5. variants

1. uzdevums

$$\begin{aligned}\operatorname{Ln}(-3+i) = \ln|-3+1| + i(\arg(-3+i)+2k\pi) &= \ln\sqrt{10} + i\left(\arctan\left(-\frac{1}{3}\right)+\pi+2k\pi\right)\\ &= \ln\sqrt{10} + i\left[(2k+1)\pi - \arctan\left(\frac{1}{3}\right)\right]\\ &\approx 1.151 + i\left[(2k+1)\pi - 0.322\right],\ k\in\mathbb{Z}.\end{aligned}$$

$$\begin{aligned}\tanh\left(\ln 3+\frac{\pi i}{4}\right) &= \frac{\sinh\left(\ln 3+\frac{\pi i}{4}\right)}{\cosh\left(\ln 3+\frac{\pi i}{4}\right)} = \frac{\sinh(\ln 3)\cos\left(\frac{\pi}{4}\right)+i\cosh(\ln 3)\sin\left(\frac{\pi}{4}\right)}{\cosh(\ln 3)\cos\left(\frac{\pi}{4}\right)+i\sinh(\ln 3)\sin\left(\frac{\pi}{4}\right)} =\\ &= \frac{\sinh(\ln 3)+i\cosh(\ln 3)}{\cosh(\ln 3)+i\sinh(\ln 3)} = \frac{\frac{4}{3}+i\frac{5}{3}}{\frac{5}{3}+i\frac{4}{3}} = \frac{4+5i}{5+4i} = \frac{(4+5i)(5-4i)}{41} =\\ &= \frac{40}{41}+i\frac{9}{41}.\end{aligned}$$

$$i^{2i} = e^{2i\operatorname{Ln} i} = \exp\left[2i(\ln 1 + i\frac{\pi}{2}+i2k\pi)\right] = \exp\left[-(4k+1)\pi\right],\ k\in\mathbb{Z}.$$

$$\begin{aligned}\arctan(1+2i) &= \frac{1}{2i}\operatorname{Ln}\frac{i-1}{3-i} = -\frac{i}{2}\left(\operatorname{Ln}(i-1)-\operatorname{Ln}(3-i)\right) =\\ &= -\frac{i}{2}\left(\ln\sqrt{2}+i\left(\frac{7\pi}{4}+2k\pi\right)-\ln\sqrt{10}-i(2p+1)\pi+i\arctan\left(\frac{1}{3}\right)\right)\\ &= \frac{\left(\frac{7\pi}{4}+2k\pi\right)-(2k+1)\pi+\arctan\left(\frac{1}{3}\right)}{2}+i\frac{\ln\sqrt{5}}{2}\\ &= \frac{\arctan\left(\frac{1}{3}\right)+\frac{3\pi}{4}+2k\pi}{2}+i\frac{\ln\sqrt{5}}{2},\ k\in\mathbb{Z}\end{aligned}$$

2. uzdevums

Nepieciešams atrast tādus φ, lai u būtu harmonisks jeb tam jāapmierina Laplasa vienādojums.

$a^2\frac{\partial^2\varphi}{\partial x^2} = -b^2\frac{\partial^2\varphi}{\partial y^2}$

Ja $a=0$ un $b\neq 0$, tad $\varphi(y)=\alpha y+\beta$, kur $\alpha,\beta\in\mathbb{C}$ Analogi, ja $b=0$ un $a\neq 0$, tad $\varphi(x)=\alpha x+\beta$. Ja $a=b=0$, tad $\varphi\equiv\text{const}$.

$a^2\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{b^2}{a^2}\frac{\partial^2\varphi}{\partial y^2}\right)=0.$

Ja a un b saista sakarība $|a|=|b|$, tad par φ var izvēlēties jebkuru divargument harmonisku funkciju. Pretējā gadījumā nevar garantēt, ka u būs harmoniska, pat ja φ ir harmoniska. Būtu nepieciešami kādi robežnosacījumi, lai konkrētāk kaut ko teikt ar, piemēram, Furjē transformāciju.

3. uzdevums

Der funkcija $f(z)=\frac{(1+2i)z+5}{(2-i)z+2-i}$. $f(\infty)=i, f(-1)=\infty, f(i)=1$.