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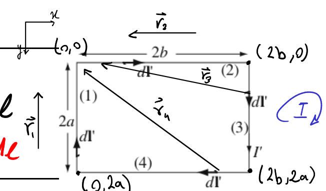
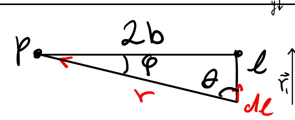
CIE 328 Assignment #3
($CLO_2$ & $CLO_3$ & $CLO_4$)

***Answer the following questions, assume any missing data:***

1. ***(Exercise 8.2, page 393)*** Consider again Example 8.4 (page 391) and calculate the magnetic field intensity at one of the corners of the loop in Figure 8.7a (page 392).




$$dH = \frac{I\, d\vec{l} \times \hat{r}}{4\pi r^2}, \quad H_1 = H_2 = 0,$$

$$dH_3 = \frac{I\, dl}{4\pi r^2}\sin\theta = \frac{I\, dl}{4\pi r^2}\cos\varphi = \frac{I\, \cancel{2b}\, \cancel{\sec^2\varphi}}{\cancel{4}\pi\,(\cancel{4}b^{\cancel{2}}\, \cancel{\sec^2\varphi})}\cos\varphi\, d\varphi$$

$8$

$$= \frac{I}{8\pi b}\cos\varphi \Rightarrow H = \frac{I}{8\pi b}\int_0^{\varphi_f}\cos\varphi\, d\varphi = \frac{I}{8\pi b}\sin\varphi_f = \frac{I}{8\pi b}\frac{a}{\sqrt{a^2+b^2}}$$

$\sin\varphi = \frac{l}{r}$

$\cos\varphi = \frac{2b}{r} \Rightarrow r = 2b\sec\varphi$

$\tan\varphi = \frac{l}{2b} \Rightarrow l = 2b\tan\varphi$

$\sin\varphi_f = \frac{a}{\sqrt{a^2+b^2}}$

$$\text{Similarly, } H_4 = \frac{I}{8\pi a}\frac{b}{\sqrt{a^2+b^2}}$$

$$H_t = \frac{I}{8\pi\sqrt{a^2+b^2}}\left(\frac{a}{b} + \frac{b}{a}\right) = \frac{I}{8\pi\cancel{\sqrt{a^2+b^2}}}\frac{(a^2+b^2)}{ab} = \boxed{\frac{I}{8\pi ab}\sqrt{a^2+b^2}}$$

2. Two semi-infinite filaments on the z axis lie in the regions $-\infty < z < -a$ and $a < z < \infty$. Each carries a current I in the $\mathbf{a}_z$ direction.
   A. Calculate H as a function of ρ and φ at z = 0.
   B. What value of "**a**" will cause the magnitude of **H** at ρ = 1, z = 0, to be one-half the value obtained for an infinite filament?

a)

$$dH_1 = \frac{I\, d\ell \times \hat{r}}{4\pi r^2} = \frac{I}{4\pi r^2}\sin\theta\, d\ell = \frac{I}{4\pi r^2}\cos\varphi\, d\ell$$

$$= \frac{-I}{4\pi r}\cos^2\varphi\, d\varphi = \frac{-I}{4\pi\rho}\cos^3\varphi\, d\varphi$$

$$H_1 = \int_{\varphi_0}^{\pi/2} \frac{-I}{4\pi\rho}\cos^3\varphi\, d\varphi = \frac{-I}{4\pi\rho}\left[\sin\varphi - \frac{\sin^3\varphi}{3}\right]$$

$\sin\varphi = \ell/r$

$\cos\varphi = \rho/r$

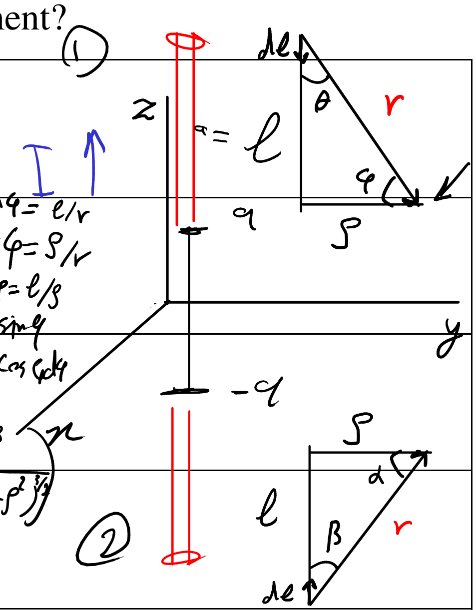
$\tan\varphi = \ell/\rho$

$\ell = r\sin\varphi$

$d\ell = -r\cos\varphi\, d\varphi$

$$\sin\varphi_0 = a/\sqrt{a^2+\rho^2} \Rightarrow H_1 = \frac{-I}{4\pi\rho}\left(\frac{a}{(a^2+\rho^2)^{1/2}} - \frac{a^3}{3(a^2+\rho^2)^{3/2}}\right)$$

$$\vec{H}_1 = \hat{\phi}\frac{I}{4\pi\rho}\left(\frac{a^3}{3(a^2+\rho^2)^{3/2}} - \frac{a}{(a^2+\rho^2)^{1/2}}\right)$$, Similarly is $H_2$




**H$_t$=** $\hat{\phi}\frac{I}{2\pi\rho}\left(\frac{a^3}{3(a^2+\rho^2)^{3/2}} - \frac{a}{(a^2+\rho^2)^{1/2}}\right)$

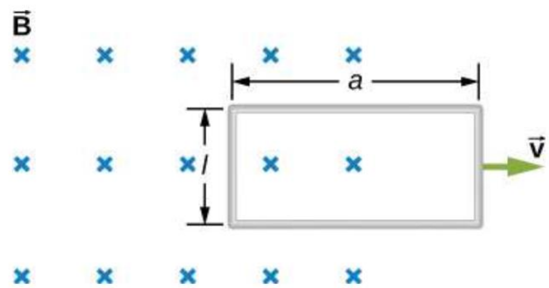
b)

$$\cancel{\frac{I}{2\pi\rho}}\left(\frac{a^3}{3(a^2+\rho^2)^{3/2}} - \frac{a}{(a^2+\rho^2)^{1/2}}\right) = 0$$

$$\frac{a^3}{3(a^2+1)^{3/2}} = \frac{a}{(a^2+1)^{1/2}} \Rightarrow a = 0$$

**a=** 0

3. The rectangular loop of $N$ turns shown below moves to the right with a constant velocity $\mathbf{v}$ while leaving the poles of a large electromagnet.
   A. Assuming that the magnetic field is uniform between the pole faces and negligible elsewhere, determine the induced emf in the loop.
   B. What is the type of this emf?

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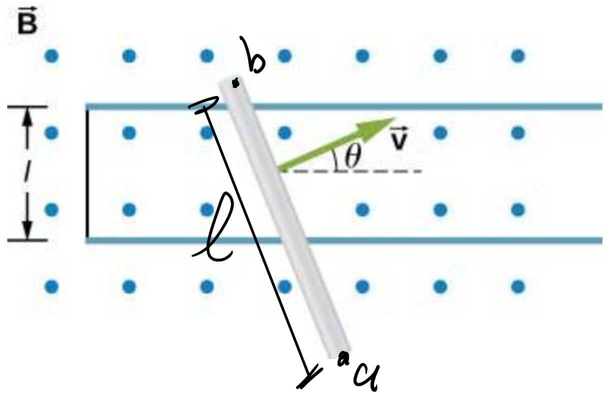
|---|---|
| a) | $\mathcal{E} = -N\frac{d\phi}{dt}$, $\phi = BA$, $A = \ell a = \ell x \Rightarrow \mathcal{E} = -NB\ell\cancel{\frac{dx}{dt}}^{\,v}$ |
| | $= NB\ell v$ |
| b) | Motional emf. |

4. As shown below, the rod slides along the conducting rails at a constant velocity **v**. the velocity is in the same plane as the rails and directed at an angle $\theta$ to them. A uniform magnetic field **B** is directed out of the page. What is the emf induced in the rod?




| |
|---|
| $\mathcal{E} = \int_a^b (V \times B)\, d\ell = \int_a^b VB\sin\theta\, d\ell = VBL\sin\theta$ |
| |
| |
| |
| **Emf=** $VBL\sin\theta$ |

5. ***(Exercise 11.2, page 582)*** The configuration in Example 11.5 (page 581) is given. The electric field intensity in material (1) is $\mathbf{E}_1(x,y,z,t) = k(\hat{\mathbf{x}} + \hat{\mathbf{y}}2 - \hat{\mathbf{z}}3)\cos 377t$ [V/m], where k is a constant. Calculate the electric field intensity and electric flux density in material (2). Assume there are no charges on the interface.

$$\Delta E_{\parallel} = 0, \quad \Delta D_{\perp} = 0 \Longrightarrow \epsilon_1 E_{1,\perp} = \epsilon_2 E_{2,\perp}$$

$$E_{1x} = k\cos 377t, \quad E_{1y} = 2k\cos 377t, \quad E_{1z} = -3k\cos 377t$$

$$E_2 = k(\hat{x} + 2\hat{y} - 3)\frac{\epsilon_1}{\epsilon_2}\sin(377t)$$

$$D_2 = k(\hat{x} + 2\hat{y} - 3)\,\epsilon_1 \sin(377t)$$

6. ***(Exercise 12.2, page 604)*** Obtain the homogeneous wave equation in terms of the electric flux density **D**.

$$\nabla \times E = \frac{-\partial B}{\partial t} = -\mu\frac{\partial H}{\partial t}, \quad \nabla \times H = \frac{\partial D}{\partial t}$$

$$-\nabla^2 H = \nabla \times \left(\frac{\partial D}{\partial t}\right) = -\mu\frac{\partial^2 H}{\partial t^2} \Longrightarrow \nabla^2 D - \mu\epsilon\frac{\partial^2 D}{\partial t^2} = 0$$

7. ***(Exercise 12.4, page 606)*** Find the time-harmonic, source-free, wave equation in terms of the electric scalar potential in lossless media.

$\nabla \cdot E = 0$

$\nabla \times E = -\frac{\partial B}{\partial t}$ , $B = \mu H$, $D = \varepsilon E$ $\Longrightarrow$ $E = -\nabla \phi - \cancel{\frac{\partial A}{\partial t}}$ (source-free)

$\nabla \times H = \frac{\partial D}{\partial t}$

$\Longrightarrow \nabla^2 \phi = 0$

$\vec{\phi} = \phi e^{-j\omega t}$ (time-harmonic)

$\Rightarrow \boxed{\nabla^2 \phi - \omega^2 \mu \epsilon \phi = 0}$