

DE PARCIAL \rightarrow FUNCIÓN POTENCIAL

4 ④ Hallar la función potencial del campo $\vec{f}(x,y) = (\overbrace{y \cdot e^{xy} + 1}^P; \overbrace{x \cdot e^{xy} + 2}^Q)$ si el potencial vale 10 en el punto (0,2).



$$\left. \begin{array}{l} P'_y = e^{xy} + xy \cdot e^{xy} \\ Q'_x = e^{xy} + xy \cdot e^{xy} \end{array} \right\} \Rightarrow \vec{f}(x,y) \text{ es un campo de gradiente.}$$

$$\nabla \varphi(x,y) = \vec{f}(x,y)$$

$$(\varphi'_x(x,y); \varphi'_y(x,y)) = (y \cdot e^{xy} + 1; x \cdot e^{xy} + 2)$$

$$\varphi'_x(x,y) = y \cdot e^{xy} + 1$$

$$\int \varphi'_x(x,y) dx = \int (y \cdot e^{xy} + 1) dx$$

$$\varphi(x,y) = e^{xy} + x + \alpha(y)$$

$$\rightarrow \varphi'_y(x,y) = x \cdot e^{xy} + \alpha'(y)$$

$$x \cdot e^{xy} + \alpha'(y) = x \cdot e^{xy} + 2$$

$$\alpha'(y) = 2$$

$$\int \alpha'(y) dy = \int 2 dy$$

$$\alpha(y) = 2y + c$$

$$\varphi(x,y) = e^{xy} + x + 2y + c$$

$$\varphi(0,2) = 10$$

$$\varphi(0,2) = e^{0 \cdot 2} + 0 + 2 \cdot 2 + c$$

$$10 = 1 + 4 + c$$

$$c = 5$$

$$\boxed{\varphi(x,y) = e^{xy} + x + 2y + 5}$$