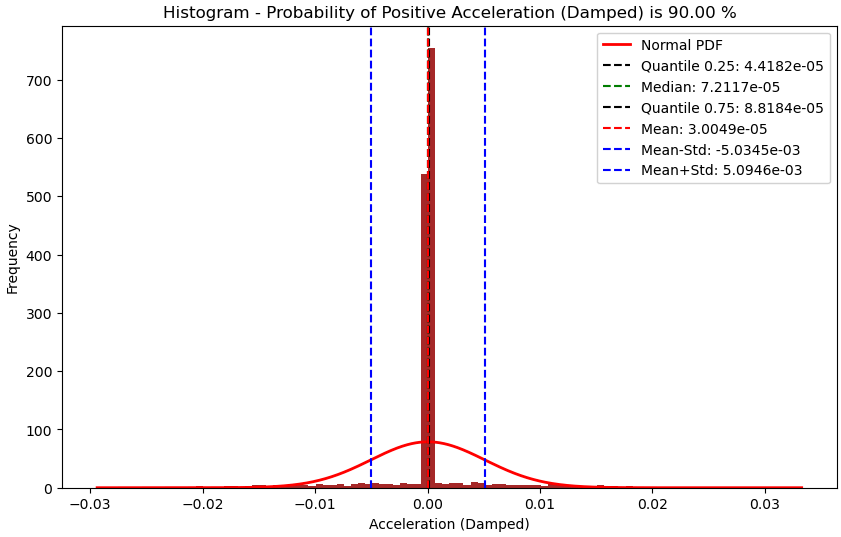
In [59]:
```
HISROGRAM_BOXPLOT(ACCELERATION_undamped, HISTO_COLOR='gold', LABEL='Acceleration (Undamped)')
HISROGRAM_BOXPLOT(ACCELERATION_damped, HISTO_COLOR='brown', LABEL='Acceleration (Damped)')
#HISTOGRAM_BOXPLOT_PLOTLY(ACCELERATION_undamped, XLABEL='Acceleration (Undamped)', TITLE='Acceleration (Undamped)', COLOR='gold')
#HISTOGRAM_BOXPLOT_PLOTLY(ACCELERATION_damped, XLABEL='Acceleration (Damped)', TITLE='Acceleration (Damped)', COLOR='brown')
```

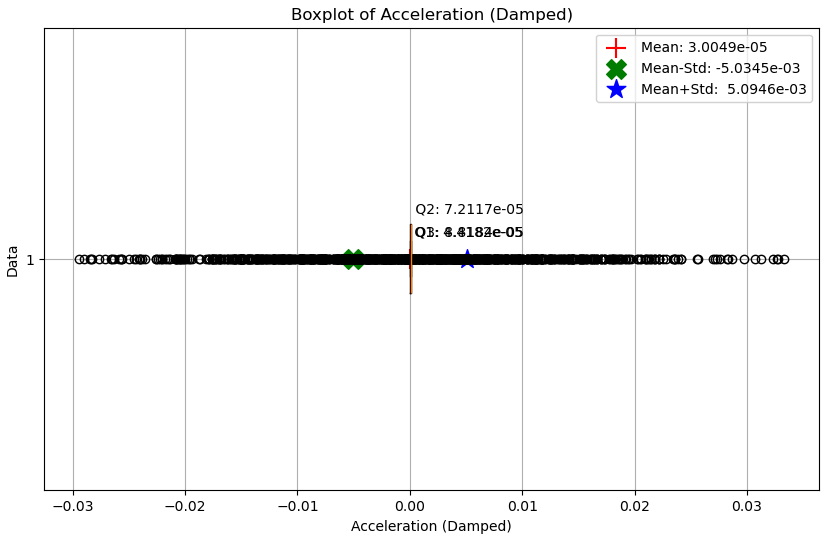
```
-------------------------
Box-Chart Datas:
Minimum: -2.9386e-02
First quartile: 4.6591e-05
Median: 7.5539e-05
Mean: 3.1995e-05
Std: 5.0646e-03
Third quartile: 9.2173e-05
Maximum: 3.3276e-02
Skewness: 7.3052e-02
kurtosis: 1.1642e+01
90% Confidence Interval: (-7.6494e-03, 6.9992e-03)
-------------------------
```






```
-------------------------
Box-Chart Datas:
Minimum: -2.9389e-02
First quartile: 4.4182e-05
Median: 7.2117e-05
Mean: 3.0049e-05
Std: 5.0646e-03
Third quartile: 8.8184e-05
Maximum: 3.3280e-02
Skewness: 7.4283e-02
kurtosis: 1.1643e+01
90% Confidence Interval: (-7.6465e-03, 6.9998e-03)
-------------------------
```

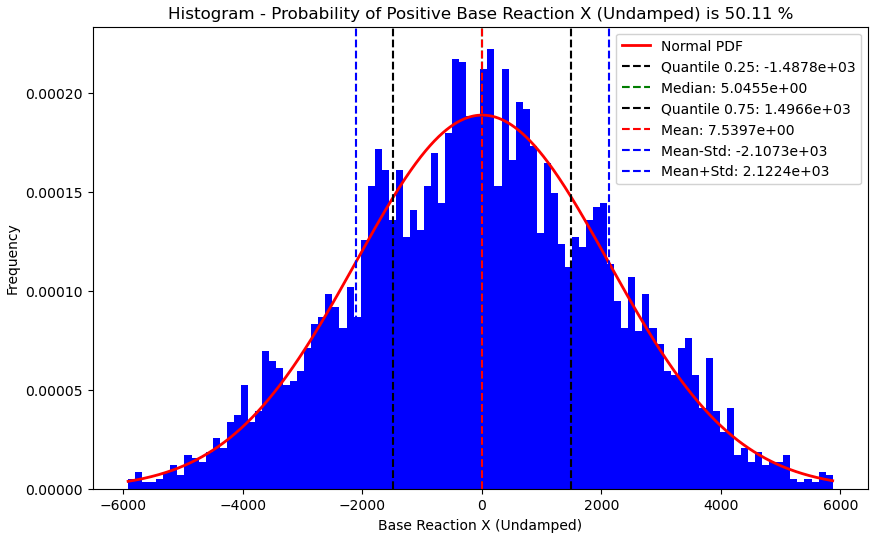


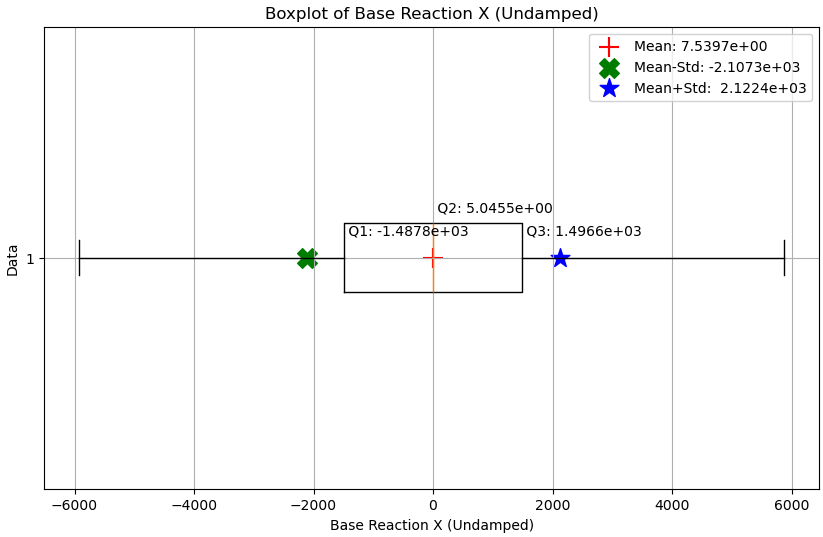


In [60]:
```
HISROGRAM_BOXPLOT(BASE_REACTION_X_undamped, HISTO_COLOR='blue', LABEL='Base Reaction X (Undamped)')
HISROGRAM_BOXPLOT(BASE_REACTION_X_damped, HISTO_COLOR='cyan', LABEL='Base Reaction X (Damped)')
```

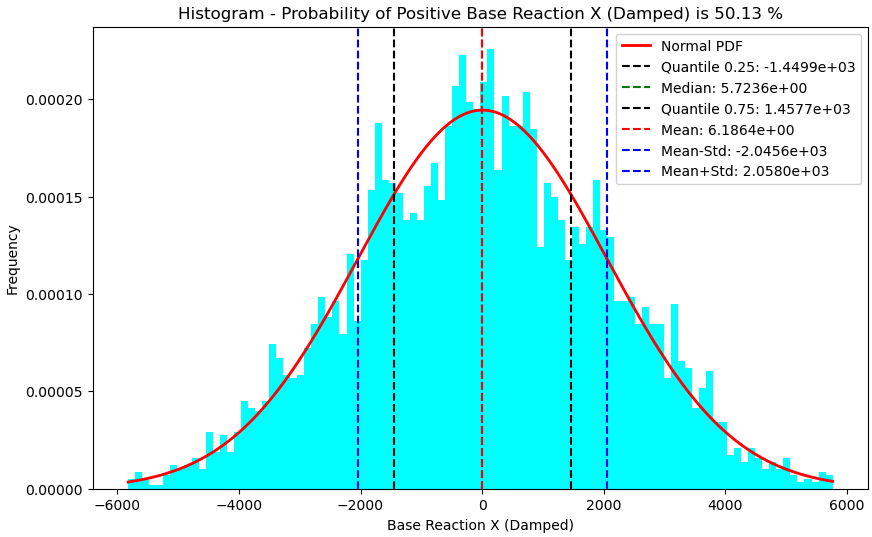
```
#HISTOGRAM_BOXPLOT_PLOTLY(BASE_REACTION_X_undamped, XLABEL='Base Reaction X (Undamped)', TITLE='Base Reaction X (Undamped)', COLOR='blue')
#HISTOGRAM_BOXPLOT_PLOTLY(BASE_REACTION_X_damped, XLABEL='Base Reaction X (Damped)', TITLE='Base Reaction X (Damped)', COLOR='cyan')
```

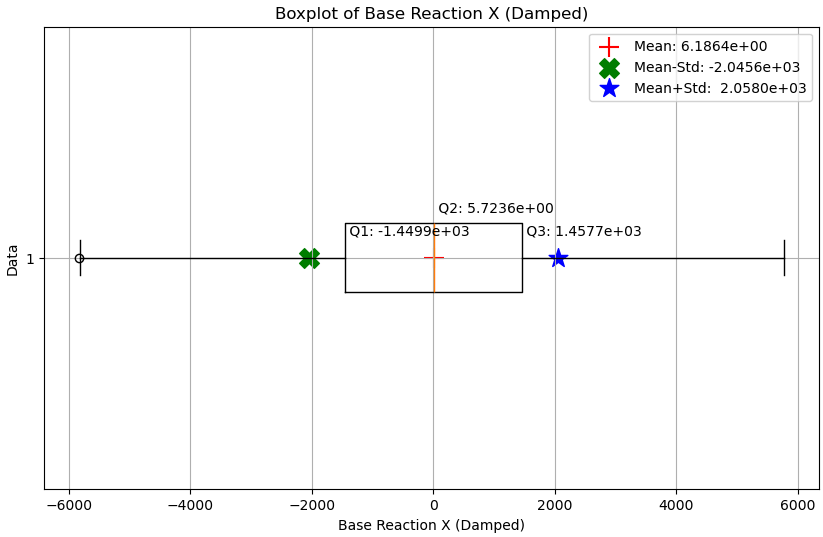
```
-------------------------
Box-Chart Datas:
Minimum: -5.9191e+03
First quartile: -1.4878e+03
Median: 5.0455e+00
Mean: 7.5397e+00
Std: 2.1149e+03
Third quartile: 1.4966e+03
Maximum: 5.8726e+03
Skewness: -1.0914e-02
kurtosis: -3.3292e-01
90% Confidence Interval: (-3.5753e+03, 3.5248e+03)
-------------------------
```





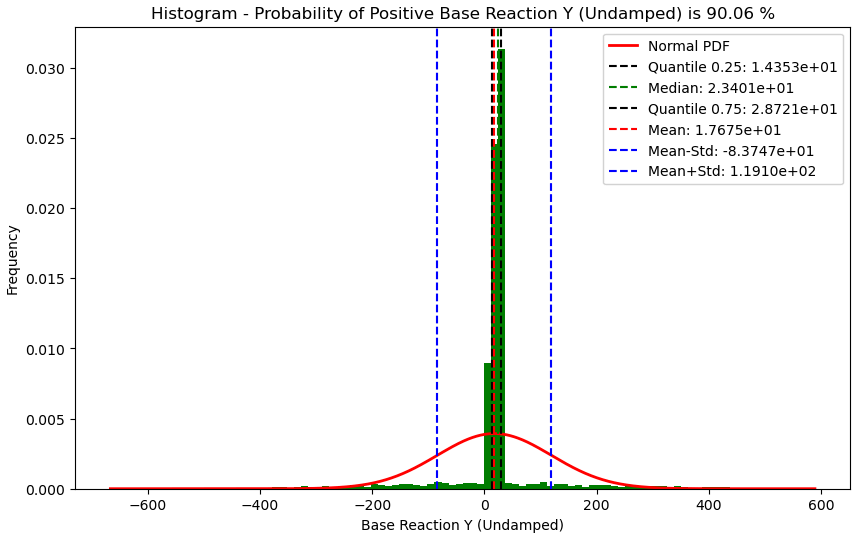

```
-------------------------
Box-Chart Datas:
Minimum: -5.8241e+03
First quartile: -1.4499e+03
Median: 5.7236e+00
Mean: 6.1864e+00
Std: 2.0518e+03
Third quartile: 1.4577e+03
Maximum: 5.7685e+03
Skewness: -1.2907e-02
kurtosis: -3.2647e-01
90% Confidence Interval: (-3.4306e+03, 3.3849e+03)
-------------------------
```

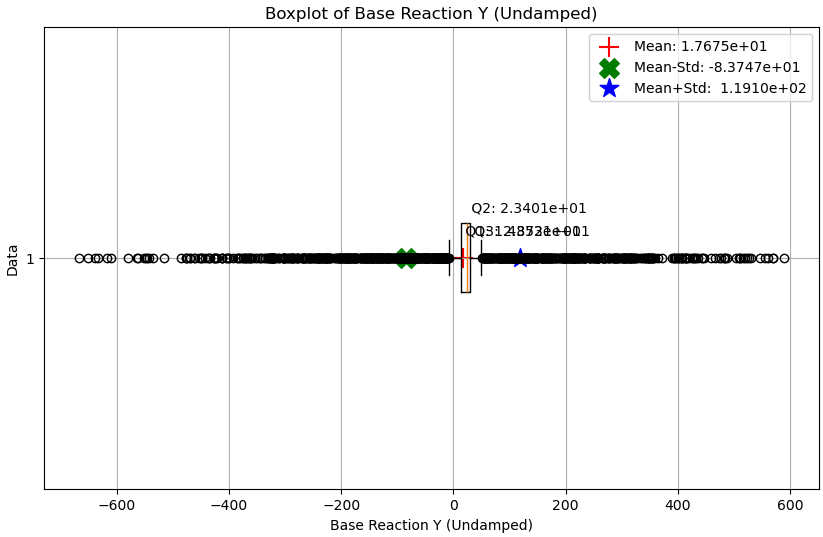




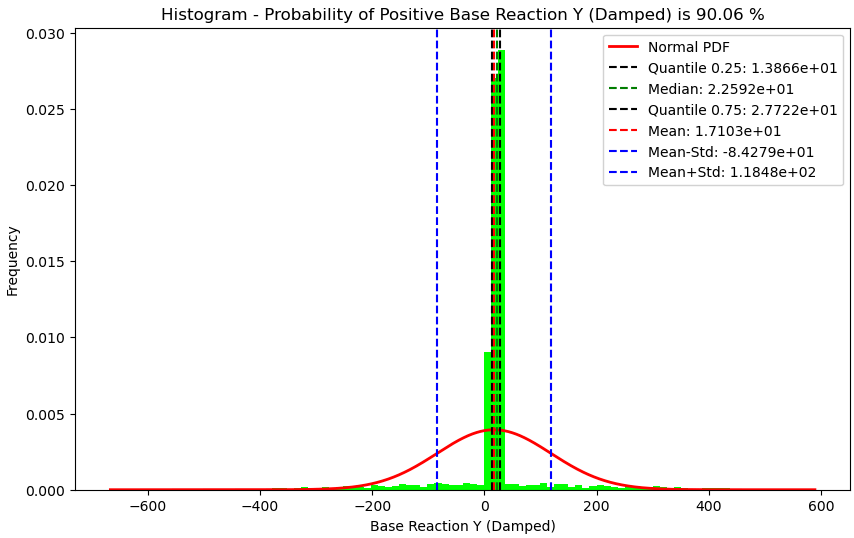
In [61]:
```
HISROGRAM_BOXPLOT(BASE_REACTION_Y_undamped, HISTO_COLOR='green', LABEL='Base Reaction Y (Undamped)')
HISROGRAM_BOXPLOT(BASE_REACTION_Y_damped, HISTO_COLOR='lime', LABEL='Base Reaction Y (Damped)')
#HISTOGRAM_BOXPLOT_PLOTLY(BASE_REACTION_Y_undamped, XLABEL='Base Reaction Y (Undamped)', TITLE='Base Reaction Y (Undamped)', COLOR='green')
#HISTOGRAM_BOXPLOT_PLOTLY(BASE_REACTION_Y_damped, XLABEL='Base Reaction Y (Damped)', TITLE='Base Reaction Y (Damped)', COLOR='lime')
```

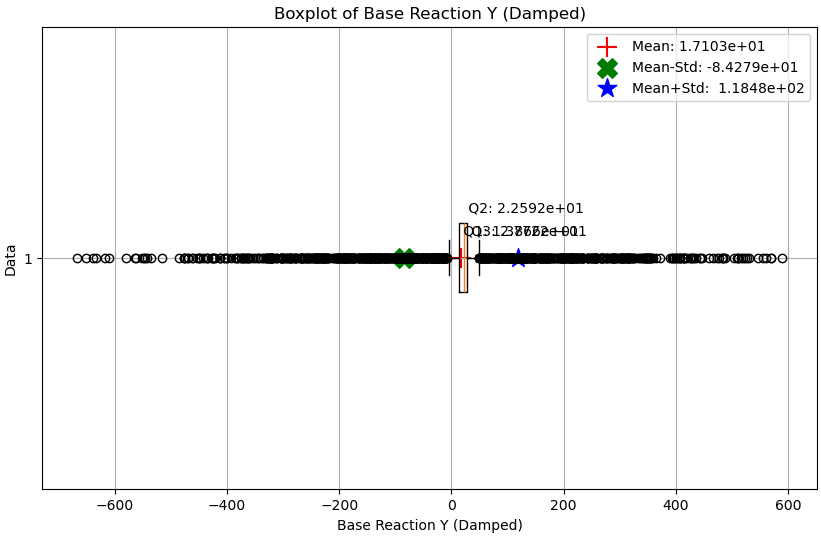
```
-------------------------
Box-Chart Datas:
Minimum: -6.6622e+02
First quartile: 1.4353e+01
Median: 2.3401e+01
Mean: 1.7675e+01
Std: 1.0142e+02
Third quartile: 2.8721e+01
Maximum: 5.8865e+02
Skewness: -5.9676e-01
kurtosis: 1.1418e+01
90% Confidence Interval: (-1.4462e+02, 1.5239e+02)
-------------------------
```






```
-------------------------
Box-Chart Datas:
Minimum: -6.6622e+02
First quartile: 1.3866e+01
Median: 2.2592e+01
Mean: 1.7103e+01
Std: 1.0138e+02
Third quartile: 2.7722e+01
Maximum: 5.8865e+02
Skewness: -5.8063e-01
kurtosis: 1.1428e+01
90% Confidence Interval: (-1.4462e+02, 1.5239e+02)
-------------------------
```





In [ ]:

In [ ]:

In [ ]:

```
###### BEAM ######

def PUSHOVER_ANALYSIS_BEAM(span, height, A_cable, num_nodes, MAX_DISP, MAX_ITERATIONS, TOLERANCE):
    import openseespy.opensees as ops
    import numpy as np
    import matplotlib.pyplot as plt

    # Define model builder
    ops.wipe()
    ops.model('basic', '-ndm', 2, '-ndf', 3)
    dx = span / (num_nodes - 1)

    # Create nodes
    for i in range(num_nodes):
        x = i * dx
        y = -height * np.sin(np.pi * x / span)
        ops.node(i + 1, x, y)
    for j in range(num_nodes):
        ops.node(num_nodes + j + 1, j * dx, -7000)

    # Define boundary conditions (fixed at both ends)
    ops.fix(1, 1, 1)              # TOP CABLE
    ops.fix(num_nodes, 1, 1)      # TOP CABLE
    ops.fix(num_nodes + 1, 1, 1)  # BOTTOM CABLE - DECK
    ops.fix(2 * num_nodes, 1, 1)  # BOTTOM CABLE - DECK

    # Define material properties
    Fy_cable = 355   # [N/mm^2] Yield strength of cable
    E_cable = 210e5  # [N/mm^2] Modulus of elasticity Cable
    E_beam = 210e5   # [N/mm^2] Modulus of elasticity Beam
    I_beam = 100000  # [mm^4] Moment of inertia of the beam
    A_beam = 2000    # [mm^2] Cross-sectional area of the beam
    b0 = 0.01
    ops.uniaxialMaterial('Elastic', 1, E_cable)
    ops.uniaxialMaterial('Elastic', 2, E_beam)

    # Define truss elements for the cable
    for i in range(num_nodes - 1):
        ops.element('corotTruss', i + 1, i + 1, i + 2, A_cable, 1)  # Cable element

    # Define geometric transformation
    ops.geomTransf('Linear', 1)
    # Define beam elements for the beam
    for i in range(num_nodes - 1):
        # element elasticBeamColumn $eleTag $iNode $jNode $A $E $Iz $transfTag <-mass $massDens> <-cMass>
        ops.element('elasticBeamColumn', num_nodes + i + 1, num_nodes + i + 1, num_nodes + i + 2, A_beam, E_beam, I_beam, 1)  # Beam element

    # Connect each node of the cable with the corresponding node of the beam using truss elements
    for i in range(num_nodes):
        ops.element('corotTruss', 2 * num_nodes + i + 1, i + 1, num_nodes + i + 1, A_cable, 1)

    mid_node = num_nodes // 2 + 1
    # Define load pattern with displacement at the middle span
    ops.timeSeries('Linear', 1)
    ops.pattern('Plain', 1, 1)
    ops.load(mid_node, 0.0, 1)  # Apply a horizontal load at node
    disp_incr = 0.01    # [mm] Incremental Displacement
    n_steps = int(np.abs(MAX_DISP / disp_incr))  # Analysis Steps

    # Define analysis parameters
    ops.system('BandGeneral')
    ops.numberer('Plain')
    ops.constraints('Plain')
    ops.integrator('DisplacementControl', mid_node, 2, disp_incr)
    ops.test('EnergyIncr', TOLERANCE, MAX_ITERATIONS)
    ops.algorithm('ModifiedNewton')
    ops.analysis('Static')

    DISPLACEMENTS = []
    DISP_X , DISP_Y = [], []
    BASE_REACTION_X, BASE_REACTION_Y = [], []

    # Perform the analysis with increments
    for i in range(n_steps):
        OK = ops.analyze(1)
```

```
        ANALYSIS(OK, 1, TOLERANCE, MAX_ITERATIONS)
        displacements = [ops.nodeDisp(j + 1) for j in range(num_nodes * 2)]
        DISPLACEMENTS.append([disp[:2] for disp in displacements])  # Remove the rotational component
        DISP_X.append(ops.nodeDisp(mid_node, 1))
        DISP_Y.append(ops.nodeDisp(mid_node, 2))
        BASE_REACTION_X.append(ops.eleResponse(1, 'force')[0])
        BASE_REACTION_Y.append(ops.eleResponse(1, 'force')[1])
        print(f'STEP: {i+1}')

    # Get initial and final node coordinates for plotting
    initial_coords = np.array([ops.nodeCoord(i + 1) for i in range(num_nodes * 2)])
    deformed_coords = initial_coords + np.array(DISPLACEMENTS[-1])

    # Output all unconstrained node displacements
    for i in range(num_nodes * 2):
        disp = ops.nodeDisp(i + 1)
        print(f"Node {i + 1}: X Displacement = {disp[0]}, Y Displacement = {disp[1]}")

    # Plot initial and deformed shape
    plt.figure(figsize=(12, 8))
    plt.plot(initial_coords[:num_nodes, 0], initial_coords[:num_nodes, 1], 'bo-', label='Initial Cable Shape')
    plt.plot(deformed_coords[:num_nodes, 0], deformed_coords[:num_nodes, 1], 'ro-', label='Deformed Cable Shape')
    plt.plot(initial_coords[num_nodes:, 0], initial_coords[num_nodes:, 1], 'go-', label='Initial Beam Shape')
    plt.plot(deformed_coords[num_nodes:, 0], deformed_coords[num_nodes:, 1], 'yo-', label='Deformed Beam Shape')

    # Plot connecting elements between the cable and the beam
    for i in range(num_nodes):
        plt.plot([initial_coords[i, 0], initial_coords[num_nodes + i, 0]],
                 [initial_coords[i, 1], initial_coords[num_nodes + i, 1]], 'c--', label='Initial Connecting Elements' if i == 0 else "")
        plt.plot([deformed_coords[i, 0], deformed_coords[num_nodes + i, 0]],
                 [deformed_coords[i, 1], deformed_coords[num_nodes + i, 1]], 'm--', label='Deformed Connecting Elements' if i == 0 else "")

    # Mark simply supported nodes with red triangles
    support_nodes = [0, num_nodes - 1, num_nodes, BEAM_NODE - 1]
    plt.plot(initial_coords[support_nodes, 0], initial_coords[support_nodes, 1], 'r^', markersize=20, label='Simply Supported Nodes')

    plt.xlabel('X [mm]')
    plt.ylabel('Y [mm]')
    plt.legend()
    plt.title('Pushover Analysis: Cable and Beam Deformed Shapes')
    plt.show()

    return initial_coords, deformed_coords, DISPLACEMENTS, DISP_X, DISP_Y, BASE_REACTION_X, BASE_REACTION_Y
```

In [ ]:
```
# Define geometry and nodes
span = 10000.0  # [mm]
height = 5000.0  # [mm]
A_cable = 200  # [mm^2]
num_nodes = 51
MAX_DISP = -10.0  # [mm]
MAX_ITERATIONS = 5000
TOLERANCE = 1.0e-10

# Run the analysis and plot results
initial_coords, deformed_coords, DISPLACEMENTS, DISP_X, DISP_Y, BASE_REACTION_X, BASE_REACTION_Y = PUSHOVER_ANALYSIS_BEAM(span, height, A_cable, num_nodes, MAX_DISP, MAX_ITERATIONS, TO
plot_reactions(DISP_X, BASE_REACTION_X, DISP_Y, BASE_REACTION_Y)
```

In [ ]:
```

```

In [ ]:
```

```