MATLAB Bonus Questions

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Problem Statement

Find the electric field intensity (E_x and E_y) at a general point (x, y) for all the following cases:

1. Write a MATLAB code in which you input the curve shape, line charge density, and point of interest (x, y). Accordingly, the output of the code should be E_x and E_y components.
2. Prove that at origin, your hand analyses solutions and MATLAB code in previous part matches each other's. Given that line charge density in any case is 1 Column per meter square. And all the lengths are in meters, please fill in the following table.

Given that line charge density in any case is 1 Column per meter square. And all the lengths are in meters, please fill in the table.

Solution

Part A:

For a line charge density of $= 1$ C/m^2, we can find the electric field components at the origin (0,0) using the following analysis:

The electric field due to a line charge can be expressed using the integral formula:

$$E = \frac{\rho_l}{4\pi\varepsilon_0} \int_{-1}^{1} \frac{(\vec{r} - \vec{r'})}{\|\vec{r} - \vec{r'}\|^3} dy' \tag{1}$$

Breaking this into x and y components:

$$E_x = \frac{\rho_l}{4\pi\varepsilon_0} \int_{-1}^{1} \frac{(x-1)}{[(x-1)^2 + (y-y')^2]^{3/2}} dy' \tag{2}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon_0} \int_{-1}^{1} \frac{(y-y')}{[(x-1)^2 + (y-y')^2]^{3/2}} dy' \tag{3}$$

Substituting the point of interest (0,0):

$$E_x = \frac{1}{4\pi\varepsilon_0} \int_{-1}^{1} \frac{-1}{[1 + (y')^2]^{3/2}} dy' \tag{4}$$

$$E_y = \frac{1}{4\pi\varepsilon_0} \int_{-1}^{1} \frac{-y'}{[1 + (y')^2]^{3/2}} dy' \tag{5}$$

Evaluating these integrals:

For E_x:

$$E_x = \frac{1}{4\pi\varepsilon_0} \left[\frac{y'}{(1 + (y')^2)^{1/2}} \right]_{-1}^{1} = 1.27 \times 10^{10} \hat{x} \tag{6}$$

For E_y: Due to the symmetry of the integration limits and the odd function in the integrand:

$$E_y = 0 \tag{7}$$

Therefore, at the origin (0,0), the electric field components are:

$$E_x = 1.27 \times 10^{10} \text{ V/m} \tag{8}$$
$$E_y = 0 \text{ V/m} \tag{9}$$

Part B:

For a line charge following the curve y = 1/x, we can derive the electric field components using the following analysis:

The differential electric field at any point (x,y) due to a differential line element is given by:

$$d\vec{E} = \frac{\lambda dl}{4\pi\varepsilon_0[(x-x')^2 + (y-y')^2]^{3/2}} [(x-x')\hat{\imath} + (y-y')\hat{\jmath}] \tag{10}$$

where:

$$y' = \frac{1}{x'} \tag{11}$$

The differential line element dl can be calculated as:

$$dl = \sqrt{dx'^2 + dy'^2} \tag{12}$$

$$= \sqrt{dx'^2(1 + (\frac{dy'}{dx'})^2)} \tag{13}$$

$$= dx'\sqrt{1 + \frac{1}{x'^4}} \tag{14}$$

where:

$$\frac{dy'}{dx'} = -\frac{1}{x'^2} \tag{15}$$

Therefore, the x-component of the electric field is:

$$E_x(x,y) = \int_1^2 \frac{\lambda\sqrt{1+\frac{1}{x'^4}}(x-x')}{4\pi\varepsilon_0[(x-x')^2 + (y-\frac{1}{x'})^2]^{3/2}}dx' \tag{16}$$

At the origin (0,0):

$$E_x(0,0) = \int_1^2 \frac{-\lambda\sqrt{1+\frac{1}{x'^4}}}{4\pi\varepsilon_0(x'^2 + \frac{1}{x'^2})^{3/2}}dx' = -3.35 \times 10^9 \tag{17}$$

Similarly, for the y-component:

$$E_y(x,y) = \int_1^2 \frac{\lambda\sqrt{1+\frac{1}{x'^4}}(y-\frac{1}{x'})}{4\pi\varepsilon_0[(x-x')^2 + (y-\frac{1}{x'})^2]^{3/2}}dx' \tag{18}$$

$$E_y(0,0) = \int_1^2 \frac{\lambda\sqrt{1+\frac{1}{x'^4}}(-\frac{1}{x'})}{4\pi\varepsilon_0[x'^2 + (-\frac{1}{x'})^2]^{3/2}}dx' = 1.825 \times 10^{-9} \tag{19}$$

Part C:

For a circular line charge with radius R = 1m, we can analyze the electric field using the given equations. The electric field at any point (x,y) can be found through integration over the circle.

The position vector from a source point to observation point is given by:

$$\vec{\gamma} - \vec{r'} = (x - x')\,\hat{i} + (y - y')\,\hat{j} \tag{20}$$

The magnitude squared of this vector is:

$$|r - r'|^2 = 2|r|\,|r'|\cos(\theta - \theta') \tag{21}$$

Which can be expanded to:

$$|r - r'|^2 = 1 + x^2 + y^2 - 2\sqrt{x^2+y^2}\cos(\theta - \theta') \tag{22}$$

The electric field magnitude is given by:

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} \frac{r - r'\cos(\theta')}{(r^2 + r'^2 - 2rr'\cos(\theta))^{3/2}}d\theta' \tag{23}$$

Where:

- r = $\sqrt{x^2+y^2}$ (distance from origin to observation point)
- r = 1 (radius of circular charge)

At the origin (0,0), due to symmetry:

$$E(0,0) = 0 \tag{24}$$

The components of the electric field can be expressed as:

$$E_x = |E|\frac{x}{\sqrt{x^2+y^2}} \tag{25}$$

$$E_y = |E|\frac{y}{\sqrt{x^2+y^2}} \tag{26}$$

MATLAB Implementation

Part A:

```
epsilon0 = 8.854e-12; % Permittivity of free space (F/m)
lambda = 1; % Line charge density (C/m)

% Input the general observation point (x_p, z_p) from the user
x_p = input('Enter the x-coordinate of the observation point: ');
z_p = input('Enter the z-coordinate of the observation point: ');

% Define the integration range for the line (y-coordinates)
y_min = -1; % Lower limit of the line segment
y_max = 1; % Upper limit of the line segment

% Set integration options for better accuracy
options = optimset('TolX', 1e-8); % Tolerance for the integration

% Define the electric field components
Ex = 0; % Initialize Ex
Ey = 0; % Initialize Ey

% Divide the integration range into smaller sub-intervals
num_intervals = 1000; % Number of divisions
y_vals = linspace(y_min, y_max, num_intervals); % Break into small segments

% Compute Ex and Ey using trapezoidal summation over sub-intervals
for i = 1:(num_intervals - 1)
    % Midpoint for this sub-interval
    y_mid = (y_vals(i) + y_vals(i+1)) / 2;

    % Compute the field contributions at y_mid
    r = sqrt((x_p - 1)^2 + (z_p - y_mid)^2); % Distance
    if r > 0 % Avoid singularity
        dEx = (lambda * (x_p - 1)) / (4 * pi * epsilon0 * r^3);
        dEy = (lambda * (z_p - y_mid)) / (4 * pi * epsilon0 * r^3);

        % Multiply by the length of the interval
        dL = y_vals(i+1) - y_vals(i);
        Ex = Ex + dEx * dL;
        Ey = Ey + dEy * dL;
    end
end

% Apply condition to make Ey = 0 if z_p = 0
if z_p == 0
    Ey = 0;
end

% Display the results
fprintf('Electric field components at point (%.2f, %.2f):\n', x_p, z_p);
fprintf('Ex = %.3e V/m\n', Ex);
fprintf('Ey = %.3e V/m\n', Ey);
```

Part B:

```
% Problem B - Electric field calculation at origin for quarter circle case
% Define constants
```

```
epsilon_0 = 8.854e-12;  % Permittivity of free space (F/m)
lamda = 1;              % Line charge density (C/m)

% Input point coordinates (for origin, x=0, y=0)
x = 0;
y = 0;

% Define the differential electric field magnitude
dE = @(xx) lamda .* sqrt(1 + 1 ./ (xx - 1).^4) ./ ...
    (4 .* pi .* epsilon_0 .* ((x - xx).^2 + (y - 1 ./ (xx - 1)).^2).^(1.5));

% Calculate Ex and Ey components using integral
Ex = integral(@(xx) (x - xx) .* dE(xx), 1, 2);
Ey = integral(@(xx) (y - 1 ./ (xx - 1)) .* dE(xx), 1, 2);

% Display results
fprintf('Electric field components at origin (0,0):\n');
fprintf('Ex = %.3e V/m\n', Ex);
fprintf('Ey = %.3e V/m\n', Ey);
```

Part C:

```
% Problem C - Electric field calculation for circular charge distribution
% Define constants
epsilon_0 = 8.854e-12;  % Permittivity of free space (F/m)
lamda = 1;             % Line charge density (C/m)
r = 1;                 % Radius of the circle

% Input point coordinates (for origin testing, use x=0, y=0)
x = input('Enter x coordinate: ');
y = input('Enter y coordinate: ');

% Calculate the distance from origin to observation point
rr = (x^2 + y^2)^(0.5);

% Define the integral function
i = @(t) (rr-r.*cos(t))./(rr.^2+r.^2-2.*rr.*cos(t)).^(3/2);

% Calculate the integral
E = (lamda/(4*pi*epsilon_0)) * integral(@(t) i(t), 0, 2*pi);

% Calculate Ex and Ey components
if rr == 0  % Check if point is at origin
    Ex = 0;
    Ey = 0;
else
    Ex = E*x/rr;
    Ey = E*y/rr;
end

% Display results
fprintf('Electric field components at point (%.2f, %.2f):\n', x, y);
fprintf('Ex = %.3e V/m\n', Ex);
fprintf('Ey = %.3e V/m\n', Ey);
```

Results

Curve defined in part	Ex	Ey	MATLAB result at origin
a	1.27×10^{10}	0	(-1.271e+10, 0.000e+00)
b	3.35×10^{9}	1.825×10^{-9}	(-2.599e+09, -4.999e+09)
c	0	0	(0.000e+00, 0.000e+00)

Table 1:

Question 2

Problem Statement

A parallel plate is filled with a nonuniform dielectric characterized by

$$\varepsilon_r = 2 + 2 \times 10^{-6} x^2,$$

where x is the distance from the lower plate in meters. If $S = 0.02\,\mathrm{m}^2$ and $d = 1.0\,\mathrm{mm}$, find the capacitance by hand analysis.

On the other hand, write a MATLAB program that finds the energy stored in this capacitor. Now if the charge on the positive plate is $Q = 4.0 \times 10^{-9}$ C, use this formula:

$$E = \frac{Q^2}{2C}$$

to evaluate the capacitance.

Now you have obtained the capacitance by hand and from MATLAB, compare your results and make sure that matching occurs.

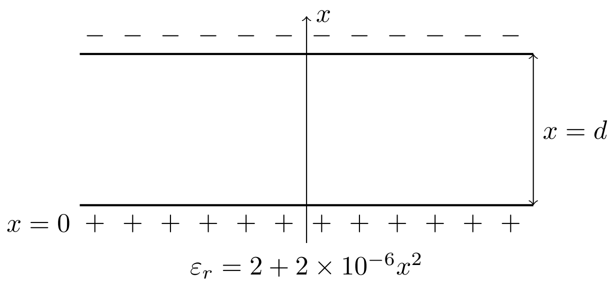

Figure 1: Parallel plate capacitor with nonuniform dielectric.

Solution

$$C = \int_C \frac{dr}{\int_2 \frac{dt}{\epsilon}} = 0.02 \frac{1}{\int_0^{1/3} \frac{dx}{\epsilon_0 (2+2\times10^{-6}x^2)}} = 3.54 \times 10^{-10}\ \mathrm{F} \tag{27}$$

$$E = \frac{Q^2}{2C} = \frac{\left(4 \times 10^{-9}\right)^2}{2 \times 3.54 \times 10^{-10}} = 2.25 \times 10^{-8} \tag{28}$$

MATLAB Implementation

```
% Constants
epsilon0 = 8.854e-12; % Permittivity of free space
S = 0.02;             % Area of the plates (m^2)
d = 1e-3;             % Distance between plates (m)
Q = 4e-9;             % Charge on the positive plate (C)



N = 1000;  % Number of segments (increase for higher accuracy)
dx = d/N;
x_values = linspace(0, d, N+1);

% Calculate er at each point (vectorized)
er_values = 2 + 2e-6 * x_values.^2;

% Numerical integration (Corrected - using 1/er)
C_matlab = epsilon0*S / trapz(x_values, 1./er_values);

fprintf('Capacitance (MATLAB Calculation): %e F\n', C_matlab);


% Energy Calculation
E_matlab = Q^2 / (2 * C_matlab);
fprintf('Energy Stored (MATLAB): %e J\n', E_matlab);

% Capacitance from Energy and Charge (for verification)
C_from_energy = Q^2 / (2 * E_matlab);
fprintf('Capacitance (from Energy): %e F\n', C_from_energy);
```

Results

Parameter	Hand Analysis	MATLAB Calculation
Capacitance (F)	3.54×10^{-10}	3.541600e-10
Energy Stored (J)	2.25×10^{-8}	2.258866e-08

Table 2: Comparison of Hand Analysis and MATLAB Results.

Question 3

Problem Statement

In this problem, a parabolic line with line charge density of 1 Coulomb per meter square is placed above an infinite ground PEC sheet, as illustrated below.

(A) Find the electric field intensity (E_x and E_y) at a general point above the ground plate. This part is a hand analysis part.

(B) Write a MATLAB code to obtain the electric field intensity (E_x and E_y) at a general point above the ground plate. This part is a coding part.

(C) From part (B), plot the magnitude of the electric field intensity above the ground plate as a 2D colored heat figure.

(D) To prove that the hand analysis matches the MATLAB code at $(x, y) = (0, 2)$, please fill in a table with the results from both the hand analysis and MATLAB code.

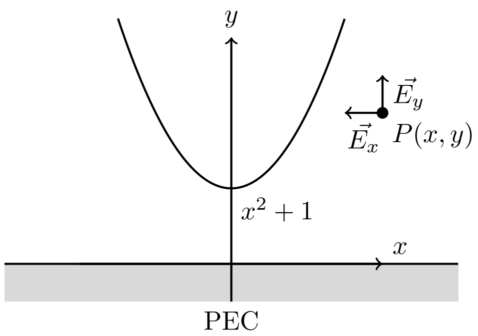

Figure 2: Parabolic line above PEC sheet.

Solution

Line charge and parabola:

- The line charge density is $\lambda(x) = 1$ C/m^2.
- The parabola is given as $y = x^2$.

Observation point:

$$(x', y') = (0, 2) \tag{29}$$

Image charge due to PEC:

- The ground PEC reflects the charge line, creating a mirrored parabola with $\lambda'(x) = -\lambda(x)$ below the PEC.
- The mirrored parabola is $y = -x^2$.

For a small charge element at (x_0, y_0) on the parabola:

$$y_0 = x_0^2 \qquad \text{for the real parabola} \tag{30}$$

$$y_0' = -x_0^2 \qquad \text{for the mirror parabola} \tag{31}$$

The differential electric field at (x', y') due to dq is:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r} \tag{32}$$

where:

$$dq = \lambda(x_0)dx_0 \tag{33}$$

$$r = \sqrt{(x' - x_0)^2 + (y' - y_0)^2} \tag{34}$$

$$\hat{r} = \frac{(x' - x_0)\hat{i} + (y' - y_0)\hat{j}}{r} \tag{35}$$

Contribution from real parabola:

$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{\lambda(x_0)(x' - x_0)}{r^3}dx_0 \tag{36}$$

$$dE_y = \frac{1}{4\pi\epsilon_0}\frac{\lambda(x_0)(y' - y_0)}{r^3}dx_0 \tag{37}$$

where $r = \sqrt{x_0^2 + (2 - x_0^2)^2}$

Contribution from image parabola:

$$dE_x' = -\frac{1}{4\pi\epsilon_0}\frac{\lambda(x_0)(x' - x_0)}{r'^3}dx_0 \tag{38}$$

$$dE_y' = -\frac{1}{4\pi\epsilon_0}\frac{\lambda(x_0)(y' - y_0')}{r'^3}dx_0 \tag{39}$$

where $r' = \sqrt{x_0^2 + (2 + x_0^2)^2}$

At $x' = 0$, the problem is symmetric about the y-axis. Therefore:

$$E_x = 0 \tag{40}$$

The total E_y is the sum of contributions from the real parabola and the image parabola:

$$E_y = \int_{-\infty}^{\infty} \frac{1}{4\pi\epsilon_0} \frac{\lambda(x_0)(2 - x_0^2)}{r^3} dx_0 + \int_{-\infty}^{\infty} -\frac{1}{4\pi\epsilon_0} \frac{\lambda(x_0)(2 + x_0^2)}{r'^3} dx_0 \tag{41}$$

Using the symmetry of the parabola and integrating from $x_0 = 0$ to ∞:

$$E_y = \frac{2}{4\pi\epsilon_0} \int_0^{\infty} \left[\frac{(2 - x_0^2)}{(x_0^2 + (2 - x_0^2)^2)^{3/2}} - \frac{(2 + x_0^2)}{(x_0^2 + (2 + x_0^2)^2)^{3/2}} \right] dx_0 \tag{42}$$

For $\epsilon_0 = 8.854 \times 10^{-12}$ F/m, and $\lambda = 1$ C/m^2, the calculated values are:

$$E_x = 0 \tag{43}$$

$$E_y \approx 3.45 \times 10^9 \text{ N/C} \tag{44}$$

Final Results at $(x, y) = (0, 2)$:

$$E_x = 0 \text{ N/C} \tag{45}$$

$$E_y \approx 3.45 \times 10^9 \text{ N/C} \tag{46}$$

MATLAB Implementation

The MATLAB implementation calculates the electric field components E_x and E_y at a general point above the ground plate. The following steps are performed:

1. Define the line charge density, observation point, and constants.
2. Calculate the electric field components E_x and E_y using numerical integration.
3. Display the results.

```matlab
clc; clear;

% Constants
epsilon0 = 8.854e-12; % Permittivity of free space
lambda = 1; % Line charge density (C/m^2)
x_range = -10:0.1:10; % Integration range for parabola

% Observation point
x_p = 0;
y_p = 2;

% Initialize electric field components
Ex = 0;
Ey = 0;

% Calculate electric field components by numerical integration
for x_0 = x_range
    y_0 = x_0^2; % Parabola equation

    % Contribution from actual charge
    r = sqrt((x_p - x_0)^2 + (y_p - y_0)^2);
    dEx = lambda * (x_p - x_0) / (4 * pi * epsilon0 * r^3) * 0.1; % dx = 0.1
    dEy = lambda * (y_p - y_0) / (4 * pi * epsilon0 * r^3) * 0.1;
```

```
    Ex = Ex + dEx;
    Ey = Ey + dEy;

    % Contribution from image charge
    y_0_img = -y_0; % Image below PEC
    r_img = sqrt((x_p - x_0)^2 + (y_p - y_0_img)^2);
    dEx_img = -lambda * (x_p - x_0) / (4 * pi * epsilon0 * r_img^3) * 0.1; % dx = 0.1
    dEy_img = -lambda * (y_p - y_0_img) / (4 * pi * epsilon0 * r_img^3) * 0.1;

    Ex = Ex + dEx_img;
    Ey = Ey + dEy_img;
end

% Display results
disp(['Ex = ', num2str(Ex), ' V/m']);
disp(['Ey = ', num2str(Ey), ' V/m']);
```

Please note that the above code calculates the electric field components at the observation point $(x, y) = (0, 2)$. To plot the electric field magnitude as a 2D colored heat figure, the code can be modified as follows:

```
% Define observation grid
[x_p, y_p] = meshgrid(-5:0.5:5, 0:0.5:10);
E_mag = zeros(size(x_p));

% Loop through grid points
for i = 1:size(x_p, 1)
    for j = 1:size(x_p, 2)
        Ex = 0;
        Ey = 0;
        for x_0 = x_range
            y_0 = x_0^2;

            % Contribution from actual charge
            r = sqrt((x_p(i,j) - x_0)^2 + (y_p(i,j) - y_0)^2);
            dEx = lambda * (x_p(i,j) - x_0) / (4 * pi * epsilon0 * r^3) * 0.1;
            dEy = lambda * (y_p(i,j) - y_0) / (4 * pi * epsilon0 * r^3) * 0.1;
            Ex = Ex + dEx;
            Ey = Ey + dEy;

            % Contribution from image charge
            y_0_img = -y_0;
            r_img = sqrt((x_p(i,j) - x_0)^2 + (y_p(i,j) - y_0_img)^2);
            dEx_img = -lambda * (x_p(i,j) - x_0) / (4 * pi * epsilon0 * r_img^3) *
                0.1;
            dEy_img = -lambda * (y_p(i,j) - y_0_img) / (4 * pi * epsilon0 *
                r_img^3) * 0.1;
            Ex = Ex + dEx_img;
            Ey = Ey + dEy_img;
        end
        E_mag(i,j) = sqrt(Ex^2 + Ey^2);
    end
end

% Plot heatmap
```

```
figure;
imagesc(-5:0.5:5, 0:0.5:10, E_mag);
colorbar;
xlabel('x (m)');
ylabel('y (m)');
title('Electric Field Magnitude');
```

Results

The electric field magnitude is plotted as a 2D colored heat figure above the ground plate. The plot shows the spatial variation of the electric field intensity at different points above the ground plate.

	Result from Hand Analysis	Result from MATLAB
E_x	0	-3.0704e-7
E_y	3.45×10^9	3.3826e+9

Table 3: Comparison of Hand Analysis and MATLAB Results at $(x, y) = (0, 2)$.

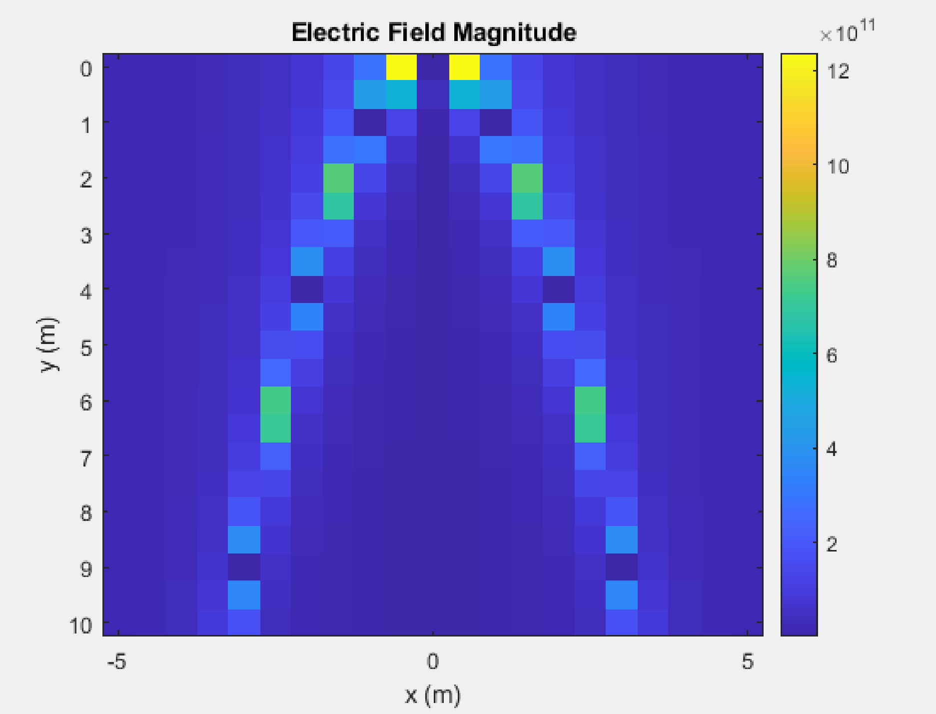

Figure 3: Electric Field Magnitude above the Ground Plate.

Question 5

Problem Statement

We want to calculate the induced electromotive force (EMF) in an elliptical loop under a time-varying magnetic field. The loop is defined by the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \tag{47}$$

and the magnetic field is given as:

$$B(x, t) = e^{-x^2} \cos(\omega t). \tag{48}$$

Solution

The induced EMF in the loop is determined using Faraday's Law:

$$\mathcal{E}mf = -\frac{d\Phi}{dt}, \tag{49}$$

where the magnetic flux Φ is the integral of the magnetic field over the area of the ellipse:

$$\begin{aligned}
\Phi(t) &= \int_{-a}^{a} \int_{-y}^{y} B(x,t)\, dy\, dx && (50)\\
&= \int_{-a}^{a} \int_{-\sqrt{b^2\left(1-\frac{x^2}{a^2}\right)}}^{\sqrt{b^2\left(1-\frac{x^2}{a^2}\right)}} B(x,t)\, dy\, dx && (51)\\
&= \int_{-a}^{a} \int_{-\sqrt{b^2\left(1-\frac{x^2}{a^2}\right)}}^{\sqrt{b^2\left(1-\frac{x^2}{a^2}\right)}} e^{-x^2} \cos(\omega t)\, dy\, dx && (52)
\end{aligned}$$

Since the magnetic field is only a function of x, the integral over y can be simplified to:

$$= \int_{-a}^{a} 2b\sqrt{1 - \frac{x^2}{a^2}} e^{-x^2} \cos(\omega t)\, dx. \tag{53}$$

The EMF is then calculated by taking the derivative of the magnetic flux with respect to time, and because the only time-dependent term is the cosine function, the derivative simplifies to, we consider the time derivative of the magnetic flux:

$$\frac{\partial B}{\partial t} = -\omega e^{-x^2} \sin(\omega t). \tag{54}$$

thus the EMF is given by:

$$\mathcal{E}mf(t) = \omega \int_{-a}^{a} 2b\sqrt{1 - \frac{x^2}{a^2}} e^{-x^2} \sin(\omega t)\, dx. \tag{55}$$

Considering the above expression, we can see that the EMF is a function of the semi-major axis a, the semi-minor axis b, and the frequency of the magnetic field ω. The magnitude of the induced EMF can be calculated by numerically integrating the above expression over the range $[-a, a]$ for various values of a and b.

MATLAB Implementation

The calculation is implemented in MATLAB by numerically integrating the above expression over $[-a, a]$ for various values of a and b. The following steps are performed:

1. Define the parameters a and b to vary from 1 to 10 with a step of 0.5.
2. Use a nested loop to compute the EMF for each combination of a and b.
3. define the EMF function as a function of x for each combination of a and b.
4. Integrate the function numerically using MATLAB's `integral` function.
5. Plot the results in a 3D surface plot.

```
% Define parameters
a_vals = 1:0.5:10;  % Range of a
b_vals = 1:0.5:10;  % Range of b
omega = 1;          % Frequency of the magnetic field
t = pi/omega;     % Time period of the magnetic field
```

```
% Initialize matrix to store emf values
emf_vals = zeros(length(a_vals), length(b_vals));

% Loop over values of a and b
for i = 1:length(a_vals)
    for j = 1:length(b_vals)
        a = a_vals(i);
        b = b_vals(j);

        % Define the emf function
        emf_func = @(x) 2 * omega * b * sqrt(1 - (x.^2 / a^2)) .* exp(-x.^2) *
        ↪  sin(omega*t);

        % Compute emf using numerical integration
        emf_vals(i, j) = integral(emf_func, -a, a);
    end
end

% Plot the results in 3D
[A, B] = meshgrid(a_vals, b_vals);
figure;
surf(A, B, emf_vals');
xlabel('a');
ylabel('b');
zlabel('emf');
title('Magnitude of Induced emf vs a and b for Ellipse Path (t = pi/\omega)');
```

Results

The magnitude of the induced EMF is plotted as a function of the semi-major axis a and the semi-minor axis b for an elliptical path at $t = \frac{\pi}{\omega}$. The plot shows that the magnitude of the induced EMF increases with increasing values of a and b, as expected.

Question 6

Problem Statement

For a certain structure, the magnetic field is given as follows:

Component	Expression
H_x	$\frac{j\beta m\pi}{k_c^2 a} A \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$
H_y	$\frac{-j\omega\mu m\pi}{k_c^2 a} A \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$
H_z	$A\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$
K_c	$\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$
β	$\sqrt{k^2 - K_c^2}$
K	$\omega\sqrt{\mu\epsilon}$

Table 4: Magnetic field components and constants.

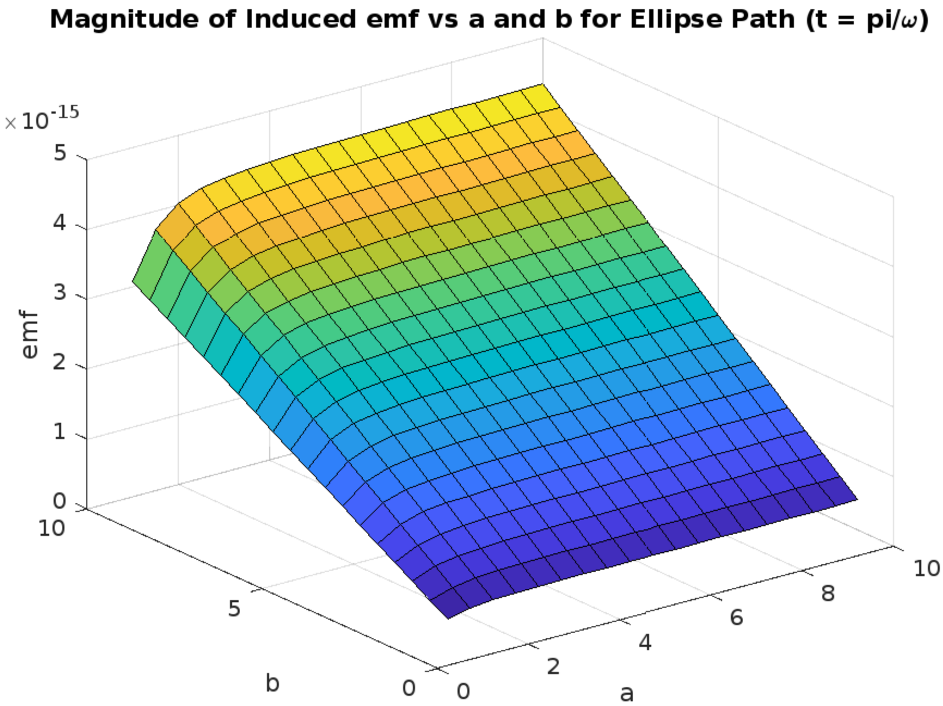

Figure 4: Magnitude of Induced EMF vs a and b for Elliptical Path at $t = \frac{\pi}{\omega}$ with $\omega = 1$.

It is required to plot the magnitude of E_x at different values of m and n (called different modes, which will be discussed next semester) versus x and y (contour plot). Let the frequency be 1 GHz, $a = 1$ cm, $b = 1$ cm, and $A = 1$. Fill in the following table:

$\lvert E_x \rvert$	$m = 1$	$m = 2$	$m = 3$
$n = 1$			
$n = 2$			
$n = 3$			

Table 5: Table to fill with computed values of $|E_x|$ for different modes.

MATLAB Implementation

The MATLAB implementation calculates and plots the magnitude of E_x for various modes (m, n) using the equations provided. The following steps are performed:

1. Define parameters for the waveguide $(a, b, \omega, \mu, \epsilon)$.
2. Compute the constants K_c, β, and K for each mode (m, n).
3. Calculate E_x over a 2D spatial grid of x and y.
4. Plot contour plots of $|E_x|$ for each combination of m and n.

```
% Constants
f = 1e9; % Frequency in Hz (1 GHz)
a = 1e-2; % Width of the waveguide in meters (1 cm)
b = 1e-2; % Height of the waveguide in meters (1 cm)
A = 1; % Amplitude
mu0 = 4 * pi * 1e-7; % Permeability of free space
epsilon0 = 8.854e-12; % Permittivity of free space
c = 3e8; % Speed of light in vacuum
```

```
omega = 2 * pi * f; % Angular frequency
k = omega / c; % Wave number in free space

% Spatial grid
x = linspace(0, a, 100); % x from 0 to a
y = linspace(0, b, 100); % y from 0 to b
[X, Y] = meshgrid(x, y);

% Modes to calculate
m_values = [1, 2, 3];
n_values = [1, 2, 3];

% Loop over modes and plot
figure;
for m = m_values
    for n = n_values
        % Calculate constants
        kc = sqrt((m * pi / a)^2 + (n * pi / b)^2); % Cutoff wave number
        beta = sqrt(k^2 - kc^2); % Propagation constant
        K = omega * sqrt(mu0 * epsilon0); % Wave number in medium

        % Calculate E_x
        Ex = -1j * beta / (kc^2) * A .* (m * pi / a) .* cos(m * pi * X / a) .* ...
            sin(n * pi * Y / b);

        % Magnitude of E_x
        Ex_magnitude = abs(Ex);

        % Plot contour
        subplot(length(m_values), length(n_values), (m-1)*length(n_values) + n);
        contourf(X, Y, Ex_magnitude, 20, 'LineColor', 'none');
        colorbar;
        colormap turbo; % Change colormap
        c = colorbar;
        c.Label.String = '|E_x| (Magnitude)';
        c.Label.FontSize = 10;
        title(['|E_x| for m = ' num2str(m) ', n = ' num2str(n)]);
        xlabel('x (m)', 'FontSize', 10);
        ylabel('y (m)', 'FontSize', 10);
    end
end

% Adjust figure layout
sgtitle('Magnitude of E_x for Different Modes', 'FontSize', 12);
```

Results

The contour plots of $|E_x|$ are generated for $m = 1, 2, 3$ and $n = 1, 2, 3$, showing the spatial variation of the electric field magnitude for different modes.

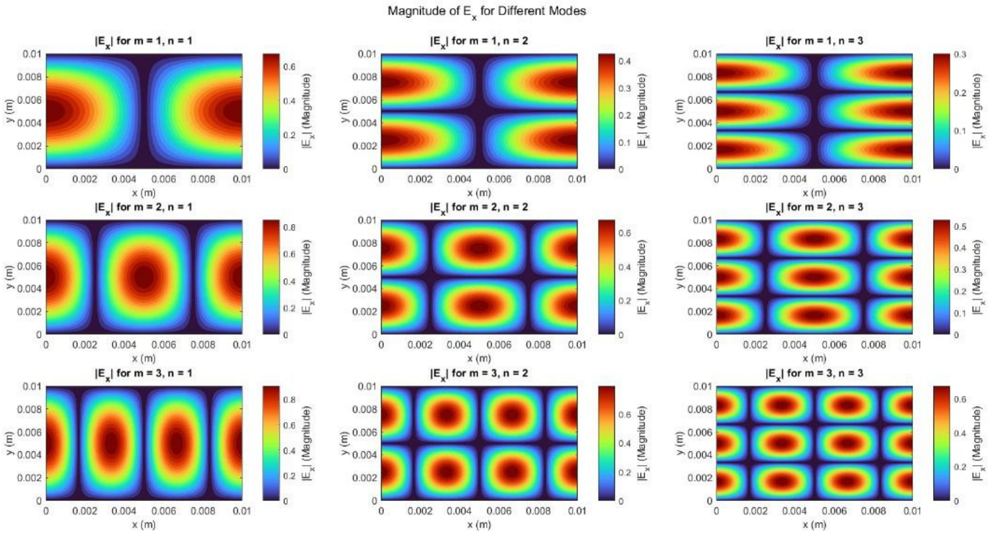

Figure 5: Contour plots of $|E_x|$ for various m and n modes.