

Coulomb's Law: $\mathbf{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\mathbf{r}|^2} \hat{\mathbf{r}}$

Electric Field: $\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{|\mathbf{r}|^2} \hat{\mathbf{r}}$

Gauss's Law: $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enc}}{\epsilon_0}$

Electric Potential: $V = - \int \mathbf{E} \cdot d\mathbf{l}$

E and V: $\mathbf{E} = -\nabla V$

Laplace's Equation: $\nabla^2 V = 0$

Poisson's Equation: $\nabla^2 V = -\frac{\rho_v}{\epsilon}$

Work Done: $W = -Q \int_a^b \mathbf{E} \cdot d\mathbf{l}$

V and W: $V_{ba} = \frac{W_{ba}}{Q}$

Electric Displacement: $\mathbf{D} = \epsilon \mathbf{E}$

Current Density: $\mathbf{J} = \sigma \mathbf{E}$

Continuity Eq: $\nabla \cdot \mathbf{J} = -\partial_t \rho_v$

Displacement Current Density: $\mathbf{J}_d = \partial_t \mathbf{D}$

Time-Harmonic Wave: $\mathbf{E}(z, t) = \text{Re}\{\mathbf{E}_0 e^{j(\omega t - \beta z)}\}$

Exponentials Form: $\mathbf{E}(z) = E_0^+ e^{-jkz} + E_0^- e^{jkz}$

For H: $\mathbf{H}(z) = \frac{k}{\omega\mu} E^+(z)$

Wavenumber: $k = \omega\sqrt{\mu\epsilon} = \frac{\omega}{v}$

Intrinsic Impedance: $\eta = \frac{|E|}{|H|} = \sqrt{\frac{\mu}{\epsilon}}$

MEs Differential Form: $\nabla \cdot \mathbf{D} = \rho_v, \nabla \cdot \mathbf{B} = 0, \nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \nabla \times \mathbf{H} = \mathbf{J} + \partial_t \mathbf{D}$

MEs Integral Form: $\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{enc}, \oint_S \mathbf{B} \cdot d\mathbf{s} = 0, \oint_C \mathbf{E} \cdot d\mathbf{l} = -\partial_t \oint_S \mathbf{B} \cdot d\mathbf{s}, \oint_C \mathbf{H} \cdot d\mathbf{l} = \oint_S \mathbf{J} \cdot d\mathbf{s} + \partial_t \oint_S \mathbf{D} \cdot d\mathbf{s}$

General Interface Conditions: $E_{1t} = E_{2t}, D_{1n} - D_{2n} = \rho_s; H_{1t} - H_{2t} = J_s^*, B_{1n} = B_{2n}$

* Dielectric-Dielectric $\Rightarrow \rho_s = J_s = 0$. Dielectric-Conductor $\Rightarrow E_t = B_n = 0, D_{1n} = \rho_s, H_{1t} = J_s^*$.

$$\int \frac{1}{(a^2 \pm x^2)^{3/2}} dx = \frac{x}{a^2 \sqrt{a^2 \pm x^2}}, \int \frac{xdx}{(a^2 + x^2)^{3/2}} = \frac{-1}{\sqrt{a^2 + x^2}}, \int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right), \int \frac{x^2 dx}{(a^2 + x^2)^{3/2}} = \frac{-x}{\sqrt{a^2 + x^2}} + \ln(x + \sqrt{a^2 + x^2})$$

Cylindrical Coordinates: **Spherical Coordinates:**

$$\nabla f = \partial_r r \hat{\mathbf{r}} + \frac{1}{r} \partial_\theta \hat{\theta} + \partial_z \hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{F} = \frac{1}{r} \partial_r (r F_r) + \frac{1}{r} \partial_\theta F_\theta + \partial_z F_z$$

$$\nabla \times \mathbf{F} = \frac{1}{r} \begin{vmatrix} \hat{\mathbf{r}} & r \hat{\theta} & \hat{\mathbf{z}} \\ \partial_r & \partial_\theta & \partial_z \\ F_r & r F_\theta & F_z \end{vmatrix}$$

$$\nabla^2 f = \frac{1}{r} \partial_r (r \partial_r f) + \frac{1}{r^2} \partial_\theta^2 f + \partial_z^2 f$$

$$dV = r dr d\phi dz$$

$$d\mathbf{l} = dr \hat{\mathbf{r}} + r d\phi \hat{\theta} + dz \hat{\mathbf{z}}$$

$$d\mathbf{S}_r = rd\phi dz \hat{\mathbf{r}}, d\mathbf{S}_\phi = dr dz \hat{\theta}$$

$$d\mathbf{S}_z = rd\phi dr \hat{\mathbf{z}}$$

Chapter 12

time-harmonic

solve eq

sol 1D: $E_x(t) = E_0^+ e^{-jkz} + E_0^- e^{jkz}$

for H: $H_y(t) = \frac{j}{\omega\mu} E_x(t)$

Permittivity: $\epsilon = \epsilon_r \epsilon_0$

Ex Pwr: $P_{ex} = \frac{1}{2} \int_V (E_x^2 + H_y^2) dV$

$\epsilon_r = \epsilon_0 / \epsilon$

in free space tot = ∞

in complex domain $\rightarrow P_{av} = \frac{1}{2} \operatorname{Re} \{ E_x H_y^* \}$

Lossy materials: $\epsilon = \epsilon_0 (1 + j\omega\tau)$

time domain $E(t) = E_0 e^{-\alpha z} \cos(\omega t - Bz)$

low loss approx: $\alpha = \frac{\sigma}{2} \sqrt{\epsilon} = \frac{\sigma}{2} \gamma$

conductor

high-loss approx: $\alpha = \frac{\sigma}{2} \sqrt{\epsilon} = \frac{\sigma}{2} \gamma$

$\epsilon = \epsilon_0 (1 + j\omega\tau)$

skin-depth / penetration depth ~ distance till \rightarrow wave amp = $(\frac{1}{e}) / (\text{wave amp})$

Current: $I = \frac{dQ}{dt}$

Total Current: $I = \iint_S \mathbf{J} \cdot d\mathbf{A}$

Ohm's Law: $V = IR$

Capacitance: $C = \frac{Q}{V} = \iint_S \frac{1}{\epsilon \epsilon_0} dL \cdot dS$

Parallel Plate Capacitor: $C = \frac{\epsilon A}{d}$

Current in Capacitor: $I = C \frac{dV}{dt}$

Capacitor Energy: $U = \frac{1}{2} CV^2$

Capacitor Electric Field: $E = \frac{V}{d}$

Work: $W = QV = \frac{CV^2}{2} = \frac{QV}{2} = \frac{Q^2}{2C}$

Work Density: $W_E = \frac{P_v V}{2} = \frac{D^2}{2\epsilon}$

Biot-Savart Law: $\mathbf{B} = \frac{\mu_0}{4\pi} \int \frac{Idl \times \hat{\mathbf{r}}}{|r|^2}$

Ampere's Law: $\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc}$

Magnetic Flux Density: $\mathbf{B} = \mu \mathbf{H}$

Magnetic Flux: $\Phi_B = \iint_S \mathbf{B} \cdot d\mathbf{S}$

Faraday's Law: $\mathcal{E} = -N \partial_t \Phi_B$

Lorentz force: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

Wave Equation: $\nabla^2 \mathbf{E} - \mu\epsilon \partial_t^2 \mathbf{E} = 0$

Wave Speed: $v = \sqrt{\frac{1}{\mu\epsilon}}$

Normal Reflection: $\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$

Transmission Coeff: $T = 1 - \Gamma$

Impedance: $Z = \sqrt{\frac{\mu}{\epsilon}}$

Snell's Law: $\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}$

Poynting Vector: $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

Time Derivative-Complex Domain: $\partial_t \equiv j\omega$

Wave Equation: $\nabla^2 \mathbf{E} = \nabla \left(\frac{\partial \mathbf{E}}{\partial t} \right) + \partial_t \mu (\mathbf{J}_s + \sigma \mathbf{E} + \partial_t \epsilon \mathbf{E})$

Linear Polarization: $E(z, t) = E_y e^{-j\omega t} \cos(\omega t + \beta z + \phi)$

Elliptical Polarization: $E(z, t) = \hat{\mathbf{x}} E_1 \cos(\omega t - \beta z) + \hat{\mathbf{y}} E_2 \cos(\omega t - \beta z + \phi)$

Wave Power: $P = \oint_A \mathbf{S} \cdot d\mathbf{A} = -\partial_t \int_V \frac{\mu H^2}{2} + \frac{\epsilon E^2}{2} dV - \int_V \mathbf{E} \cdot \mathbf{J} dV$

Vector Theorems:

$$\iiint_V (\nabla \cdot \mathbf{F}) dV = \iint_S \mathbf{F} \cdot \hat{\mathbf{n}} dA$$

$$\iint_S (\nabla \times \mathbf{F}) \cdot \hat{\mathbf{n}} dA = \oint_C \mathbf{F} \cdot d\mathbf{r}$$

$$\oint_C P dx + Q dy = \iint_S \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} dx dy$$

Vector Identities:

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0, \nabla \times (\nabla f) = 0$$

$$\nabla \cdot (\nabla f) = \nabla^2 f$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla (\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$$

$$\nabla (UQ) = U(\nabla Q) + Q(\nabla U)$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = -\mathbf{A} \cdot (\nabla \times \mathbf{B}) + (\nabla \times \mathbf{A}) \cdot \mathbf{B}$$

$$\nabla \times (U \mathbf{A}) = U(\nabla \times \mathbf{A}) + (\nabla U) \times \mathbf{A}$$

$\sigma = \text{conductivity}, \circ = \text{lossless}$
if source free $\rightarrow \nabla^2 \mathbf{E} = \omega^2 \epsilon \mathbf{E} = 0$

GREEK LETTERS
 $\lambda = \frac{2\pi}{\omega}$

$\text{lossless: } V_p = \frac{1}{\sqrt{\mu\epsilon}}; K = \omega \sqrt{\epsilon\mu}$
 $\lambda = \frac{2\pi}{\omega} = \frac{2\pi V_p}{\sqrt{\mu\epsilon}}$

$\lambda = \frac{\omega}{\sqrt{\mu\epsilon}} = \frac{K}{V_p}$ for pwr ... if S.F.

$\text{lossy: } \epsilon_c = \frac{\sigma + j\omega\tau}{j\omega}; \gamma = \sqrt{\mu\epsilon}(\sigma + j\omega\tau)$

$\lambda_c = \frac{1}{\sqrt{\mu\epsilon_c}} = \frac{1}{\sqrt{\mu(\sigma + j\omega\tau)}} = \sqrt{\frac{\sigma}{\mu} + \frac{\omega^2}{\mu}}$

$\lambda = \frac{j\omega\tau}{\gamma} = \sqrt{\mu\epsilon(\sigma + j\omega\tau)} = \sqrt{\sigma + j\omega\tau}$

$V_p = \frac{\omega}{B}; \lambda = \frac{2\pi}{B}$

high loss $\rightarrow \delta = \frac{1}{\lambda} \sqrt{\frac{\mu}{\epsilon}}$

$P = E \times H$

$\delta = \frac{1}{\lambda} \sqrt{\frac{\mu}{\epsilon}}$

Chapter 13

reflected transmitted +2 propagation z

$$\text{normal incidence} \rightarrow \theta_i = \theta_r = \theta_t = 0 \Rightarrow E_i = \hat{x} E_{ii} e^{-j k z}, E_r = \hat{x} E_{ri} e^{+j k z}, H_i = \hat{y} (E_{ii} e^{-j k z} - E_{ri} e^{+j k z}) \frac{1}{\eta_1}$$

reflection coeff $\Gamma = \frac{E_r}{E_{ii}}$; transmission coeff $T = \frac{E_t}{E_{ii}}$

$$E_i + E_r = E_{ii} \Rightarrow \Gamma + T = 1$$

$$H_i + H_r = H_{ii} \Rightarrow (E_{ii} - \Gamma E_{ii}) \frac{1}{\eta_1} = (T E_{ii}) \frac{1}{\eta_1} \Rightarrow \frac{1 - \Gamma}{\eta_1} = \frac{T}{\eta_1} \Rightarrow T = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\text{general case } E_{ii} = \hat{x} E_{ii} (T e^{-j k z} - \Gamma j 2 \sin(j k z)) \quad E_{ri} = \hat{x} T E_{ii} e^{-j k z} \quad H_{ii} = \hat{y} \frac{E_{ii}}{\eta_1} (T e^{-j k z} - j 2 \cos(j k z)) \quad H_{ri} = \hat{y} T \frac{E_{ii}}{\eta_1} e^{-j k z}$$

normal incidence w/ conductor $\Rightarrow T=0 \& \Gamma=-1 \Rightarrow E_i(z) = \hat{x} j 2 E_{ii} \sin(B_i z), H_i(z) = \hat{y} 2 \frac{E_{ii}}{\eta_1} \cos(B_i z) \sim P_{av} = \frac{1}{2} \operatorname{Re} [E_i(z) \times H_i(z)^*]$

$$E_i(z,t) = -\hat{x} 2 E_{ii} \sin(B_i z) \sin(\omega t) \quad // \quad E_{ri}(z) = \hat{x} T E_{ii} e^{-j k z} \quad H_{ri}(z) = \hat{y} T \frac{E_{ii}}{\eta_1} e^{-j k z}$$

$$\Gamma = \frac{1 + j - \sigma_2 \delta_2 \eta_2}{1 + j + \sigma_2 \delta_2 \eta_2}; T = \frac{2(1+j)}{\sigma_2 \delta_2 \eta_2 + (1+j)}; \alpha = B = \sqrt{\pi f \mu_0 \sigma}; \delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}$$

oblique incidence perp polariz. $\rightarrow E_i(x,z) = \hat{y} E_{ii} e^{-j B_i (x \sin(\theta_i) \pm z \cos(\theta_i))}$ make -ve for H_r & E_r

$$H_i(x,z) = \frac{E_{ii}}{\eta_1} (-\hat{x} \cos(\theta_i) + \hat{z} \sin(\theta_i)) e^{-j B_i (x \sin(\theta_i) \pm z \cos(\theta_i))} \rightarrow \frac{\omega \sqrt{\epsilon_1 \mu_1} \sin(\theta_i)}{(\sin(\theta_i)/\sin(\theta_i))} = \frac{\eta_1}{\eta_2} = \sqrt{\mu_2/\mu_1}$$

Parallel polarization $\rightarrow E_r(x,z) = E_{ii} (\hat{x} \cos(\theta_i) - \hat{z} \sin(\theta_i)) e^{-j B_i (x \sin(\theta_i) + z \cos(\theta_i))} \quad H_r(x,z) = \hat{y} \frac{E_{ii}}{\eta_1} e^{-j B_i (x \sin(\theta_i) + z \cos(\theta_i))}$

$$E_{ii}(x,z) = E_{ii} (\hat{x} \cos(\theta_i) - \hat{y} \sin(\theta_i)) e^{-j B_i (x \sin(\theta_i) + z \cos(\theta_i))} \quad H_{ii}(x,z) = \hat{y} (\epsilon_i^* / \eta_2) e^{-j B_i (x \sin(\theta_i) + z \cos(\theta_i))}$$

$$\Gamma_{||} = -\frac{E_{ri}}{E_{ii}} = \frac{\eta_2 \cos(\theta_i) - \eta_1 \sin(\theta_i)}{\eta_2 \cos(\theta_i) + \eta_1 \sin(\theta_i)} \quad T_{||} = \frac{E_{ri}}{E_{ii}} = \frac{2 \eta_2 \cos(\theta_i)}{\eta_2 \cos(\theta_i) + \eta_1 \sin(\theta_i)} \rightarrow E_{ii} \cos(\theta_i) - E_{ri} \cos(\theta_i) = E_{ri} \cos(\theta_i)$$

Brewster's angle (zero polarization, perfect transmission parallel & ref = 0) $\rightarrow \eta_1 \cos(\theta_i) = \eta_2 \cos(\theta_i) \Rightarrow \sin(\theta_b) = \sqrt{\frac{\epsilon_2 (\mu_2 \epsilon_1 - \mu_1 \epsilon_2)}{\mu_1 (\epsilon_1^2 - \epsilon_2^2)}} \quad \eta_1 \cos(\theta_i) = \eta_1 \sin(\theta_t) \rightarrow \sin(\theta_b) = \sqrt{\frac{\mu_2 (\epsilon_1 \mu_2 - \epsilon_2 \mu_1)}{\epsilon_1 (\mu_1^2 - \mu_2^2)}}$

total int. ref. $\Rightarrow \Gamma = -1 \quad T = 1 \quad \text{at } \theta_t = 90^\circ \Rightarrow \sin(\theta_t) = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}} \sin(\theta_i)$