Wave Equation

frequency

$$\nabla^2 E + \omega^2 \mu \epsilon E = 0 \Longrightarrow E_x(z) = E_0^+ e^{-j\overset{2\pi/\lambda}{k} z} + E_0^- e^{jkz}$$

$$P_{av} = \frac{1}{2} \operatorname{Re}\{E \times H\} = \frac{1}{2} E^2 , \quad P_{rad} = \int P_{av} \cdot dS$$

$$k = \omega\sqrt{\mu_0 \epsilon_0} \Rightarrow \boxed{\omega^2 = \frac{k^2}{\mu \epsilon}}$$

$$\omega / k = C \Rightarrow \boxed{f = ck / 2\pi}$$

Propagation in $+\hat{x}$, $E_z(x=0) = 5\cos(10^9 \pi t)$

$$E_z(x) = 5\cos(\omega t - kx)$$

3×10^8

$$k = \omega c = \frac{10\pi}{3}$$

$$E_z(x=20)?? \Rightarrow E_z(20) = 5\cos\left(10^9 \pi t - \frac{200}{3}\pi\right)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & E_z \end{vmatrix} = \langle -\partial_t B_x , -\partial_t B_y , -\partial_t B_z \rangle$$

$$\partial_y E_z = -\partial_t B_x \Rightarrow B_x = -\int \partial_y E_z \, dt$$

$$\eta^2 = \frac{\mu}{\varepsilon}, \quad \eta^2 = \frac{E^2}{H^2} \qquad r, \theta, \varphi$$

$$E = \hat{\theta}\,\frac{12\pi}{r}\,e^{-j2\pi r}\sin\theta \Rightarrow H = \hat{\varphi}\,\frac{12}{12\pi r}\,e^{-j2\pi r}\sin\theta$$

$$P_{av} = \frac{1}{2\eta}E^2 = \frac{1}{2\eta}\cdot\left(\frac{12\pi}{r}\right)^2 \sin^2\theta$$

$$\left|e^{-j2\pi r}\right|^2 = \sin^2(\cancel{-2\pi r}) + \cos^2(\cancel{-2\pi r}) = 1$$

$$P = \int_0^{\pi}\int_0^{2\pi} P_{av}\, r^2 \sin\theta \, d\varphi\, d\theta$$

$\sigma = 4$

$f = 100\,Hz$	$f = 100\,MHz$	$f = 10\,GHz$
$\varepsilon_r = 80$	$\varepsilon_r = 32$	$\varepsilon_r = 24$
$\frac{\sigma}{\omega\varepsilon} = 9\times10^6$	$\frac{\sigma}{\omega\varepsilon} = 22.\sim$	$\frac{\sigma}{\omega\varepsilon} = 0.3$
good conductor	less lossy (semi-conductor)	dielectric and conductor

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$$H = \hat{x} H_0 e^{j\beta y} + \hat{z} H_1 e^{j\beta y}, \quad \vec{S} = \vec{E} \times \vec{H}$$

General Law: $\hat{q}$ - direction $\Leftrightarrow \cos(\omega t - \beta q)$

$-\hat{y}$ - $\sim$ $\Leftrightarrow \cos(\omega t + \beta q)$

$$\vec{E} \times \vec{H} = -\hat{y} \Rightarrow \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ & 0 & \\ & & \end{vmatrix}$$