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Digital Design and Computer Architecture (CIE 239)

Assignment 1

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1. How many different numbers can be represented with 16 bits?

$$2^{16} = 65536. \tag{1}$$

2. Convert the following unsigned binary numbers to decimal. Show your work.

   (a) 1110_2

$$1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 14_{10}. \tag{2}$$

   (b) 100100_2

$$1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 36_{10}. \tag{3}$$

3. Convert the following hexadecimal numbers to decimal. Show your work.

   (a) $4E_{16}$

$$4 \times 16^1 + 14 \times 16^0 = 78_{10}. \tag{4}$$

   (b) $7C_{16}$

$$7 \times 16^1 + 12 \times 16^0 = 124_{10}. \tag{5}$$

4. Convert the following decimal numbers to 8-bit two's complement numbers or indicate that the decimal number would overflow the range.

   (a) 42_{10}

$$42/2 = 21 \text{ r } 0 \tag{6}$$
$$21/2 = 10 \text{ r } 1 \tag{7}$$
$$10/2 = 5 \text{ r } 0 \tag{8}$$
$$5/2 = 2 \text{ r } 1 \tag{9}$$
$$2/2 = 1 \text{ r } 0 \tag{10}$$
$$1/2 = 0 \text{ r } 1. \tag{11}$$

$$42_{10} = 00101010_2. \tag{12}$$

(b) -63_{10}

$$63/2 = 31 \text{ r } 1 \tag{13}$$
$$31/2 = 15 \text{ r } 1 \tag{14}$$
$$15/2 = 7 \text{ r } 1 \tag{15}$$
$$7/2 = 3 \text{ r } 1 \tag{16}$$
$$3/2 = 1 \text{ r } 1 \tag{17}$$
$$1/2 = 0 \text{ r } 1. \tag{18}$$

$$00111111'_2 = 11000000_2 \tag{19}$$
$$11000000_2 + 1 = 11000001_2 \tag{20}$$

$$-63_{10} = 11000001_2. \tag{21}$$

5. Convert each of the following octal numbers to binary, hexadecimal, and decimal.

(a) 23_8

$$2_{10} \times 8^1_{10} + 3_{10} \times 8^0_{10} = 19_{10}. \tag{22}$$

$$19/2 = 9 \text{ r } 1 \tag{23}$$
$$9/2 = 4 \text{ r } 1 \tag{24}$$
$$4/2 = 2 \text{ r } 0 \tag{25}$$
$$2/2 = 1 \text{ r } 0 \tag{26}$$
$$1/2 = 0 \text{ r } 1. \tag{27}$$

$$23_8 = 0010011_2. \tag{28}$$

$$19/16 = 1 \text{ r } 3 \tag{29}$$
$$1/16 = 0 \text{ r } 1. \tag{30}$$

$$23_8 = 13_{16}. \tag{31}$$

(b) 45_8

$$4_{10} \times 8^1_{10} + 5_{10} \times 8^0_{10} = 37_{10}. \tag{32}$$

$$37/2 = 18 \text{ r } 1 \tag{33}$$
$$18/2 = 9 \text{ r } 0 \tag{34}$$
$$9/2 = 4 \text{ r } 1 \tag{35}$$
$$4/2 = 2 \text{ r } 0 \tag{36}$$
$$2/2 = 1 \text{ r } 0 \tag{37}$$
$$1/2 = 0 \text{ r } 1. \tag{38}$$

$$45_8 = 0100101_2. \tag{39}$$

$$37/16 = 2 \text{ r } 5 \tag{40}$$
$$2/16 = 0 \text{ r } 2. \tag{41}$$

$$45_8 = 25_{16}. \tag{42}$$

6. Perform the following additions of signed binary numbers. Indicate whether or not the sum overflows a 4-bit result in both cases.

   (a) $1001_2 + 0100_2$

$$1001_2 + 0100_2 = 1101_2 \tag{43}$$
$$= -3_{10}. \tag{44}$$

   (b) $1101_2 + 1011_2$

$$1101_2 + 1011_2 = 1\underline{1000}_2. \tag{45}$$

Overflows. □

7. Convert the following decimal numbers to 6-bit two's complement binary numbers and add them. Indicate whether or not the sum overflows a 6-bit result in both cases.

   (a) $7_{10} + 13_{10}$

$$7/2 = 3 \text{ r } 1 \tag{46}$$
$$3/2 = 1 \text{ r } 1 \tag{47}$$
$$1/2 = 0 \text{ r } 1. \tag{48}$$

$$7_{10} = 000111_2. \tag{49}$$

$$13/2 = 6 \text{ r } 1 \tag{50}$$
$$6/2 = 3 \text{ r } 0 \tag{51}$$
$$3/2 = 1 \text{ r } 1 \tag{52}$$
$$1/2 = 0 \text{ r } 1. \tag{53}$$

$$13_{10} = 001101_2. \tag{54}$$

$$000111_2 + 001101_2 = 010100_2 \tag{55}$$
$$= 20_{10}. \tag{56}$$

   (b) $17_{10} + 25_{10}$

$$17/2 = 8 \text{ r } 1 \tag{57}$$
$$8/2 = 4 \text{ r } 0 \tag{58}$$
$$4/2 = 2 \text{ r } 0 \tag{59}$$
$$2/2 = 1 \text{ r } 0 \tag{60}$$
$$1/2 = 0 \text{ r } 1. \tag{61}$$

$$17_{10} = 010001_2. \tag{62}$$

$$25/2 = 12 \text{ r } 1 \tag{63}$$
$$12/2 = 6 \text{ r } 0 \tag{64}$$
$$6/2 = 3 \text{ r } 0 \tag{65}$$
$$3/2 = 1 \text{ r } 1 \tag{66}$$
$$1/2 = 0 \text{ r } 1. \tag{67}$$

$$25_{10} = 011001_2. \tag{68}$$

$$010001_2 + 011001_2 = 101010_2 \tag{69}$$
$$= -10_{10}. \tag{70}$$

Overflows. □

8. Perform the following additions of signed hexadecimal numbers. Indicate whether or not the sum overflows an 8-bit (two hex digit) result.

   Since it is signed 8-bit so the range is -128_{10} to 127_{10} so anything greater than $7F_{16}$ overflows.

   (a) $22_{16} + 8_{16}$

   $$22_{16} + 8_{16} = 2A_{16}. \tag{71}$$

   (b) $73_{16} + 2C_{16}$

   $$73_{16} + 2C_{16} = 9F_{16} \tag{72}$$
   $$9F_{16} > 7F_{16} \implies -1F_{16}. \tag{73}$$

   Overflows. □

9. Represent the following decimal number in BCD.

   (a) 13597

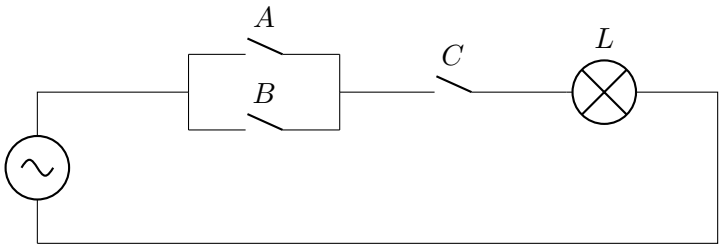
1	3	5	9	7
0001	0011	0101	1001	0111

   (b) 93286

9	3	2	8	6
1001	0011	0010	1000	0110

10. Express the switching circuit shown in the figure in binary logic notation

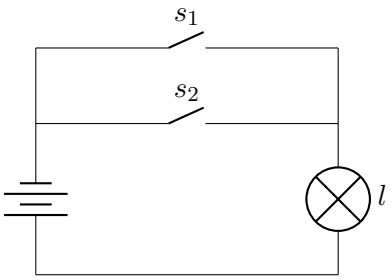
(a)



$$(A + B)C = L. \tag{74}$$

$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{C}$	$\boldsymbol{L}$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

(b)



$$s_1 + s_2 = l. \tag{75}$$

$\boldsymbol{s_1}$	$\boldsymbol{s_2}$	$\boldsymbol{l}$
0	0	0
0	1	1
1	0	1
1	1	1