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# Thermodynamics, Wave Motion and Optics

Assignment 4

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1. For a compound lens system of two thin lenses,

   (a) Show that the image distance, $S_i$ takes the following form:

   $$S_i = \frac{f_2 d - [f_1 f_2 S_0/(S_0 - f_1)]}{d - f_2 - [f_1 S_0/(S_0 - f_1)]},$$

   where $S_0$, $f_1$ and $f_2$ are the object distance and the focal lengths of the system

   *Solution.*

   $$\frac{1}{S_1} + \frac{1}{S_0} = \frac{1}{f_1} \implies S_1 = \frac{S_0 f_1}{S_0 - f_1} \tag{1}$$

   $$\frac{1}{S_2} + \frac{1}{S_3} = \frac{1}{f_2} \tag{2}$$

   From figure $S_2 = d - S_1$ and $S_3 = S_i$:

   $$\frac{1}{d - S_1} + \frac{1}{S_i} = \frac{1}{f_2} \implies S_i = \frac{(d - S_1) f_2}{(d - S_1) - f_2} \tag{3}$$

   $$\tag{4}$$

   $$\begin{aligned} S_i &= \frac{(d - \frac{S_0 f_1}{S_0 - f_1}) f_2}{(d - \frac{S_0 f_1}{S_0 - f_1}) - f_2} \qquad (5) \\ &= \frac{f_2 d - \frac{f_1 f_2 S_0}{S_0 - f_1}}{d - f_2 - \frac{S_0 f_1}{S_0 - f_1}}. \qquad (6) \end{aligned}$$

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2. Show that the total magnification of the system takes the explicit form:

   $$M_T = \frac{f_2 S_i}{d(S_0 - f_1) - S_0 f_1},$$

   *Solution.*

   $$\begin{aligned} M_1 &= -\frac{S_1}{S_0} \qquad (7) \\ M_2 &= -\frac{S_3}{S_2} \qquad (8) \\ M_T &= M_1 M_2 \qquad (9) \\ &= \frac{S_1 S_3}{S_0 S_2}. \qquad (10) \end{aligned}$$

Using $S_2 = d - S_1$, $S_3 = S_i$, and formulas from previous part:

$$
\begin{aligned}
M_T &= \frac{S_1 S_i}{S_0(d - S_1)} && (11)\\
&= \frac{\frac{S_0 f_1}{S_0 - f_1} S_i}{S_0(d - \frac{S_0 f_1}{S_0 - f_1})} && (12)\\
&= \frac{S_0 f_1 S_i}{(S_0 - f_1) S_0 (d - \frac{S_0 f_1}{S_0 - f_1})} && (13)\\
&= \frac{S_0 f_1 S_i}{(S_0 - f_1)(S_0 d - S_0 \frac{S_0 f_1}{S_0 - f_1})} && (14)\\
&= \frac{S_0 f_1 S_i}{(S_0 - f_1) S_0 d - S_0 S_0 f_1} && (15)\\
&= \frac{f_1 S_i}{(S_0 - f_1) d - S_0 f_1}. && (16)
\end{aligned}
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3. Show that for a thin lens immersed in a medium of refractive index nm, the Lensmaker equation takes the following form.

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

where $n_l$ is the index of refraction of glass.

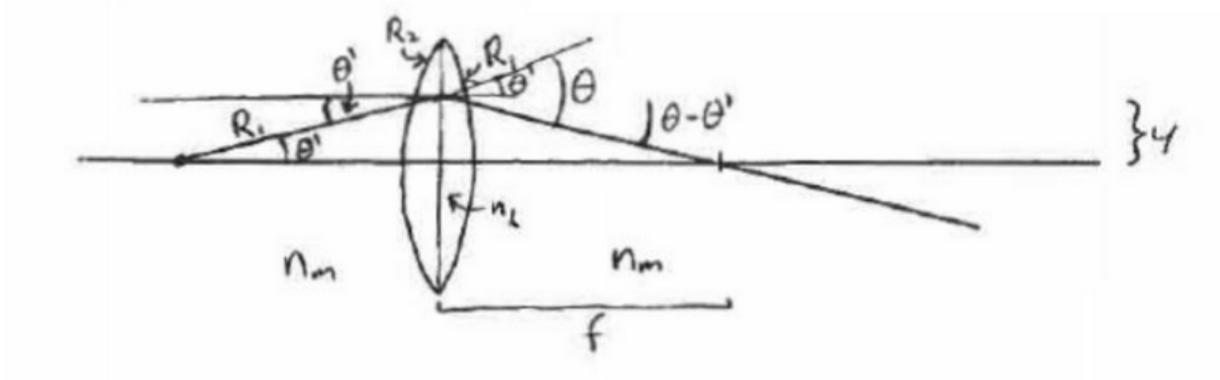


Figure 1

*Solution.* For $\theta \ll 1$

$$\frac{y}{R_1} = \sin\theta' \approx \theta' \tag{17}$$

$$n_l \sin\theta' = n_m \sin\theta \implies n_l\theta' = n_m\theta \tag{18}$$

$$\theta - \theta' = \left(\frac{n_l}{n_m} - 1\right)\theta' = \left(\frac{n_l}{n_m} - 1\right)\frac{y}{R_1} \tag{19}$$

$$f = \frac{y}{\tan(\theta-\theta')} \approx \frac{y}{\theta-\theta'} = \frac{R_1}{\frac{n_l}{n_m} - 1} = \frac{n_m R_1}{n_l - n_m} \tag{20}$$

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\frac{1}{R_1} \tag{21}$$

The other lens will add linearly

$$\frac{1}{f} = \frac{n_l - n_m}{n_m}\left(\frac{1}{R_1} + \frac{1}{R_2}\right). \tag{22}$$

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4. The passenger side rear view mirror of a car has a sign "Objects in the mirror are closer than they appear". Can we conclude if the mirror is convex or concave? If an object located 100 m away appears 125 m away, what is the radius of the mirror?

   *Solution.* Concave, since it forms a virtual large image for objects between focus and mirror, so for mirrors with large focal lengths it will preform the desired effect.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{23}$$
$$f = \frac{ss'}{s+s'} \tag{24}$$
$$= \frac{-100 * 125}{100 - 125} \tag{25}$$
$$= 500. \tag{26}$$

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5. Imagine a stratified system consisting of $m$ planar layers of transparent materials of different thickness. Show that the propagation direction of the emerging beam is determined by only the incident direction and the refractive indices of the initial and final layers, i.e., $n_1$ and $n_m$

   *Solution.*

$$n_1 \sin\theta_{i1} = n_2 \sin\theta_{t2} \tag{27}$$
$$n_1 \sin\theta_{i2} = n_2 \sin\theta_{t3} \tag{28}$$
$$\vdots \tag{29}$$
$$n_\ell \sin\theta_{i\ell} = n_m \sin\theta_m \tag{30}$$

   Notice $\theta_{t2} = \theta_{i2}, \theta_{t3} = \theta_{i3}$

$$n_1 \sin\theta_{i1} = n_2 \sin\theta_{i2} = \ldots = n_\ell \sin\theta_{i\ell} = n_m \sin\theta_m. \tag{31}$$

   Therefore $n_1 \sin\theta_{i1} = n_m \sin\theta_{tm}$ ■

6. A small fish, four feet below the surface of Lake Mendota is viewed through a simple thin converging lens with focal length 30 feet. H the lens is 2 feet above the water surface (as shown in Fig.), where is the image of the fish seen by the observer? Assume the fish lies on the optical axis of the lens and that $n_{\text{air}} = 1.00$, $n_{\text{water}} = 1.33$.

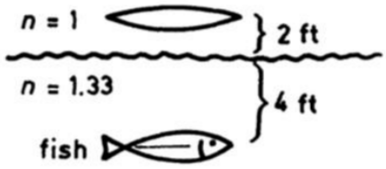


Figure 2

*Solution.*

$$1.33 \sin\theta_1 = \sin\theta_2 \tag{32}$$

For $\theta \ll 1$

$$\begin{aligned}
1.33\theta_1 &= \theta_2 && (33)\\
1.33\frac{y}{R_1} &= \frac{y}{R_2} && (34)\\
1.33\frac{1}{R_1} &= \frac{1}{R_2} && (35)\\
R_2 &= \frac{R_1}{1.33} && (36)\\
&= 3 && (37)\\
u &= 2 + R_2 && (38)\\
&= 5 && (39)\\
\frac{1}{f} &= \frac{1}{u} + \frac{1}{v} && (40)\\
v &= -6. && (41)
\end{aligned}$$

The image of the fish will appear as is. ■

7. It is said that: "In normal use, the magnitude of the magnification of an imaging system increases as its equivalent focal length is increased." Find an expression for the magnification in terms of the focal length that demonstrates the truth or falsehood of this statement. State any conditions that must be satisfied in the mathematical expression (e.g., why does the sentence include the caveat "in normal use"?).

   *Solution.*

$$M_T = -\frac{s'}{s} \tag{42}$$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \tag{43}$$

$$s' = \frac{sf}{s-f} \tag{44}$$

$$M_T = -\frac{\left(\frac{sf}{s-f}\right)}{s} \tag{45}$$

$$= \frac{f}{f-s} \tag{46}$$

$$= -\frac{f}{s}\left(\frac{1}{1-\frac{f}{s}}\right) \tag{47}$$

   For $f < s$ we can use $\frac{1}{1-t} = \sum_{n=0}^{\infty} t^n$ where $t < 1$

$$M_T = -\sum_{n=1}^{\infty}\left(\frac{f}{s}\right)^n \tag{48}$$

   For $f \ll s$

$$M_T \approx -\frac{f}{s} - \frac{f^2}{s^2}. \tag{49}$$

   Therefore $M_T \propto f, f^2$ for $f \ll s$. ∎

8. As shown in the figure below, a thin converging lens of focal length 14 cm forms an image of the square $abcd$, which is $hc = hb = 10$ cm high and lies between distances of $pd = 20$ cm and $pa = 30$ cm from the lens.
Let $a', b', c'$, and $d'$ represent the respective corners of the image. Let $qa$ represent the image distance for points $a'$ and $b'$, $qd$ represent the image distance for points $c'$ and $d'$, $h'b$ represents the distance from point $b'$ to the axis, and $h'c$ represent the height of $c'$.

(a) Evaluate each of the quantities written in bold.

*Solution.*

$$\frac{1}{p_a} + \frac{1}{q_a} = \frac{1}{f} \tag{50}$$

$$\frac{1}{30} + \frac{1}{q_1} = \frac{1}{14} \tag{51}$$

$$q_1 = 26.2 \tag{52}$$

$$h'_b = hM_a = h\left(-\frac{q_a}{p_a}\right) \tag{53}$$

$$= (10.0\text{ cm})(-0.875) = -8.75\text{ cm} \tag{54}$$

$$\frac{1}{20} + \frac{1}{q_d} = \frac{1}{14} q_d \qquad = 46.7\text{ cm} \tag{55}$$

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(b) Make a sketch of the image.

*Solution.* ■

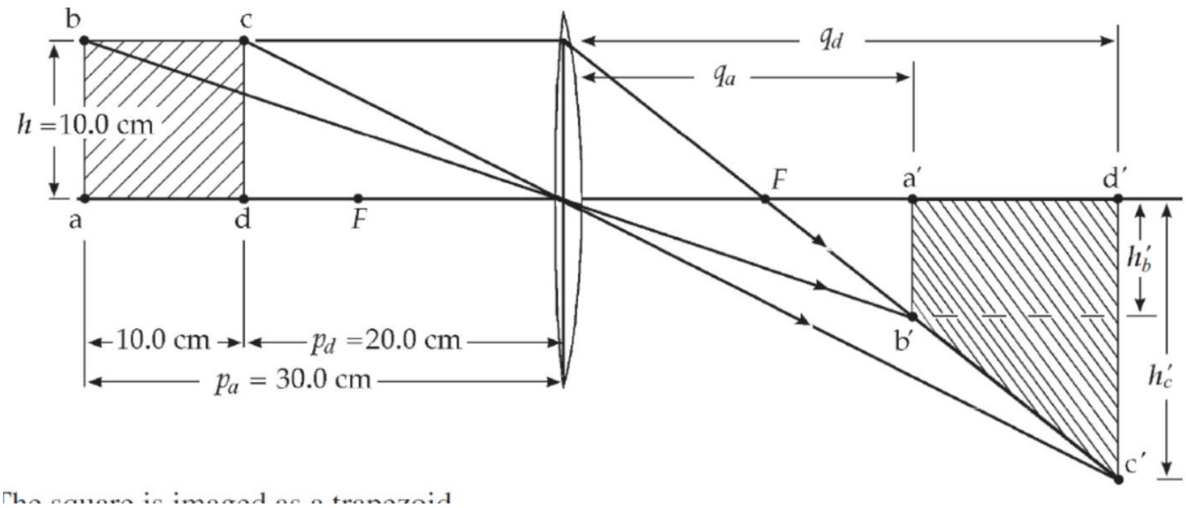


Figure 3

(c) Calculate the area of the image using the results of (a) and (b).

*Solution.*

$$\frac{(h'_c + h'_b)(qd - qa)}{2} = 238\text{ cm}^2. \tag{56}$$

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(d) Let $q$ represent the image distance of any point between $a'$ and $d'$, for which the object distance is $p$. Let $h'$ represent the distance from the axis to the point at the edge of the image between $b'$ and $c'$ at image distance $q$. Show that

$$|h'| = (10 \text{ cm})q\left(\frac{1}{14 \text{ cm}} - \frac{1}{q}\right).$$

*Solution.*

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \tag{57}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{14} \tag{58}$$

$$|h'| = |hM| = |h\left(-\frac{q}{p}\right) \tag{59}$$

$$= (10.0 \text{ cm})q\left(\frac{1}{14 \text{ cm}} - \frac{1}{q}\right). \tag{60}$$

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(e) Explain why the geometric area of the image can be given by

$$\int_{q_a}^{q_d} |h^*|dq.$$

*Solution.* Because integration gives the area under the region. ■

(f) Calculate the area of the image using integration above.

*Solution.*

$$\int_{q_a}^{q_d} |h'|\, dq = \int_{q_a}^{q_d} (10.0 \text{ cm})\left(\frac{q}{14 \text{ cm}} - 1\right) dq = (10.0 \text{ cm})\left(\frac{q^2}{28 \text{ cm}} - q\right)_{26.2 \text{ cm}}^{46.7 \text{ cm}} \tag{61}$$

$$\text{Area } = (10.0 \text{ cm})\left(\frac{46.7^2 - 26.2^2}{28} - 46.7 + 26.2\right) \text{cm} = 328 \text{ cm}^2. \tag{62}$$

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9. A thin equi-convex lens of glass of refractive index ng= 3/2 and of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water nw = 4/3. On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens as shown in the figure below. The separation between the lens and the mirror is 0.8 m. A small object is placed at a distance of 0.9 m from the lens as shown in the figure.

   Find the position (relative to the lens) of the image of the object formed by the system.

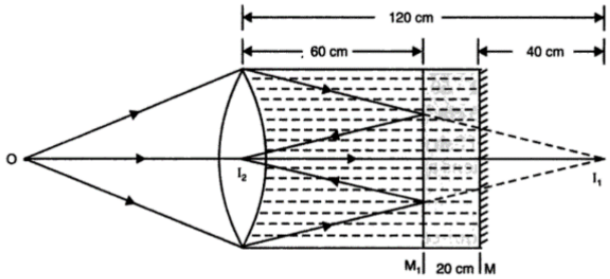


Figure 4

*Solution.*

$$\frac{n1}{s} + \frac{n2}{s'} = \frac{n2 - n1}{R} \tag{63}$$

$$\frac{3/2}{s'} + \frac{1}{0.9} = \frac{0.5}{0.3} \tag{64}$$

$$s' = 2.7 \text{ m} \tag{65}$$

$$\frac{\frac{4}{3}}{s''} - \frac{\frac{3}{2}}{2.7} = \frac{\frac{4}{3} - \frac{3}{2}}{-0.3} \tag{66}$$

$$\frac{4}{3s''} = \frac{1}{1.8} + \frac{1}{1.8} = \frac{1}{0.9} \tag{67}$$

$$s'' = 1.2 \text{ m} \tag{68}$$

$$x = \left(1 - \frac{1}{n_{\text{water}}}\right) \tag{69}$$

$$x_0 = \left(1 - \frac{3}{4}\right) 0.8. \tag{70}$$

$$= 0.2 \text{ m}. \tag{71}$$

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