1. Determine $i(t)$ for $t > 0$ in the circuit of Fig. 1.

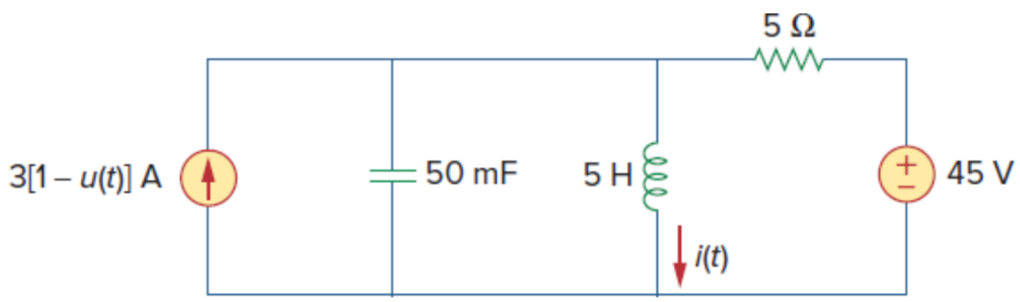
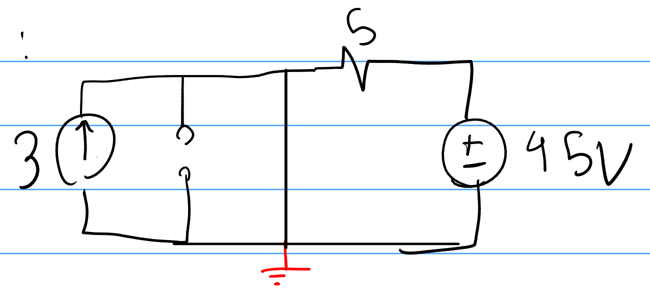


Figure 1

$t<0$:

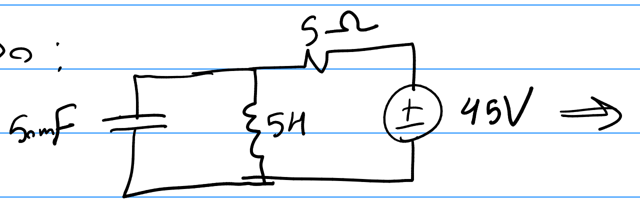


$i(0) = 3 + \frac{45}{5} = 12A$

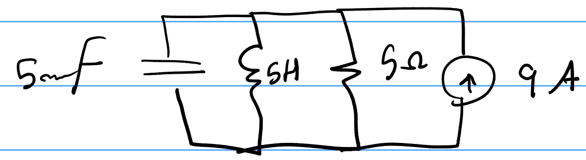
$V(0) = 0$

$V = IR$

$t>0$:



$\Rightarrow$



$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 5 \times 50m} = 2$, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 50m}} = 2$

$\alpha = \omega_0 \Rightarrow$ Critically Damped : $i(t) = i_s + (A_1 + A_2 t) e^{-\alpha t}$

$= 9 + (A_1 + A_2 t) e^{-2t}$

$i(0) = 9 + (A_1 + \cancel{A_2 \times 0}) \cancel{e^{-2 \times 0}}$

$12 = 9 + A_1$

$A_1 = 3$

$V_0 = L \frac{di(0)}{dt}$, $\frac{di}{dt} = -2A_1 e^{-2t} + A_2 (e^{-2t} - 2e^{-2t} t)$

$= [-2A_1 + A_2(1-2t)] e^{-2t}$

$0 = 5 \times (A_2 - 6) \Rightarrow A_2 = 6$; $\boxed{i(t) = 9 + (3 + 6t) e^{-2t}}$

2. Find $v_c(t)$ for $t > 0$ for the circuit of Fig. 2. Assume steady-state conditions exist at $t = 0^-$.

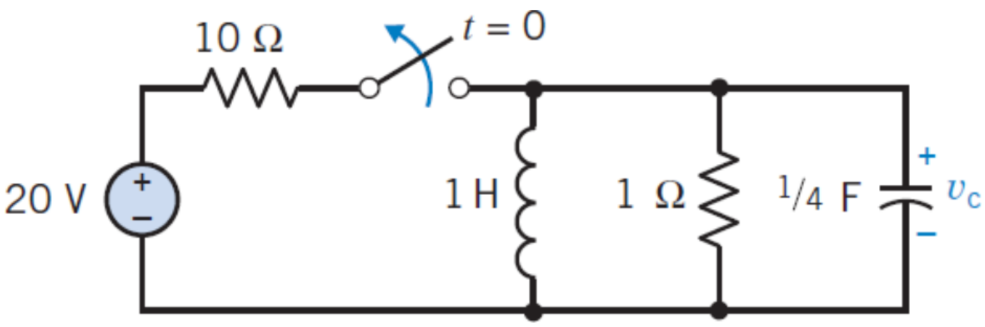
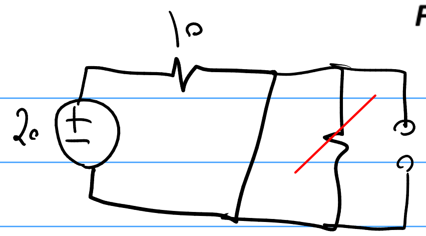


Figure 2

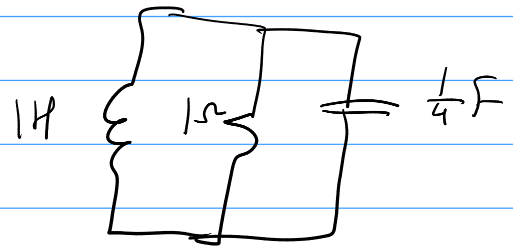
$t<0:$



$v_0 = 0V$

$i_0 = 2A$

$t>0:$



$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 1\times \frac{1}{4}} = 2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times \frac{1}{4}}} = 2$$

$\alpha = \omega_0 \Rightarrow$ Critically Damped : $v(t) = (A_1 + A_2 t)e^{-\alpha t}$

$$= (A_1 + A_2 t)e^{-2t}$$

$A_1 = 0 \Rightarrow V(t) = Ate^{-2t}$

$$\frac{dV(0)}{dt} = -\frac{V_0 + I_0 R}{RC} \Rightarrow A = -8$$

$$\boxed{V(t) = -8\, e^{-2t}}$$

3. Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit of Fig. 2.

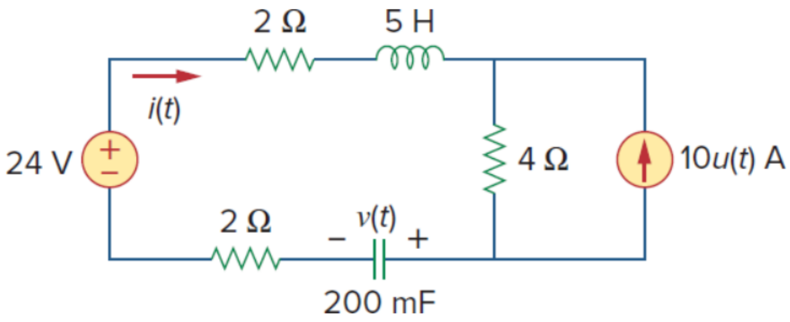
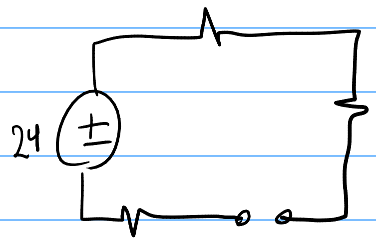


Figure 2

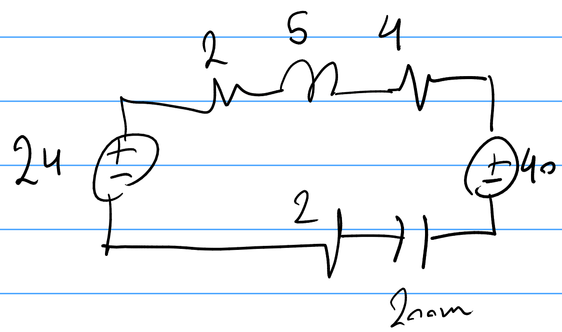
$t<0$:



$V_0 = 24$

$I_0 = 0$

$t>0$



$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 8\times 200m} = 0.3125$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times 200m}} = 1$$

$\alpha < \omega_0 \Rightarrow$ Underdamped

$$V(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 0.95$$

$A_1 = 24$, $\frac{dV}{dt} = -\alpha \cancel{e^{-\alpha t}}(A_1 \cos \cancel{\omega_d t} + \cancel{A_2 \sin\omega_d t}) + \cancel{e^{-\alpha t}}(A_2 \omega_d \cancel{\cos\omega_d t} - \cancel{A_1 \omega_d \sin\omega_d t})$

$\frac{dv_0}{dt} = -\frac{V_0 + I_0 R}{RC} \Rightarrow \quad -15 = -\alpha A_1 + \omega_d A_2$

$= -0.3125 \times 24 + 0.95 A_2 \Rightarrow \quad A_2 = -7.9$

$$\boxed{V(t) = e^{-0.3125t}(24\cos 0.95t - 7.9 \sin 0.95t)}$$

$$i(t) = e^{-0.312s}\left(3\cos 0.95t - 0.99\sin 0.95t\right)$$

4. In the circuit of Fig. 3, find $i(t)$ for $t > 0$.

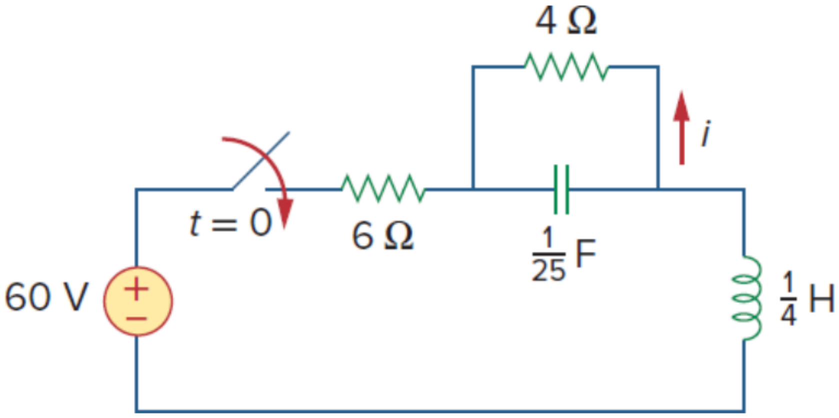
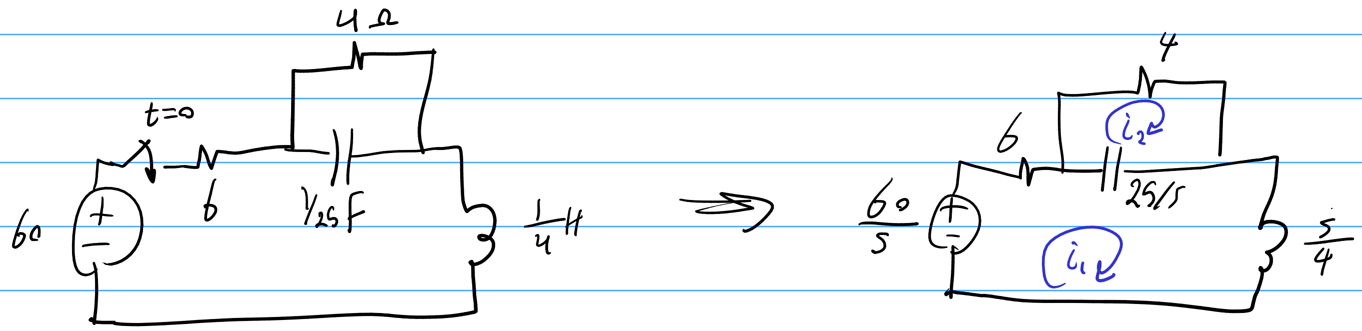


Figure 3



$$I_1\left(6 + \frac{25}{s} + \frac{s}{4}\right) + I_2\left(-\frac{25}{s}\right) = \frac{60}{s}$$

$$I_1\left(s^2 + 24s + 100\right) + I_2(-25) = 240 \quad (1)$$

$$I_1\left(-\frac{25}{s}\right) + I_2\left(\frac{25}{s} + 4\right) = 0$$

$$I_1(-25) + I_2(4s + 25) = 0 \quad (2) \Rightarrow I_1 = I_2\frac{4s + 25}{25}$$

$$I_2(0.16s + 1)(s^2 + 24s + 100) + I_2(-25) = 240$$

$$I_2\left[(0.16s + 1)(s^2 + 24s + 100) - 25\right] = 240$$

$$I_2\left(0.16s^3 + 3.84s^2 + 16s + s^2 + 24s + 100 - 25\right) = 240$$

$$I_2\left(0.16s^3 + 4.84s^2 + 40s + 75\right) = 240$$

$$I_2\left(s^3 + 30.25s^2 + 250s + 468.75\right) = 1500$$

$$I_2 = \frac{1500}{s^3 + 30.25 s^2 + 250 s + 468.75}$$

$$= \frac{1500}{(s+10.1136)(s+17.4858)(s+2.65064)}$$

$$= \frac{13.5484}{s+2.65064} + \frac{13.7153}{s+17.4858} - \frac{27.2638}{s+10.1136}$$

by partial fraction decomposition (approximately)

$$i_2(t) = 13.55 e^{-2.65t} + 13.72 e^{-17.49t} - 27.26 e^{-10.11t}$$