Newtonian Mechanics in Geometric Algebra

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Lecture 1: Introduction

1.1 Physics as a tool

It is frequently the case that we forget that science as a whole is no more than a mere model that tries to reflect what reality is through our own perspective. It does not represent the truth nor claim to. It tries its best to be a useful tool for the human kind to progress and improve their own lives.

Physics, therefore, is no exception. It is a collection of theories which failed to be proven wrong based on the empirical data we have. It is a tool that we use to understand the world around us and to predict the future; a *tool*, not a *truth*.

Thus no one should think too highly of physics as something holy and true in some absolute sense. Thus, we should think of our models as a distorted view of reality, and we should keep in mind that the hypothetical statements they make are not representative of the *true* nature of *existence* itself.

Debates on the true nature of existence is more of a philosophical thing. Physics, even if theoretical, is an experimental science, not humanities. It does not try to answer questions like dilemmas of god, free will, ideologies, etc. We only think of how we can improve our current theories to fit the reality we observe.

Furthermore, absurd theories (e.g string theory, quantum gravity, etc) are not profound physics theories. They are more of a set of arbitrary hypothesis bunched together to form a model that seem coherent yet does not new scarily conform to our reality. It is not to say that such theories are pseudoscience or they are not worth studying–even the theory of relativity was in such a state when it was first published. It is just that they are not *real* within our studying of the physical nature. They are more like philosophical speculations of the world; they may be true, but, for the most part, they are unreliable perspectives.

This section is not needed, it is more of a conceptual overlook of physics as a science.

This is more of my personal rant than an educated opinion.

1.2 Formulation of a Mathematical Structure

Mathematical structures are made to help us, not to be enslaved to. Their formulation is like a game; it consists of a hierarchy of:

1. Goals
2. Rules
3. Definitions

Example. Cards Game

- Goals: winning (through gaining most cards).
- Rules:
  - Joker takes all, number takes its combination, etc.
  - Poker
  - etc
- Definitions: Ace of hearts, Knight of diamonds, etc.

Note:-

We can have the same game definitions with a different set of rules.

Definition 1.1: Logic

The process of using a set of definitions within a set of rules/constrains to reach a desired set of goals.

Note:-

Mathematical structures are not a truth within themselves; they are just tools to help us represent phenomenons in the real life efficiently.

Example. Useful Mathematical Structures

- $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ is a useful mathematical construct in some cases
  - You worked half half an hour yesterday and worked third of an hour today, **then** you worked a total of 50 minutes out of 60 minutes.
  - You won 1 out of 2 games yesterday and won 1 out of 3 games today, **then** you won a total of 2 out of 5 games
- $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$ is a plausible construct in most cases, but–clearly–not in all cases.

Definition 1.2: Truth

The combination of a set of *definitions* and a set of *rules*.

Definition 1.3: Validity

The inescapable conclustions formed by a given set of truth.

Example. Truths

- $\vec{V_1} \times \vec{V_2} = n|\vec{V_1}||\vec{V_2}| \sin\theta$ (We define the cross product as the perpendicular vector of the area formed by a parallelogram whose sides are formed from the vectors since it is a useful physical quantity).
- $\vec{V_1} \times \vec{V_2} = -\vec{V_2} \times \vec{V_1}$ (We define the anti-commutative property of cross product since it represents rotational quantities).

Example. Validities

- $(\vec{V_1} \times \vec{V_2}) \times \vec{V_3} \neq \vec{V_1} \times (\vec{V_2} \times \vec{V_3})$ (We can prove this using already defined proprieties and rules of the cross product).

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Lecture 2: Vectors

2.1 Formulation

Definition 2.4: Vector

A mathematical object that has a:

- Magnitude
- Orientation
- Sense (sign)

We use arrows to represent vectors graphically, the choice of arrows is very similar to the choice of lengths to represent numbers.

Definition 2.5: Vector equality

For $\vec{a}$ to be equal to $\vec{b}$

- $|\vec{a}| = |\vec{b}|$
- $\vec{a} \parallel \vec{b}$
- Sense of $\vec{a}$ is the same as that of $\vec{b}$

The choice of such an equality, although looks natural, is a made construct. This specific equality is useful for our calculations. However, it is as real as $\frac{1}{2} = \frac{3}{6}$, which, even though represents the notion of ratios, is not *real* since, clearly, the process of cutting a pizza into 2 pieces is different from cutting it into 6 piece.

Note:-

Notice how the equality is defined in terms of the original definition.

2.2 Arithmetic

2.2.1 Addition

Definition 2.6: Vector addition

Vector $\vec{a} + \vec{b}$ starts at the end of $\vec{A}$ and ends at the end of $\vec{B}$

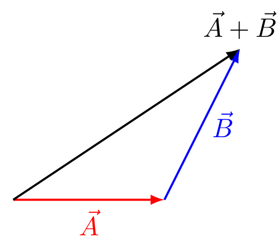

Figure 1: Sum of two vectors

This definition is based on the practicality of such a geometrical construct according to real life observations. Moreover, this enables commutative and associative.

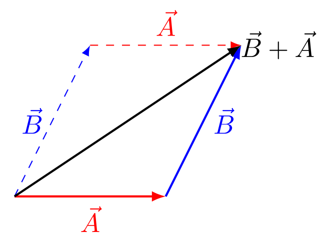

Figure 2: Commutative property of vector sum

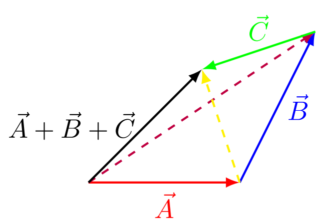

Figure 3: Associative property of vector sum

We define the zero vector due to the usefulness of zero in normal arithmetic.

Definition 2.7: Zero Vector

$$\vec{v} + \vec{0} = \vec{v}.$$

Note:-

$$\vec{0} \neq 0.$$
$$|\vec{0}| = 0.$$

2.3 Cartesian coordinates

Add operations before this

To use vectors in a more practical context, we put them onto the Cartesian coordinates where:

- All vectors starts from the origin point of the coordinate system we chose.
- The tip of the vector lies inside the coordinate space we chose.

It is not physically helpful to use vectors starting from different origin points, and when this is needed, it helps to clarify the difference between various systems for calculations to be accurate.

In order to actually use the proprieties of the Cartesian coordinate, we have to associate the vector's tip with some point (a, b). Sometimes vectors and points are used interchangeably. Whenever they are used in such manner, the physical context clarifies this confusion, but in some cases, they both are used within the same system where the distinction between them is needed.

Note:-

Vectors are not points, neither are equivalent to them.

2.3.1 Unit vectors

To both avoid the confusion with points and make calculations more computable we use unit vectors. Unite vectors are like a reference point for a given system. Their physical value is completely arbitrary, although usually used with real life units (meter, km, mile, etc).

Definition 2.8: Unit vectors

Vectors of length 1, orthogonal to each other, and parallel to the axes of the coordinate system, such that:

- the tip of e_1 is on the point $(1, 0, \ldots)$.
- the tip of e_2 is on the point $(0, 1, \ldots)$.

  $\vdots$
- the tip of e_n is on the point $(0, 0, \ldots, 1)$.

We usually refer to the first unit vectors as:

- $e_1 = \hat{i}$
- $e_2 = \hat{j}$
- $e_3 = \hat{k}$

$\hat{i}$ and $\hat{j}$ forms the relation $\vec{A} = a\hat{i} + b\hat{j}$.

Notes