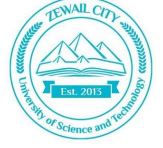


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Date: October 29, 2023



Linear Algebra and Vector Geometry (MATH 201)

Bonus 1

Prove that $(AB)^\top = B^\top A^\top$

Proof. Let A be an $m \times n$ matrix and B be an $n \times p$ matrix.

$$A_{ij} = \sum_{i=1}^m \sum_{j=1}^n a_{ij} \quad (1)$$

$$B_{ij} = \sum_{i=1}^n \sum_{j=1}^p b_{ij}. \quad (2)$$

Using the definition of matrix multiplication:

$$AB_{ij} = \sum_{k=1}^n a_{ik} b_{kj}. \quad (3)$$

Using definition of transpose:

$$(AB)_{ij}^\top = AB_{ji} \quad (4)$$

$$= \sum_{k=1}^n a_{jk} b_{ki}. \quad (5)$$

Then:

$$(B^\top A^\top)_{ij} = \sum_{k=1}^n b_{ik}^\top a_{kj}^\top \quad (6)$$

$$= \sum_{k=1}^n b_{ki} a_{jk} \quad (7)$$

$$= \sum_{k=1}^n a_{jk} b_{ki}. \quad (8)$$

From (5) and (8) we conclude that $(AB)^\top = B^\top A^\top$. □

Notice that $(AB)^\top \neq A^\top B^\top$.