Name: SalahDin Ahmed Salh Rezk
ID: 202201079
Date: March 1, 2024

Signals and Systems (CIE 227)

Mini-Assignment 1

Simplify the following expressions. Give the answers in both Cartesian form $(x + jy)$ and polar form $\left(re^{j\theta}\right)$.

1. $3e^{j\pi/3} + 4e^{-j\pi/6}$

Solution.

$$3e^{j\pi/3} + 4e^{-j\pi/6} \tag{1}$$

$$3\cos\left(\frac{\pi}{3}\right) + 3j\sin\left(\frac{\pi}{3}\right) + 4\cos\left(-\frac{\pi}{6}\right) + 4j\sin\left(-\frac{\pi}{6}\right) \tag{2}$$

$$3\cos\left(\frac{\pi}{3}\right) + 4\cos\left(-\frac{\pi}{6}\right) + j\left[3\sin\left(\frac{\pi}{3}\right) + 4\sin\left(-\frac{\pi}{6}\right)\right] \tag{3}$$

$$3\cdot\frac{1}{2} + 4\cdot\frac{\sqrt{3}}{2} + j\left(3\cdot\frac{\sqrt{3}}{2} + 4\cdot-\frac{1}{2}\right) \tag{4}$$

$$\frac{3}{2} + 2\sqrt{3} + j\left(\frac{3\sqrt{3}}{2} - 2\right) \tag{5}$$

$$\boxed{\frac{3+4\sqrt{3}}{2} + j\frac{3\sqrt{3}-4}{2}} \tag{6}$$

$$\sqrt{\left(\frac{3+4\sqrt{3}}{2}\right)^2 + \left(\frac{3\sqrt{3}-4}{2}\right)^2}\cdot\exp\left[j\tan^{-1}\left(\frac{3\sqrt{3}-4}{2}\div\frac{3+4\sqrt{3}}{2}\right)\right] \tag{7}$$

$$5\cdot\exp\left[j\tan^{-1}\left(\frac{3\sqrt{3}-4}{3+4\sqrt{3}}\right)\right] \tag{8}$$

$$5\cdot\exp\left[j\tan^{-1}\left(\frac{48-25\sqrt{3}}{39}\right)\right] \tag{9}$$

$$\boxed{\approx 5e^{0.1199j}} \tag{10}$$

■

2. $(\sqrt{3} - j3)^{10}$

Solution.

$$\left(\sqrt{3} - 3j\right)^{10} \tag{11}$$

$$\left(2\sqrt{3}\cdot e^{-\frac{1}{3}\pi j}\right)^{10} \tag{12}$$

$$\left(2\sqrt{3}\right)^{10} e^{-\frac{10}{3}\pi j} \tag{13}$$

$$\boxed{248832e^{\frac{2}{3}\pi j}} \tag{14}$$

(15)

$$248832\left[\cos\left(\frac{2}{3}\pi\right)+j\sin\left(\frac{2}{3}\pi\right)\right] \tag{16}$$

$$248832\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right) \tag{17}$$

$$\boxed{-124416+j124416\sqrt{3}} \tag{18}$$

$$\tag{19}$$

■

3. $(\sqrt{3}-j3)^{-1}$

Solution.

$$\left(\sqrt{3}-j3\right)^{-1} \tag{20}$$

$$\left(2\sqrt{3}\cdot e^{-\frac{1}{3}\pi j}\right)^{-1} \tag{21}$$

$$\boxed{\frac{1}{2\sqrt{3}}e^{\frac{1}{3}\pi j}} \tag{22}$$

$$\frac{1}{2\sqrt{3}}\left[\cos\left(\frac{1}{3}\pi\right)+j\sin\left(\frac{1}{3}\pi\right)\right] \tag{23}$$

$$\frac{1}{2\sqrt{3}}\left[\frac{1}{2}+j\frac{\sqrt{3}}{2}\right] \tag{24}$$

$$\frac{1}{4\sqrt{3}}+j\frac{1}{4} \tag{25}$$

$$\boxed{\frac{\sqrt{3}}{12}+\frac{1}{4}j} \tag{26}$$

■

4. $(\sqrt{3}-j3)^{1/3}$

Solution.

$$\left(\sqrt{3}-j3\right)^{1/3} \tag{27}$$

$$\left(2\sqrt{3}\cdot e^{-\frac{\pi}{3}j}\right)^{1/3} \tag{28}$$

$$\boxed{\sqrt[3]{2\sqrt{3}}\cdot e^{-\frac{\pi}{9}j}} \tag{29}$$

$$\boxed{1.422-0.5175j} \tag{30}$$

■

5. $\mathfrak{R}\left\{je^{-j\pi/3}\right\}$

Solution.

$$\mathfrak{R}\left\{je^{-j\pi/3}\right\} \tag{31}$$

$$\mathfrak{R}\left\{j\left[\cos\left(-\frac{\pi}{3}\right)+j\sin\left(-\frac{\pi}{3}\right)\right]\right\} \tag{32}$$

$$\mathfrak{R}\left\{j\cos\left(-\frac{\pi}{3}\right)-\sin\left(-\frac{\pi}{3}\right)\right\} \tag{33}$$

$$\mathfrak{R}\left\{j\frac{1}{2}+\frac{\sqrt{3}}{2}\right\} \tag{34}$$

$$\boxed{\frac{\sqrt{3}}{2}} \tag{35}$$

$$\boxed{\frac{\sqrt{3}}{2}\cdot e^{0j}} \tag{36}$$

■