CIE 328 Assignment #1 [CLO-1]

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Answer the following questions, assume any missing data:

1. A cube is defined by $1 \le x \le 1.2$, $1 \le y \le 1.2$, and $1 \le z \le 1.2$, where the limits are in meters. The electric flux density in the cube is given by $\mathbf{D} = 2x^2y\,\mathbf{a_x} + 3x^2y^2\,\mathbf{a_y}$ [C/m²]. The medium is a dielectric of relative permittivity $\varepsilon_r = 4$.

   a) Find the total electric flux ψ leaving the closed surface of the cube.

$$\Psi = \oint\!\!\oint_S \vec{D}\cdot d\vec{S} = \iiint_V \vec{\nabla}\cdot\vec{D}\,dV \quad ; \quad \vec{\nabla}\cdot\vec{D} = 4xy + 6x^2y$$

$$= \iiint_V (4xy + 6x^2y)\,dV = \int_1^{1.2}\int_1^{1.2}\int_1^{1.2} (4xy + 6x^2y)\,dx\,dy\,dz$$

$$= \int_1^{1.2} dz \int_1^{1.2} (4x + 6x^2) \int_1^{1.2} y\,dy = 0.09344$$

a) $\boldsymbol{\psi} =$ $0.09344\ Nm^2/C$

b) Find the charge density ρ_v as a function of position (x,y,z). Evaluate ρ_v at the center of the cube.

$$\rho_v(x,y,z) = \vec{\nabla} \cdot \vec{D} = 4xy + 6x^2y$$

$$\rho_v\left(\frac{1+1.2}{2}, \frac{1+1.2}{2}, \frac{1+1.2}{2}\right) = 4(1.1)(1.1) + 6(1.1)^2(1.1)$$

$$= 12.826$$

center of the cube

b) ρ_v(x,y,z)= $4xy + 6x^2y$

At center of cube ρ_v= $12.826 \; C/N$

c) Find the total charge enclosed by the cube.

$$\iiint_V \rho_v \, dv = \int_1^{1.2}\int_1^{1.2}\int_1^{1.2} (4xy + 6x^2y)\, dx\, dy\, dz = 0.09344$$

$\mathbf{Q}_{en} =$ $0.09344 \; C$

d) Find the electric field vector **E** in the cube as a function of position (x,y,z).

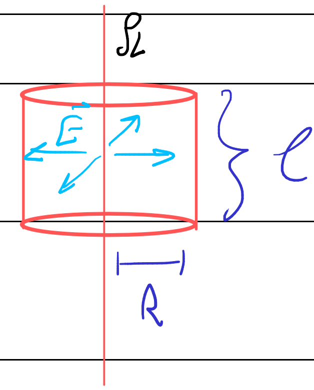
$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} \Rightarrow \vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r} = \frac{\vec{D}}{4\varepsilon_0} = \frac{2x^2y\, a_x + 3x^2y^2 a_y}{4\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}\left(\frac{1}{2} x^2 y\, a_x + \frac{3}{4} x^2 y^2\, a_y\right)$$

E(x,y,z)= $\frac{1}{\varepsilon_0}\left(\frac{1}{2} x^2 y\, a_x + \frac{3}{4} x^2 y^2\, a_y\right)$

2. A uniform line charge of ρ_L = 3 [μC/m] lies along the z-axis, and a coaxial circular cylinder of radius 2m has ρ_s = -1.5/4π [μC/m²]. Both distributions are infinite in extent with z. Use Gauss's law to find ***D*** in all regions.

D at first region 0<R<2



$$\oint_S \vec{D}\cdot d\vec{A} = Q_{enc}$$

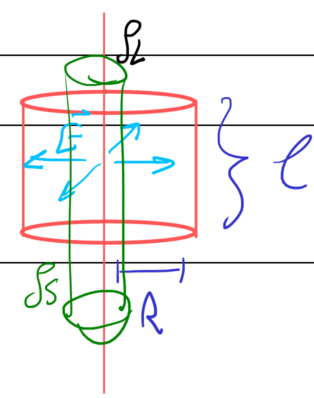
$$D_R\, 2\pi R \ell = \rho_L \ell$$

$$D_R = \frac{\rho_L}{2\pi R}$$

$$\vec{D} = \frac{\rho_L}{2\pi r}\hat{r} = \frac{3}{2\pi r}\hat{r}$$

D= $\frac{3\mu}{2\pi r}\hat{r}$ C/N

D at second region R>2



$$D_R\, 2\pi R \ell = \rho_L \ell + \rho_s 2\pi a \ell$$

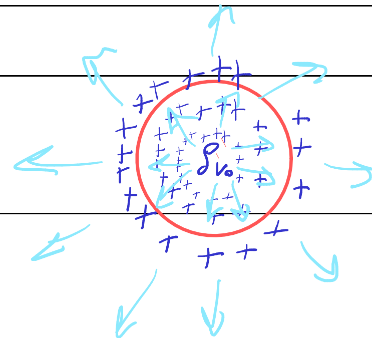
$$D_R = \frac{\rho_L}{2\pi R} + \frac{\rho_s a}{R}$$

$$= \frac{3}{2\pi R} + \frac{-1.5(2)}{4\pi R} = \frac{3}{4\pi R}$$

D= $\frac{3\mu}{4\pi R}$ C/N

3. Charge is distributed uniformly within a sphere of radius a, with volume charge density ρ_{vo} [C/m^3]. Determine **E** inside and outside the sphere.

Inside



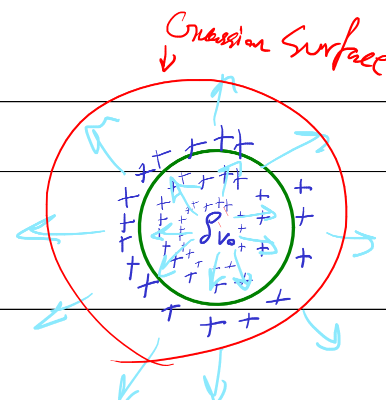
$$\oiint_S \vec{D} \cdot d\vec{A} = Q_{enc}$$

$$D_R \, 4\pi R^2 = \rho_{vo} \frac{4}{3} \pi R^3$$

$$D_R = \frac{\rho_{vo} R}{3} \Rightarrow \vec{E} = \frac{\rho_{vo} \vec{r}}{3 \varepsilon_o}$$

E= $\dfrac{\rho_{vo} \vec{r}}{3 \varepsilon_o}$

outside



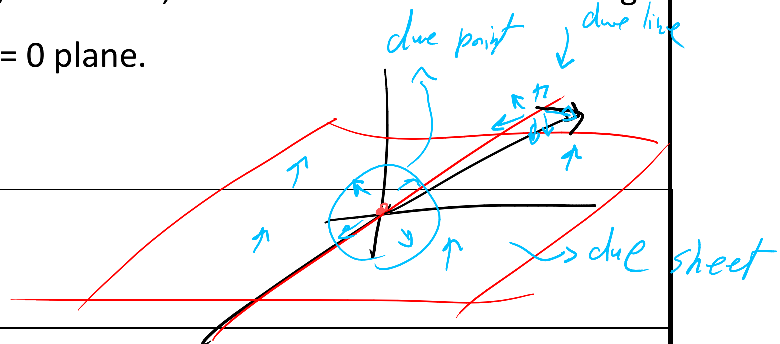
$$D_R \, 4\pi R^2 = \rho_{vo} \frac{4}{3} \pi a^3$$

$$D_R = \frac{\rho_{vo} a^3}{3 R^2} \Rightarrow \vec{E} = \frac{\rho_{vo} a^3}{3 \varepsilon_o r^2} \hat{r}$$

E= $\dfrac{\rho_{vo} a^3}{3 \varepsilon_o r^2} \hat{r}$

4. A point charge of 6 [μC] is located at the origin, a uniform line charge density of 180 [nC/m] lies along the x-axis, and a uniform sheet of charge equal to 25 [nC/m²], lies in the z = 0 plane.

(a) Find the **D** at A (0, 0, 4).



$$\vec{D}_Q = \frac{6\mu}{4\pi r^3}\vec{r}$$

$$\vec{D}_L = \frac{180n}{2\pi r^2}\langle 0, y, z\rangle \qquad \vec{D}_S = \frac{25n}{2}\langle 0,0,1\rangle \;;\; r=\sqrt{x^2+y^2+z^2}$$

$$\sum D = \left\langle \frac{6\mu}{4\pi r^3}x \,,\, \left(\frac{6\mu}{4\pi r^3}+\frac{180n}{2\pi r^2}\right)y \,,\, \left(\frac{6\mu}{4\pi r^3}+\frac{180n}{2\pi r^2}\right)z+\frac{25n}{2}\right\rangle$$

D$_t$= $\left\langle \frac{6\mu}{4\pi r^3}x \,,\, \left(\frac{6\mu}{4\pi r^3}+\frac{180n}{2\pi r^2}\right)y \,,\, \left(\frac{6\mu}{4\pi r^3}+\frac{180n}{2\pi r^2}\right)z+\frac{25n}{2}\right\rangle$

At A (0, 0, 4)

D$_t$= $\langle 0, 0, 49.5n\rangle$ N/C

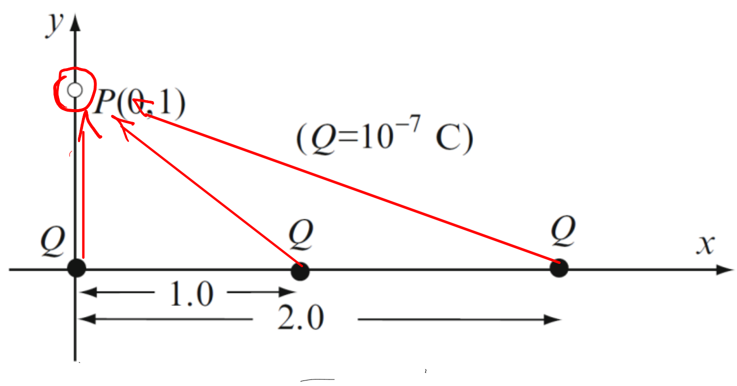
(b) Calculate the total electric flux leaving the surface of a sphere of 4 [m] radius centered at the origin.

$$\psi = \frac{Q_{enc}}{\varepsilon_0} = 6\mu + 180n\cdot(8) + 25n(\pi 4^2)$$

$$= 8.69\mu \ Nm^2/C$$

$\boldsymbol{\psi} = 8.69\mu \ Nm^2/C$

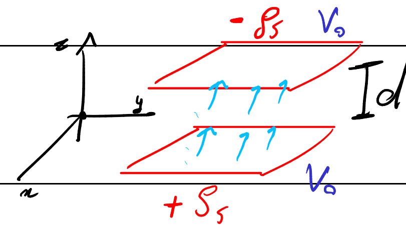
5. Three charges, each equal to Q =10^{-7} C, are located on the x axis at x = 0, x =1 [m], and x = 2 [m], as shown in Figure. Calculate the potential at (x = 0, y = 1).



$$V = \frac{10^{-7}}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{5}}\right) = 1936\ V$$

V= 1936 V

6. Two parallel plates are very large in extent (infinite), separated a distance d [m], in air, and charged with equal but opposite charge density ρ_s [C/m^2]. Calculate the potential difference between the two plates.



$$\vec{E} = \frac{\rho_s}{\varepsilon_0}\hat{k}$$

$$V = -\int_C \vec{E}\cdot d\vec{l} = -\int_{r_0}^{r} \frac{\rho_s}{\varepsilon_0}\hat{k}\cdot d\vec{r} = \frac{\rho_s}{\varepsilon_0} z_0 - \frac{\rho_s}{\varepsilon_0} z = \frac{\rho_s}{\varepsilon_0}(z_0 - z)$$

In case the plate is our reference point r_0: $V = V_0 - \frac{\rho_s}{\varepsilon_0} z$

V= $V_0 - \frac{\rho_s}{\varepsilon_0} z$

7. Two parallel plates are very large in extent (infinite), separated a distance d and connected to a potential difference V. Calculate the electric field intensity between the two plates.

$$E = -\nabla V$$
$$= \frac{\partial}{\partial z}\left(V - \frac{\rho_s}{\varepsilon_0} z\right)\hat{k}$$
$$= \rho_s/\varepsilon_0\, \hat{k}$$

E= $\rho_s/\varepsilon_0\, \hat{k}$