Student Name: SalahDin Ahmed Salh Rezk
Student ID: 202201079
Date: October 23, 2022

Introduction to Classical Mechanics (PHYS 101)

Assignment 1

---

1. $\vec{A} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

   (a) Let the unit vector in the direction of $\vec{A}$ be $\hat{A}$.

$$\begin{aligned}
\hat{A} &= \frac{\vec{A}}{|\vec{A}|} && (1)\\
&= \frac{\hat{\imath} + 2\hat{\jmath} + 3\hat{k}}{\sqrt{1^2 + 2^2 + 3^2}} && (2)\\
&= \frac{\hat{\imath} + 2\hat{\jmath} + 3\hat{k}}{\sqrt{14}} && (3)\\
&= \frac{1}{\sqrt{14}}\hat{\imath} + \frac{2}{\sqrt{14}}\hat{\jmath} + \frac{3}{\sqrt{14}}\hat{k} && (4)\\
&= \frac{\sqrt{14}}{14}\hat{\imath} + \frac{\sqrt{14}}{7}\hat{\jmath} + \frac{3\sqrt{14}}{14}\hat{k} && (5)\\
\hat{A} &\approx 0.267\hat{\imath} + 0.535\hat{\jmath} + 0.802\hat{k} && (6)
\end{aligned}$$

   (b) Since $\vec{A}$ is in 3-dimensional space, it has an infinite number of perpendicular vectors in infinitly different directions. Therefore, the x, y, and z values of the perpendicular vector can be arbitrarily assumed.
   Let $\vec{B}$ be a perpendicular vector on vector $\vec{A}$ and $\hat{B}$ be the unit vector in its direction.

$$\begin{aligned}
&\because \vec{B} \perp \vec{A} && (7)\\
&\therefore \vec{B} \cdot \vec{A} = 0 && (8)\\
&\vec{B}_x\vec{A}_x + \vec{B}_y\vec{A}_y + \vec{B}_z\vec{A}_z = 0 && (9)\\
&\vec{B}_x + 2\vec{B}_y + 3\vec{B}_z = 0 && (10)\\
&\text{let } \vec{B}_x = 1, \vec{B}_y = \frac{1}{2}, \text{ and } \vec{B}_z = -\frac{2}{3} && (11)\\
&\text{then } \vec{B} = \vec{\imath} + \frac{1}{2}\vec{\jmath} - \frac{2}{3}\vec{k} && (12)\\
&\hat{B} = \frac{6\sqrt{61}}{61}\hat{\imath} + \frac{3\sqrt{61}}{61}\hat{\jmath} - \frac{4\sqrt{61}}{61}\hat{k} && (13)
\end{aligned}$$

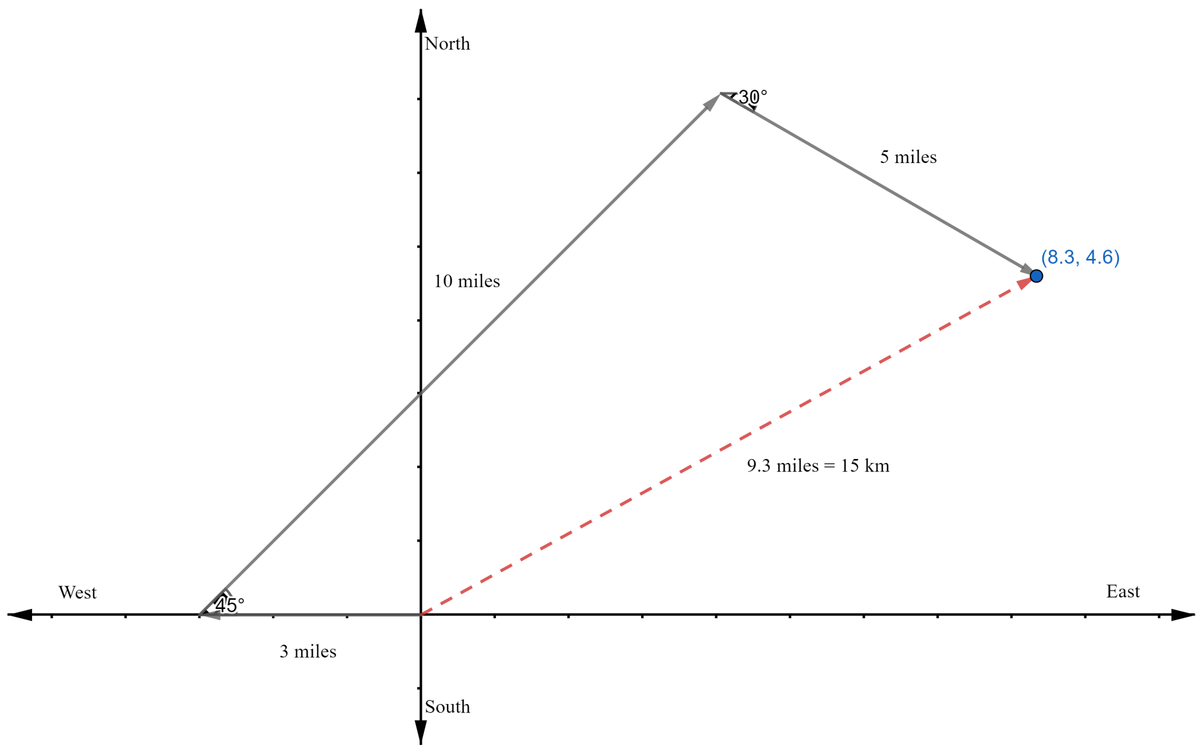
   (c) For a right-handed system to be constituted, all of its unit vectors have to be perpendicular.

$$\begin{aligned}
&\because \hat{C} \perp \hat{B} \perp \hat{A} && (14)\\
&\therefore \hat{C} = \hat{B} \times \hat{A} && (15)
\end{aligned}$$

$$
\begin{aligned}
\hat{C} &= \hat{B} \times \hat{A} && (16)\\
&= \frac{6\sqrt{61}}{61}\hat{i} + \frac{3\sqrt{61}}{61}\hat{j} - \frac{4\sqrt{61}}{61}\hat{k} \times \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k} && (17)\\
&= \frac{17}{\sqrt{854}}\hat{i} - \frac{11\sqrt{2}}{\sqrt{427}}\hat{j} + \frac{9}{\sqrt{854}}\hat{k} && (18)
\end{aligned}
$$

2. 3.0 miles west, 10 miles northeast, and 5 miles at 30° south of east.

   (a)



   (b) Let the first vector be $\vec{V_1}$, the second $\vec{V_2}$, the third $\vec{V_3}$, and the resultant displacement $\vec{S}$.

$$
\begin{aligned}
\vec{V_1} &= -3\hat{i} && (19)\\
\vec{V_2} &= 10\cos(45°)\hat{i} + 10\sin(45°)\hat{j} && (20)\\
& && (21)\\
&= \frac{10}{\sqrt{2}}\hat{i} + \frac{10}{\sqrt{2}}\hat{j} && (22)\\
\vec{V_3} &= 5\cos(-30)\hat{i} + 5\sin(-30)\hat{j} && (23)\\
&= \frac{5\sqrt{3}}{2}\hat{i} - \frac{5}{2}\hat{j} && (24)\\
\vec{S} &= \vec{V_1} + \vec{V_2} + \vec{V_3} && (25)\\
&= \left(-3 + \frac{10}{\sqrt{2}} + \frac{5\sqrt{3}}{2}\right)\hat{i} + \left(\frac{10}{\sqrt{2}} - \frac{5}{2}\right)\hat{j} && (26)\\
|\vec{S}| &\approx 9.6 miles = 15.4 km && (27)
\end{aligned}
$$

3.

$$\vec{r_1} = 2\hat{i} - \hat{j} + \hat{k}$$
$$\vec{r_2} = \hat{i} + 3\hat{j} - 2\hat{k}$$
$$\vec{r_3} = -2\hat{i} + \hat{j} - 3\hat{k}$$
$$\vec{r_4} = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{r_4} = a\vec{r_1} + b\vec{r_2} + c\vec{r_3} \tag{28}$$
$$3\hat{i} + 2\hat{j} + 5\hat{k} = a(2\hat{i} - \hat{j} + \hat{k}) + b(\hat{i} + 3\hat{j} - 2\hat{k}) + c(-2\hat{i} + \hat{j} - 3\hat{k}) \tag{29}$$
$$= 2a\hat{i} - a\hat{j} + a\hat{k} + b\hat{i} + 3b\hat{j} - 2b\hat{k} + -2c\hat{i} + c\hat{j} - 3c\hat{k} \tag{30}$$
$$= (2a + b - 2c)\hat{i} + (-a + 3b + c)\hat{j} + (a - 2b - 3c)\hat{k} \tag{31}$$

$$3 = 2a + b - 2c \tag{32}$$
$$2 = -a + 3b + c \tag{33}$$
$$5 = a - 2b - 3c \tag{34}$$
$$\tag{35}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 & 1 & -2 \\ -1 & 3 & 1 \\ 1 & -2 & -3 \end{bmatrix}^{-1} \times \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} \tag{36}$$
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{7} & \frac{2}{7} & 0 \\ \frac{1}{14} & -\frac{5}{14} & -\frac{1}{2} \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} \tag{37}$$
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} \tag{38}$$

4. $\vec{A} = \cos\theta_1\hat{i} + \sin\theta_1\hat{j}$ and $\vec{B} = \cos\theta_2\hat{i} + \sin\theta_2\hat{j}$.

$$\vec{A} \cdot \vec{B} = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 \tag{39}$$
$$|\vec{A}||\vec{B}|\cos\theta = \tag{40}$$
$$|\vec{A}| = |\vec{B}| = 1 \implies \cos\theta = \tag{41}$$
$$\cos(\theta_1 - \theta_2) = \tag{42}$$
$$\therefore \cos(\theta_1 \pm \theta_2) = \cos\theta_1\cos\theta_2 \mp \sin\theta_1\sin\theta_2 \tag{43}$$

$$\vec{A}\times\vec{B} = (\vec{A}_y\vec{B}_z - \vec{A}_z\vec{B}_y)\hat{i} - (\vec{A}_x B_z - \vec{A}_z B_x)\hat{j} + (\vec{A}_x B_y - \vec{A}_y B_x)\hat{k} \tag{44}$$
$$= (\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)\hat{k} \impliedby \vec{A}_z = \vec{B}_z = 0 \tag{45}$$
$$|\vec{A}||\vec{B}|\sin\theta n = \tag{46}$$
$$\sin\theta\hat{k} = \tag{47}$$
$$\sin(\theta_1 - \theta_2)\hat{k} = \tag{48}$$
$$\therefore \sin(\theta_1 \pm \theta_2) = \sin\theta_1\cos\theta_2 \pm \cos\theta_1\sin\theta_2 \tag{49}$$

5. $\vec{r} = x(t)\hat{i} + y(t)\hat{j}$ where $x(t) = 2\alpha t - \sin(\alpha t)$ and $y(t) = 1 - \cos(\alpha t)$.

(a)

$$\vec{v} = \vec{v}_x\hat{i} + \vec{v}_y\hat{j} \tag{50}$$
$$= \frac{d\vec{r}_x}{dt}\hat{i} + \frac{d\vec{r}_y}{dt}\hat{j} \tag{51}$$
$$= \frac{d[2\alpha t - \sin(\alpha t)]}{dt}\hat{i} + \frac{d[1-\cos(\alpha t)]}{dt}\hat{j} \tag{52}$$
$$= [2\alpha - \alpha\cos(\alpha t)]\hat{i} + \alpha\sin(\alpha t)\hat{j} \tag{53}$$

(b)

$$\vec{v} = \vec{v}_r + \vec{v}_n \tag{54}$$
$$\vec{v}_r = [2\alpha t - \sin(\alpha t)]\hat{i} + [1-\cos(\alpha t)]\hat{j} \tag{55}$$
$$\vec{v}_n = \frac{1}{2\alpha t - \sin(\alpha t)}\hat{i} - \frac{1}{1-\cos(\alpha t)}\hat{j} \tag{56}$$
$$\tag{57}$$

$$\vec{v} = \begin{bmatrix} 2\alpha t - \sin(\alpha t) & 1-\cos(\alpha t) \\ \frac{1}{2\alpha t - \sin(\alpha t)} & -\frac{1}{1-\cos(\alpha t)} \end{bmatrix}^{-1} \times \begin{bmatrix} 2\alpha - \alpha\cos(\alpha t) \\ \alpha\sin(\alpha t) \end{bmatrix} \tag{58}$$
$$= \begin{bmatrix} \frac{-(-\sin(\alpha t)+2\alpha t)(2\alpha - \alpha\cos(\alpha t)) - \alpha\sin\alpha t(-\sin(\alpha t)+2\alpha t)(1-\cos(\alpha t))^2}{-4\alpha^2t^2 + 4\alpha t\sin(\alpha t) - 2} \\ \frac{-(1-\cos(\alpha t))(2\alpha - \alpha\cos(\alpha t)) + \alpha\sin\alpha t(2\alpha t - \sin(\alpha t))^2(1-\cos(\alpha t))}{-4\alpha^2t^2 + 4\alpha t\sin(\alpha t) - 2} \end{bmatrix} \tag{59}$$

Made with heart in LaTeX