مدينة زويل للعلوم والتكنولوچيا
Zewail City of Science and Technology

# MATH 201

Linear Algebra and Vector Geometry

**Open Courseware**
Lecture #5

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# 1 Operations of Matrices

**Definition 1.1: Matrix**

It is basically a table of numbers arranged in rows and columns.

**Example** (Matrix).

$$A_{2\times 3} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$$

If we have a matrix in which the number of rows is equal to the number of columns, then it is called a square matrix.

**Note:-**

For symbol notation, we usually use the capital letters for matrices and the small ones for the entries:

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

## 1.1 Addition of Matrices

We can only add two matrices if they have the same dimensions.

**Example.**

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}.$$

## 1.2 Scalar Multiplication of a Matrix

$$k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}.$$

**Example.**

$$2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}.$$

## 1.3 Multiplication of Matrices

To be able to multiply two matrices, the number of columns of the first matrix must be equal to the number of rows of the second matrix.

$$A_{m\times n} \times B_{n\times p} = C_{m\times p}.$$

**Example** (Compute AB).

$$A = \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}, B = \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} 4 & 3 & 6 \\ 1 & -2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2\times 4 + 3\times 1 & 2\times 3 + 3\times(-2) & 2\times 6 + 3\times 3 \\ 1\times 4 + (-5)\times 1 & 1\times 3 + (-5)\times(-2) & 1\times 6 + (-5)\times 3 \end{bmatrix} \\ &= \begin{bmatrix} 10 & -4 & 18 \\ 1 & 11 & 3 \end{bmatrix}. \end{aligned}$$

### 1.3.1 Row-vector rule

$$(AB)_{ij} = \text{row}_i(A) \cdot \vec{b}_j.$$

**Example.** If we want to find the 3rd element in the 1st row of $AB$, we dot product $A$'s 1st row with $B$'s 3rd column.

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For $A_{m\times n}$ and $B_{n\times p}$,

- $AB$ exist if $n = p$.
- $AB$ exist if $m \times q$ matrix.
- $(AB)_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$.

**Example** (Find $AB$ and $BA$).

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 4 & -3 \end{bmatrix} \\ BA &= \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 7 \\ 4 & -2 \end{bmatrix}. \end{aligned}$$

**Note:-**

$$AB \neq BA.$$

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**Example** ($AB = 0$ while $A \neq 0$ and $B \neq 0$).

$$AB = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

**Example** (Find $(A+B)^2$, $A^2 + 2AB + B^2$).

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}.$$

$$\begin{aligned} (A+B)^2 &= (A+B)(A+B) \\ &= \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 13 & 8 \\ 8 & 5 \end{bmatrix}. \end{aligned}$$

$$\begin{aligned} A^2 &= \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 0 & 7 \end{bmatrix} \\ B^2 &= \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 0 & 4 \end{bmatrix} \\ AB &= \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 4 & -4 \end{bmatrix} \\ 2AB &= \begin{bmatrix} 4 & 10 \\ 8 & -8 \end{bmatrix}. \end{aligned}$$

$$A^2 + B^2 + 2AB = \begin{bmatrix} 15 & 6 \\ 8 & -3 \end{bmatrix}.$$

**Note:-**

$$(A+B)^2 \neq A^2 + 2AB + B^2.$$
$$(A+B)^2 = A^2 + AB + BA + B^2.$$

**Example** ($AC = BC$ does not imply $A = B$).

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}.$$

$$\begin{aligned} AC &= \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} \\ BC &= \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}. \end{aligned}$$