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Date: October 29, 2023

Linear Algebra and Vector Geometry (MATH 201)
Bonus 1

Prove that $(AB)^\intercal = B^\intercal A^\intercal$

Proof. Let A be an $m \times n$ matrix and B be an $n \times p$ matrix.

$$A_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \tag{1}$$

$$B_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{p} b_{ij}. \tag{2}$$

Using the definition of matrix multiplication:

$$AB_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}. \tag{3}$$

Using definition of transpose:

$$(AB)^\intercal_{ij} = AB_{ji} \tag{4}$$

$$= \sum_{k=1}^{n} a_{jk} b_{ki}. \tag{5}$$

Then:

$$(B^\intercal A^\intercal)_{ij} = \sum_{k=1}^{n} b^\intercal_{ik} a^\intercal_{kj} \tag{6}$$

$$= \sum_{k=1}^{n} b_{ki} a_{jk} \tag{7}$$

$$= \sum_{k=1}^{n} a_{jk} b_{ki}. \tag{8}$$

From (5) and (8) we conclude that $(AB)^\intercal = B^\intercal A^\intercal$. $\square$

Notice that $(AB)^\intercal \neq A^\intercal B^\intercal$.