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Date: October 26, 2023

Thermodynamics, Wave Motion and Optics (PHYS 201)
Study Assignment 2

Problem (I)

Imagine you are part of a rescue team on board a submarine that has lost control of its propulsion and steering. The submarine is drifting at an unknown speed, v_{sub}, towards an underwater canyon wall. The sonar system sends out a series of sound waves with a frequency of f_0. The speed of sound in water is represented by c. These sound waves bounce off the canyon wall and return to the submarine after a certain time interval, Δt. When the reflected signal combines with the original signal, it creates a pattern of beats with a frequency of f_{beat}.

(a) What is the speed of the submarine?

$$\begin{aligned} f_{beat} &= \Delta f && (1)\\ &= \left| f' - f_0 \right|. && (2)\end{aligned}$$

$$\begin{aligned} f' &= f_0 \frac{c \pm v_o}{c \pm v_s} && (3)\\ v_o &= v_s = v_{sub} && (4)\\ f' &= f_0 \frac{c + v_{sub}}{c - v_{sub}}. && (5)\end{aligned}$$

$$\begin{aligned} f_{beat} &= f_0 \frac{c + v_{sub}}{c - v_{sub}} - f_0 && (6)\\ f_{beat} + f_0 &= f_0 \frac{c + v_{sub}}{c - v_{sub}} && (7)\\ \frac{f_{beat} + f_0}{f_0} &= \frac{c + v_{sub}}{c - v_{sub}} && (8)\\ \frac{f_{beat} + f_0}{f_0} (c - v_{sub}) &= c + v_{sub} && (9)\\ \frac{f_{beat} + f_0}{f_0} c - \frac{f_{beat} + f_0}{f_0} v_{sub} &= c + v_{sub} && (10)\\ \frac{f_{beat} + f_0}{f_0} c - c &= \frac{f_{beat} + f_0}{f_0} v_{sub} + v_{sub} && (11)\\ \frac{f_{beat} + f_0}{f_0} c - c &= \left(\frac{f_{beat} + f_0}{f_0} + 1 \right) v_{sub} && (12)\\ \frac{f_{beat} + f_0}{f_0} c - c &= \left(\frac{f_{beat} + f_0 + f_0}{f_0} \right) v_{sub} && (13)\\ \frac{f_{beat} + f_0}{f_0} c - c &= \left(\frac{f_{beat} + 2f_0}{f_0} \right) v_{sub} && (14)\\ \left(\frac{f_{beat} + f_0}{f_0} c - c \right) \left(\frac{f_0}{f_{beat} + 2f_0} \right) &= v_{sub}. && (15)\end{aligned}$$

$$
\begin{align}
v_{sub} &= \left(\frac{f_{beat}+f_0}{f_0}c - c\right)\left(\frac{f_0}{f_{beat}+2f_0}\right) \tag{16}\\
&= \frac{f_{beat}+f_0}{f_0}\left(\frac{f_0}{f_{beat}+2f_0}\right)c - \left(\frac{f_0}{f_{beat}+2f_0}\right)c \tag{17}\\
&= \frac{f_{beat}+f_0}{f_{beat}+2f_0}c - \left(\frac{f_0}{f_{beat}+2f_0}\right)c \tag{18}\\
&= c\left(\frac{f_{beat}+f_0}{f_{beat}+2f_0} - \frac{f_0}{f_{beat}+2f_0}\right) \tag{19}\\
&= c\left(\frac{f_{beat}+f_0-f_0}{f_{beat}+2f_0}\right) \tag{20}\\
&= c\left(\frac{f_{beat}}{f_{beat}+2f_0}\right). \tag{21}
\end{align}
$$

$$\boxed{v_{sub} = \frac{cf_{beat}}{f_{beat}+2f_0}.}$$

(b) When is the collision with the canyon wall expected if the submarine maintains its current trajectory?

$$t_{\text{collision}} = \frac{d}{v_{sub}}. \tag{22}$$

$$
\begin{align}
d + d - \Delta t v_{sub} &= \Delta t c \tag{23}\\
2d - \Delta t v_{sub} &= \Delta t c. \tag{24}
\end{align}
$$

$$
\begin{align}
2d &= \Delta t v_{sub} + \Delta t c \tag{25}\\
d &= \frac{\Delta t v_{sub} + \Delta t c}{2} \tag{26}\\
d &= \frac{\Delta t}{2}\left(v_{sub}+c\right) \tag{27}\\
\frac{d}{v_{sub}} &= \frac{\Delta t}{2}\left(1+\frac{c}{v_{sub}}\right) \tag{28}\\
t_{\text{collision}} &= \frac{\Delta t}{2}\left(1+\frac{c}{v_{sub}}\right) \tag{29}\\
&= \frac{\Delta t}{2}\left(1+c\frac{f_{beat}+2f_0}{cf_{beat}}\right) \tag{30}\\
&= \frac{\Delta t}{2}\left(1+\frac{f_{beat}+2f_0}{f_{beat}}\right) \tag{31}\\
&= \frac{\Delta t}{2}\left(\frac{f_{beat}+2f_0+f_{beat}}{f_{beat}}\right) \tag{32}\\
&= \frac{\Delta t}{2}\left(\frac{2\left(f_{beat}+f_0\right)}{f_{beat}}\right) \tag{33}\\
&= \Delta t\left(\frac{f_{beat}+f_0}{f_{beat}}\right). \tag{34}
\end{align}
$$

$$\boxed{t_{\text{collision}} = \Delta t\left(\frac{f_{beat}+f_0}{f_{beat}}\right).}$$

Problem (II)

During the building of the Admin and Culture Complex in ZC, a deep well is formed in the con- struction area. A Phy201 student wanted to use a frequency generator of adjustable frequency to measure the depth of the well. If the student reported hearing two successive resonances at 51.87 Hz and 59.85 Hz:

$$v = 346 \text{ m/s} \quad f_n = \frac{(2n-1)v}{4L} \quad f_n = 51.87 \text{ Hz} \quad f_{n+1} = 59.85 \text{ Hz}. \tag{35}$$

$$L = \frac{nv}{4f_n} \tag{36}$$

$$\frac{(2n+1)v}{4f_{n+1}} = \frac{(2n-1)v}{4f_n} \tag{37}$$

$$\frac{2n+1}{f_{n+1}} = \frac{2n-1}{f_n} \tag{38}$$

$$\frac{2n+1}{59.85} = \frac{2n-1}{51.87} \tag{39}$$

$$51.87(2n+1) = 59.85(2n-1) \tag{40}$$

$$103.74n + 51.87 = 119.7n - 59.85 \tag{41}$$

$$15.96n = 111.72 \tag{42}$$

$$n = 7. \tag{43}$$

(a) How deep is the well?

$$L = \frac{(2n-1)v}{4f_n} \tag{44}$$

$$L = \frac{(2 \times 7 - 1) \times 346}{4 \times 51.87} = 21.68 \text{ m} \tag{45}$$

$$= \frac{(2 \times 8 - 1) \times 346}{4 \times 59.85} = 21.68 \text{ m}. \tag{46}$$

$$\boxed{L = 21.68 \text{ m.}}$$

(b) How many antinodes are in the standing wave at 51.87 Hz?

$$\boxed{n = 7.}$$

Problem (III)

In designing the sound system in one of ZC parties, suppose you put two speakers in a configuration as shown below. In your testing, the two speakers were emitting equal amplitude waves with a frequency of 343 Hz. If speaker II is half of a cycle ahead of speaker I, and you are initially standing 1.25 m from speaker I and 2.75 m from speaker II on a line connecting the two sources.

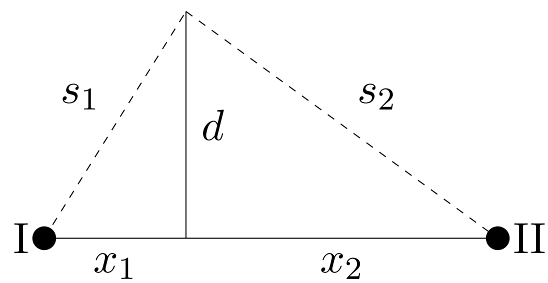


Figure 1

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda} \quad \phi_0 = \pi \quad v = 343 \text{ m/s} \quad f = 343 \text{ Hz} \quad x_1 = 1.25 \text{ m} \quad x_2 = 2.75 \text{ m}. \tag{47}$$

(a) What is the phase difference at your initial position? Does this correspond to maximum constructive, maximum destructive, or other interference?

$$\begin{aligned}
\Delta\phi &= 2\pi\frac{\Delta x}{\lambda} - \phi_0 && (48)\\
&= 2\pi\frac{x_2 - x_1}{\lambda} - \phi_0 && (49)\\
&= 2\pi\frac{2.75 - 1.25}{\lambda} - \pi && (50)\\
&= 2\pi\frac{1.5}{\lambda} - \pi && (51)\\
&= 2\pi\frac{1.5}{\frac{v}{f}} - \pi && (52)\\
&= 2\pi\frac{1.5}{\frac{343}{343}} - \pi && (53)\\
&= 2\pi\frac{1.5}{1} - \pi && (54)\\
&= 2\pi \times 1.5 - \pi && (55)\\
&= 3\pi - \pi && (56)\\
&= 2\pi. && (57)
\end{aligned}$$

The phase difference is 2π, which corresponds to maximum constructive interference. □

(b) What is the shortest distance d that you must walk along a line perpendicular to the line connecting the two speakers in order to hear no sound?

$$\begin{aligned} \Delta\phi - \phi_0 &= \pi && (58)\\ 2\pi\Delta x - \pi &= \pi && (59)\\ 2\pi\Delta x &= 2\pi && (60)\\ \Delta x &= 1. && (61) \end{aligned}$$

$$\begin{aligned} \Delta x &= s_2 - s_1 && (62)\\ &= \sqrt{x_2^2 + d^2} - \sqrt{x_1^2 + d^2}. && (63) \end{aligned}$$

$$\begin{aligned} \sqrt{x_2^2 + d^2} - \sqrt{x_1^2 + d^2} &= 1 && (64)\\ \sqrt{2.75^2 + d^2} - \sqrt{1.25^2 + d^2} &= 1 && (65)\\ \sqrt{2.75^2 + d^2} &= 1 + \sqrt{1.25^2 + d^2} && (66)\\ &. && (67) \end{aligned}$$

$$\begin{aligned} 2.75^2 + d^2 &= 1 + 2\sqrt{1.25^2 + d^2} + 1.25^2 + d^2 && (68)\\ 2.75^2 - 1.25^2 &= 2\sqrt{1.25^2 + d^2} && (69)\\ \frac{2.75^2 - 1.25^2}{2} &= \sqrt{1.25^2 + d^2} && (70)\\ \left(\frac{2.75^2 - 1.25^2}{2}\right)^2 &= 1.25^2 + d^2 && (71)\\ \left(\frac{2.75^2 - 1.25^2}{2}\right)^2 - 1.25^2 &= d^2 && (72)\\ \sqrt{\left(\frac{2.75^2 - 1.25^2}{2}\right)^2 - 1.25^2} &= d && (73)\\ \frac{5\sqrt{3}}{4} &= d. && (74) \end{aligned}$$

$$\boxed{d = \frac{5\sqrt{3}}{4} \approx 2.17 \text{ m.}}$$

Problem (IV)

After one of the final exams, while you are enjoying an organ recital at Cairo Opera, the air compressor that drives the organ pipes suddenly fails. This is sad, but as a ZC nerdy in the audience you embarked to help by replacing the compressor with a tank of a pressurized tank of nitrogen gas.

$$v \propto \frac{1}{\sqrt{M}} \quad f \propto \frac{1}{v} \implies f \propto \sqrt{M} \tag{75}$$

(a) What effect, will the nitrogen gas have on the frequency output of the organ pipes?

$$M_{\mathrm{N}} = 28 \text{ g/mol} \quad M_{\mathrm{air}} = 29 \text{ g/mol}. \tag{76}$$

Frequency will decrease by a factor of $\sqrt{28/29} \approx 0.98$. □

(b) When you are back to the dorms, your ZC roommate nerd told you that helium would have been a better option. Investigate his clam.

$$M_{\mathrm{He}} = 4 \text{ g/mol}. \tag{77}$$

Frequency will decrease by a factor of $\sqrt{4/29} \approx 0.37$. His claim is incorrect. □

Problem (V)

Trains A and B are approaching each other on parallel tracks. Train A's locomotive continuously sounds a whistle. A passenger in train B perceives a frequency of 307 Hz as they approach train A, a frequency of 256 Hz when they are precisely parallel to train A's locomotive, and a frequency of 213 Hz as the trains move away from each other. If the speed of sound in the air is 340 m/s, what are the speeds of both trains?

$$f' = f\frac{v \pm v_B}{v \pm v_A} \quad v = 340 \text{ m/s} \quad f_0 = 307 \text{ Hz} \quad f_1 = 256 \text{ Hz} \quad f_2 = 213 \text{ Hz}. \tag{78}$$

$$f_0 = f\frac{v + v_B}{v - v_A} \tag{79}$$
$$f_1 = f \tag{80}$$
$$f_2 = f\frac{v - v_B}{v + v_A} \tag{81}$$
$$. \tag{82}$$

$$f_0 = 256\frac{340 + v_B}{340 - v_A} \tag{83}$$
$$307 = 256\frac{340 + v_B}{340 - v_A} \tag{84}$$
$$\frac{307}{256} = \frac{340 + v_B}{340 - v_A} \tag{85}$$
$$307(340 - v_A) = 256(340 + v_B) \tag{86}$$
$$104380 - 307v_A = 87040 + 256v_B \tag{87}$$
$$17340 = 256v_B + 307v_A. \tag{88}$$

$$f_2 = 256\frac{340 - v_B}{340 + v_A} \tag{89}$$
$$213 = 256\frac{340 - v_B}{340 + v_A} \tag{90}$$
$$\frac{213}{256} = \frac{340 - v_B}{340 + v_A} \tag{91}$$
$$213(340 + v_A) = 256(340 - v_B) \tag{92}$$
$$72420 + 213v_A = 87040 - 256v_B \tag{93}$$
$$213v_A + 256v_B = 14620. \tag{94}$$

$$307v_A + 256v_B = 17340 \tag{95}$$
$$213v_A + 256v_B = 14620. \tag{96}$$

$$\begin{bmatrix} 307 & 256 \\ 213 & 256 \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 17340 \\ 14620 \end{bmatrix}. \tag{97}$$

$$\begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 307 & 256 \\ 213 & 256 \end{bmatrix}^{-1} \begin{bmatrix} 17340 \\ 14620 \end{bmatrix}. \tag{98}$$

$$\begin{bmatrix} v_A \\ v_B \end{bmatrix} \approx \begin{bmatrix} 29.0 \\ 33.0 \end{bmatrix}. \tag{99}$$

$$\boxed{v_A \approx 29.0 \text{ m/s} \quad v_B \approx 33.0 \text{ m/s}.}$$

Problem (VI)

Two speakers, A and B, each with a power output of 100.0 watts, are positioned at a separation distance of $D = 3.60$ meters. These speakers produce sound waves at a frequency of $f = 10,000.0$ Hz in phase. Point P_1 is situated at coordinates $x_1 = 4.50$ meters and $y_1 = 0$ meters, while point P_2 is positioned at $x_2 = 4.50$ meters with a vertical offset of $-\Delta y$.

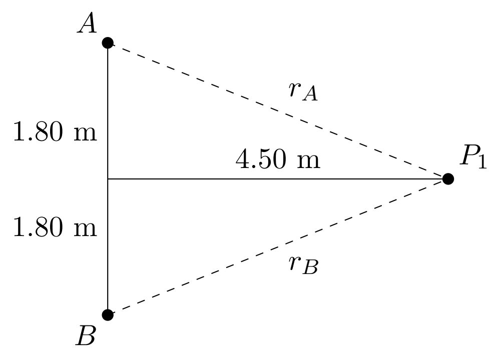


Figure 2

(a) Neglecting speaker B, what is the intensity, I_{A_1} (in W/m^2), of the sound at point P_1 due to speaker A? Assume that the sound from the speaker is emitted uniformly in all directions.

$$I = \frac{P}{4\pi r^2}. \tag{100}$$

$$\begin{aligned} r_A &= \sqrt{4.5^2 + 1.8^2} &(101)\\ &= \sqrt{23.5} &(102)\\ &\approx 4.85. &(103)\end{aligned}$$

$$\begin{aligned} I_A &= \frac{P_A}{4\pi r_A^2} &(104)\\ &= \frac{100}{4\pi(4.85)^2} &(105)\\ &\approx .338. &(106)\end{aligned}$$

$$\boxed{I_{A_1} \approx 0.338 \text{ W/m}^2.}$$

(b) What is this intensity in terms of decibels (sound level, β_{A_1})?

$$\beta = 10\log_{10}\frac{I}{I_0}. \tag{107}$$

$$\begin{aligned} \beta_{A_1} &= 10\log_{10}\frac{I_{A_1}}{I_0} &(108)\\ &= 10\log_{10}\frac{0.338}{10^{-12}} &(109)\\ &\approx 115. &(110)\end{aligned}$$

$$\boxed{\beta_{A_1} \approx 115 \text{ dB.}}$$

(c) When both speakers are turned on, there is a maximum in their combined intensities at P_1. As one moves toward P_2, this intensity reaches a single minimum and then becomes maximized again at P_2. How far is P_2 from P_1; that is, what is Δy? Solve for the case where $L \gg \Delta y$

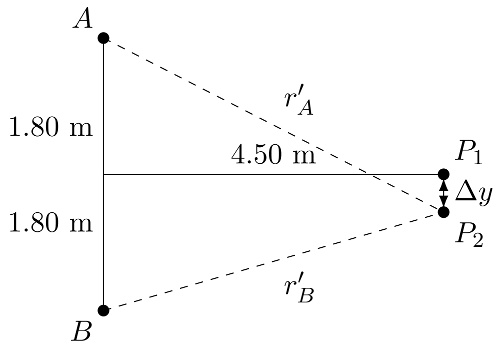


Figure 3

$$\Delta d = \frac{n\lambda}{2} \quad f = 10,000.0 \text{ Hz} \quad v = 343 \text{ m/s} \quad \lambda = \frac{v}{f} = \frac{343}{10,000.0} = 0.0343. \tag{111}$$

$$\begin{aligned} \Delta r &= \frac{n \times 0.0343}{2} && (112) \\ 0 &= \frac{0 \times 0.0343}{2} && (113) \\ \Delta d &= \frac{1 \times 0.0343}{2} && (114) \\ &= 0.01715. && (115) \end{aligned}$$

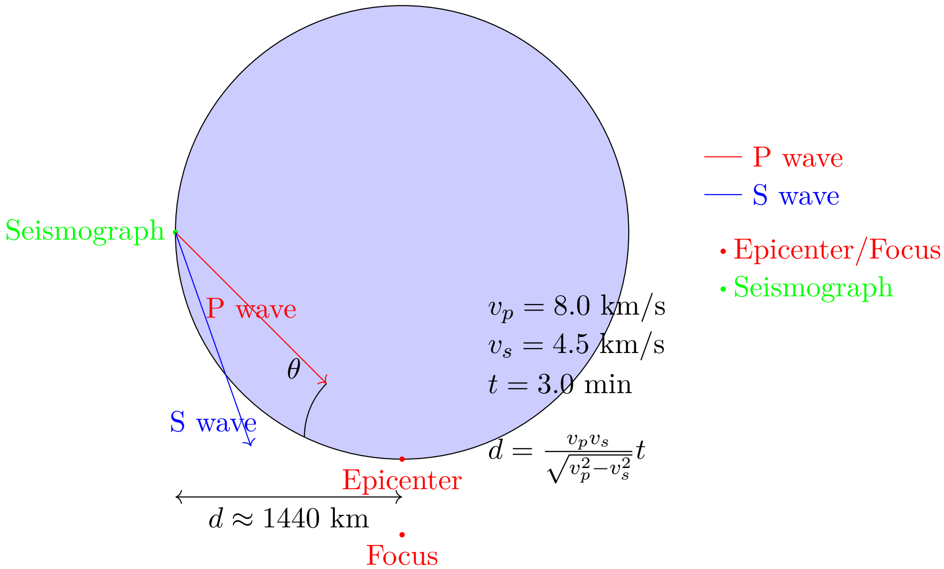
$$\begin{aligned} \Delta d &= |r'_A - r'_B| && (116) \\ 0.01715 &= \sqrt{(4.5)^2 + (1.8 + \Delta y)^2} - \sqrt{(4.5)^2 + (1.8 - \Delta y)^2}. && (117) \end{aligned}$$

$$\Delta y \approx 0.0230. \tag{118}$$

$$\boxed{\Delta y \approx 0.0230 \text{ m.}}$$

Problem (VII)

Earthquakes produce seismic waves within the Earth. Unlike gases, Earth can support both transverse (S) and longitudinal (P) seismic waves. Normally, S waves travel at a speed of approximately 4.5 km/s, while P waves travel at 8.0 km/s. A seismograph detects and records both P and S waves when an earthquake occurs. Interestingly, the first P waves are detected 3.0 minutes before the initial arrival of S waves. Assuming that these waves travel in a straight line, what is the distance to the earthquake's epicenter?



$$\Delta t = 3.0 \text{ min} = 180 \text{ sec} \quad v_P = 8.0 \text{ km/s} \quad v_S = 4.5 \text{ km/s}. \tag{119}$$

$$\begin{aligned} \Delta t &= |t_P - t_S| & (120)\\ &= \frac{d}{v_P} - \frac{d}{v_S} & (121)\\ &= d\left(\frac{1}{v_P} - \frac{1}{v_S}\right) & (122)\\ &= d\left(\frac{v_S - v_P}{v_P v_S}\right). & (123) \end{aligned}$$

$$\begin{aligned} d &= \Delta t \frac{v_S v_P}{v_P - v_S} & (124)\\ &= 180 \frac{4.5 \times 8.0}{8.0 - 4.5} & (125)\\ &\approx 1.9 \times 10^3. & (126) \end{aligned}$$

$$\boxed{d \approx 1.9 \times 10^3 \text{ km.}}$$