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Date: January 22, 2024

Thermodynamics, Wave Motion and Optics

Assignment 10

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1. The Figure below shows a hypothetical speed distribution for a sample of N gas particles.

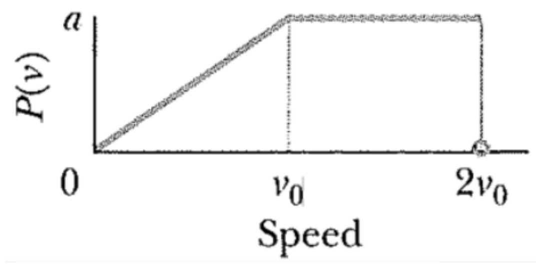

Figure 1

Let $P(v) = 0$ for speed $v > 2v_o$. What are the values of

(a) $a * v_o$

Solution.

$$\int P(v)dv = 1 \tag{1}$$

$$= \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0 \tag{2}$$

$$\frac{3}{2}av_0 = 1 \tag{3}$$

$$av_0 = \frac{2}{3} \tag{4}$$

■

(b) v_{avg}/v_o

Solution.

$$v_{\text{avg}} = \int vP(v)dv \tag{5}$$

For the triangular portion $P(v) = \frac{av}{v_0}$:

$$v_{\text{avg\_tri}} = \frac{a}{v_0}\int_0^{v_0} v^2 dv \tag{6}$$

$$= \frac{a}{3v_0}v_0^3 \tag{7}$$

$$= \frac{av_0^2}{3} \tag{8}$$

Use $av_0 = \frac{2}{3}$:

$$= \frac{2}{9}v_0 \tag{9}$$

For the rectangular portion $P(v) = av$:

$$\begin{aligned} v_{\text{avg\_rec}} &= a\int_{v_0}^{2v_0} v dv &(10)\\ &= \frac{a}{2}\left(4v_0^2 - v_0^2\right) &(11)\\ &= \frac{3a}{2}v_0 2 = v_0 &(12)\end{aligned}$$

Substituting:

$$\begin{aligned} v_{\text{avg}} &= \frac{2}{9}v_0 + v_0 = \frac{11}{9}v_0 &(13)\\ \frac{v_{\text{avg}}}{v_0} &= \frac{11}{9} &(14)\end{aligned}$$

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(c) v_{rms}/v_o

Solution.

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\int v^2 P(v) dv} &(15)\\ &= \sqrt{\frac{a}{v_0}\int_0^{v_0} v^3 dv + a\int_{v_0}^{2v_0} v^2 dv} &(16)\\ &= \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} &(17)\\ &= \sqrt{\frac{31}{18}v_0^2} &(18)\\ &= \frac{\sqrt{62}}{6}v_0 &(19)\\ \frac{v_{\text{rms}}}{v_0} &= \frac{\sqrt{62}}{6} &(20)\end{aligned}$$

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(d) What fraction of the particles has a speed between $1.5v_o$ and $2.0v_o$?

Solution.

$$N\int_{1.5v_0}^{2v_0} P(v)\, dv \tag{21}$$

Using $P(v) = a$ in the range:

$$Na(2.0v_0 - 1.5v_0) = 0.5Nav_0 = \frac{N}{3} \tag{22}$$

The fraction of particles in this range is $\frac{1}{3}$. ■

2. Consider a system of one particle with 128 possible discrete locations in space. Assuming that all locations are equally probable:

   (a) Determine the lack of information in bits

   Solution.

$$S = -\Sigma p_i \log_2 p_i \tag{23}$$

   Using $p_i = \frac{1}{128}$

$$S = -\Sigma \frac{1}{128} \log_2 \left(\frac{1}{128}\right) = 7 \tag{24}$$

$$127 = 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 \tag{25}$$

   Therefore if all digits are zeros, the particle resides in the state 128. ■

   (b) What if you know that 64 of which are forbidden states for the particle?

   Solution.

$$S = -\Sigma \frac{1}{64} \log_2 \left(\frac{1}{64}\right) = 6 \tag{26}$$

$$63 = 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 \tag{27}$$

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3. An isolated Thermos contains 130 g of water at 80 °C. You put in a 12 g ice cube at 0 °C to form a system of ice + original water ($L_f = 333 \times 10^3$ J/Kg, c = 4190 J/(Kg.C))

   (a) What is the equilibrium temperature of the system?

   Solution.

$$\Sigma Q = 0 \implies L_f m + cm(T_f - 0) + cm'(T_f - 80) = 0 \tag{28}$$

   Substituting values:

$$T_f = 339.67 \text{ K} \tag{29}$$

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   (b) What are the entropy changes of the water that was originally the ice cube as it melts? and

   Solution.

$$\frac{Q}{T} = \frac{L_f m}{273.15} = 14.6 \text{ J/K} \tag{30}$$

■

   (c) as it warms to the equilibrium temperature?

   Solution.

$$\int_{273.15}^{339.67} \frac{cm}{T}\, dT = cm \ln\left(\frac{339.67}{273.15}\right) = 11.0 \text{ J/K} \tag{31}$$

■

(d) What is the entropy change of the original water as it cools to the equilibrium temperature?

Solution.

$$\int_{353.15}^{339.67} \frac{cm'}{T} dT = cm' \ln\left(\frac{339.67}{353.15}\right) = -21.2 \text{ J/K} \tag{32}$$

■

(e) What is the net entropy change of the ice + original water system as it reaches the equilibrium temperature?

Solution.

$$\Delta S_{\text{net}} = 14.6 + 11.0 - 21.2 = 4.39 \text{ J/K} \tag{33}$$

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4. A multi-cylinder gasoline engine in an airplane, operating at 2500 rev/min, takes in energy 7.89×10^3 J and exhausts 4.58×10^3 J for each revolution of the crankshaft.

(a) How many liters of fuel does it consume in 1.0 h of operation if the heat of combustion is 4.03×10^7 J/L?

Solution.

$$7.89 \times 10^3 \cdot 2500 \times \frac{60}{1} = 1.18 \times 10^9 \text{ J/h} \tag{34}$$

$$\frac{1.18 \times 10^9}{4.03 \times 10^7} = 29.4 \text{ L/h.} \tag{35}$$

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(b) What is the mechanical power output of the engine? Ignore friction.

Solution.

$$W_{\text{eng}} = Q_h - Q_c \tag{36}$$

$$\frac{W_{\text{eng}}}{dt} = \frac{Q_h}{dt} - \frac{Q_c}{dt} \tag{37}$$

$$= \left(7.89 \times 10^3 - 4.58 \times 10^3\right) \cdot \frac{2500}{60} \tag{38}$$

$$= 1.38 \times 10^5 \text{ W.} \tag{39}$$

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(c) What power must the exhaust and cooling system transfer out of the engine?

Solution.

$$\frac{Q_c}{dt} = 4.58 \times 10^3 \cdot \frac{2500}{60} = 1.91 \times 10^5 \text{ W.} \tag{40}$$

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(d) Bonus: What is the torque exerted by the crankshaft on the load?

Solution.

$$\tau = \frac{W_{\text{eng}}}{\omega} = \frac{1.38 \times 10^5}{2500/60} \cdot \frac{1}{2\pi} = 527 \text{ N.m.} \tag{41}$$

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5. Suppose a heat engine working with a vdW gas with a known constant b and a temperature-independent heat capacity c_V. If the gas undergoes a cycle that consists of two isochors (V_1 and V_2) and two adiabats processes. Deduce the efficiency of the heat engine.

Solution.

$$\begin{aligned}
e &= 1 - \frac{\Delta Q_c}{\Delta Q_h} && (42)\\
&= 1 - \frac{T_B - T_C}{T_A - T_D} && (43)\\
S_{vdW} &= \frac{f}{2} N k_B \ln T + N k_B \ln(V - Nb) + const && (44)\\
S_{vdW} &= \frac{f}{2} N k_B \ln\left(\frac{T_f}{T_i}\right) + N k_B \ln\left(\frac{V_f - Nb}{V_i - Nb}\right) && (45)
\end{aligned}$$

For adiabatic process:

$$0 = \frac{f}{2} N k_B \ln\left(\frac{T_f}{T_i}\right) + N k_B \ln\left(\frac{V_f - Nb}{V_i - Nb}\right) \iff T^{\frac{f}{2}}(V - Nb) = conste \quad = 1 - \left(\frac{V_1 - Nb}{V_2 - Nb}\right)^{\frac{R}{c_V}}. \tag{46}$$

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6. For a Carnot cycle where the working substance is a Van der Waals gas and the cycle processes are as shown in Figure 1-2 and 3-4 are isotherms, 2-3 and 4-1 – adiabats. The temperatures of the hot and cold reservoirs are T_H and TC, respectively.

   (a) Draw the cycle.

   Solution. ■

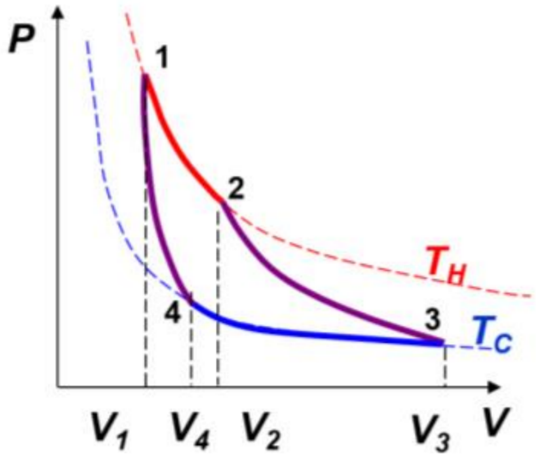

Figure 2

   (b) Deduce the efficiency of this heat engine.

   Solution.

$$e = 1 - \frac{\Delta Q_c}{\Delta Q_h} \tag{47}$$

1-2 and 3-4 isothermal expansion

$$\Delta U = \Delta Q + \Delta W \tag{48}$$

$$\Delta U_{vdW}|_T = N^2 a(\frac{1}{v_i} - \frac{1}{V_f}) \tag{49}$$

$$\Delta W = -\int_{V_i}^{V_f} (\frac{Nk_B T}{V - Nb} - \frac{N^2 a}{V^2})\, dV \tag{50}$$

$$= -Nk_b T \ln\left(\frac{V_f - Nb}{V_i - Nb}\right) + N^2 a(\frac{1}{V_i} - \frac{1}{V_f}) \tag{51}$$

$$\Delta Q_{1-2} = Nk_B T_H \ln\left(\frac{V_2 - Nb}{V_1 - Nb}\right) \tag{52}$$

$$\Delta Q_{3-4} = Nk_B T_H \ln\left(\frac{V_2 - Nb}{V_1 - Nb}\right) \tag{53}$$

$$e = 1 - \frac{T_C \ln\left(\frac{V_3 - Nb}{V_4 - Nb}\right)}{T_H \ln\left(\frac{V_2 - Nb}{V_1 - Nb}\right)} \tag{54}$$

2-3 and 4-1 adiabatic and quasistatic

$$\Delta Q = 0 \tag{55}$$
$$\Delta U = \Delta W \tag{56}$$
$$S_{vdW} = \frac{f}{2} N k_B \ln T + N k_B \ln(V - Nb) + const \tag{57}$$
$$S_{vdW} = \frac{f}{2} N k_B \ln\left(\frac{T_f}{T_i}\right) + N k_B \ln\left(\frac{V_f - Nb}{V_i - Nb}\right) \tag{58}$$

For adiabatic process:

$$0 = \frac{f}{2} N k_B \ln\left(\frac{T_f}{T_i}\right) + N k_B \ln\left(\frac{V_f - Nb}{V_i - Nb}\right) \tag{59}$$
$$T^{\frac{f}{2}}(V - Nb) = const \tag{60}$$
$$V_4 - Nb = (V_1 - Nb)\left(\frac{T_H}{T_C}\right)^{\frac{f}{2}} \tag{61}$$
$$V_3 - Nb = (V_2 - Nb)\left(\frac{T_H}{T_C}\right)^{\frac{f}{2}} \tag{62}$$
$$e = 1 - \frac{T_C}{T_H}. \tag{63}$$

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(c) Compare the result in (b) with that for the Carnot cycle with an ideal gas.

Solution. The same as of the cycle of an ideal gas. ■

7. Suppose you are given 1 kg of water at temperature 1000C and a block of ice at temperature 00C. If a reversible heat engine absorbs heat from the water and expels heat to the ice until that point at which work can no longer be extracted from the system. Given that the heat capacity of water is 4.2 J/g · K and the heat of fusion of ice is 333 J/g. When the process is complete:

   (a) Calculate the temperature of the water.

   Solution.

$$e = 1 - \frac{T_{\text{ice}}}{T_{\text{water}}} = 0 \tag{64}$$

$$T_{\text{water}} = T_{\text{ice}} = 0^\circ \text{ C}. \tag{65}$$

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   (b) Calculate the heat absorbed by the block of ice in the process.

   Solution.

$$e = 1 - \frac{\Delta Q_C}{\Delta Q_H} \tag{66}$$

$$Q_C = \int_{T_i}^{T_f} (1-e) m_w c_w dT = m_w c_w \int_{273}^{373} \frac{273}{T} dT \tag{67}$$

$$= 1 \times 4.2 \times 273 \times \ln\left(\frac{373}{273}\right) \tag{68}$$

$$= 357.9 \text{ kJ} \tag{69}$$

$$. \tag{70}$$

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   (c) Deduce how much ice has been melted.

   Solution.

$$M_{\text{ice}} = \frac{Q_C}{L} = \frac{357.9}{333} = 1.07 \text{ kg}. \tag{71}$$

■

   (d) Calculate the work done by the engine.

   Solution.

$$W = Q_H - Q_C = 1 \times 4.2 \times 100 - 357.9 = 62.1 \text{ kJ}.$$

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