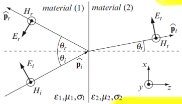
Chapter 13 refle.

reflected, transmitted, +z propagation

normal incidence $\rightarrow \theta_i = \theta_r = \theta_t = 0 \Rightarrow E_i = \hat{x} E_{i1} e^{-\gamma_1 z}$, $E_r = \hat{x} E_{r1} e^{+\gamma_1 z}$, $\gamma_1 = \alpha_1 + j\beta_1$ & $H_1 = \hat{y}\left(E_{i1} e^{-\gamma_1 z} - E_{r1} e^{+\gamma_1 z}\right)\frac{1}{\eta_1}$



reflection coeff $\rightarrow \Gamma = \frac{E_r}{E_{i1}}$; $E_r = \hat{x}\Gamma E_{i1} e^{+\gamma_1 z}$

transmission coeff $\rightarrow T = \frac{E_t}{E_{i1}}$; $E_t = \hat{x} T E_{i1} e^{-\gamma_2 z}$

$\rightarrow E_i + E_r = E_2 \equiv E_{i1} + \Gamma E_{i1} = T E_{i1}$

$H_i + H_r = H_2 \equiv (E_{i1} - \Gamma E_{i1})\frac{1}{\eta_1} = (T E_{i1})\frac{1}{\eta_2}$

$\rightarrow 1 + \Gamma = T$, $\frac{1}{\eta_1} - \frac{\Gamma}{\eta_1} = \frac{T}{\eta_2}$ $\rightarrow \Gamma = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$, $T = \frac{2\eta_2}{\eta_1 + \eta_2}$

gen. case $\rightarrow E_1 z = \hat{x} E_{i1}\left(Te^{-\gamma_1 z} - \Gamma j 2\sin(j\gamma_1 z)\right)$; $E_2 z = \hat{x} T E_{i1} e^{-\gamma_2 z}$ & $H_1(z) = \hat{y}\frac{E_{i1}}{\eta_1}\left(Te^{-\gamma_1 z} - \Gamma 2\cos(j\gamma_1 z)\right)$; $H_2(z) = \hat{y} T \frac{E_{i1}}{\eta_2} e^{-\gamma_2 z}$

normal incidence w/ conductor $\Rightarrow T = 0$ & $\Gamma = -1 \Rightarrow E_1(z) = \hat{x} j 2E_{i1}\sin(\beta_1 z)$, $H_1(z) = \hat{y} 2\frac{E_{i1}}{\eta_1}\cos(\beta_1 z)$ — $P_{av_1} = \frac{1}{2}\mathrm{Re}\{\vec{E}_1(z) \times \vec{H}_1(z)^*\}$

$E_1(z,t) = -\hat{x}\, 2E_{i1}\sin(\beta_1 z)\sin(\omega t)$ /// $E_2(z) = \hat{x} T E_{i1} e^{-\gamma_2 z}$ $H_2(z) = \hat{y} T\left(\frac{1}{\eta_0}\right) E_{i1} e^{-\gamma_2 z}$

$\Gamma = \frac{1 + j - \sigma_2 \delta_2 \eta_2}{1 + j + \sigma_2 \delta_2 \eta_2}$; $T = \frac{2(1+j)}{\sigma_2 \delta_2 \eta_2 + (1+j)}$; $\alpha = \beta = \sqrt{\pi f \mu \sigma}$ $\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}$

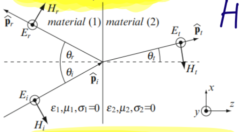
oblique Incidence perp. polariz. $\rightarrow E_i(x,z) = \hat{y} E_{i1} e^{-j\beta_1(x\sin(\theta_i) \pm z\cos(\theta_i))}$

make -ve for H_r & E_r

$H_i(x,z) = \frac{E_{i1}}{\eta_1}\left(\underline{-}\hat{x}\cos\theta_i + \hat{z}\sin\theta_i\right) e^{-j\beta_1(x\sin(\theta_i) \pm z\cos(\theta_i))}$

$\rightarrow \omega\sqrt{\varepsilon_1\mu_1}\,\sin\theta_i = \omega\sqrt{\varepsilon_2\mu_2}\,\sin\theta_t$

$(\sin\theta_t / \sin\theta_i) = n_1 / n_2 = V_{p2}/V_{p1}$



$\Gamma_\perp = \frac{E_r}{E_{i1}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$ $T_\perp = \frac{E_{t2}}{E_{i1}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$ $\rightarrow$ **Perpendicular polarization**

$E_i(x,z) = E_{i1}(\hat{x}\cos\theta_i - \hat{z}\sin\theta_i) e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}$ & $H_i(x,z) = \hat{y}\frac{E_{i1}}{\eta_1} e^{-j\beta(x\sin\theta_i + z\cos\theta_i)}$

Parallel polarization $\rightarrow E_r(x,z) = E_{i1}(-\hat{x}\cos\theta_i - z\sin\theta_i) e^{-j\beta(x\sin\theta_i - z\cos\theta_i)}$ & $H_r(x,z) = -\hat{y}(E_{i1}/\eta_1) e^{-j\beta(x\sin\theta_i - z\cos\theta_i)}$

$\hookrightarrow E_t(x,z) = E_{i1}(\hat{x}\cos(\theta_t) - \hat{y}\sin(\theta_t)) e^{-j\beta(x\sin\theta_t + z\cos\theta_t)}$ & $H_t(x,z) = \hat{y}(E_t/\eta_2) e^{-j\beta(x\sin\theta_t + z\cos\theta_t)}$

$\Gamma_{\parallel} = -\frac{E_{r1}}{E_{i1}} = \frac{\eta_2\cos\theta_t - \eta_1\sin\theta_i}{\eta_2\cos\theta_t + \eta_1\sin\theta_i}$ $T_{\parallel} = \frac{E_{t2}}{E_{i1}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$ $\rightarrow E_{i1}\cos(\theta_i) - E_{r1}\cos(\theta_i) = E_{t2}\cos(\theta_t)$

$E_{i1}/\eta_1 + E_{r1}/\eta_1 = E_{t2}/\eta_2$

Brewster's angle (zero polarization, perfect transmission & ref = 0)

parallel $\parallel$: $\eta_1\cos(\theta_t) = \eta_2\cos(\theta_i) \rightarrow \sin(\theta_b) = \sqrt{\frac{\varepsilon_2(\mu_2\varepsilon_1 - \mu_1\varepsilon_2)}{\mu_1(\varepsilon_1^2 - \varepsilon_2^2)}}$

perpendicular $\perp$: $\eta_2\cos\theta_i = \eta_1\sin\theta_t \rightarrow \sin(\theta_b) = \sqrt{\frac{\mu_2(\varepsilon_1\mu_2 - \varepsilon_2\mu_1)}{\varepsilon_1(\mu_2^2 - \mu_1^2)}}$

total int ref. $\Rightarrow \Gamma = -1$ $T = 1$ at $\theta_t = 90 \Rightarrow \sin\theta_t = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}\,\sin\theta_i$