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Assignment: CS325 HW6

1. *Shortest Path using LP. Using linear programming to answer the questions below. Submit a copy of the LP code and output*

   a. *Find the distance of the shortest path from G to C*

   I originally was going to try and complete this function in Matlabs but in reading the documentation I found it a bit confusing and honestly a little difficult. As a result I entered the entered the values of the edges in LINDO as shown in video 3. Below is my input code:

```
max dc
ST
    dg = 0
    dd - dg <= 2
    dh - dg <= 3
    da - dh <= 4
    dg - de <= 7
    db - dh <= 9
    de - db <= 10
    db - da <= 8
    dd - de <= 9
    dd - dc <= 3
    df - dd <= 18
    dc - db <= 4
    de - dd <= 25
    dc - df <= 3
    db - df <= 7
    df - da <= 10
    da - df <= 5
    de - df <= 2
END
```

   And below was my output code:

```
 LP OPTIMUM FOUND AT STEP      5

       OBJECTIVE FUNCTION VALUE

       1)      16.00000

  VARIABLE        VALUE          REDUCED COST
```

DC	16.000000	0.000000
DG	0.000000	0.000000
DD	0.000000	0.000000
DH	3.000000	0.000000
DA	4.000000	0.000000
DE	16.000000	0.000000
DB	12.000000	0.000000
DF	14.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	2.000000	0.000000
4)	0.000000	1.000000
5)	3.000000	0.000000
6)	23.000000	0.000000
7)	0.000000	1.000000
8)	6.000000	0.000000
9)	0.000000	0.000000
10)	25.000000	0.000000
11)	19.000000	0.000000
12)	4.000000	0.000000
13)	0.000000	1.000000
14)	9.000000	0.000000
15)	1.000000	0.000000
16)	9.000000	0.000000
17)	0.000000	0.000000
18)	15.000000	0.000000
19)	0.000000	0.000000

NO. ITERATIONS= 5

Answer: the shortest distance from G -> C is 16.

b. Find the distances of the shortest paths from G to all other vertices

As suggested in video 3 of the lectures, I used LINDO to enter the values to find the "max" distance from G to all the remaining vertices in the graph. Below is my input code:

(NOTE: the only value we do not insert in the "max" command is the value of dg because we are starting in vertex G.

Input Code:

```
max dc + da + db + dd + de + df
ST
     dg = 0
     dd - dg <= 2
     dh - dg <= 3
     da - dh <= 4
     dg - de <= 7
     db - dh <= 9
     de - db <= 10
     db - da <= 8
     dd - de <= 9
     dd - dc <= 3
     df - dd <= 18
     dc - db <= 4
     de - dd <= 25
     dc - df <= 3
     db - df <= 7
     df - da <= 10
     da - df <= 5
     de - df <= 2
END
```

Below is the output code that shows the shortest paths from G -> to other vertices :

```
 LP OPTIMUM FOUND AT STEP      8

      OBJECTIVE FUNCTION VALUE

      1)      73.00000

 VARIABLE        VALUE          REDUCED COST
      DC        16.000000         0.000000
      DA         7.000000         0.000000
      DB        12.000000         0.000000
      DD         2.000000         0.000000
      DE        19.000000         0.000000
      DF        17.000000         0.000000
      DG         0.000000         0.000000
      DH         3.000000         0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
      2)         0.000000         6.000000
```

3)	0.000000	1.000000
4)	0.000000	5.000000
5)	0.000000	3.000000
6)	26.000000	0.000000
7)	0.000000	2.000000
8)	3.000000	0.000000
9)	3.000000	0.000000
10)	26.000000	0.000000
11)	17.000000	0.000000
12)	3.000000	0.000000
13)	0.000000	1.000000
14)	8.000000	0.000000
15)	4.000000	0.000000
16)	12.000000	0.000000
17)	0.000000	2.000000
18)	15.000000	0.000000
19)	0.000000	1.000000

NO. ITERATIONS= 8

Answer:

G -> C = 16 (as noted in answer for 1a.)
G-> A = 7
G-> B = 12
G-> D = 2
G-> E = 19
G-> F = 17
G-> G = 0
G -> H = 3

2. *Product Mix: Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is maximize profit:*

   Profit per tie = selling price – labor cost – material cost

   Labor cost = $0.75 per tie for all 4 types of ties

   a. Formulate the problem as linear program w/ an objective function and all constraints

   Taking into account the 4th table in the question, the profit for each tie is arrived to by the following:

Profit per tie = selling price – labor – material cost

We arrive to the material costs by multiplying the volume of material needed times (*) the cost per yard of that material (ex: Silk Material Cost = .125 * 20 = $2.50). As a result, below are the functions for each of the four types of ties:

Silk tie, s, profit:

6.70 - .75 – 2.5 = $3.45

Polyester tie, pt, profit:

3.55 - .75 - .48 = $2.32

Blend 1, bt1, profit:

4.31 - .75 - .75 = $2.81

Blend 2, bt2, profit:

4.81 - .75 - .81 = $3.25

As a result our objective is:

MAX $3.45s + $2.32pt + $2.81bt1 + $3.25bt2

And the **constraints** to our problem are the following:

- The available yards per month for the materials used

  Silk tie material limit:
  0.125s <= 1000

  Polyester tie material limit, keeping in mind it's used in the ties p, bt1, and bt2:
  .08pt + .05bt1 + .03bt2 <= 2000

  Cotton tie material limit, keeping in mind it's used in the ties bt1 and bt2:
  .05bt1 + .07bt2 <= 1250

- Another constraint is the minimum and maximum number of tie we need to sell:

  Silk tie production:
  s >= 6000 & s <= 7000

  Polyester tie production:

pt >= 10,000 & pt <= 14,000

Blend 1 tie production:
Bt1 >= 13,000 & bt1 <= 16,000

Blend 2 tie production:
Bt2 >= 6000 & bt2 <= 8,500

b. *The optimal solution for the linprog using any software.*

Again, used LINDO I felt it was easier when compared to MATHWORKS

Below was my **input** code:

```
MAX 3.45s + 2.32pt + 2.81bt1 + 3.25bt2
ST
        0.125s <= 1000
        .08pt + .05bt1 + .03bt2 <= 2000
        .05bt1 + .07bt2 <= 1250
        s >= 6000
        s <= 7000
        pt >= 10000
        pt <= 14000
        bt1 >= 13000
        bt1 <= 16000
        bt2 >= 6000
        bt2 <= 8500
END
```

Below is my output code:

```
LP OPTIMUM FOUND AT STEP      4

    OBJECTIVE FUNCTION VALUE

    1)      120196.0

 VARIABLE        VALUE          REDUCED COST
       S      7000.000000         0.000000
      PT     13625.000000         0.000000
     BT1     13100.000000         0.000000
     BT2      8500.000000         0.000000
```

```
ROW   SLACK OR SURPLUS     DUAL PRICES
 2)       125.000000          0.000000
 3)         0.000000         29.000000
 4)         0.000000         27.200001
 5)      1000.000000          0.000000
 6)         0.000000          3.450000
 7)      3625.000000          0.000000
 8)       375.000000          0.000000
 9)       100.000000          0.000000
10)      2900.000000          0.000000
11)      2500.000000          0.000000
12)         0.000000          0.476000

NO. ITERATIONS=       4
```

c. *What are the optimal numbers of ties of each type to maximize profit?*

So per the calculations, I need to sell:

Type of Tie	***Optimal # of Units to Sell***	***Total Profit***
Silk	7000	7000 * 3.45 = $24,150
Polyester	13625	13625 * 2.32 = $31610
Blend 1	13100	13100 * 2.81 = $36811
Blend 2	8500	8500 3.25 = $27625

Total Profit: $120,196.00

3. Transshipment Model

   a. Formulate the objective function and the constraints

      Objective is to minimize the cost of shipping. Per the tables provided below is the **objective function** when in relation to the plant -> warehouse -> retailer:

      Min 10p1w1 + 15p1w2 + 11p2w1 + 8p2w2 + 13p3w1 + 8p3w2 + 9p3w3 + 14p4w2 + 8p4w3 + 5w1r1 + 6w1r2 + 7w1r3 + 10w1r4 + 12w2r3 + 8w2r4 + 10w2r5 + 14w2r6 + 14w3r4 + 12w3r5 + 12w3r6 +6w3r7

      The **constraints** are as follows:

      We know there is a constraint in relationship to the supply regarding capacity at each of the plants:

P1w1 + p1w2 <= 150
P2w1 + p2w2 <= 450
P3w1 + p3w2 + p3w3 <= 250
P4w2 + p4w3 <= 150

Constraint regarding demand at each of the retailers:

W1r1 >= 100
W1r2 >= 150
W1r3 + w2r3 >= 100
W1r4 + w2r4 + w3r4 >= 200
W2r5 + w3r5 >= 200
W2r6 + w3r6 >= 150
W3r7 >= 100

The final constraint is ensuring we deliver more than 0 refrigerators from our plants:

P1w1 + p2w1 + p3w1 – w1r1 – w1r2 – w1r3 – w1r4 >= 0
P1w2 + p2w2 + p3w2 + p4w2 – w2r3 – w2r4 – w2r5 – w2r6 >= 0
P3w3 + p4w3 – w3r4 – w3r5 – w3r6 – w3r7 >= 0

b. Determine the optimal solution for the linprog

Again I used LINDO to find the optimum solution.

Below is my **input** code:

```
MIN 10P1W1 + 15P1W2 + 11P2W1 + 8P2W2 + 13P3W1 + 8P3W2 + 9P3W3 + 14P4W2 +
8P4W3 + 5W1R1 + 6W1R2 + 7W1R3 + 10W1R4 + 12W2R3 + 8W2R4 + 10W2R5 +
14W2R6 + 14W3R4 + 12W3R5 + 12W3R6 + 6W3R7
ST
  P1W1 + P1W2 <= 150
  P2W1 + P2W2 <= 450
  P3W1 + P3W2 + P3W3 <= 250
  P4W2 + P4W3 <= 150

  W1R1 >= 100
  W1R2 >= 150
  W1R3 + W2R3 >= 100
  W1R4 + W2R4 + W3R4 >= 200
  W2R5 + W3R5 >= 200
  W2R6 + W3R6 >= 150
  W3R7 >= 100
```

```
P1W1 + P2W1 + P3W1 - W1R1 - W1R2 - W1R3 - W1R4 >= 0
P1W2 + P2W2 + P3W2 + P4W2 - W2R3 - W2R4 - W2R5 - W2R6 >= 0
P3W3 + P4W3 - W3R4 - W3R5 - W3R6 - W3R7 >= 0

END
```

Below is my **output**:

```
LP OPTIMUM FOUND AT STEP     13

  OBJECTIVE FUNCTION VALUE

  1)     17100.00

VARIABLE       VALUE          REDUCED COST
   P1W1      150.000000        0.000000
   P1W2        0.000000        8.000000
   P2W1      200.000000        0.000000
   P2W2      250.000000        0.000000
   P3W1        0.000000        2.000000
   P3W2      150.000000        0.000000
   P3W3      100.000000        0.000000
   P4W2        0.000000        7.000000
   P4W3      150.000000        0.000000
   W1R1      100.000000        0.000000
   W1R2      150.000000        0.000000
   W1R3      100.000000        0.000000
   W1R4        0.000000        5.000000
   W2R3        0.000000        2.000000
   W2R4      200.000000        0.000000
   W2R5      200.000000        0.000000
   W2R6        0.000000        1.000000
   W3R4        0.000000        7.000000
   W3R5        0.000000        3.000000
   W3R6      150.000000        0.000000
   W3R7      100.000000        0.000000


  ROW   SLACK OR SURPLUS     DUAL PRICES
   2)        0.000000          1.000000
   3)        0.000000          0.000000
   4)        0.000000          0.000000
   5)        0.000000          1.000000
```

6) 0.000000 -16.000000
7) 0.000000 -17.000000
8) 0.000000 -18.000000
9) 0.000000 -16.000000
10) 0.000000 -18.000000
11) 0.000000 -21.000000
12) 0.000000 -15.000000
13) 0.000000 -11.000000
14) 0.000000 -8.000000
15) 0.000000 -9.000000

NO. ITERATIONS= 13

c. What are the optimal shipping routes and minimum costs:

Route	***Number of Fridges***
P1 -> w1	150
P2 -> w1	200
P2 -> w2	250
P3 -> w2	150
P3 -> w3	100
P4 -> w3	150
W1 -> r1	100
W1 -> r2	150
W1 -> r3	100
W2 -> r4	200
W2 -> r5	200
W3 -> r6	150
W3 -> r7	100

The optimum solution to the minimum cost is: $17,100.