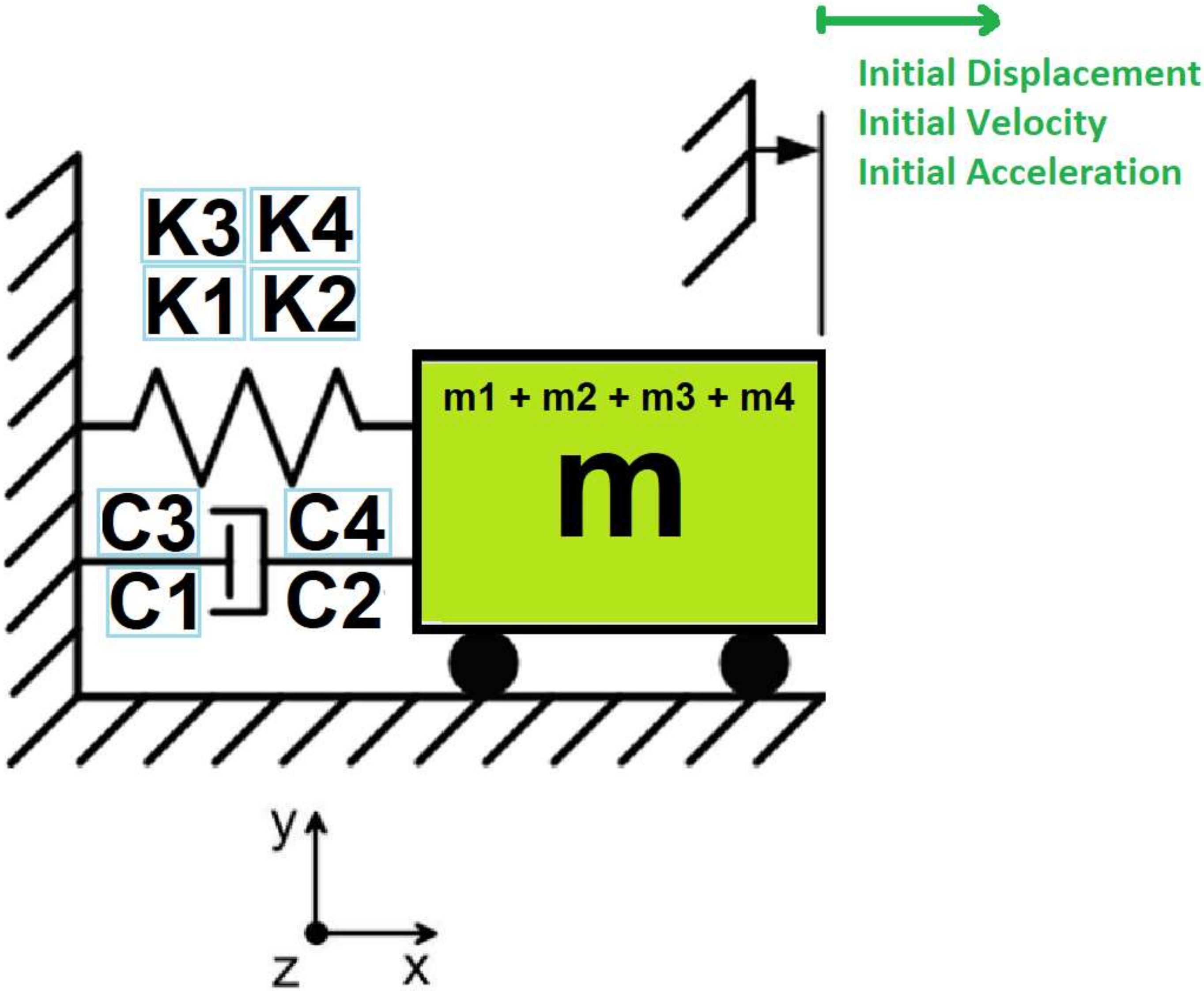
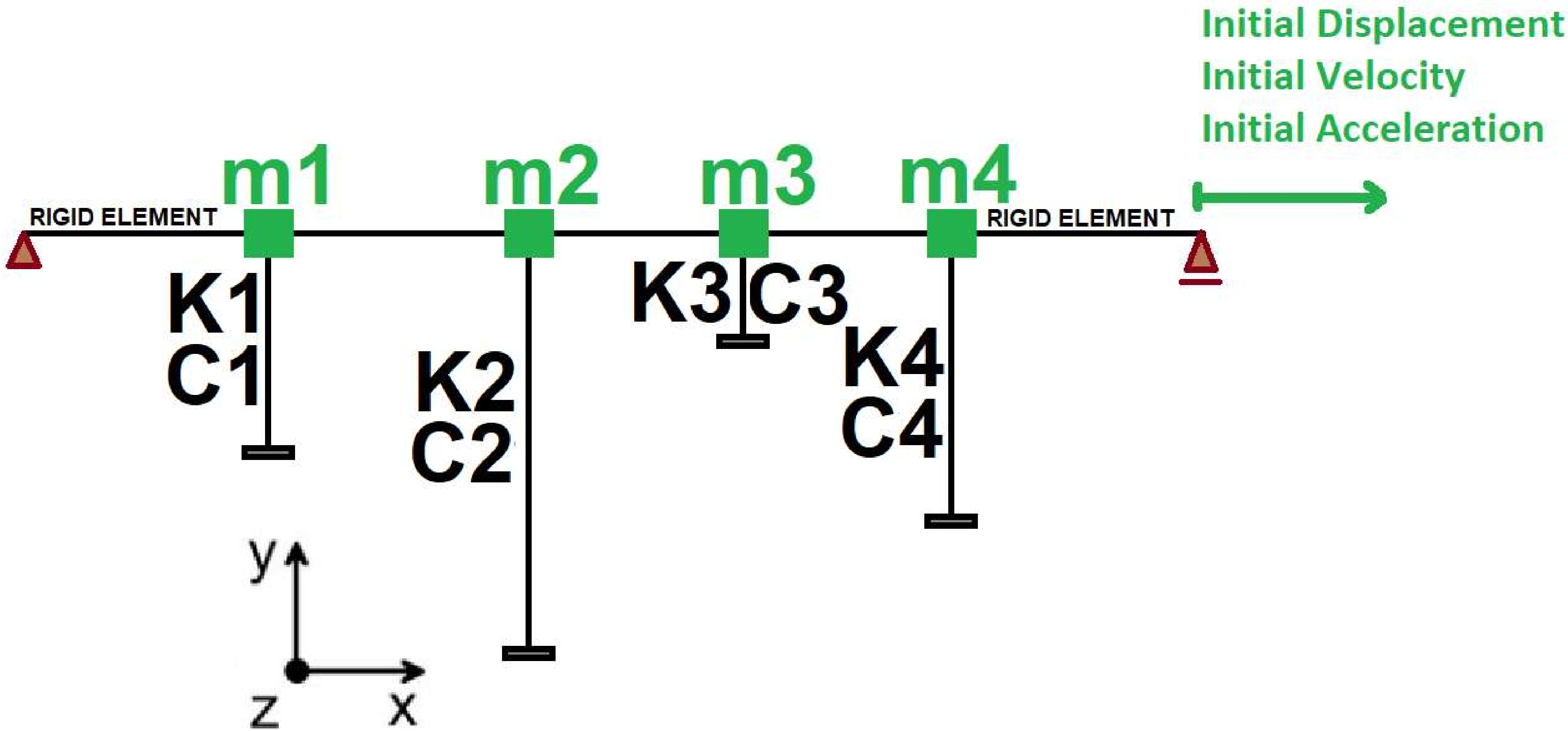
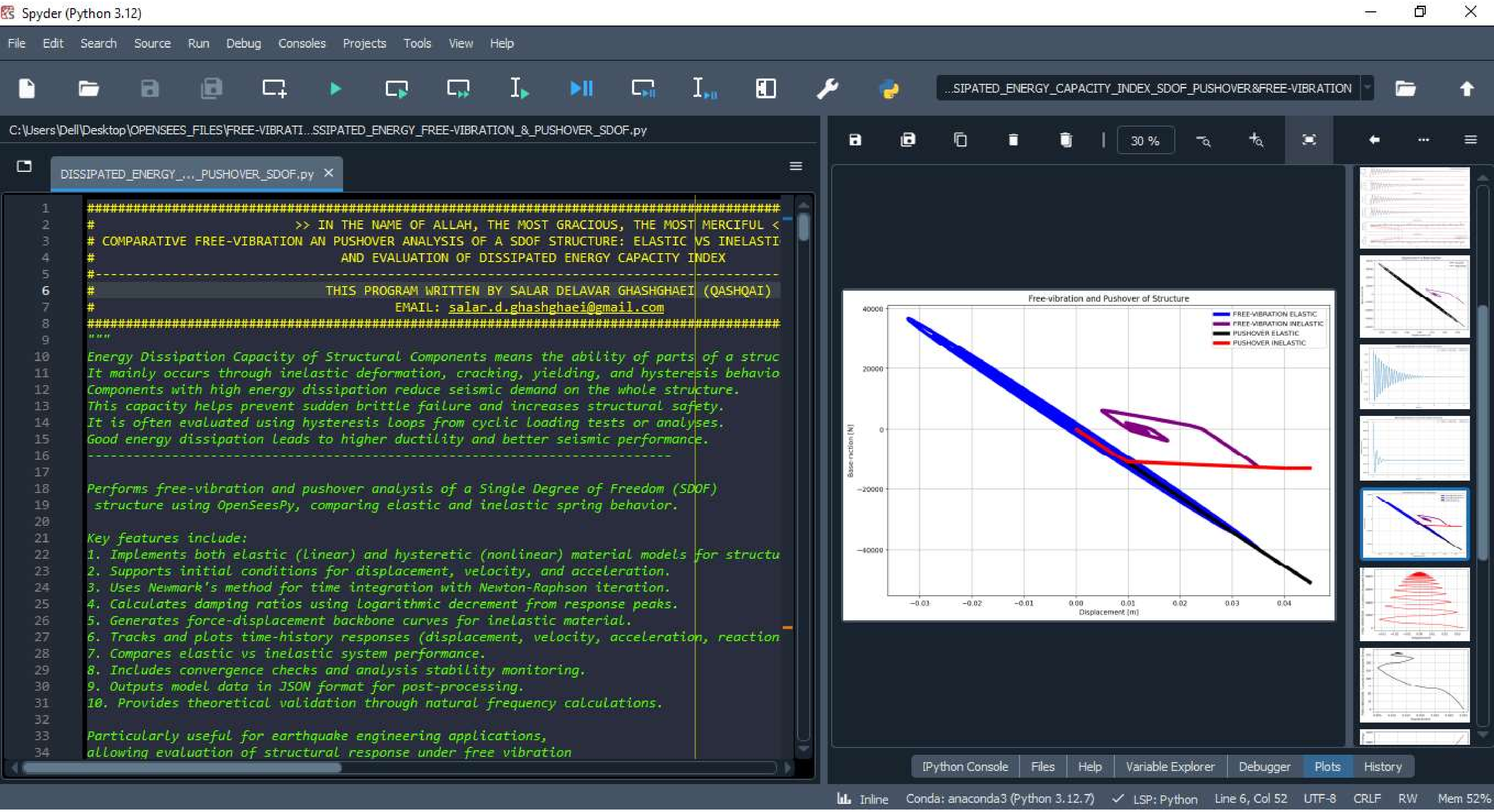
>> IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL <<

COMPARATIVE FREE-VIBRATION AND PUSHOVER ANALYSIS OF A SDOF STRUCTURE: ELASTIC VS INELASTIC RESPONSE USING OPENSEES AND EVALUATION OF DISSIPATED ENERGY CAPACITY INDEX

WRITTEN BY SALAR DELAVAR GHASHGHAEI (QASHQAI)


Initial Displacement
Initial Velocity
Initial Acceleration
K3
K4
K1
K2
m1 + m2 + m3 + m4
m
C3
C4
C1
C2
y
z
x


Initial Displacement
Initial Velocity
Initial Acceleration
m1
m2
m3
m4
RIGID ELEMENT
RIGID ELEMENT
K1
C1
K2
C2
K3
C3
K4
C4
y
z
x



Spyder (Python 3.12)
File Edit Search Source Run Debug Consoles Projects Tools View Help
...SIPATED_ENERGY_CAPACITY_INDEX_SDOF_PUSHOVER&FREE-VIBRATION
C:\Users\Dell\Desktop\OPENSEES_FILES\FREE-VIBRATI...SSIPATED_ENERGY_FREE-VIBRATION_&_PUSHOVER_SDOF.py
DISSIPATED_ENERGY_..._PUSHOVER_SDOF.py
##
>> IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL <
COMPARATIVE FREE-VIBRATION AN PUSHOVER ANALYSIS OF A SDOF STRUCTURE: ELASTIC VS INELASTI
AND EVALUATION OF DISSIPATED ENERGY CAPACITY INDEX
#---
THIS PROGRAM WRITTEN BY SALAR DELAVAR GHASHGHAEI (QASHQAI)
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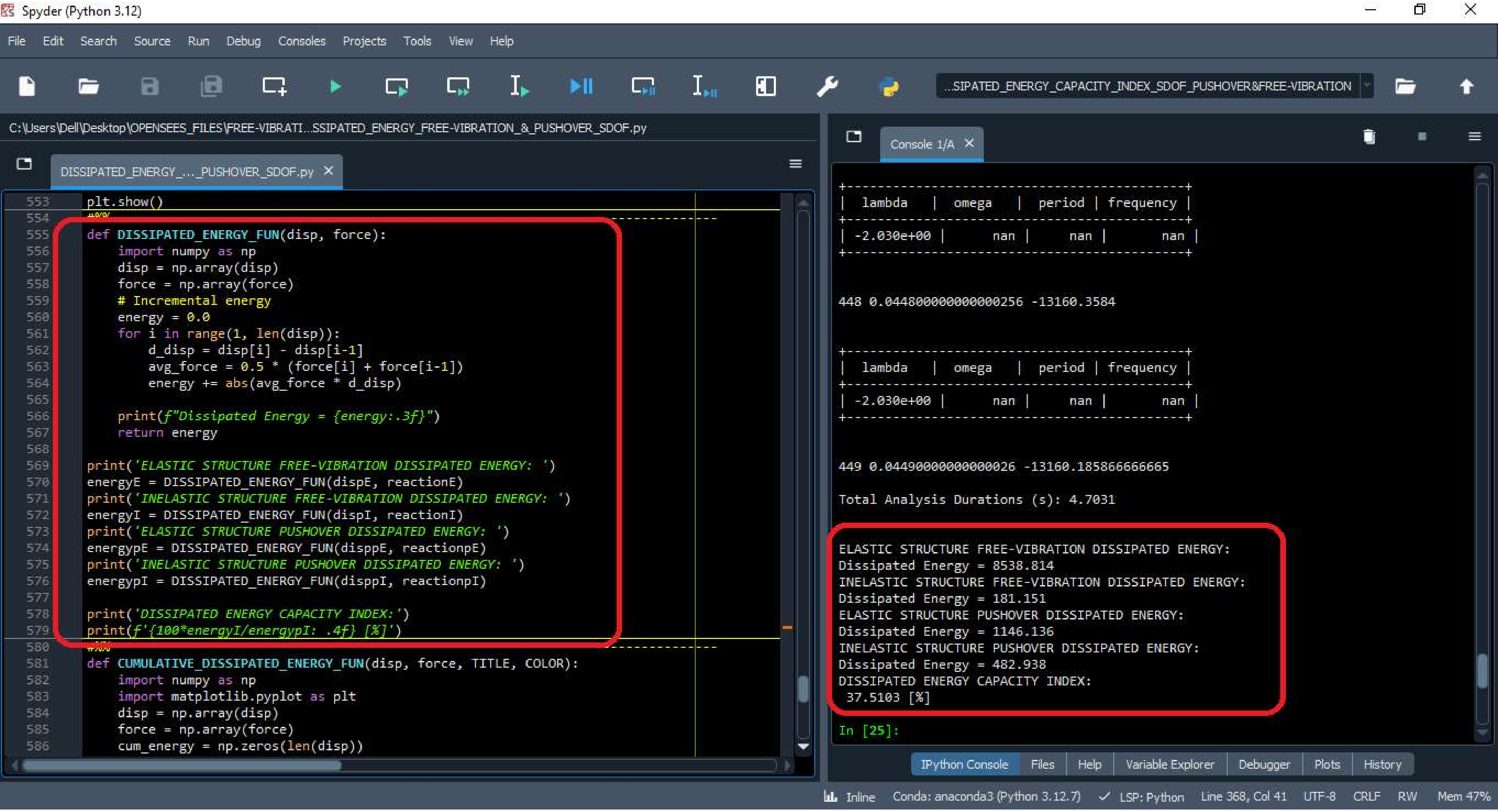
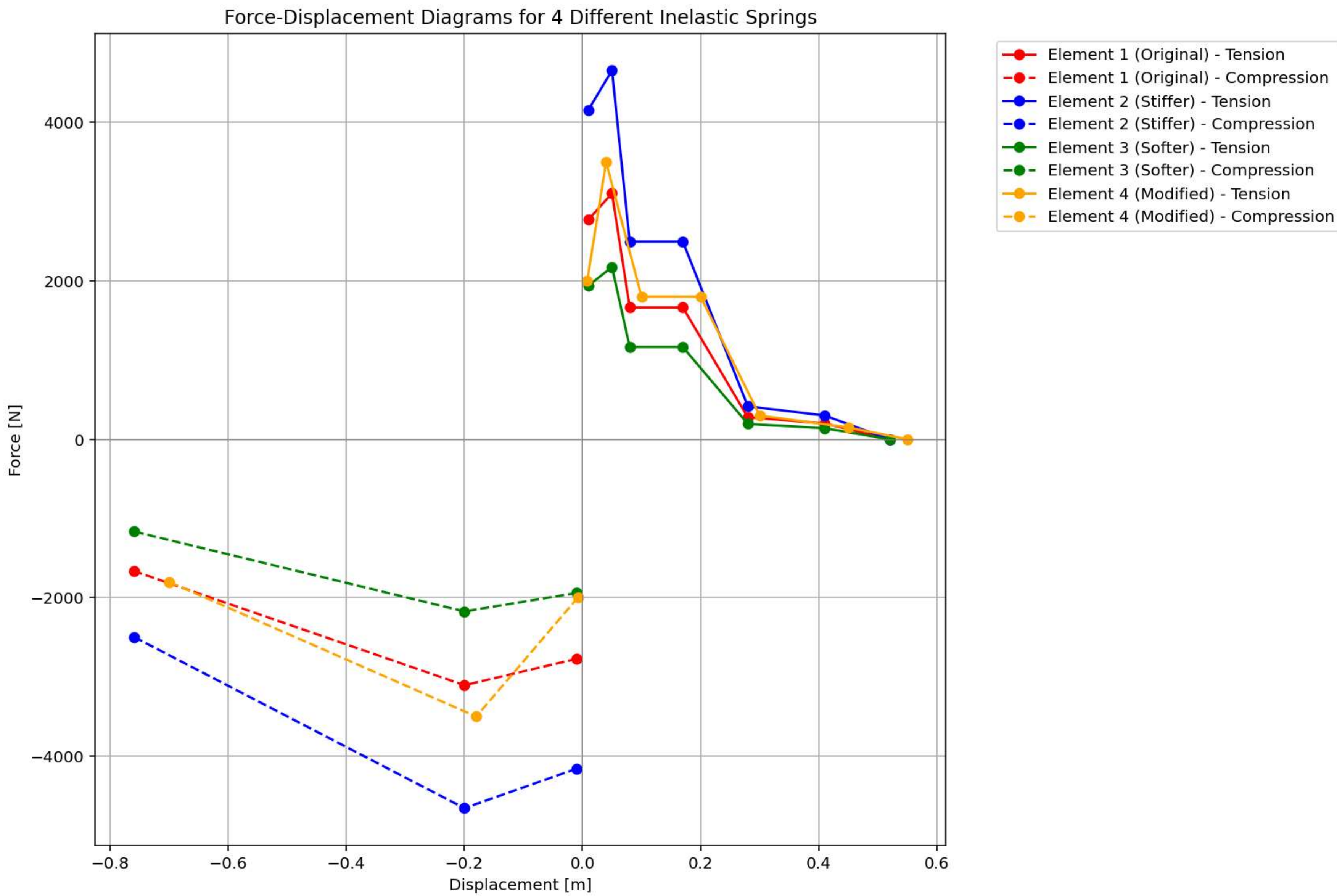
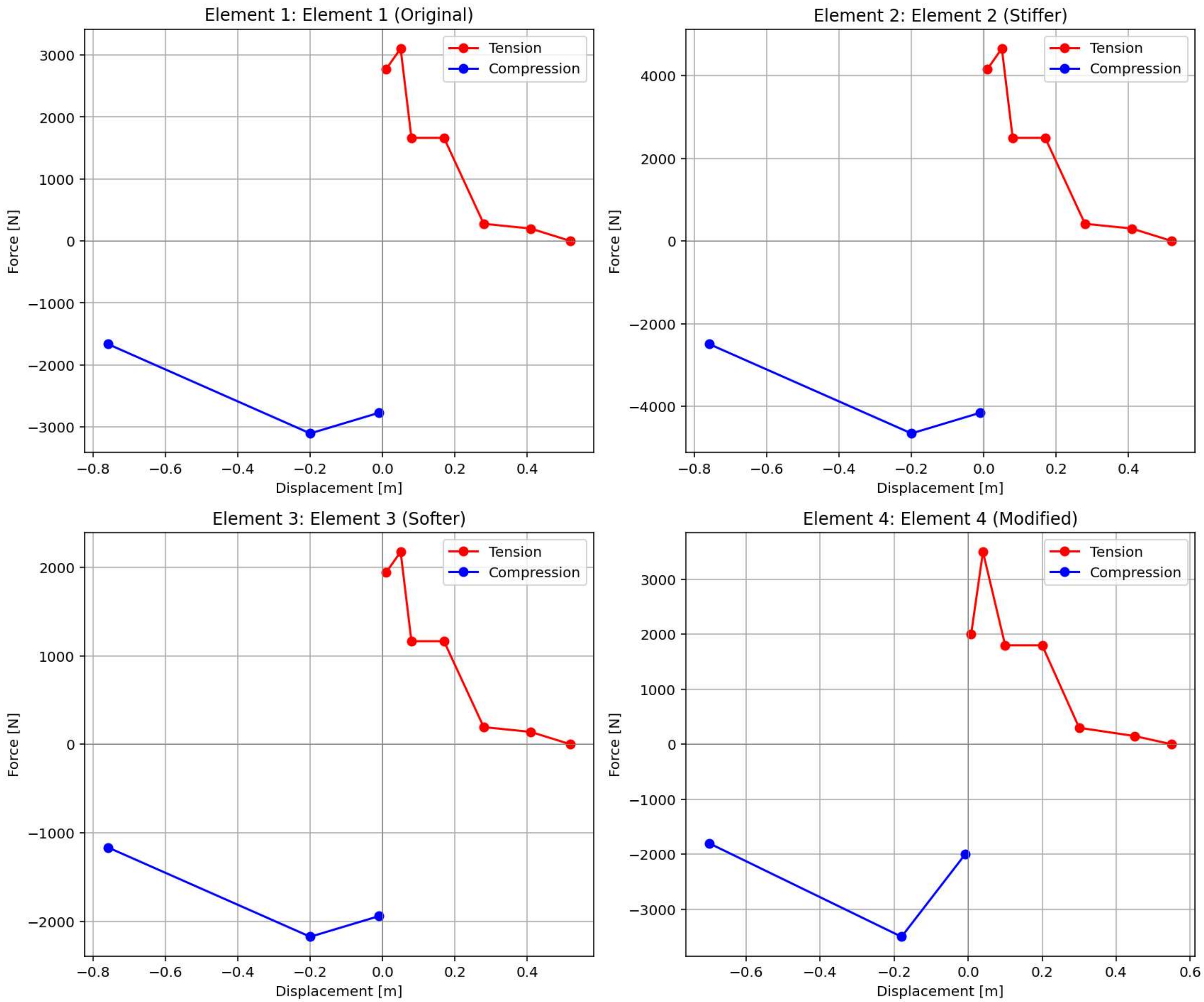
##
"""
Energy Dissipation Capacity of Structural Components means the ability of parts of a struc
It mainly occurs through inelastic deformation, cracking, yielding, and hysteresis behavio
Components with high energy dissipation reduce seismic demand on the whole structure.
This capacity helps prevent sudden brittle failure and increases structural safety.
It is often evaluated using hysteresis loops from cyclic loading tests or analyses.
Good energy dissipation leads to higher ductility and better seismic performance.

Performs free-vibration and pushover analysis of a Single Degree of Freedom (SDOF)
structure using OpenSeesPy, comparing elastic and inelastic spring behavior.
Key features include:
1. Implements both elastic (linear) and hysteretic (nonlinear) material models for structu
2. Supports initial conditions for displacement, velocity, and acceleration.
3. Uses Newmark's method for time integration with Newton-Raphson iteration.
4. Calculates damping ratios using logarithmic decrement from response peaks.
5. Generates force-displacement backbone curves for inelastic material.
6. Tracks and plots time-history responses (displacement, velocity, acceleration, reaction
7. Compares elastic vs inelastic system performance.
8. Includes convergence checks and analysis stability monitoring.
9. Outputs model data in JSON format for post-processing.
10. Provides theoretical validation through natural frequency calculations.
Particularly useful for earthquake engineering applications,
allowing evaluation of structural response under free vibration
30 %
Free-vibration and Pushover of Structure
FREE-VIBRATION ELASTIC
FREE-VIBRATION INELASTIC
PUSHOVER ELASTIC
PUSHOVER INELASTIC
Base-raction [N]
Displacement [m]
IPython Console Files Help Variable Explorer Debugger Plots History
Inline Conda: anaconda3 (Python 3.12.7) LSP: Python Line 6, Col 52 UTF-8 CRLF RW Mem 52%


Spyder (Python 3.12)
File Edit Search Source Run Debug Consoles Projects Tools View Help
..SIPATED_ENERGY_CAPACITY_INDEX_SDOF_PUSHOVER&FREE-VIBRATION
C:\Users\Dell\Desktop\OPENSEES_FILES\FREE-VIBRATI...SSIPATED_ENERGY_FREE-VIBRATION_&_PUSHOVER_SDOF.py
DISSIPATED_ENERGY_..._PUSHOVER_SDOF.py
plt.show()
#%%
def DISSIPATED_ENERGY_FUN(disp, force):
    import numpy as np
    disp = np.array(disp)
    force = np.array(force)
    # Incremental energy
    energy = 0.0
    for i in range(1, len(disp)):
        d_disp = disp[i] - disp[i-1]
        avg_force = 0.5 * (force[i] + force[i-1])
        energy += abs(avg_force * d_disp)
    print(f"Dissipated Energy = {energy:.3f}")
    return energy
print('ELASTIC STRUCTURE FREE-VIBRATION DISSIPATED ENERGY: ')
energyE = DISSIPATED_ENERGY_FUN(dispE, reactionE)
print('INELASTIC STRUCTURE FREE-VIBRATION DISSIPATED ENERGY: ')
energyI = DISSIPATED_ENERGY_FUN(dispI, reactionI)
print('ELASTIC STRUCTURE PUSHOVER DISSIPATED ENERGY: ')
energypE = DISSIPATED_ENERGY_FUN(disppE, reactionpE)
print('INELASTIC STRUCTURE PUSHOVER DISSIPATED ENERGY: ')
energypI = DISSIPATED_ENERGY_FUN(disppI, reactionpI)
print('DISSIPATED ENERGY CAPACITY INDEX:')
print(f'{100*energyI/energypI: .4f} [%]')
#%%
def CUMULATIVE_DISSIPATED_ENERGY_FUN(disp, force, TITLE, COLOR):
    import numpy as np
    import matplotlib.pyplot as plt
    disp = np.array(disp)
    force = np.array(force)
    cum_energy = np.zeros(len(disp))
Console 1/A
| lambda | omega | period | frequency |
| -2.030e+00 | nan | nan | nan |
448 0.04480000000000256 -13160.3584
| lambda | omega | period | frequency |
| -2.030e+00 | nan | nan | nan |
449 0.04490000000000026 -13160.185866666665
Total Analysis Durations (s): 4.7031
ELASTIC STRUCTURE FREE-VIBRATION DISSIPATED ENERGY:
Dissipated Energy = 8538.814
INELASTIC STRUCTURE FREE-VIBRATION DISSIPATED ENERGY:
Dissipated Energy = 181.151
ELASTIC STRUCTURE PUSHOVER DISSIPATED ENERGY:
Dissipated Energy = 1146.136
INELASTIC STRUCTURE PUSHOVER DISSIPATED ENERGY:
Dissipated Energy = 482.938
DISSIPATED ENERGY CAPACITY INDEX:
37.5103 [%]
In [25]:
IPython Console Files Help Variable Explorer Debugger Plots History
Inline Conda: anaconda3 (Python 3.12.7) LSP: Python Line 368, Col 41 UTF-8 CRLF RW Mem 47%



Force-Displacement Diagrams for 4 Different Inelastic Springs
Element 1 (Original) - Tension
Element 1 (Original) - Compression
Element 2 (Stiffer) - Tension
Element 2 (Stiffer) - Compression
Element 3 (Softer) - Tension
Element 3 (Softer) - Compression
Element 4 (Modified) - Tension
Element 4 (Modified) - Compression
4000
2000
0
−2000
−4000
Force [N]
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
Displacement [m]



Element 1: Element 1 (Original)
Element 2: Element 2 (Stiffer)
Element 3: Element 3 (Softer)
Element 4: Element 4 (Modified)
Tension
Compression
Force [N]
Displacement [m]

FREE-VIBRATION ANALYSIS

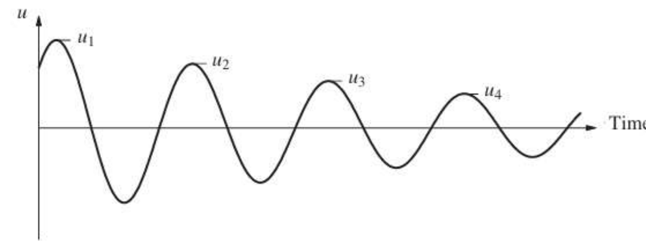
VISCOUSLY DAMPED FREE VIBRATION

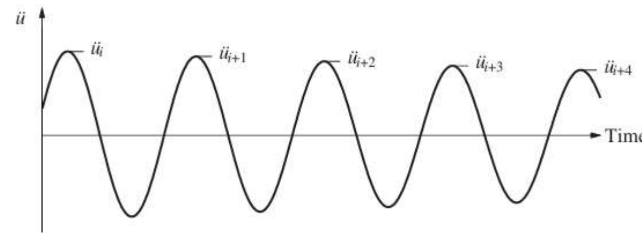
$$m\ddot{u} + c\dot{u} + ku = 0$$

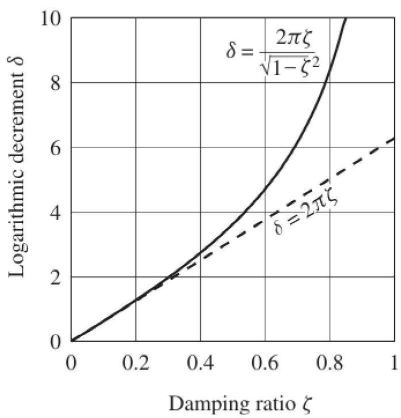
$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = 0$$

$$\omega_n = \sqrt{k/m} \quad \zeta = \frac{c}{2m\omega_n} = \frac{c}{c_{\mathrm{cr}}} \quad \omega_D = \omega_n\sqrt{1-\zeta^2}$$

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos\omega_D t + \frac{\dot{u}(0) + \zeta\omega_n u(0)}{\omega_D}\sin\omega_D t\right]$$





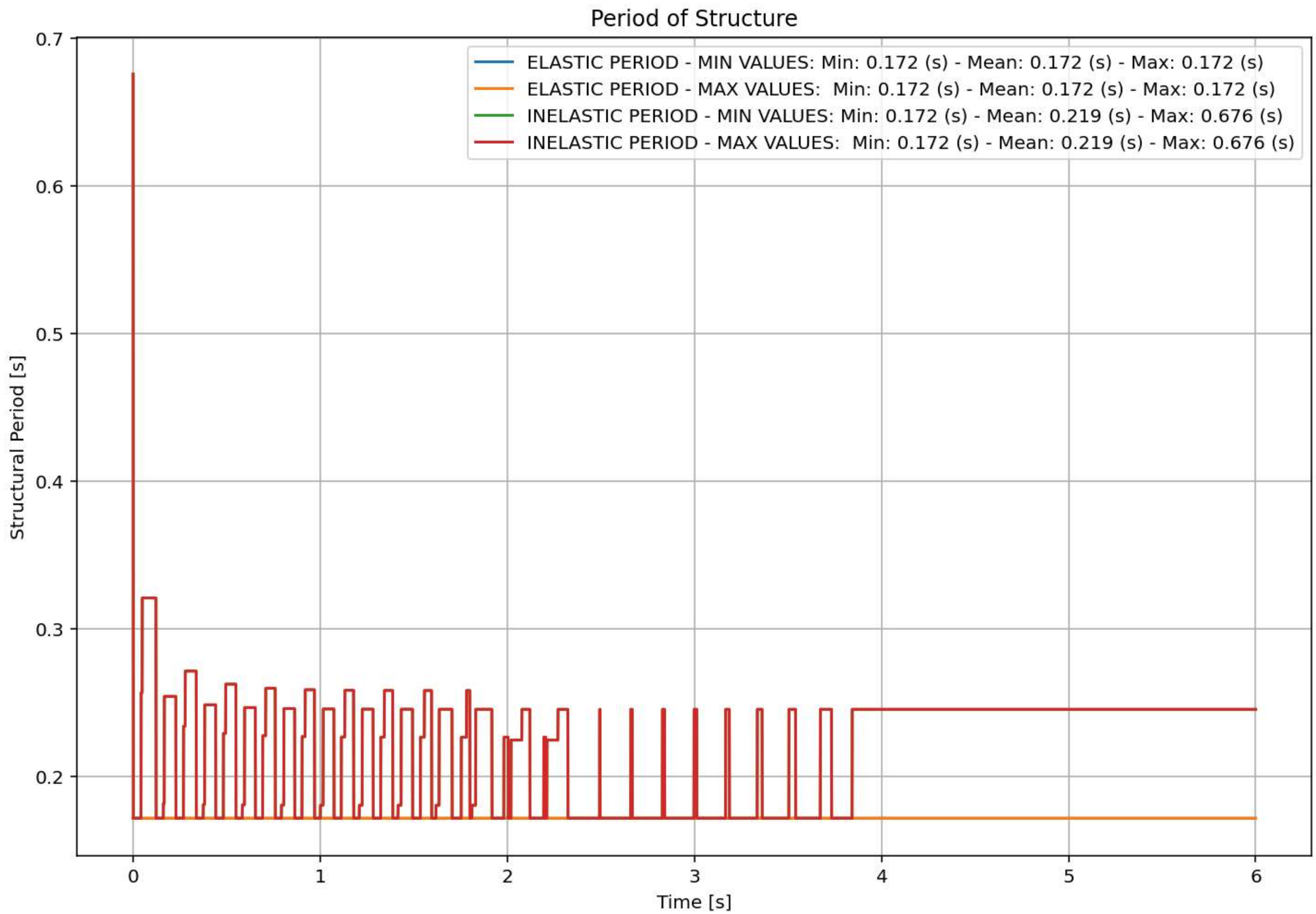


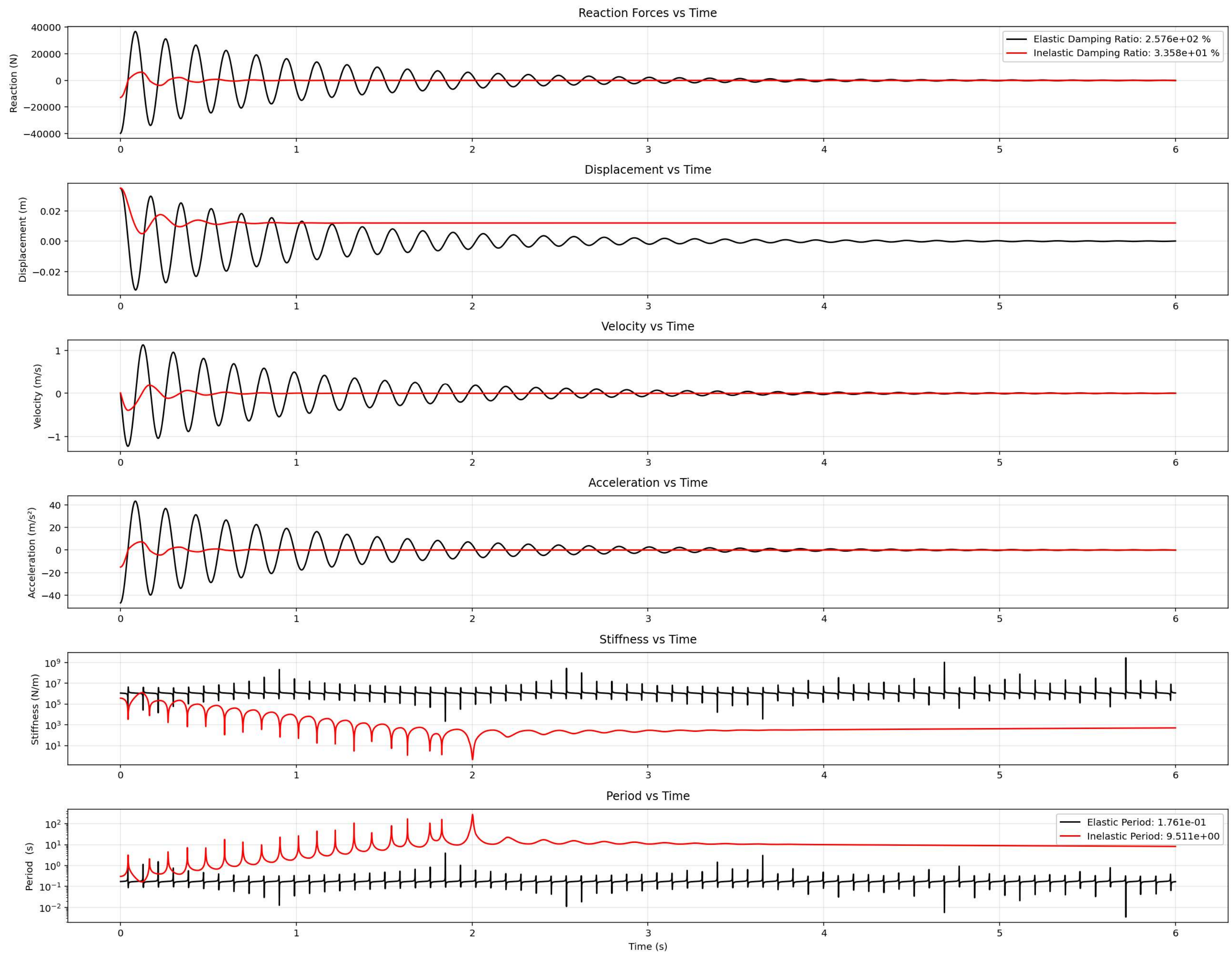
Decay of Motion

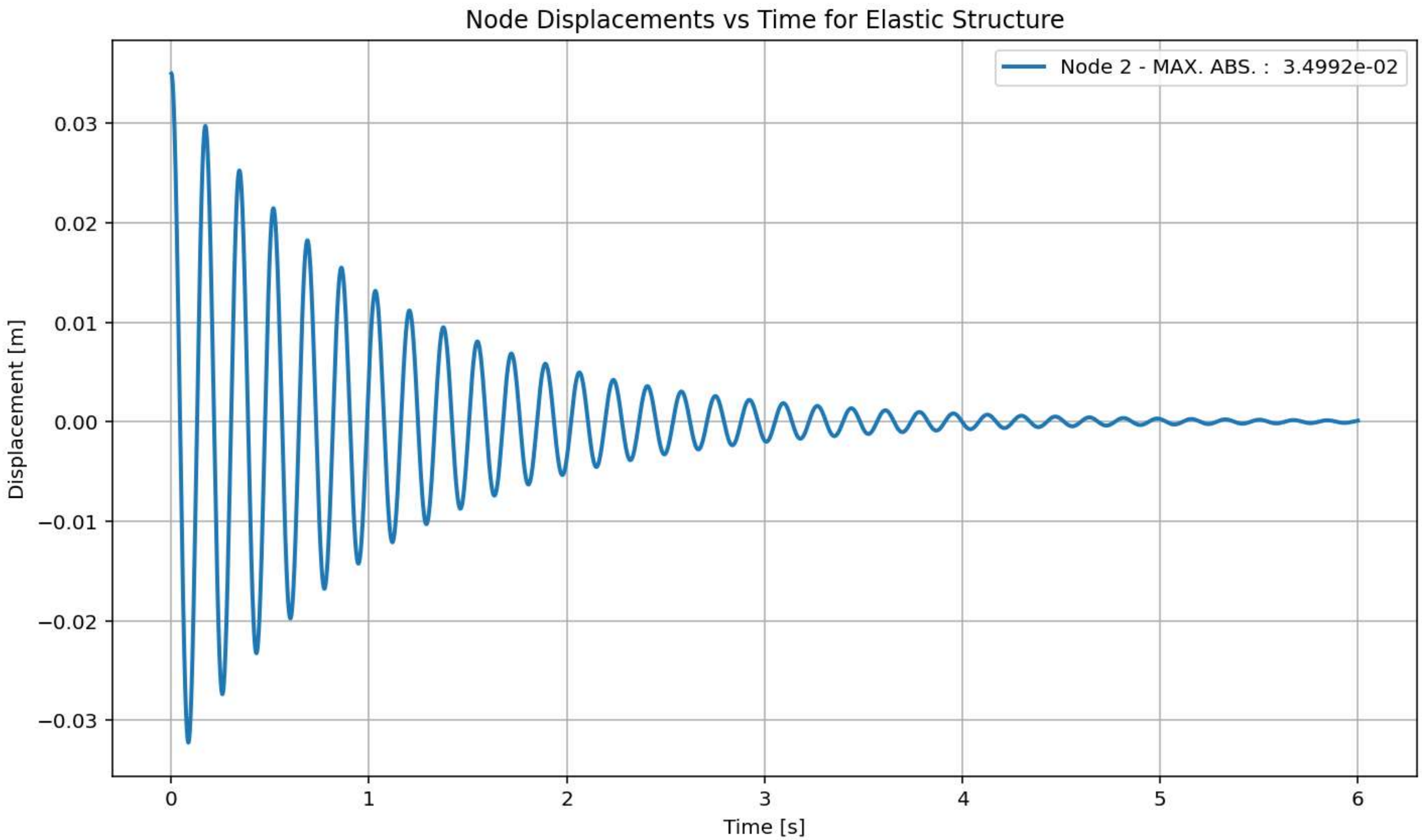
$$\delta = \ln\frac{u_i}{u_{i+1}} = 2\pi\zeta$$ **(APPROXIMATE RELATION)**

$$\delta = \ln\frac{u_i}{u_{i+1}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$ **(EXACT RELATION)**

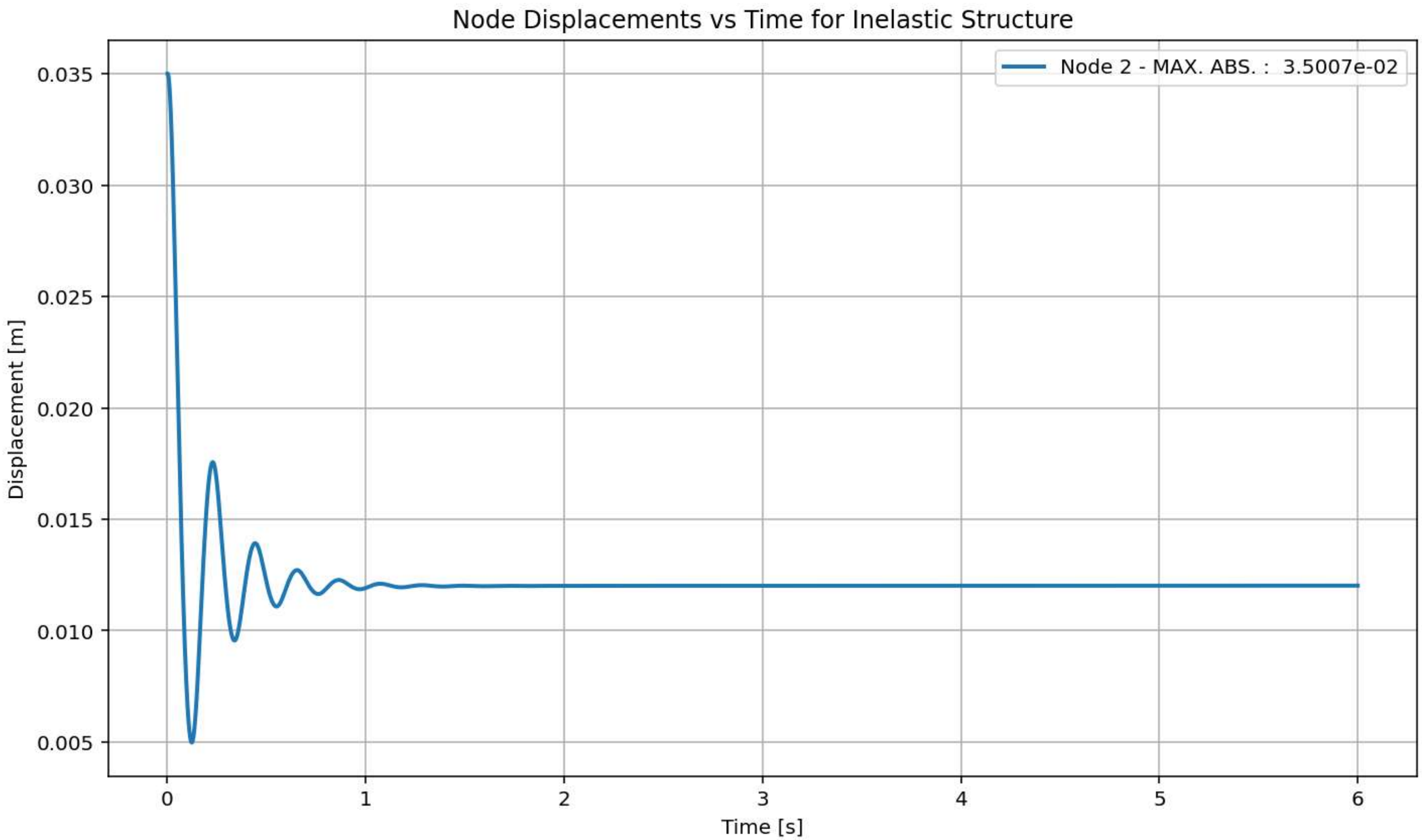
EXACT AND APPROXIMATE RELATIONS BETWEEN LOGARITHMIC DECREMENT AND DAMPING RATIO

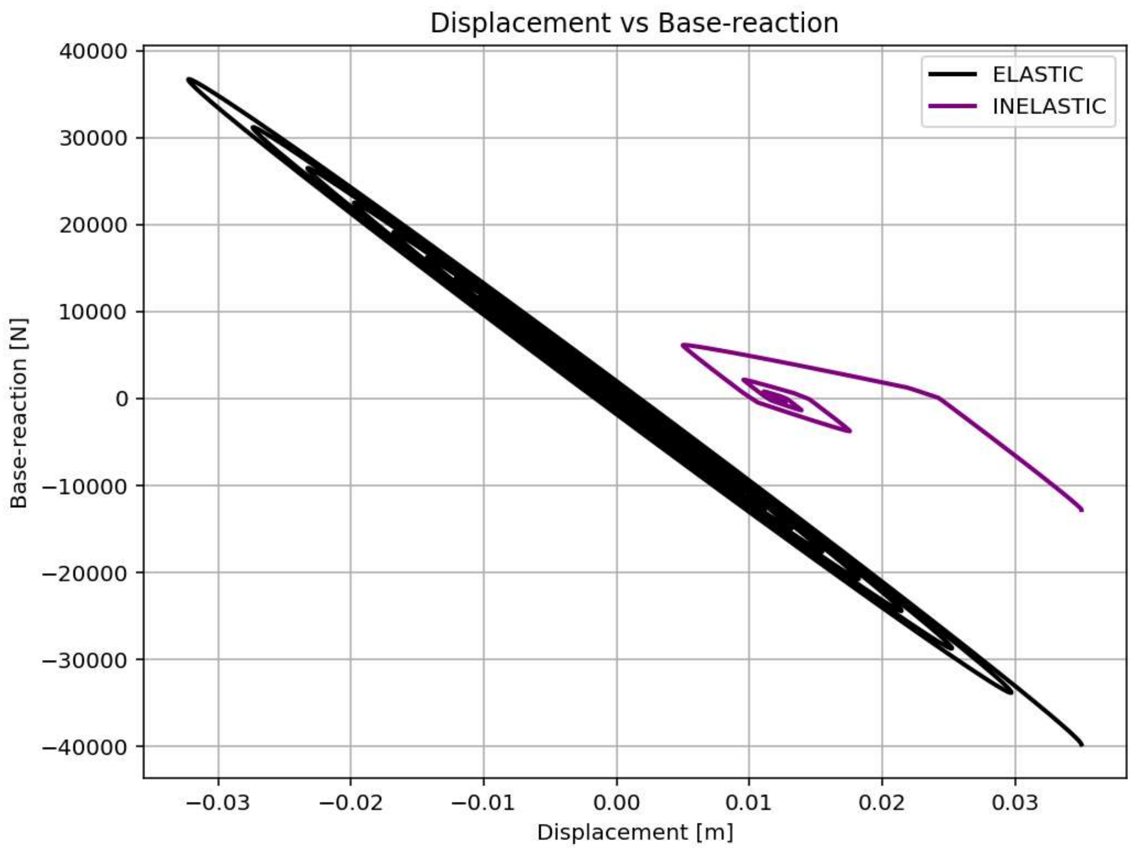

Period of Structure
ELASTIC PERIOD - MIN VALUES: Min: 0.172 (s) - Mean: 0.172 (s) - Max: 0.172 (s)
ELASTIC PERIOD - MAX VALUES: Min: 0.172 (s) - Mean: 0.172 (s) - Max: 0.172 (s)
INELASTIC PERIOD - MIN VALUES: Min: 0.172 (s) - Mean: 0.219 (s) - Max: 0.676 (s)
INELASTIC PERIOD - MAX VALUES: Min: 0.172 (s) - Mean: 0.219 (s) - Max: 0.676 (s)
Structural Period [s]
Time [s]
0.7
0.6
0.5
0.4
0.3
0.2
0
1
2
3
4
5
6


Reaction Forces vs Time
Elastic Damping Ratio: 2.576e+02 %
Inelastic Damping Ratio: 3.358e+01 %
Reaction (N)
Displacement vs Time
Displacement (m)
Velocity vs Time
Velocity (m/s)
Acceleration vs Time
Acceleration (m/s²)
Stiffness vs Time
Stiffness (N/m)
Period vs Time
Elastic Period: 1.761e-01
Inelastic Period: 9.511e+00
Period (s)
Time (s)

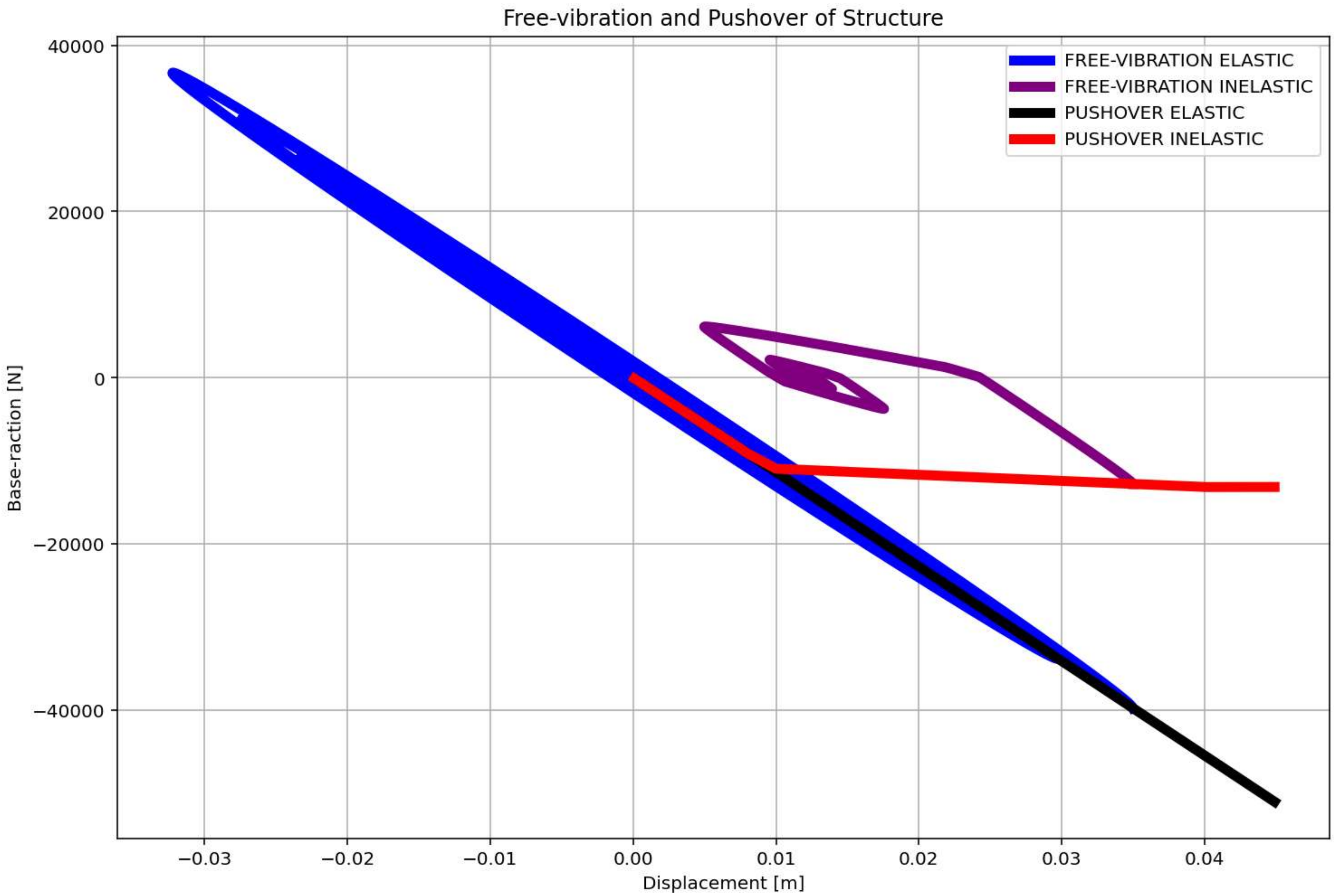


Node Displacements vs Time for Elastic Structure
Node 2 - MAX. ABS. : 3.4992e-02
Displacement [m]
0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
0
1
2
3
4
5
6
Time [s]

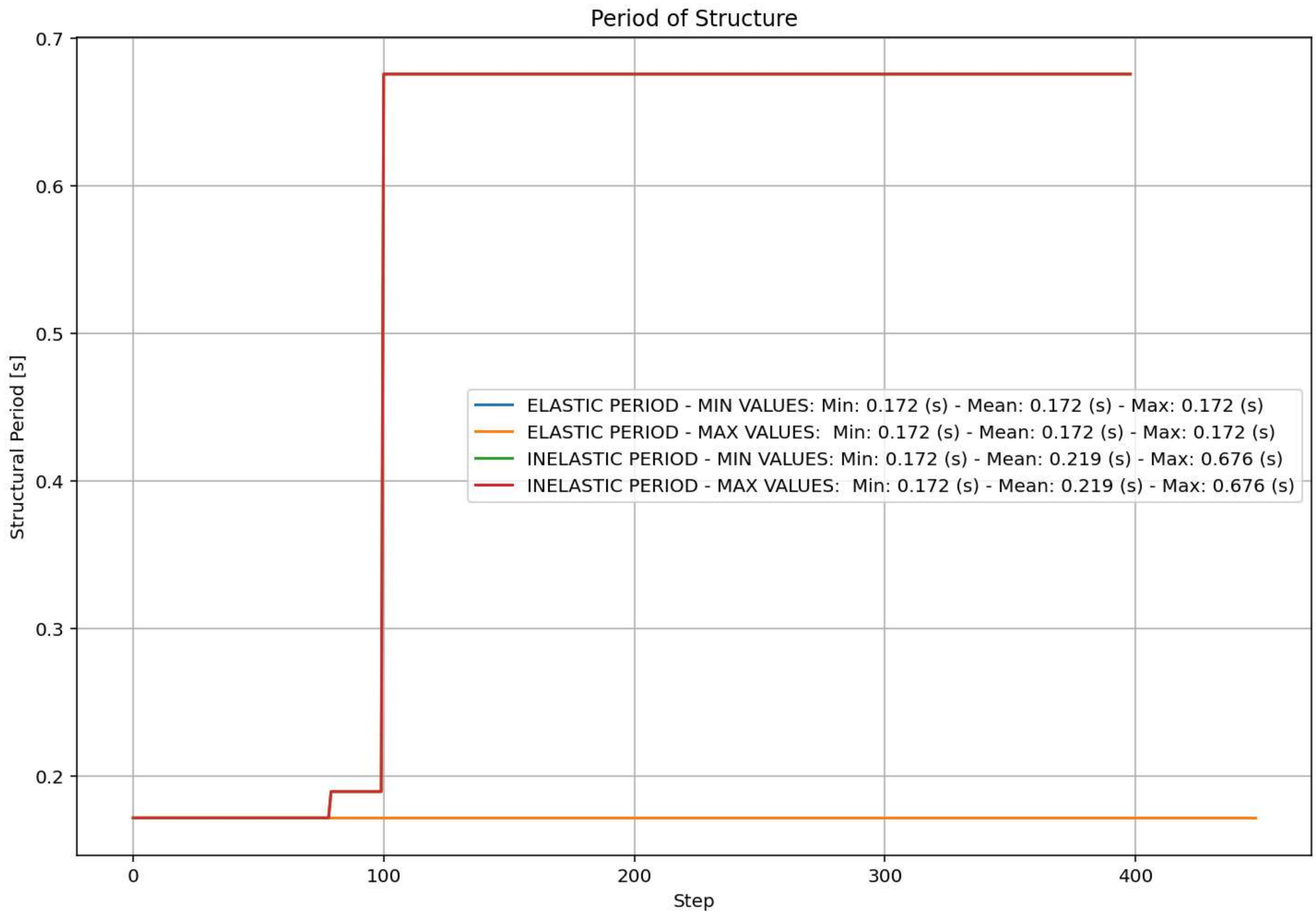

Node Displacements vs Time for Inelastic Structure
Node 2 - MAX. ABS. : 3.5007e-02
0.035
0.030
0.025
0.020
0.015
0.010
0.005
Displacement [m]
0
1
2
3
4
5
6
Time [s]

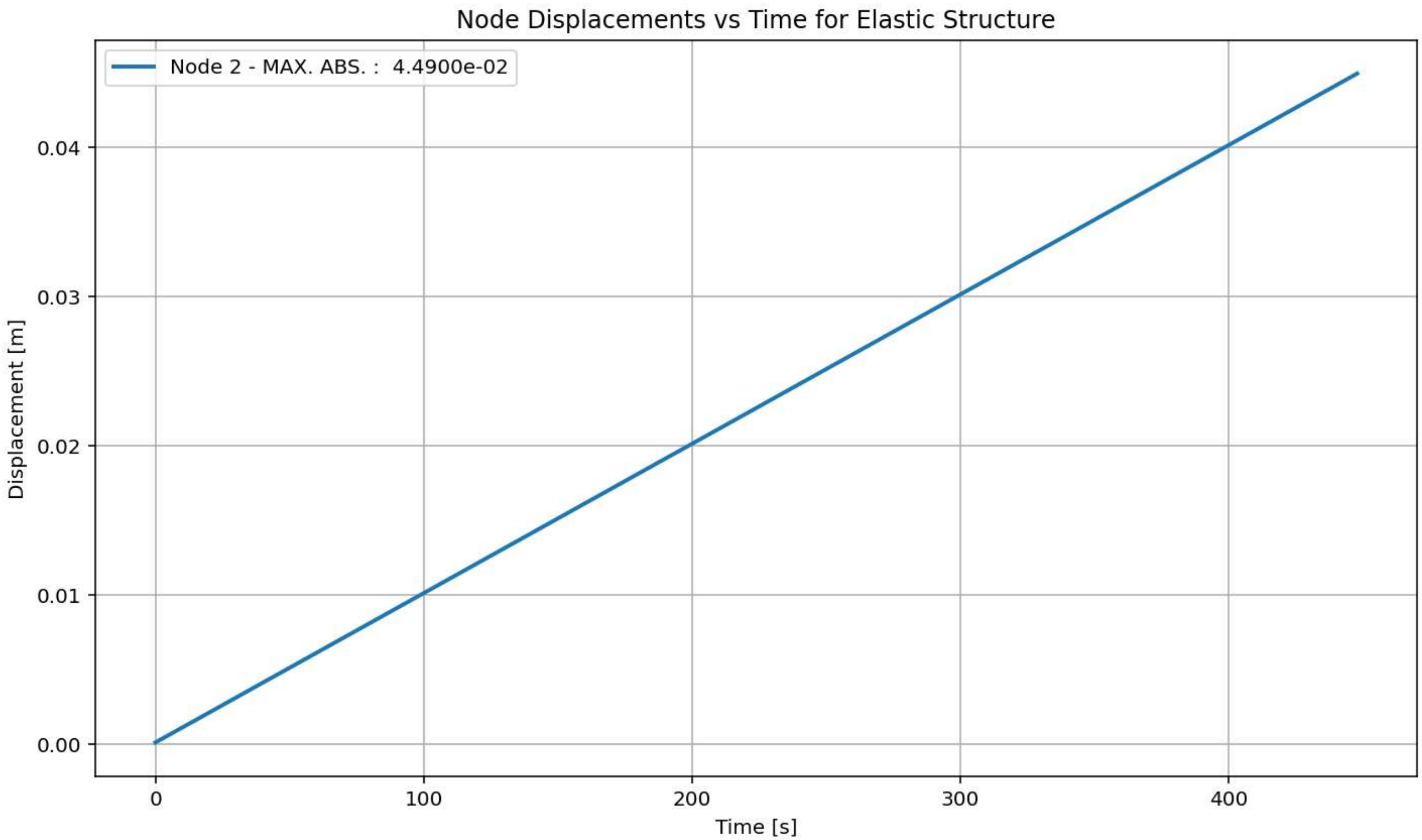

Displacement vs Base-reaction
ELASTIC
INELASTIC
Base-reaction [N]
Displacement [m]
40000
30000
20000
10000
0
−10000
−20000
−30000
−40000
−0.03
−0.02
−0.01
0.00
0.01
0.02
0.03

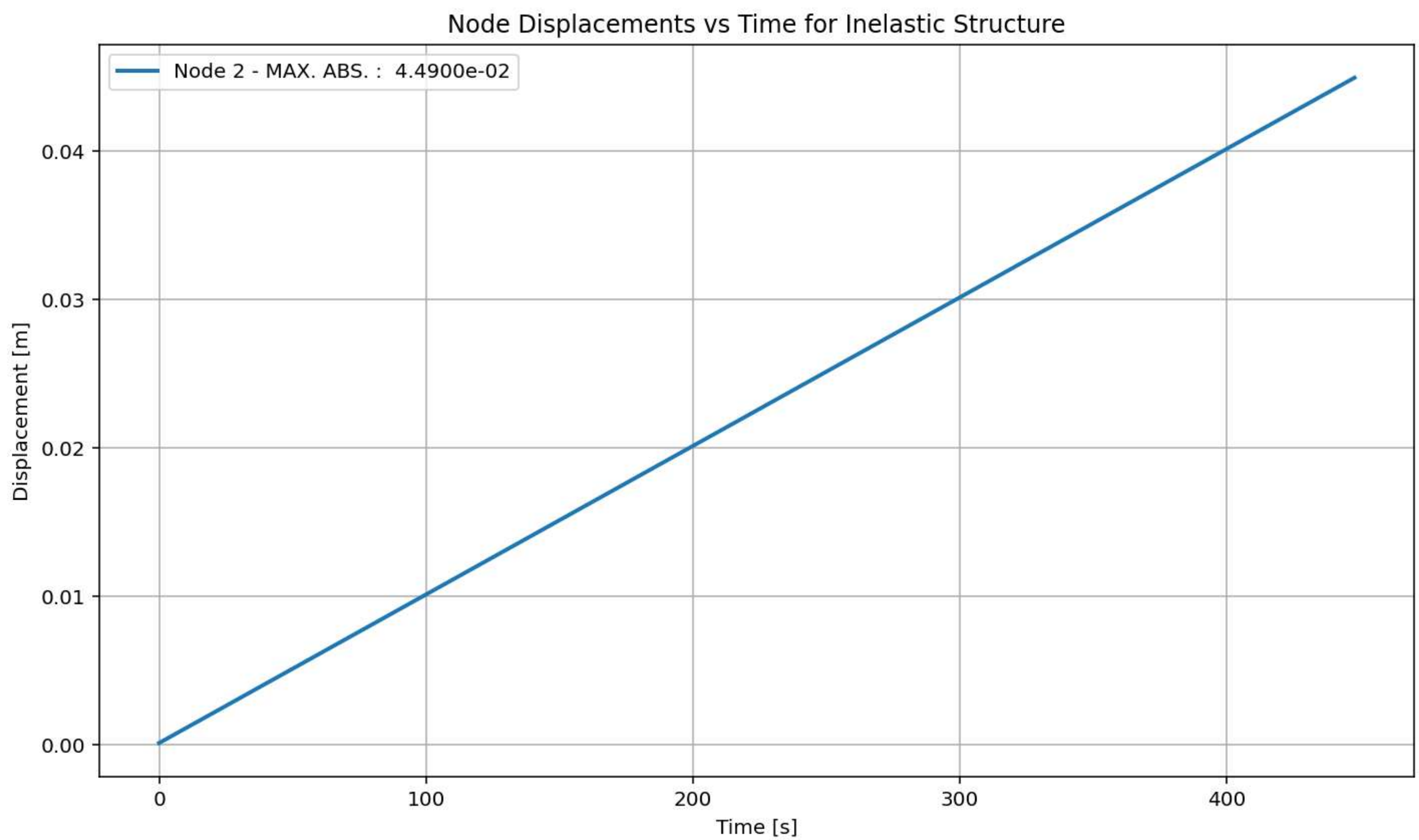
PUSHOVER ANALYSIS

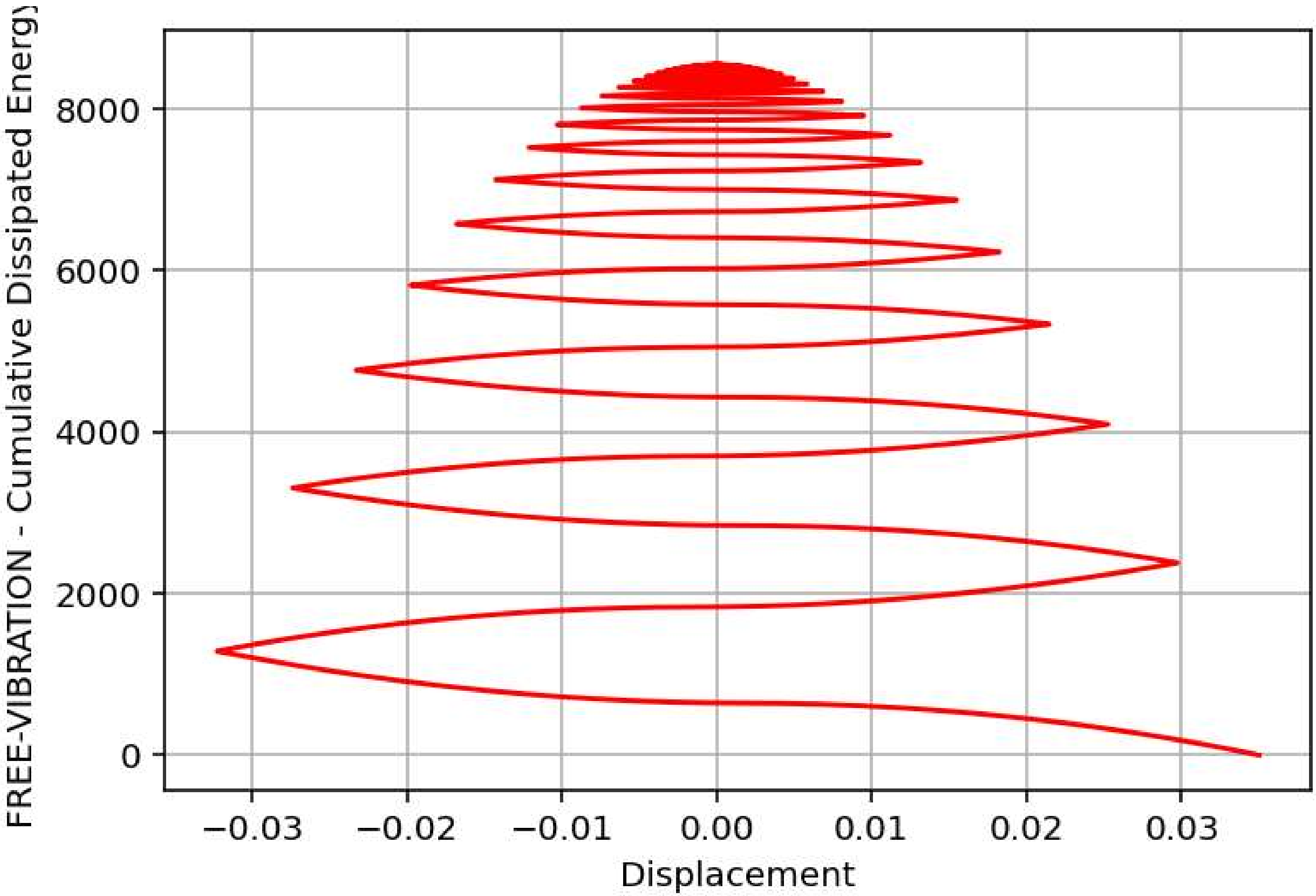


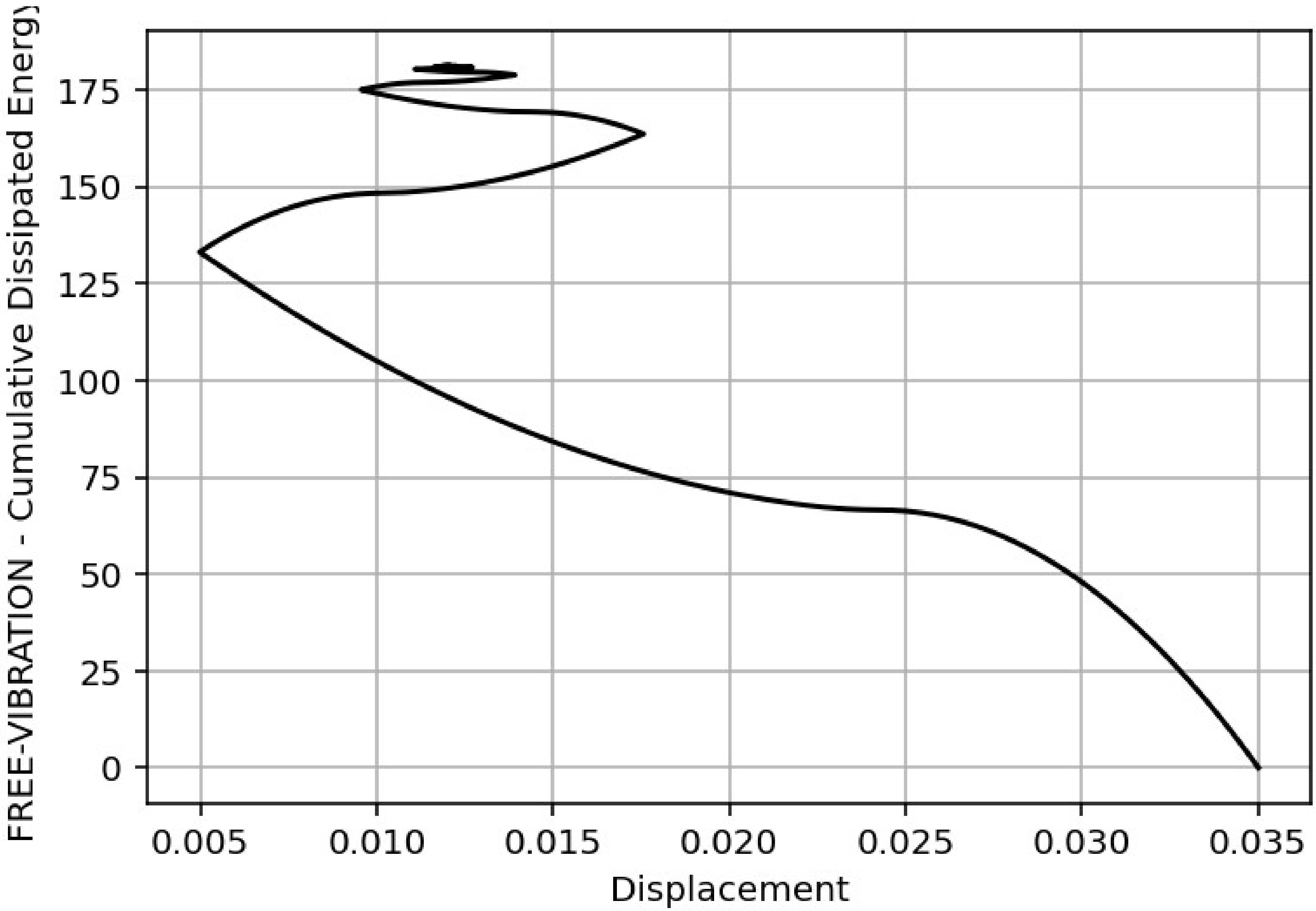
Free-vibration and Pushover of Structure
FREE-VIBRATION ELASTIC
FREE-VIBRATION INELASTIC
PUSHOVER ELASTIC
PUSHOVER INELASTIC
40000
20000
0
−20000
−40000
Base-raction [N]
−0.03
−0.02
−0.01
0.00
0.01
0.02
0.03
0.04
Displacement [m]


Period of Structure
0.7
0.6
0.5
0.4
0.3
0.2
0
100
200
300
400
Step
Structural Period [s]
ELASTIC PERIOD - MIN VALUES: Min: 0.172 (s) - Mean: 0.172 (s) - Max: 0.172 (s)
ELASTIC PERIOD - MAX VALUES: Min: 0.172 (s) - Mean: 0.172 (s) - Max: 0.172 (s)
INELASTIC PERIOD - MIN VALUES: Min: 0.172 (s) - Mean: 0.219 (s) - Max: 0.676 (s)
INELASTIC PERIOD - MAX VALUES: Min: 0.172 (s) - Mean: 0.219 (s) - Max: 0.676 (s)

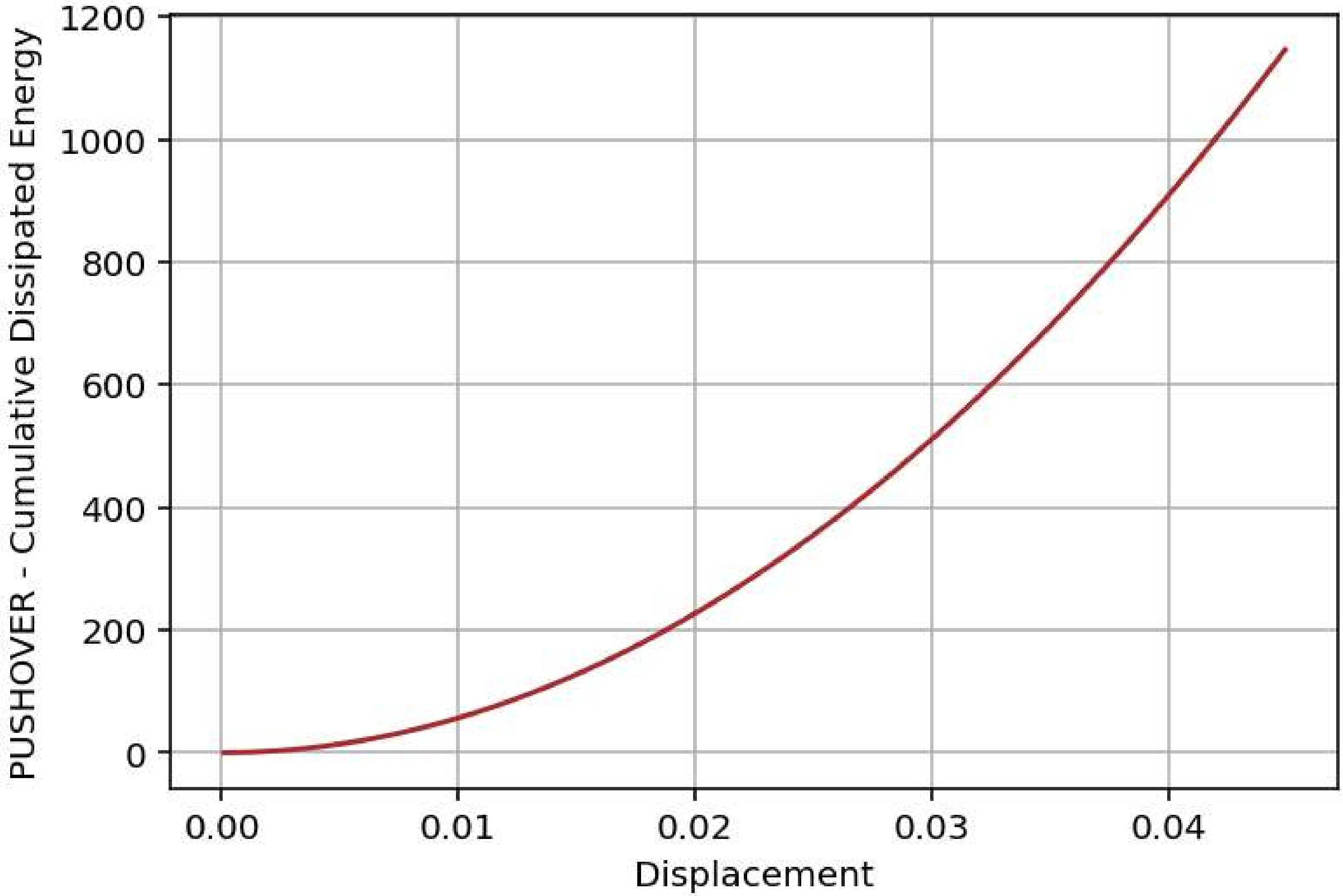

Node Displacements vs Time for Elastic Structure
Node 2 - MAX. ABS. : 4.4900e-02
0.00
0.01
0.02
0.03
0.04
Displacement [m]
0
100
200
300
400
Time [s]


Node Displacements vs Time for Inelastic Structure
Node 2 - MAX. ABS. : 4.4900e-02
0.04
0.03
0.02
0.01
0.00
Displacement [m]
0
100
200
300
400
Time [s]

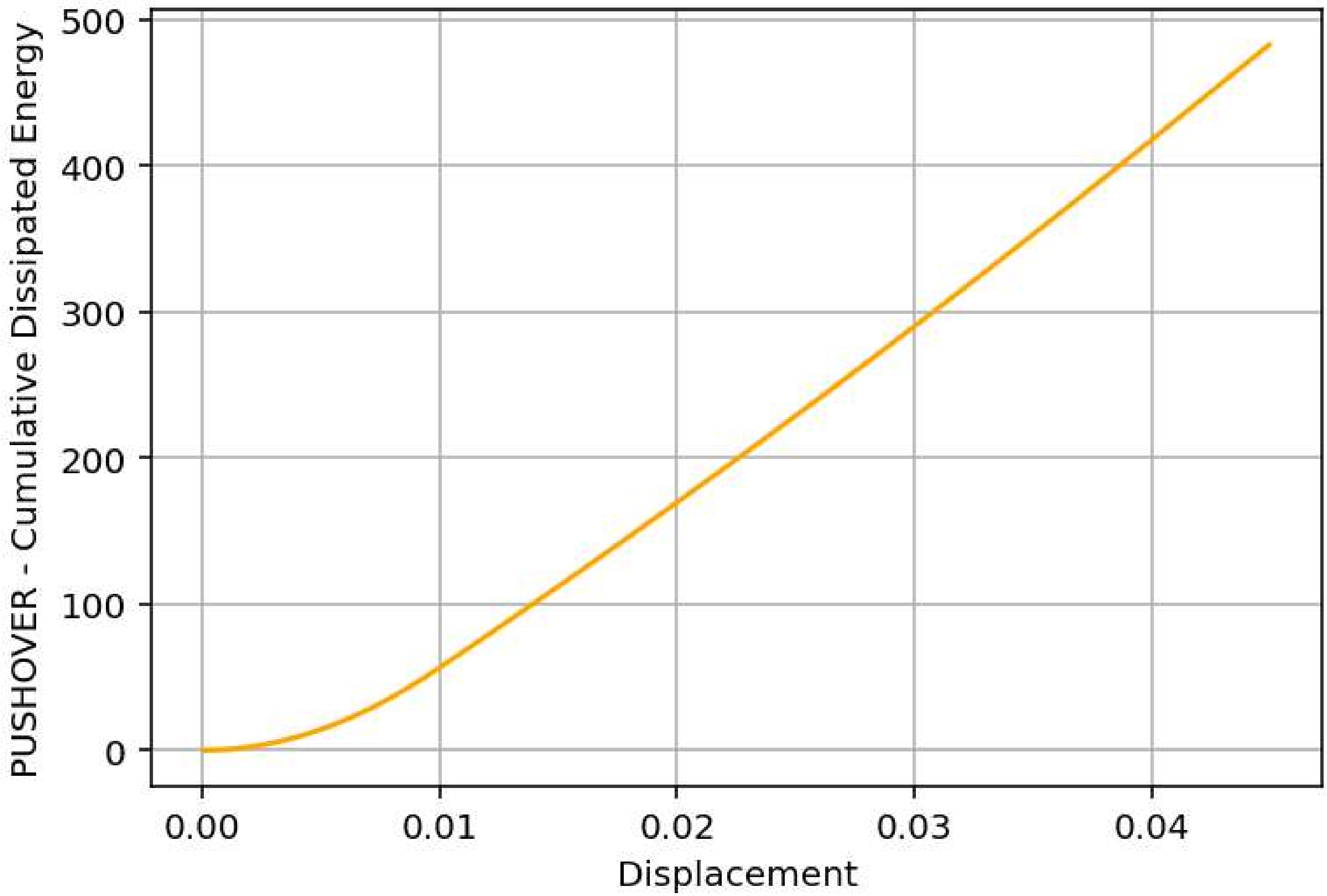

FREE-VIBRATION - Cumulative Dissipated Energy
8000
6000
4000
2000
0
−0.03
−0.02
−0.01
0.00
0.01
0.02
0.03
Displacement



FREE-VIBRATION - Cumulative Dissipated Energy
175
150
125
100
75
50
25
0
0.005
0.010
0.015
0.020
0.025
0.030
0.035
Displacement



PUSHOVER - Cumulative Dissipated Energy
1200
1000
800
600
400
200
0
0.00
0.01
0.02
0.03
0.04
Displacement



PUSHOVER - Cumulative Dissipated Energy
500
400
300
200
100
0
0.00
0.01
0.02
0.03
0.04
Displacement