ESM204 Assignment 3

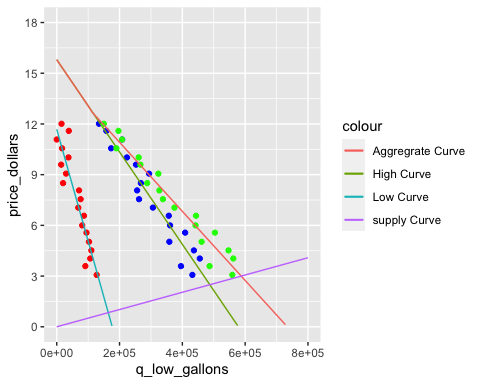
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#readin Data  
cost\_gas\_data<- read\_csv("assign\_3\_data.csv") %>%   
 clean\_names()

# Linear Regression Low Cost Consumer   
lm\_low <- lm(price\_dollars ~ q\_low\_gallons, data=cost\_gas\_data)   
#print(lm\_low) # 1.169e+01 + -6.611e-05\*q  
# Linear Regression High Cost Consumer  
lm\_high <- lm(price\_dollars ~ q\_high\_gallons, data=cost\_gas\_data)   
#print(lm\_high) # 1.580e+01 + -2.731e-05\*q  
# Linear Regression Aggregate Cost Consumer  
lm\_agg <- lm(price\_dollars ~ q\_aggregrate, data=cost\_gas\_data)   
#print(lm\_agg) # 1.500e+01 + -2.043e-05\*q

# Demand Curve Coefficients based on linear models  
I1<- lm\_low$coefficients[1]   
I2<- lm\_high$coefficients[1]  
S1<- lm\_low$coefficients[2]  
S2<- lm\_high$coefficients[2]  
  
# functional definition of high and low demand based on linear model  
low\_demand <- function(q) I1 + (S1\*q)  
high\_demand <- function(q) I2 + (S2\*q)  
  
# Determines the 'dominate' curve. This only works for linear models  
demand\_dom <- function(q){  
 if (I1==I2){0}  
 else if(I1>I2){low\_demand(q)}  
 else if(I2>I1){high\_demand(q)}  
}  
# Determines q such that high demand and low demand are equal. Used to find  
# the kink in the curve.   
intersect\_point <- abs(uniroot(function(q){high\_demand(q)-low\_demand(q)},  
 interval = c(0,1),extendInt = "yes")$root)  
  
# functional defintion of aggregate demand   
# the function itself. When q is less than the interset, returns the 'dominant'  
agg\_demand <- function(q) {ifelse(q<intersect\_point,demand\_dom(q),  
 lm\_agg$coefficients[1] + (lm\_agg$coefficients[2]\*q))  
}  
  
demand\_dom\_x\_intersect <- uniroot(demand\_dom,  
 interval = c(0,1),  
 extendInt = "yes")$root  
# finding the supply curve requires us to find the point at which the agg curve the price is   
# equal to three. We know the current price is untaxed and equal to three.  
current\_price <- 3  
supply\_agg\_intersect <- uniroot(function(q){agg\_demand(q)-current\_price}, interval = c(0,1), extendInt = "yes")$root  
supply\_slope <- current\_price/supply\_agg\_intersect  
# functional definition of Supply  
supply <- function(q){q\*supply\_slope}   
  
#functional definition of environmental costs  
local\_env\_cost\_curve<- function(q) 1.5 + (.0000\*q) # local MEC $ 1.50  
global\_env\_cost\_curve<- function(q) .50 + (.0000\*q) # global MEC $.50  
total\_env\_cost\_curve<- function(q) 2+ (.0000\*q)# total MEC $.50

ggplot()+  
 geom\_point(data = cost\_gas\_data, aes(x= q\_low\_gallons, y = price\_dollars),color="red")+  
 stat\_function(aes(x=0, color = "Low Curve"), fun = low\_demand)+   
 geom\_point(data = cost\_gas\_data, aes(x= q\_high\_gallons, y = price\_dollars),color="blue")+  
 stat\_function(aes(x=0, color = "High Curve"), fun = high\_demand)+  
 geom\_point(data = cost\_gas\_data, aes(x= q\_aggregrate, y = price\_dollars),color="green")+  
 stat\_function(aes(x=0, color = "Aggregrate Curve"), fun = agg\_demand)+  
 stat\_function(aes(x=0, color = "supply Curve"), fun = supply) +  
 xlim(0,800000)+   
 scale\_y\_continuous(breaks=c(0,3,6,9,12,15,18),limits = c(0,18))

 1. Answer the following Questions:

1. What is the aggregate daily demand curve for gasoline?
2. What is the supply curve for gasoline?
3. What is the “benefit” to consumers under the status quo?

# find intersect of aggregate curve at P = $3.00 Q = 587482.43  
agg\_curve\_intersect <- uniroot(function(q){agg\_demand(q)-supply(q)},interval = c(0,1),extendInt = "yes")$root  
  
# integrate to find total area under agg demand curve   
gross\_consumer\_benefit <- integrate(agg\_demand,lower=0,upper=agg\_curve\_intersect)$value  
  
# integrate under the supply curve  
producer\_benefit <- integrate(supply,lower=0,upper=agg\_curve\_intersect)$value  
   
# subtract gross\_consumer and producer to find net consumer benefit  
net\_consumer\_benefit <- gross\_consumer\_benefit - producer\_benefit

Consumer Benefit

1. What is the “benefit” to producers under the status quo?

Producer Benefit

1. What is the environmental cost under the status quo (locally and in the rest of the world)?

# integrate under each MEC from 0 to the agggrate curve intersect with supply curve  
local\_env\_cost <- integrate(local\_env\_cost\_curve, 0, agg\_curve\_intersect)$value # 881223.6  
global\_env\_cost <- integrate(global\_env\_cost\_curve, 0, agg\_curve\_intersect)$value # 293741.2  
total\_env\_cost <- integrate(total\_env\_cost\_curve, 0, agg\_curve\_intersect)$value # 1174965

Local Environmental Cost

Global Environmental Cost

Total Environmental Cost

1. How is the current consumer benefit divided between “High” and “Low” income consumers?

high\_curve\_intersect <- uniroot(function(q){high\_demand(q)-supply(q)},interval = c(0,1),extendInt = "yes")$root  
low\_curve\_intersect <- uniroot(function(q){low\_demand(q)-supply(q)},interval = c(0,1),extendInt = "yes")$root  
  
high\_gross\_consumer\_benefit <- integrate(high\_demand,lower=0,upper=high\_curve\_intersect)$value  
low\_gross\_consumer\_benefit <- integrate(low\_demand,lower=0,upper=low\_curve\_intersect)$value

High Consumer Benefit

Low Consumer Benefit

1. A gas tax of $1.00/gal. is proposed. What would be the effects of this tax on:
2. The amount of gasoline produced and consumed.

# Defining the tax rate  
tax <- 1  
#functional supply curve with added tax  
supply\_tax <- function(q){supply(q)+tax}  
  
#Agg curve intersect with new tax supply curve  
agg\_tax\_curve\_intersect <- uniroot(function(q){agg\_demand(q)-supply\_tax(q)},interval = c(0,1),extendInt = "yes")$root

Quantity Produced and consumed

1. The price of gasoline.

tax\_price <- supply\_tax(agg\_tax\_curve\_intersect)

Quantity Produced and consumed $=

1. Welfare of “High” income consumers.

# New curve intersects for high and low consumers  
high\_tax\_curve\_intersect <- uniroot(function(q){high\_demand(q)-supply\_tax(q)},interval = c(0,1),extendInt = "yes")$root  
low\_tax\_curve\_intersect <- uniroot(function(q){low\_demand(q)- supply\_tax(q)},interval = c(0,1),extendInt = "yes")$root  
  
# Total consumer benefit under new tax paradigm  
high\_gross\_consumer\_welfare\_tax <- integrate(high\_demand,lower=0,upper=high\_tax\_curve\_intersect)$value  
low\_gross\_consumer\_welfare\_tax <- integrate(low\_demand,lower=0,upper=low\_tax\_curve\_intersect)$value  
  
# Total Consumer taxes, incase the welfare is total benefit minus taxes...  
high\_tax\_total <- high\_tax\_curve\_intersect  
low\_tax\_total <- low\_tax\_curve\_intersect

High income consumer welfare

1. Welfare of “Low” income consumers.

Low income consumer welfare

1. Welfare of gas producers.

# Producer welfare  
producer\_tax\_welfare <- integrate(supply,lower=0,upper=agg\_tax\_curve\_intersect)$value

Producer consumer welfare

1. Local environmental damage.

# integrate under each MEC from 0 to the agggrate curve intersect with supply curve  
local\_env\_cost\_tax <- integrate(local\_env\_cost\_curve, 0, agg\_tax\_curve\_intersect)$value   
global\_env\_cost\_tax <- integrate(global\_env\_cost\_curve, 0, agg\_tax\_curve\_intersect)$value  
total\_env\_cost\_tax<- integrate(total\_env\_cost\_curve, 0, agg\_tax\_curve\_intersect)$value

Local Environmental Cost with tax

1. Rest of world environmental damage.

Global Environmental Cost with tax

1. Total revenue generated by the tax.

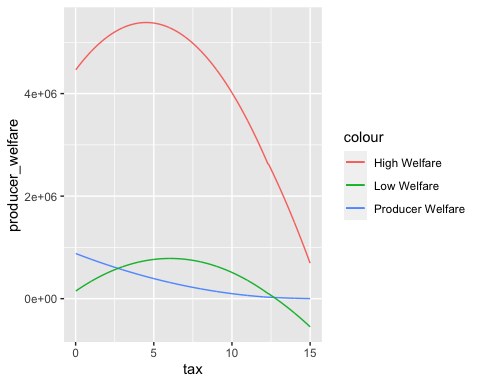
# multiply the tax by the quantity produced  
tax\_revenue <- tax\*agg\_tax\_curve\_intersect

Total tax revenue

1. Now, assume that all revenue from a tax will be redistributed to the two groups in proportion to their pre-tax consumption of gas. For example, if 80% of the gas was consumed by High income consumers, then they get 80% of the tax revenue. Also assume that “Low” income consumers bear all local environmental costs. For a range of gas taxes (ranging from $0 - $5.00/gal), calculate the effects of the tax on:
2. Overall welfare of “High” income consumers

# The overall welfare consumers will change such that:  
# - the tax in proportion to pre-tax gas use is added to the welfare  
# - the local environmental cost for the tax based consumption is subtracted from the low income users  
# To do this, we need to find:  
# - the proportion of gas consumption of high and low users  
  
low\_curve\_intersect <- uniroot(function(q){low\_demand(q)-supply(q)},interval = c(0,1),extendInt = "yes")$root  
high\_curve\_intersect <- uniroot(function(q){high\_demand(q)-supply(q)},interval = c(0,1),extendInt = "yes")$root  
low\_prop <- low\_curve\_intersect/agg\_curve\_intersect   
high\_prop <- high\_curve\_intersect/agg\_curve\_intersect   
  
# - the aggregate quantity consumed for a given tax rate to be used by finding local env cost  
# - the tax revenue for a given tax rate to find the tax redistribution  
#   
# To do this lets make a function where tax is the variable t. Then we will construct a dataframe   
# which calculates between the range of 0-5, then we can plot it. We will need a function for both  
# the high and low consumers  
  
  
tax\_fun <- function(t) {  
   
 #find the new aggregate curve intersect based on the tax  
 agg\_tax\_curve\_intersect\_var <- uniroot(function(q){agg\_demand(q)-(supply(q)+t)},interval = c(0,1),extendInt = "yes")$root  
 high\_tax\_curve\_intersect\_var <- uniroot(function(q){high\_demand(q)-(supply(q)+t)},interval = c(0,1),extendInt = "yes")$root  
 low\_tax\_curve\_intersect\_var <- uniroot(function(q){low\_demand(q)-(supply(q)+t)},interval = c(0,1),extendInt = "yes")$root  
   
 #tax times the quantity  
 total\_tax\_revenue <- t\*agg\_tax\_curve\_intersect\_var  
   
 #redistribution of taxes  
 redis\_low <- low\_prop \* total\_tax\_revenue  
 redis\_high <- high\_prop \* total\_tax\_revenue  
   
 local\_env\_cost\_tax\_var <- integrate(local\_env\_cost\_curve,lower=0,upper=agg\_tax\_curve\_intersect\_var)$value  
   
 high\_welfare\_var <- integrate(high\_demand,lower=0,upper=high\_tax\_curve\_intersect\_var)$value + redis\_high  
 low\_welfare\_var <- integrate(low\_demand,lower=0,upper=low\_tax\_curve\_intersect\_var)$value +redis\_low -local\_env\_cost\_tax\_var  
 producer\_welfare\_var <- integrate(supply,lower=0,upper=agg\_tax\_curve\_intersect\_var)$value  
   
 return(data.frame(tax=t,high\_welfare = high\_welfare\_var, low\_welfare = low\_welfare\_var, producer\_welfare = producer\_welfare\_var))  
   
 }  
   
tax\_df <- purrr::map\_dfr(seq(from=0.01,to=15,by=0.01),tax\_fun)

ggplot()+  
 geom\_line(data=tax\_df,aes(x=tax,y=producer\_welfare,color="Producer Welfare"))+  
 geom\_line(data=tax\_df,aes(x=tax,y=high\_welfare,color="High Welfare"))+  
 geom\_line(data=tax\_df,aes(x=tax,y=low\_welfare,color="Low Welfare"))



1. Overall welfare of “Low” income consumers
2. Gas producers

Simone

1. A new electric car technology is invented and it lowers the demand curves of all income groups by half (vertically). Under these new demand curves, what are the effects on:

# assert new demand curves divided by 50 vertically   
I1\_h<- lm\_low$coefficients[1]/2  
I2\_h<- lm\_high$coefficients[1]/2  
  
low\_demand\_h <- function(q) I1\_h + S1\*q  
high\_demand\_h<- function(q) I2\_h + S2\*q

# Determines the 'dominate' curve for new curves.   
demand\_dom\_h <- function(q){  
 if (I1\_h==I2\_h){0}  
 else if(I1\_h>I2\_h){low\_demand\_h(q)}  
 else if(I2\_h>I1\_h){high\_demand\_h(q)}  
}  
# Determines q such that the new high demand and low demand are equal. #Used to find the kink in the curve.   
intersect\_point\_h <- abs(uniroot(function(q){high\_demand\_h(q)-low\_demand\_h(q)},  
 interval = c(0,1),extendInt = "yes")$root)  
  
# functional defintion of new aggregate demand   
# the function itself. When q is less than the interset, returns the 'dominant'  
agg\_demand\_h <- function(q) {ifelse(q<intersect\_point\_h,demand\_dom\_h(q),  
 lm\_agg$coefficients[1]/2 + (lm\_agg$coefficients[2]\*q))  
}  
  
demand\_dom\_x\_intersect\_h <- uniroot(demand\_dom\_h,  
 interval = c(0,1),  
 extendInt = "yes")$root

Aggregate Curve when Demand is decreased by 50%

# find intersect of high & Low curves at P = $3.00   
high\_curve\_intersect\_h <- uniroot(function(q){high\_demand\_h(q)-supply(q)},interval = c(0,1),extendInt = "yes")$root  
  
low\_curve\_intersect\_h <- uniroot(function(q){low\_demand\_h(q)-supply(q)},interval = c(0,1),extendInt = "yes")$root

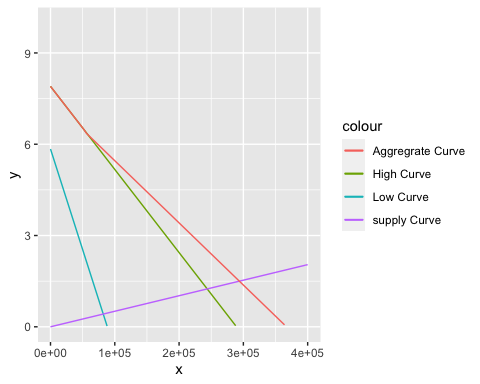
1. Gas consumption by “High” income consumers
2. Gas consumption by “Low” income consumers
3. Gas price

#New Agg curve intersect with supply curve P = 440637.96  
agg\_curve\_intersect\_h <- uniroot(function(q){agg\_demand\_h(q)-supply(q)},interval = c(0,1),extendInt = "yes")$root

# integrate under each MEC from 0 to the new agggrate curve intersect with supply curve  
local\_env\_cost\_h <- integrate(local\_env\_cost\_curve, 0, agg\_curve\_intersect\_h)$value  
  
global\_env\_cost\_h <- integrate(global\_env\_cost\_curve, 0, agg\_curve\_intersect\_h)$value

1. Local environmental damage from gasoline
2. Rest of world environmental damage from gasoline

# Visualize  
ggplot()+  
 stat\_function(aes(x=0, color = "Low Curve"), fun = low\_demand\_h)+  
 stat\_function(aes(x=0, color = "High Curve"), fun = high\_demand\_h)+  
 stat\_function(aes(x=0, color = "supply Curve"), fun = supply)+ stat\_function(aes(x=0, color = "Aggregrate Curve"), fun = agg\_demand\_h)+   
xlim(0,400000)+   
 scale\_y\_continuous(breaks=c(0,3,6,9,12,15,18),limits = c(0,10))



1. Compare two situations:
2. Total quantity of gasoline consumed with gas tax of $1/gal. but no electric car technology$=`rcomma(agg\_tax\_curve\_intersect)`$
3. Total quantity of gasoline consumed with no gas tax but with electric car technology$=`rcomma(agg\_curve\_intersect\_h)`$

consumption\_difference<- agg\_curve\_intersect\_h - agg\_curve\_intersect  
# intercept w/ electric cars = 293,741; is smaller than intercept without electric cars

Determine what value of tax makes the local environmental quality equal between these two situations. Or in other words, what tax will lower the Q\* for the no electric cars scenario to the Q\* in the electric car scenario. When the quantity consumer is the same, the environmental cost will also be equal.

# Find Price from no electric car aggregate curve when Q\*= 293,741 electric car Q\*  
p\_tax\_increase <- agg\_demand(agg\_curve\_intersect\_h)

# Solve p = supply()+ tax; for tax by rearranging now that we know our P\* and Q\* in the no electric cars scenario   
increase\_tax<- p\_tax\_increase- supply\_slope\*agg\_curve\_intersect\_h  
supply\_increase\_tax <- function(q){supply(q)+increase\_tax}  
  
#Agg curve intersect with adjusted tax supply curve  
agg\_tax\_increase\_curve\_intersect <- uniroot(function(q){agg\_demand(q)-supply\_increase\_tax(q)},interval = c(0,1),extendInt = "yes")$root  
  
# Check that environmental cost is equal for both scenarios with new\_tax  
#Environmental Without electric cars + new tax   
local\_env\_cost\_increase\_tax <- integrate(local\_env\_cost\_curve, 0, agg\_tax\_increase\_curve\_intersect)$value   
  
# 440611.82

1. Local environmental damage from gasoline
2. Rest of world environmental damage from gasoline

ggplot()+  
 stat\_function(aes(x=0, color = "Low Curve"), fun = low\_demand\_h)+  
 stat\_function(aes(x=0, color = "High Curve"), fun = high\_demand\_h)+  
 stat\_function(aes(x=0, color = "Supply Curve"), fun = supply)+ stat\_function(aes(x=0, color = "Aggregrate Curve"), fun = agg\_demand\_h)+   
xlim(0,400000)+   
 scale\_y\_continuous(breaks=c(0,3,6,9,12,15,18),limits = c(0,10))

