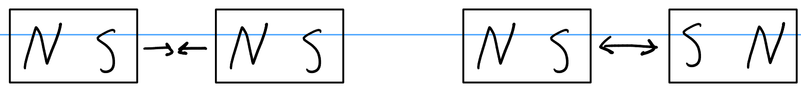
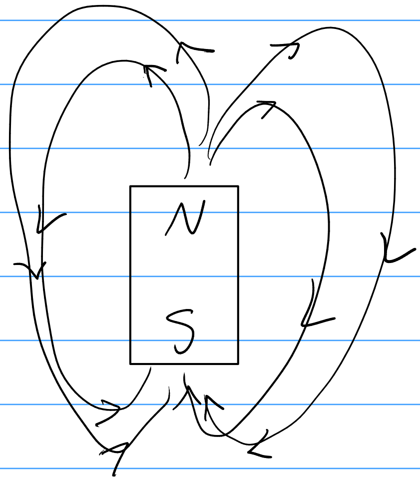
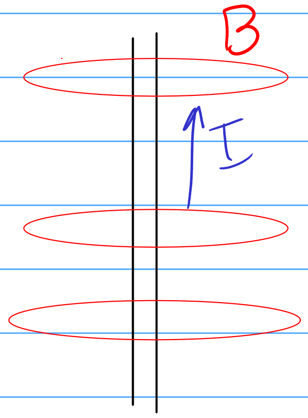
Magnetic Field







right hand rule

Lorentz Force

$$\vec{F}_B = q\vec{v} \times \vec{B} \qquad = q\frac{\partial \vec{x}}{\partial t} \times \vec{B}$$

$$\vec{F}_E = q\vec{E}$$

$$\Longrightarrow \quad \vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{B} = \mu \vec{H} \iff \vec{D} = \varepsilon \vec{E}$$

flux density

magnetic Field intensity

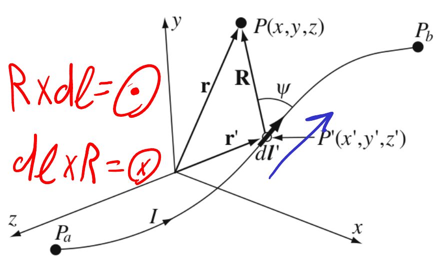
$$\left\{ \mu \equiv \frac{\text{Henry}}{\text{meter}} \right\} \quad H \equiv \frac{\text{Ampere}}{\text{meter}} \quad B \equiv \text{Tesla}$$

$$dH = \frac{I\,dl}{4\pi R^2} \Longrightarrow d\vec{H} = \frac{I\,d\vec{l} \times \hat{R}}{4\pi R^2} = \frac{I\,d\vec{l} \times \vec{R}}{4\pi R^3}$$

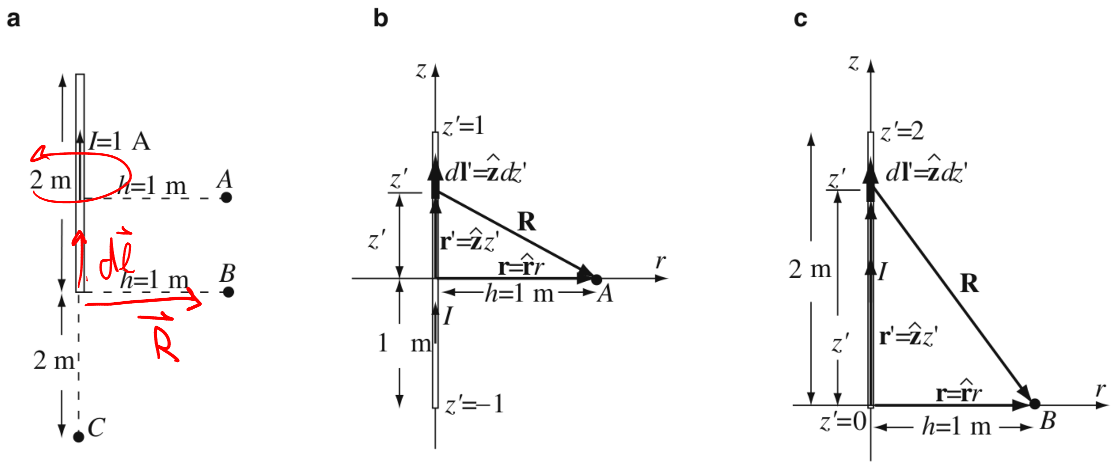
Biot-Savart

$$\vec{H} = \frac{1}{4\pi}\int_a^b \frac{I\,d\vec{l} \times \vec{R}}{R^3}$$

$$\vec{B} = \frac{\mu}{4\pi}\int_a^b \frac{I\,d\vec{l} \times \vec{R}}{R^3}$$



Magnetic field due to the wire inside the wire is zero because $d\vec{l}$ and $\vec{R}$ are perpendicular so the cross will be zero.



Example 8.2 Magnetic Field Intensity and Magnetic Flux Density Due to a Half-Loop A current I [A] flows in the circuit shown in **Figure 8.5**. Calculate the magnetic flux density and the magnetic field intensity at the center of the half-loop assuming the circuit is in free space.

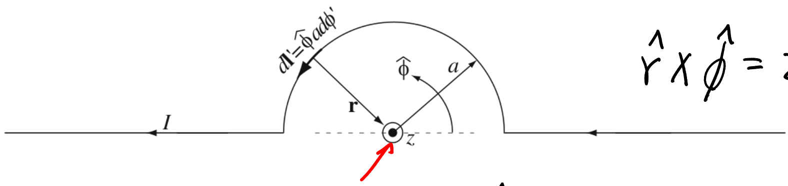


Figure 8.5 Calculation of the magnetic flux density at the center of a semicircular current loop

$\hat{r}\times\hat{\phi} = \hat{z}$

$d\vec{l} = \hat{\phi}\, a\, d\phi$, $\vec{r} = -\hat{r} a$ $\Rightarrow$ $d\vec{l}\times\vec{r} = \hat{\phi}\, a\, d\phi \times (-\hat{r} a) = \hat{z} a^2 d\phi$

$$H = \frac{1}{4\pi}\int_0^{\pi} \frac{I\,\hat{z} a^2}{a^3}\, d\phi = \frac{I\,\hat{z}}{4\pi a}\int_0^{\pi} d\phi = \boxed{\frac{I}{4a}\hat{z}}$$

Example 8.3 Magnetic Field Intensity of a Circular Loop

(a) Calculate the magnetic field intensity **H** at point P in **Figure 8.6** generated by the current I [A] in the loop. Point P is at a height h [m] along the axis of the loop.
(b) Calculate the magnetic field intensity at the center of the loop (point O).

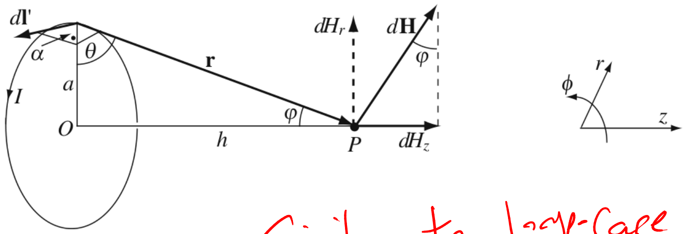
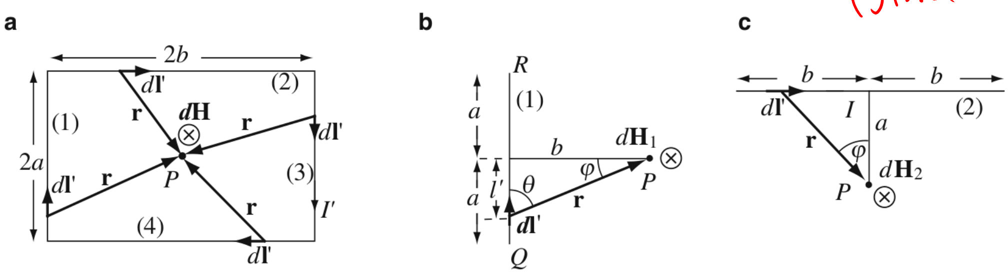


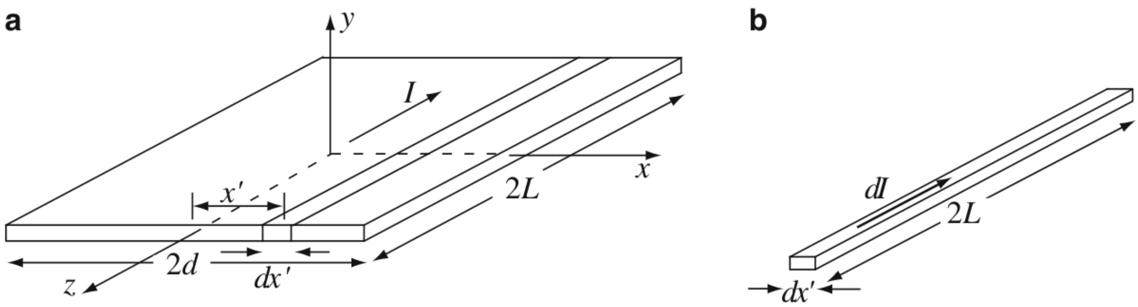
Figure 8.6 Calculation of the magnetic field intensity at height h above a current-carrying loop

Similar to loop-Case in Colomb's law

Example 8.4 Magnetic Field Intensity Due to a Rectangular Loop: Superposition of Fields A rectangular loop carries a current I [A] as shown in **Figure 8.7a**. Calculate the magnetic field intensity at the center of the loop.

Divide and Conquer.





$$dI = \frac{I}{2d}\,dx \Rightarrow d\vec{l} = \hat{x}\,dx$$

and the total contribution due to this differential wire is found using **Eq. (8.8)**:

$$d\mathbf{H}(x,y,z) = \left[\int_{z'=-L}^{z'=+L} \frac{I\,dx'}{2d}\,\frac{d\mathbf{l}' \times \hat{\mathbf{R}}}{4\pi|\mathbf{r}-\mathbf{r}'|^2}\right] \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{8.11}$$

where integration is on $d\mathbf{l}$ and $\mathbf{r}$ is the vector connecting $d\mathbf{l}$ and $P(x,y,z)$ (see **Figure 8.3**). To obtain the total field intensity, we integrate over the width of the current sheet in **Figure 8.9a**. We get

$$\mathbf{H}(x,y,z) = \int_{x'=-d}^{x'=+d}\left[\int_{z'=-L}^{z'=+L} \frac{I}{2d}\,\frac{d\mathbf{l}' \times \hat{\mathbf{R}}}{4\pi|\mathbf{r}-\mathbf{r}'|^2}\right]dx' \quad \left[\frac{\mathrm{A}}{\mathrm{m}}\right] \tag{8.12}$$

Ampere's Law

$$\oint_C \vec{H}\cdot d\vec{\ell} = I_{enc}$$

$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu I_{enc}$$

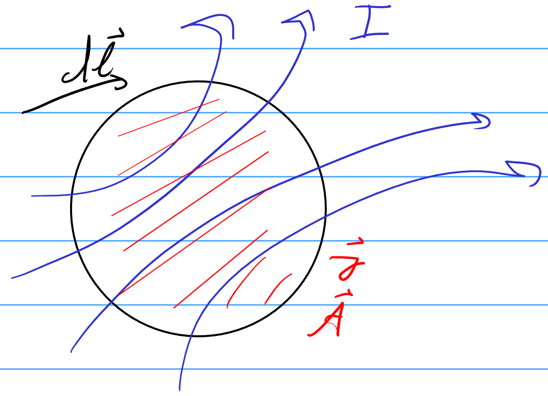
Example 8.6 Application: Field Intensity Due to a Single, Thin Wire–Magnetic Field of Overhead Transmission Lines Calculate the magnetic field intensity due to a long filamentary conductor carrying a current *I* at a distance *h* from the wire. The conductor is very long (infinite). Compare this result with the result in **Example 8.1c**.

$$\oint \vec{H}\cdot d\vec{\ell} = \oint \hat{\phi} H \cdot \hat{\phi}\, h\, d\phi = Hh\int_0^{2\pi} d\phi = 2\pi H h = I \Rightarrow H = \frac{I}{2\pi h}$$

$$\oint_C \vec{H}\cdot d\vec{\ell} = I_{enc}$$

$$\oiint \vec{\nabla}\times\vec{H} = \oiint_S \vec{J}\cdot d\vec{A}$$

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J}}$$



$$\oint$$