CIE 328 Assignment #2 (CLO-1)

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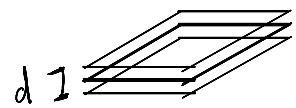
Answer the following questions, assume any missing data:

1. **(Exercise 4.8)** A parallel plate capacitor of unit area and separation d [m] is filled with two insulating layers. One layer is d_1 [m] thick and has permittivity $\varepsilon = 2\varepsilon_0$ [F/m], and the second is d_2 [m] thick ($d = d_1 + d_2$) with permittivity $\varepsilon = 3\varepsilon_0$ [F/m]. What is the capacitance of the capacitor?

$$C_1 = \frac{\varepsilon_1 A}{d_1} = \frac{2\varepsilon_0 A}{d_1}$$



$$C_2 = \frac{\varepsilon_2 A}{d_2} = \frac{3\varepsilon_0 A}{d_2}$$

$$C^{-1} = C_1^{-1} + C_2^{-1} \Rightarrow C = \frac{\varepsilon_0 A}{d_1/2 + d_2/3} = \frac{6\varepsilon_0 A}{3d_1 + 2d_2}$$

C_t= $\dfrac{6\varepsilon_0 A}{3d_1 + 2d_2}$

2. **(Exercise 4.9)** The capacitor in the above problem is connected across a voltage V. Calculate the voltage across each dielectric layer and the electric field intensity in each dielectric layer. Use the idea of capacitors in series to simplify the solution.

$$V = \frac{Q}{C} \Rightarrow Q = VC = \frac{6V\varepsilon_0 A}{3d_1 + 2d_2}$$

$$V = V_1 + V_2$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} \Rightarrow$$

$$V_1 = \frac{\overset{3}{\cancel{6}}V\cancel{\varepsilon_0}\cancel{A}}{3d_1 + 2d_2} \cdot \frac{d_1}{\cancel{2}\cancel{\varepsilon_0}\cancel{A}} = \boxed{\frac{3Vd_1}{3d_1 + 2d_2}}$$

$$V_2 = \frac{\overset{2}{\cancel{6}}V\cancel{\varepsilon_0}\cancel{A}}{3d_1 + 2d_2} \cdot \frac{d_2}{\cancel{3}\cancel{\varepsilon_0}\cancel{A}} = \boxed{\frac{2Vd_2}{3d_1 + 2d_2}}$$

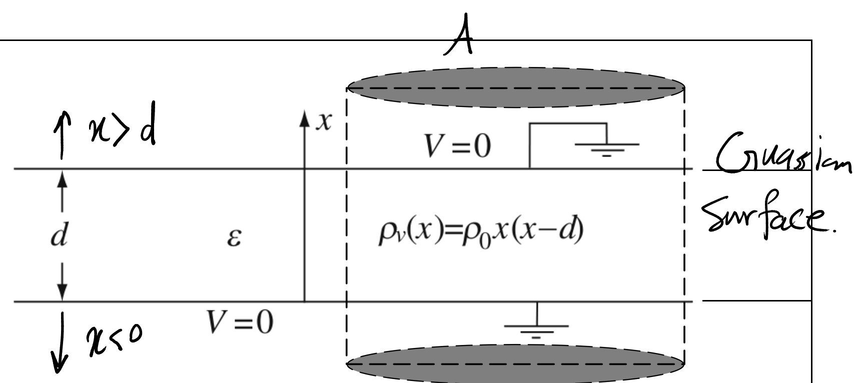
$$E = \frac{V}{d} \Rightarrow E_1 = \boxed{\frac{3V}{3d_1 + 2d_2}}$$

$$E_2 = \boxed{\frac{2V}{3d_1 + 2d_2}}$$

E₁= $\frac{3V}{3d_1+2d_2}$
E₂= $\frac{2V}{3d_1+2d_2}$

3. **(Exercise 5.1)** What is the electric field intensity and the electric potential outside the plates in Example **5.1** (Page **234**)? **Hint:** You must use Gauss's law.

Figure 5.1 A parallel plate capacitor with grounded plates and a charge density between the plates



$$\int_s \vec{E}\cdot d\vec{A} = 2EA \Rightarrow 2EA = \frac{Q_{enc}}{\varepsilon_0}$$

$$= \int \frac{\rho_v}{\varepsilon_0} dV$$

$$2E\cancel{A} = \frac{\rho_0}{\varepsilon_0}\cancel{A}\int x(x-d)\,dx$$

$$E = \frac{\rho_0}{2\varepsilon_0}\int_0^d x(x-d)\,dx$$

$$\left[\frac{x^3}{3} - \frac{d}{2}x^2\right]_0^d = \frac{-\rho_0}{12\varepsilon_0} d^3\hat{x}$$

$x<0$: $V = -\int_0^x \vec{E}\cdot d\vec{l} = \dfrac{-\rho_0 d^3 x}{12\varepsilon_0}$, $x>d$: $V = \int_d^x \vec{E}\cdot d\vec{l} = \dfrac{-\rho_0 d^3 (x-d)}{12\varepsilon_0}$

X<0

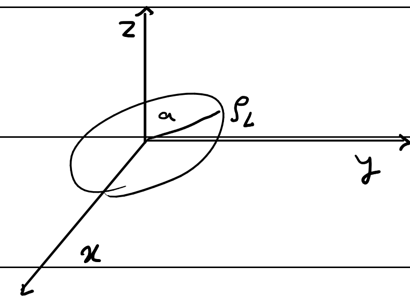
V= $\dfrac{-\rho_0 d^3 x}{12\varepsilon_0}$

X>d

V= $\dfrac{-\rho_0 d^3 (x-d)}{12\varepsilon_0}$

4. Uniform line charge of density $\rho_l = 5$ nC/m lies on a circular ring of radius $a = 3$ m in the $z = 0$ plane. The circle is centered at the origin. Find an expression for the potential V at the point P (0,0, z). What is the value of V if z = 4 m and the medium is vacuum?

$dV = \dfrac{dQ}{4\pi\varepsilon_0 r}$, $dQ = \rho_L\, dl$, $r = \sqrt{a^2+z^2}$



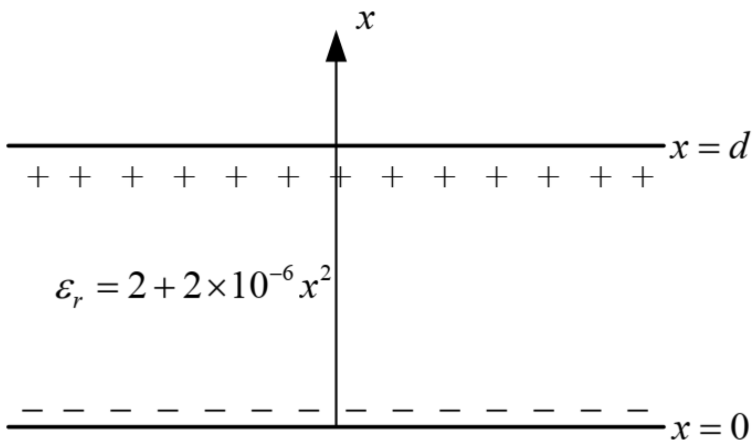
$$V = \int_0^{2\pi a} \frac{\rho_L\, dl}{4\pi\varepsilon_0\sqrt{a^2+z^2}} = \frac{2\pi a \rho_L}{4\pi\varepsilon_0\sqrt{a^2+z^2}}$$

$$= \frac{a\rho_L}{2\varepsilon_0\sqrt{a^2+z^2}}$$

$$V\big|_{z=4} = \frac{3\cdot 5n}{2\varepsilon_0\sqrt{3^2+z^2}} = 169\ V$$

V= 169 V

5. A parallel plate is filled with a non-uniform dielectric characterized by $\varepsilon_r = 2 + 2 \times 10^6 x^2$ where x is the distance from the lower plate in meters. If S = 0.02 m^2 and d = 1.0 mm, find the capacitance C. Calculate the energy stored in this capacitor if the charge on the positive plate is $Q = 4.0 \times 10^{-9}$ C.



$\varepsilon_r = 2 + 2\cdot10^6 x^2 \qquad d = 1\cdot10^{-3} \qquad A = 0.02 \qquad Q = 4.0\times10^{-9}$

$$C^{-1} = \int_0^d \frac{dx}{\varepsilon(x) A} = \frac{1}{2\varepsilon_0 A}\int_0^d \frac{dx}{1 + 10^6 x^2} = \frac{1}{2\varepsilon_0 A}\cdot\frac{1}{\sqrt{10^6}}\cdot \tan^{-1}\left(x\sqrt{10^6}\right)$$

$$= \frac{1}{40\varepsilon_0}\cdot\frac{\pi}{4}$$

C = 0.45 nF

$$W = \frac{Q^2}{2C} = 1.78\times10^{-8}$$

C= 0.45 nF