Chapter 12 Waves

$\nabla^2 A - \mu\varepsilon \frac{\partial^2 A}{\partial t^2} = -\mu J$

$\nabla^2 H - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = -\nabla \times J$

$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\varepsilon}$

time-harmonic

$\mu\varepsilon (j\omega)^2 \vec{A} = -\omega^2\mu\varepsilon A$

$\nabla^2 \vec{E} = \vec{\nabla}(\rho_v/\varepsilon) + j\omega\mu \vec{J}_s + j\omega\mu(\sigma\vec{E} + j\omega\varepsilon\vec{E})$ → σ = conductivity, 0 if lossless & source free

↳ **if source free**: $\nabla^2\vec{E} = j\omega\mu(\sigma\vec{E} + j\omega\varepsilon\vec{E})$ ↳ $\nabla^2 \vec{E} = \omega^2\mu\varepsilon\vec{E} = 0$

GREEK LETTERS

wave eq sol. 1D: $E_x(z) = E_o^+ e^{-jkz} + E_o^- e^{jkz}$ → E_o from boundry. z = prop. direction; $k = \omega\sqrt{\mu\varepsilon}$ ⇒ $E_x^+(z,t) = E_o^+\cos(\omega t - kz + \phi)$

↳ for H: $H_y^+(z) = \frac{j}{\omega\mu}\frac{\partial E_x}{\partial z} = \frac{k}{\omega\mu}E_x^+(z)$; $\eta = \frac{E_x(z)}{H_y(z)} = \sqrt{\frac{\mu}{\varepsilon}}$ = wave/intrinsic impedance

Poynting vector EM Pwr: $\oint_S (\vec{E}\times\vec{H})\cdot d\vec{s} = -\frac{\partial}{\partial t}\int_v\left(\frac{\mu H^2}{2} + \frac{\varepsilon E^2}{2}\right)dv - \int_v E\cdot J\,dv$ ~ $\vec{P}(t) = (\vec{E}\times\vec{H})$ → power density {instantanous} ~ $P_{av} = \frac{1}{T}\int_0^T P(t)\,dt$

↳ $\frac{E_o^2}{\eta} = \eta H_o^2$ in free space tot = ∞ ← $P_{tot.inst}(t) = \oint_S \vec{P}(t)\cdot d\vec{s}$ $P_{av.surface} = \oint_S \vec{P}_{av}\cdot d\vec{s}$

↳ in complex domain → $P_{av} = \frac{1}{2} Re\{E\times H^*\}$

Lossy materials: $\nabla^2\vec{E} = j\omega\mu(\sigma + j\omega\varepsilon)\vec{E}$ ~ $\nabla^2\vec{E} = \gamma^2\vec{E}$ ⇒ sol = $E_x(z) = E_o^+ e^{-\alpha z}e^{-j\beta z} + E_o^- e^{\alpha z}e^{j\beta z}$

↳ time domain & +z = $E_x(z,t) = E_o^+ e^{-\alpha z}\cos(\omega t - \beta z)$ ~ $H = \frac{1}{\eta}E_o^+ e^{-\alpha z}\cos(\omega t - \beta z)$ → prop in +ve z.

Low loss approx: $\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma}{2}\eta$; $\beta \simeq \omega\sqrt{\mu\varepsilon}\left(1 + \frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)$ ⇒ if very-low loss → $\beta \simeq \omega\sqrt{\mu\varepsilon}$

$\eta = \sqrt{\frac{\mu}{\varepsilon}}\left(1 + \frac{j\sigma}{2\omega\varepsilon}\right) = \sqrt{\frac{\mu}{\varepsilon}}\left(\left(1 - \frac{j\sigma}{\omega\varepsilon}\right)^{-1/2}\right)$

conductor high-loss approx: $\gamma = j\omega\sqrt{\mu\varepsilon}\sqrt{-\frac{j\sigma}{\omega\varepsilon}} = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}}$ → $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$

$E_x(z) = E_o^+ e^{-z/\delta}e^{-jz/\delta}$ ↳ $V = \omega/\beta = \omega\delta = \sqrt{\frac{2\omega}{\mu\sigma}}$; $\lambda = 2\pi/\beta = 2\pi\delta$

$\eta = \frac{j\omega\mu}{\gamma} = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} = (1+j)\left(\frac{1}{\sigma\delta}\right) = (1+j)\left(\frac{\omega\mu\delta}{2}\right)$

skin-depth / penetration depth ~ distance till ⇒ Wave Amp = $\left(\frac{1}{e}\right)$(wave amp)

Lossless: $V_p = \frac{1}{\sqrt{\mu\varepsilon}}$; $k = \omega\sqrt{\varepsilon\mu}$ wave num.

↳ $\lambda = V_p/f = \frac{2\pi}{k} = \frac{2\pi V_p}{\omega} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$

$\eta = \frac{\omega\mu}{k} = \sqrt{\mu/\varepsilon} = \frac{k}{\omega\varepsilon}$ ↳ for pwr... if S.F. $\int_{vol}$ → $J = \sigma E$

Lossy: $\varepsilon_c = \frac{\sigma + j\omega\varepsilon}{j\omega}$; $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

↳ $\gamma = jk_c$, $k_c = \omega\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}} = \alpha + j\beta$

$\alpha, \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\mu}\right)^2} \mp 1\right]}$

$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

$V_p = \frac{\omega}{\beta}$; $\lambda = \frac{2\pi}{\beta}$

high loss → $\delta = 1/\alpha$

$P = E\times H$