(cartesian)

$dv = dx\,dy\,dz$

$d\vec{l} = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$

$d\vec{S}_x = (\hat{x})\,dy\,dz$

$d\vec{S}_y = (\hat{y})\,dx\,dz$

$d\vec{S}_z = (\hat{z})\,dx\,dy$

cylendrical

$dv = dr\,(r\,d\phi)\,dz$

$d\vec{l} = dr + r\hat{\phi}d\phi + \hat{z}dz$

$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\phi}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z}$

$d\vec{S}_r = (\hat{r})(r\,d\phi\,dz)$

$d\vec{S}_\phi = (\hat{\phi})(dr\,dz)$

$d\vec{S}_z = (\hat{z})(r\,d\phi\,dr)$

spherical (θ = angle w/ z, ϕ = angle w/ x)

$dv = dr\,(r\,d\theta)(r\sin\theta\,d\phi)$

$d\vec{l} = \hat{r}dr + r\hat{\theta}d\theta + r\sin\theta\,\hat{\phi}\,d\phi$

$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \frac{1}{r}\hat{\theta}\frac{\partial}{\partial \theta} + \frac{1}{r\sin\theta}\hat{\phi}\frac{\partial}{\partial \phi}$

$d\vec{S}_r = (\hat{r})\,((r\sin\theta)d\phi\,(r\,d\theta))$

$d\vec{S}_\phi = (\hat{\phi})\,(r\,d\theta\,dr)$

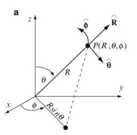
$d\vec{S}_\theta = (\hat{\theta})\,(r\sin\theta\,d\phi\,dr)$

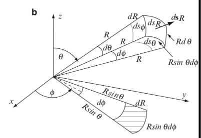
Electric Dipole ($\vec{P}$)

$\vec{P} = Q\vec{d}$

$\vec{E}_d \approx \left(\frac{1}{4\pi\varepsilon_0 R^3}\right)\left(3\frac{\vec{R}\cdot\vec{P}}{R^2}\vec{R} - \vec{P}\right)$

$\vec{d} = \hat{z}$ if charges on z-axis





cyl Vol $= 2\pi r^2 h$

cyl SA $= 2\pi r h + 2\pi r^2$

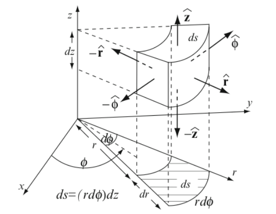
sph Vol $= \frac{4}{3}\pi r^3$

sph SA $= 4\pi r^2$

$\varepsilon_0 = 8.854 \times 10^{-12}$

sph $\to \theta\ \{0 \to \pi\}$, $\phi\ \{0 \to 2\pi\}$

cyl $\to \phi\ \{0 \to 2\pi\}$



Point charges: $\vec{F} = \left(\frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}\right)\hat{r} = \left(\frac{Q Q_2}{4\pi\varepsilon_0 r^3}\right)\vec{r}_{sf} = Q\vec{E} = \frac{QD}{\varepsilon}$; $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{Q}{4\pi\varepsilon_0 r^3}\vec{r}_{sf}$; $\varepsilon = \varepsilon_0\varepsilon_r$

line charge: $d\vec{E} = \frac{(\vec{r}_{point} - \vec{r}_{line})\rho_L\,dL}{4\pi\varepsilon_0\|\vec{r}_{point} - \vec{r}_{line}\|^3}$; $\vec{E} = \int d\vec{E} \Rightarrow$ $\left[\vec{E}_{fl} = \hat{r}_\perp\left(\frac{\rho_L a\,d}{(4\pi\varepsilon_0)a^2(a^2+d^2)^{1/2}}\right)\Big|_{l_2}^{l_1}\right.$; $\left.\vec{E}_{il} = \frac{\rho_L}{2\pi\varepsilon_0 a}\hat{r}\right] \Rightarrow$ a = distance to point, $\hat{r}$ = shortest vect to point, Z = axis line is on

r_{pnt} = point to measure at; finite line on d axis (cartesian); infinite line

r_{line} = inf. small point of line $\Rightarrow \vec{r}_{line} = (x\hat{x} + y\hat{y} + z\hat{z})$; $\vec{R}_{sf} = \vec{R}_p - \vec{R}_l$

surface charges: $d\vec{E} = \frac{\rho_s\,ds\,\vec{r}_{sf}}{4\pi\varepsilon_0\|\vec{r}_{sf}\|^3}$

Volume charges: $d\vec{E} = \frac{\rho_v\,dV\,\vec{r}_{sf}}{4\pi\varepsilon_0\|\vec{r}_{sf}\|^3}$

Polarization $\Rightarrow P = D - \varepsilon_0 E$

$\rho_{pv} = -\vec{\nabla}\cdot\vec{P}$ Dielec pol. vol density

$\rho_{psb} = \vec{P}\cdot\hat{a}_n$ Dielec pol sur density

flux density: $\vec{D} = \varepsilon\vec{E} = \frac{Q\,\vec{R}}{4\pi\|R\|^3}$ | flux $\Rightarrow \int\vec{D}\cdot d\vec{S} = Q_{enc}$ | $\vec{\nabla}\times\vec{E} = 0 \equiv \oint_c \vec{E}\cdot d\vec{l} = 0$

Gauss' law: $\oint_s E\cdot d\vec{S} = \frac{Q}{\varepsilon_0} \equiv \vec{\nabla}\cdot\vec{E} = \frac{\rho_v}{\varepsilon_0} \Leftrightarrow \int_v \vec{\nabla}\vec{E}\,dV = \frac{1}{\varepsilon_0}\int_v \rho_v\,dV$ | $\oint\vec{D}\cdot ds = Q \Leftrightarrow D\cdot A = Q_{enc}$

Work & pot: (voltage) $W = -Q\int_a^b \vec{E}\cdot d\vec{L} = W_b - W_a \Rightarrow V_{ba} = \frac{W_{ba}}{Q} = -\int_a^b \vec{E}\cdot d\vec{l}$ | Point charge: $V_{ba} = -\int_{R_a}^{R_b}\frac{Q}{4\pi\varepsilon R^2}\,dR = \frac{Q}{4\pi\varepsilon R}\Big|_a^b = V_b - V_a$

$\vec{E} = -\vec{\nabla}V \Rightarrow V = ED$

interface conditions: $E_{1t} = E_{2t}$ & $D_{1n} - D_{2n} = \rho_s$ — Dielec & Dielec w/ charged surface | $E_{1t} = E_{2t} = D_{1n} = D_{2n} = 0$ — Dielec & conductor | θ_1, ε_1, θ_2, ε_2 ; $\tan\theta_1 = \frac{E_{1t}}{E_{1n}}$, $\tan\theta_2 = \frac{E_{2t}}{E_{2n}}$ $\Rightarrow \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2}$

Capacitance: $C = Q/V = \left(\frac{\varepsilon A}{D}\right)$ (Parallel plate) $= \int_S \frac{1}{\int_L \frac{dL_{\parallel}}{\varepsilon\,dS_\perp}} \Rightarrow$ $dL_{\parallel}$ = paralell to EF $\Rightarrow$, $dS_\perp$ = Perp to EF $\Rightarrow$

example: $dL_{\parallel} = dR$, $dS_\perp = R^2\sin\theta\,d\theta\,d\phi$

Parallel cap $\Rightarrow C_t = C_1 + C_2 \cdots$

Series cap $\Rightarrow \frac{1}{C_t} = \frac{1}{C_1} + \frac{1}{C_2} \cdots$

Work pt2: $W = QV = \frac{CV^2}{2} = \frac{QV}{2} = \frac{Q^2}{2C} \Rightarrow \frac{1}{2}\sum Q_i V_i \Leftrightarrow \frac{1}{2}\int_L \rho_L\,dl \Leftrightarrow \frac{1}{2}\int_S \rho_s\,ds$ | $W = \frac{1}{2}\int D\cdot E\,dv = \frac{1}{2}\int_v \varepsilon E^2\,dV$

W_E = work density $= \frac{\rho_v V}{2} = \frac{D^2}{2\varepsilon} = \frac{D\cdot(\nabla V)}{2} = \frac{1}{2}\vec{D}\vec{E} = \frac{1}{2}\varepsilon_0\vec{E}^2 \Rightarrow \vec{E} = \frac{F}{Q} = \frac{-\vec{\nabla}W_e}{Q} \Rightarrow W = \int_v W_E\,dV$

↳ energy density; ↳ total energy stored

Common integrals or EF calculations: $\int\frac{1}{(a^2 \pm x^2)^{3/2}}dx = \frac{x}{a^2\sqrt{a^2 \pm x^2}}$; $\int\frac{x\,dx}{(a^2+x^2)^{3/2}} = \frac{-1}{\sqrt{a^2+x^2}}$; $\int\frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$\vec{E}_{inf\,plane} = \frac{\rho_s}{2\varepsilon_0}\hat{n}$ | $\vec{E}_{inf\,line} = \frac{\rho_L}{2\pi\varepsilon_0 d}$ | $\int\frac{x^2\,dx}{(a^2+x^2)^{3/2}} = \frac{-x}{\sqrt{a^2+x^2}} + \ln(x + \sqrt{a^2+x^2})$

Rectangular Coordinates

$$\nabla\Phi = \frac{\partial\Phi}{\partial x}\mathbf{a}_x + \frac{\partial\Phi}{\partial y}\mathbf{a}_y + \frac{\partial\Phi}{\partial z}\mathbf{a}_z$$

$$\nabla\cdot\mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla\times\mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{a}_z$$

Cylindrical Coordinates

$$\nabla\Phi = \frac{\partial\Phi}{\partial\rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial\Phi}{\partial\phi}\mathbf{a}_\phi + \frac{\partial\Phi}{\partial z}\mathbf{a}_z$$

$$\nabla\cdot\mathbf{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial F_\phi}{\partial\phi} + \frac{\partial F_z}{\partial z}$$

$$\nabla\times\mathbf{F} = \left(\frac{1}{\rho}\frac{\partial F_z}{\partial\phi} - \frac{\partial F_\phi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial\rho}\right)\mathbf{a}_\phi + \left(\frac{1}{\rho}\frac{\partial(\rho F_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial F_\rho}{\partial\phi}\right)\mathbf{a}_z$$

Spherical Coordinates

$$\nabla\Phi = \frac{\partial\Phi}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}\mathbf{a}_\phi$$

$$\nabla\cdot\mathbf{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(F_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{F} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(F_\phi\sin\theta) - \frac{\partial F_\theta}{\partial\phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial}{\partial r}(rF_\phi)\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial\theta}\right]\mathbf{a}_\phi$$

Poisson & Laplace

$\vec{E} = -\nabla V \Rightarrow \nabla^2 V = \frac{-\rho_v}{\varepsilon_0} \xrightarrow{\rho_v = 0} \nabla^2 V = 0$

$$\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2} = -\frac{\rho_v}{\varepsilon}$$

$I = \int_S \vec{J}\cdot d\vec{A} = -\frac{dQ}{dt}$; $\vec{J} = \sigma\vec{E}$; $V = IR$

$P = \vec{F}\cdot\vec{v} = \int_V \vec{E}\cdot\vec{J}\,dV$; $\nabla\cdot\vec{J} = -\frac{\partial\rho_v}{\partial t}$

$\oint_S \vec{J}\cdot d\vec{A} = 0$; $\frac{\partial\rho_v}{\partial t} + \frac{\sigma}{\varepsilon}\rho_v = 0$; $\oint_S \frac{\vec{J}\cdot d\vec{\ell}}{\sigma}\,dA = 0$

$\nabla\times\frac{\vec{J}}{\sigma} = 0$; $V_b = \oint_c \frac{\vec{J}}{\sigma}\cdot d\vec{\ell}$