5. variants

1. uzdevums

$$\begin{aligned}\operatorname{Ln}(-3+i) = \ln|-3+1| + i(\arg(-3+i)+2k\pi) &= \ln\sqrt{10} + i\left(\arctan\left(-\frac{1}{3}\right)+\pi+2k\pi\right)\\ &= \ln\sqrt{10} + i\left[(2k+1)\pi - \arctan\left(\frac{1}{3}\right)\right]\\ &\approx 1.151 + i\left[(2k+1)\pi - 0.322\right],\ k\in\mathbb{Z}.\end{aligned}$$

$$\begin{aligned}\tanh\left(\ln 3+\frac{\pi i}{4}\right) &= \frac{\sinh\left(\ln 3+\frac{\pi i}{4}\right)}{\cosh\left(\ln 3+\frac{\pi i}{4}\right)} = \frac{\sinh(\ln 3)\cos\left(\frac{\pi}{4}\right)+i\cosh(\ln 3)\sin\left(\frac{\pi}{4}\right)}{\cosh(\ln 3)\cos\left(\frac{\pi}{4}\right)+i\sinh(\ln 3)\sin\left(\frac{\pi}{4}\right)} =\\ &= \frac{\sinh(\ln 3)+i\cosh(\ln 3)}{\cosh(\ln 3)+i\sinh(\ln 3)} = \frac{\frac{4}{3}+i\frac{5}{3}}{\frac{5}{3}+i\frac{4}{3}} = \frac{4+5i}{5+4i} = \frac{(4+5i)(5-4i)}{41} =\\ &= \frac{40}{41}+i\frac{9}{41}.\end{aligned}$$

$$i^{2i} = e^{2i\operatorname{Ln} i} = \exp\left[2i(\ln 1 + i\frac{\pi}{2} + i2k\pi)\right] = \exp\left[-(4k+1)\pi\right],\ k\in\mathbb{Z}.$$

$$\begin{aligned}\arctan(1+2i) &= \frac{1}{2i}\operatorname{Ln}\frac{i-1}{3-i} = -\frac{i}{2}\left(\operatorname{Ln}(i-1) - \operatorname{Ln}(3-i)\right) =\\ &= -\frac{i}{2}\left(\ln\sqrt{2} + i\left(\frac{7\pi}{4}+2k\pi\right) - \ln\sqrt{10} - i(2p+1)\pi + i\arctan\left(\frac{1}{3}\right)\right)\\ &= \frac{\left(\frac{7\pi}{4}+2k\pi\right) - (2k+1)\pi + \arctan\left(\frac{1}{3}\right)}{2} + i\frac{\ln\sqrt{5}}{2}\\ &= \frac{\arctan\left(\frac{1}{3}\right)+\frac{3\pi}{4}+2k\pi}{2} + i\frac{\ln\sqrt{5}}{2},\ k\in\mathbb{Z}\end{aligned}$$

2. uzdevums

Tā kā funkcija φ netiek precīzāk aprakstīta, tad vienīgi atliek rakstīt vispārīgo atrisinājumu.

$$f(z) = f(x,y) = \varphi(ax+by) + i\int_{(x_0,y_0)}^{(x,y)} -b\frac{\partial\varphi(x,y)}{\partial y}dx + a\frac{\partial\varphi(x,y)}{\partial x}dy + C,$$

kur (x_0,y_0) jebkurš punkts, kur funkcija φ analītiska.

3. uzdevums

Der funkcija $f(z) = \frac{(1+2i)z+5}{(2-i)z+2-i}$. $f(\infty) = i, f(-1) = \infty, f(i) = 1$.