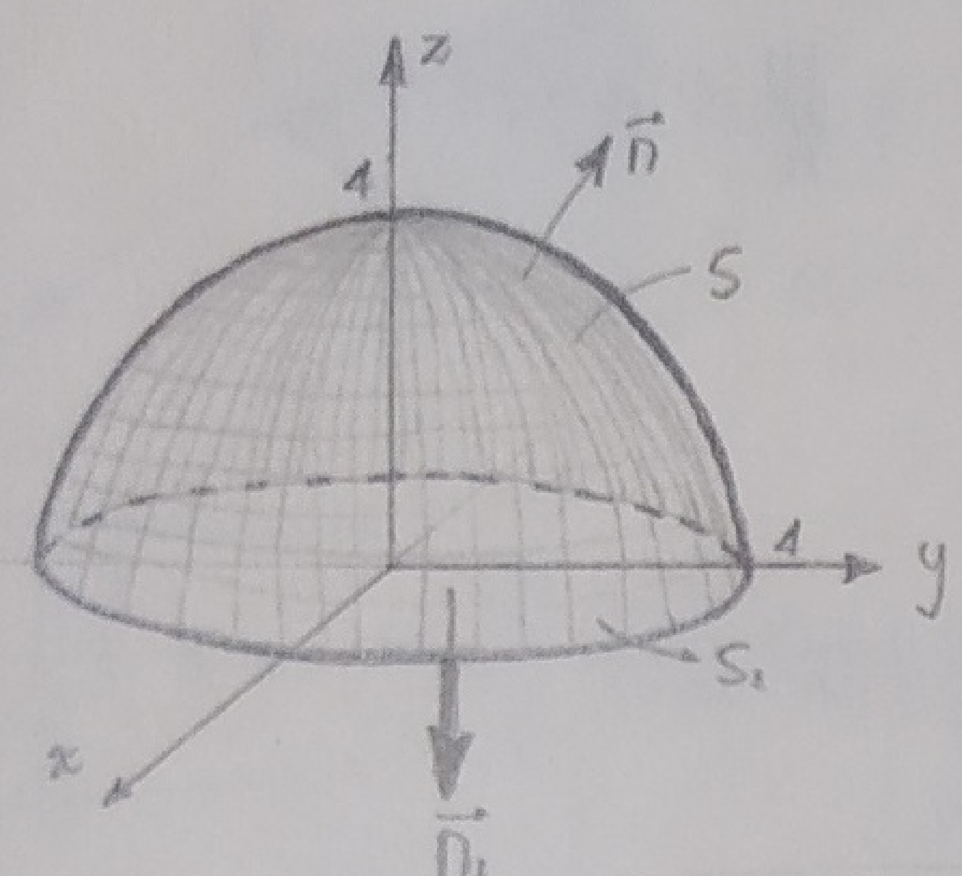
DE PARCIAL → INTEGRAL DE SUPERFICIE – FLUJO

1 (4) Calcular el flujo del campo $\bar{f}(x,y,z) = (x+e^{yz};\ y+e^{xz};\ z+x^2)$ a través de la porción de superficie abierta $z=\sqrt{16-x^2-y^2}$ mediante una conveniente aplicación del teorema de la divergencia.

Indicar en un gráfico el sentido en que calcula el flujo.



La semiesfera es una superficie abierta.
Para poder aplicar el teorema de la divergencia, necesitamos que la superficie sea cerrada.
Para "cerrarla", le agregamos una "tapa".

$\left.\begin{array}{l} \underline{S}\text{: semiesfera.} \\ \underline{S_1}\text{: la tapa de la semiesfera} \end{array}\right\} S^* = S \cup S_1$

→ $\underline{M}$: sólido encerrado entre S y S_1.

$$\operatorname{div}\bar{f} = \nabla\cdot\bar{f} = \underbrace{\frac{\partial(x+e^{yz})}{\partial x}}_{1} + \underbrace{\frac{\partial(y+e^{xz})}{\partial y}}_{1} + \underbrace{\frac{\partial(z+x^2)}{\partial z}}_{1} = 3$$

$$\iint_{S^*} \bar{f}\cdot\vec{n}\, d\sigma = \iiint_M \operatorname{div}\bar{f}\, dx\, dy\, dz = 3\underbrace{\iiint_M dx\, dy\, dz}_{\text{volumen de la semiesfera}} = \underline{128\pi}$$

$$\left(=\frac{2}{3}\pi(R^3)\right) = \frac{128}{3}\pi \qquad R = 4$$

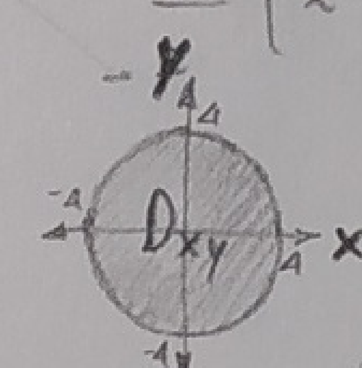
$$\iint_{S^*}(\bar{f}\cdot\vec{n})\,ds = \iint_S(\bar{f}\cdot\vec{n})\,ds + \iint_{S_1}(\bar{f}\cdot\vec{n})\,ds$$

$$128\pi = \iint_S(\bar{f}\cdot\vec{n})\,ds + (-64\pi)$$

$$\boxed{\iint_S(\bar{f}\cdot\vec{n})\,ds = 192\pi}$$

$S_1: \begin{cases} z=0 \\ x^2+y^2 \le 16 \end{cases}$ $G(x,y,z) = z$, $\nabla G = (0,0,1)$, $G'_z = 1$

es entrante, pero necesitamos que sea saliente el flujo! (que apunte hacia abajo).



$0 \le \theta \le 2\pi$

$0 \le \rho \le 4$

$$\iint_{S_1}(\bar{f}\cdot\vec{n})\,ds = \iint_{D_{xy}} \bar{f}\cdot\frac{\|\nabla G\|}{|G'_z|}\Bigg|_{z=0} dx\, dy$$

$$= \iint_{D_{xy}} \Big(\underbrace{x+e^{y0}}_{x+1};\ \underbrace{y+e^{x0}}_{y+1};\ \underbrace{0+x^2}_{x^2}\Big)\cdot\frac{(0,0,1)}{1}\, dx\, dy$$

$$= -\iint_{D_{xy}} x^2\, dx\, dy$$

aplicando polares $\begin{cases} x=\rho\cdot\cos\theta \\ y=\rho\cdot\operatorname{sen}\theta \end{cases}$ $x^2+y^2=\rho^2$, $|J(\rho,\theta)| = \rho$

$$= -\int_0^{2\pi} d\theta \underbrace{\int_0^4 \rho\,\rho^2\cdot\cos^2\theta\, d\rho}_{\cos^2\theta\left[\frac{1}{4}\rho^4\right]_0^4 = 64\cdot\cos^2\theta}$$

$$= -64\int_0^{2\pi}\cos^2\theta\, d\theta$$

$$= -64\underbrace{\left[\frac{1}{2}\theta + \frac{\operatorname{sen}(2\theta)}{4}\right]_0^{2\pi}}_{\pi} = \underline{-64\pi}$$

EL FLUJO DEBE SER SALIENTE SIEMPRE

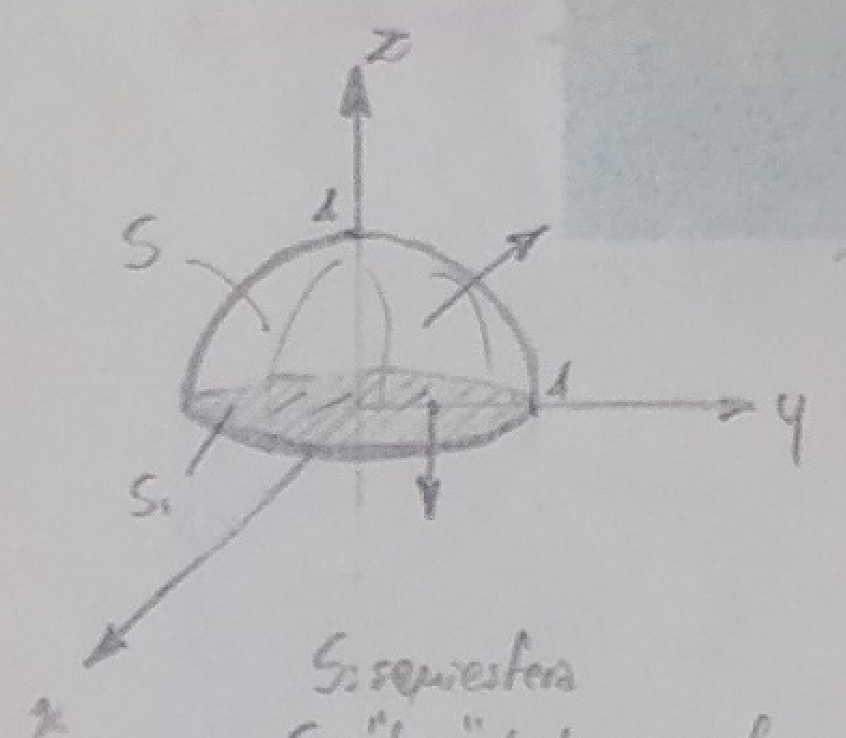
↓

las normales deben apuntar "hacia afuera"

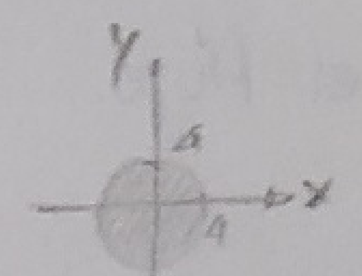
$\text{div}\,\bar{F} = 0$

2 (3) Sea $\bar{F} \in C^1$ solenoidal. Calcular el flujo de $\bar{F}$ a través de la porción de superficie $z = \sqrt{16 - x^2 - y^2}$; si $\bar{F}(x,y,0) = (y; x; x^2 - y)$.

semiesfera de radio 4. positiva



proyección en el plano xy



$0 \leq \rho \leq 4$
$0 \leq \theta \leq 2\pi$

$x = \rho\cos\theta$
$y = \rho\,\text{sen}\,\theta$
$z = 0$
$x^2 + y^2 = \rho^2$
$|J| = \rho$

S: semiesfera
S_1: "tapa" de la semiesfera
S^*: superficie cerrada / $S^* = S \cup S_1$.

$$\iint_{S^*} \bar{f}\cdot\breve{n}\cdot ds = \iiint_M \underbrace{\text{div}\,\bar{F}}_{=0} \cdot dx\,dy\,dz$$

$$= \iiint_M 0\; dx\,dy\,dz$$

$$= 0$$

$$\iint_{S^*} \bar{f}\cdot\breve{n}\cdot d\bar{s} = \iint_S \bar{f}\cdot\breve{n}\;d\bar{s} + \iint_{S_1} \bar{f}\;\breve{n}\;d\bar{s}$$

$$0 = \iint_S \bar{f}\cdot\breve{n}\;ds + (-64\pi)$$

$$\boxed{\iint_S \bar{f}\cdot\breve{n}\cdot ds = 64\pi}$$

$$\iint_{S_1} \bar{f}\cdot\breve{n}\;ds = \iint_{D_{xy}} \bar{f}\cdot\breve{n}\cdot dxdy$$

$G = z$
$\nabla G = (0,0,1)$
el flujo debe ser saliente! O sea, $(0,0,-1)$.

$$= -\iint_{D_{xy}} (y, x, x^2-y)\cdot(0,0,1)\;dx\;dy$$

$$= -\iint_{D_{xy}} x^2 - y\;dx\,dy$$

$$= \iint_{D_{xy}} -x^2 + y\;dx\,dy \qquad -\rho^2\cos^2\theta + \rho\,\text{sen}\,\theta$$

aplicando polares

$$= \int_0^{2\pi} d\theta \int_0^4 \rho\left(-\rho^2\cos^2\theta + \rho\,\text{sen}\,\theta\right) d\rho$$

$$= \int_0^{2\pi} d\theta \cdot \int_0^4 \left(-\rho^3\cos^2\theta + \rho^2\,\text{sen}\,\theta\right) d\rho$$

$$= \int_0^{2\pi} d\theta \left[-\cos^2\theta \underbrace{\int_0^4 \rho^3 d\rho}_{\frac{\rho^4}{4}\Big|_0^4 = 64} + \text{sen}\,\theta \underbrace{\int_0^4 \rho^2 d\rho}_{\frac{\rho^3}{3}\Big|_0^4 = \frac{64}{3}}\right]$$

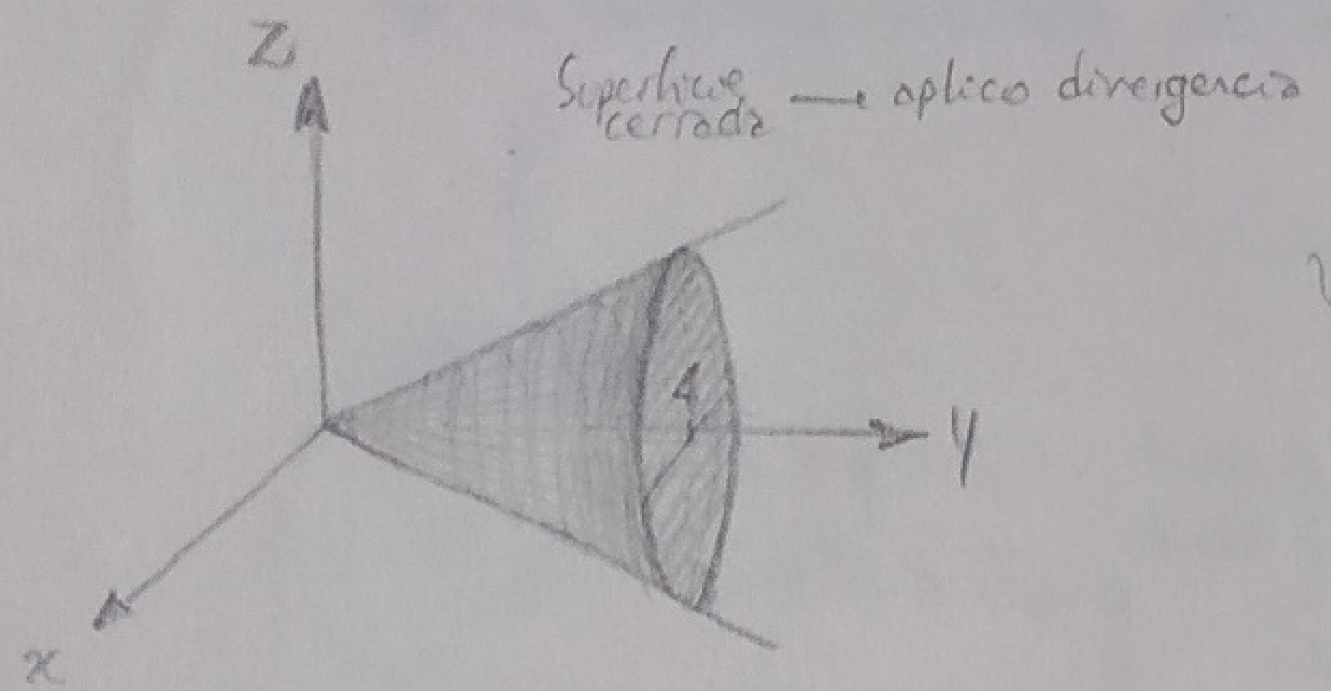
$$= -64\int_0^{2\pi} \cos^2\theta\;d\theta + \frac{64}{3}\int_0^{2\pi} \text{sen}\,\theta\;d\theta$$

$$= -64\left[\frac{\theta}{2} + \frac{\text{sen}(2\theta)}{4}\Big|_0^{2\pi}\right] + \underbrace{\cancel{\frac{64}{3}\left[-\cos\theta\Big|_0^{2\pi}\right]}}_{0}$$

$$= -64\pi$$

DE PARCIAL → INTEGRAL DE SUPERFICIE - FLUJO

3 ③ Calcular el flujo de $\bar{F}(x,y,z) = (y^2x;\ xz^2;\ z^2)$ a través de la superficie frontera de sólido que queda definida por: $\sqrt{x^2+z^2} \le y \le 4$.



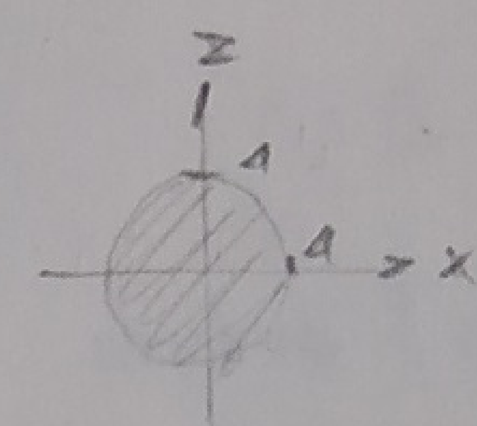
$$\operatorname{div} \bar{F} = P'_x + Q'_y + R'_z$$
$$= y^2 + \cancel{0} + 2z$$

$\frac{y=4}{z=0} \to \sqrt{x^2+z^2} = y$

$\sqrt{x^2} = 4$

$x = 4$

proyección sobre el plano xz



$$\begin{cases} x = \rho \cdot \cos\theta \\ y = y \\ z = \rho \cdot \operatorname{sen}\theta \end{cases}$$

$x^2+z^2 = \rho^2$ $\quad 0 \le \rho \le 4$

$|J| = \rho$ $\quad 0 \le \theta \le 2\pi$

$$\boxed{\iint_S \bar{f} \cdot \bar{n} \cdot ds = \iiint_M \operatorname{div} \bar{f} \cdot dx\, dy\, dz}$$

$$= \iiint_M (y^2 + 2z)\ dx\, dy\, dz$$

$y^2 + 2(\rho \cdot \operatorname{sen}\theta)$

$y^2 + 2\rho \operatorname{sen}\theta$

$\sqrt{x^2+z^2} \le y \le 4$

$\sqrt{\rho^2}$

$\rho \le y \le 4$

$$= \int_0^{2\pi} d\theta \int_0^4 \rho\, d\rho \int_\rho^4 (y^2 + 2\rho \operatorname{sen}\theta)\, dy$$

$\to \frac{y^3}{3} + 2\rho \operatorname{sen}\theta\, y \Big|_\rho^4$

$\frac{4^3}{3} + 2\rho \cdot \operatorname{sen}\theta \cdot 4 - \left(\frac{\rho^3}{3} + 2\rho \operatorname{sen}\theta\, \rho\right)$

$\frac{64}{3} + 8 \cdot \rho \cdot \operatorname{sen}\theta - \frac{\rho^3}{3} - 2 \cdot \rho^2 \cdot \operatorname{sen}\theta$

$$= \int_0^{2\pi} d\theta \int_0^4 \rho \left[\frac{64}{3} + 8\rho \operatorname{sen}\theta - \frac{\rho^3}{3} - 2\rho^2 \operatorname{sen}\theta\right] d\rho$$

$$= \int_0^{2\pi} d\theta \int_0^4 \left(\frac{64}{3}\rho + 8\rho^2 \operatorname{sen}\theta - \frac{\rho^4}{3} - 2\rho^3 \operatorname{sen}\theta\right) d\rho$$

$\int_0^4 \frac{64}{3}\rho\, d\rho = \frac{64}{3}\frac{\rho^2}{2}\Big|_0^4 = \frac{32}{3}\rho^2\Big|_0^4 = \frac{512}{3}$

$\int_0^4 8\rho^2 \operatorname{sen}\theta\, d\rho = \operatorname{sen}\theta \left[\frac{8}{3}\rho^3\Big|_0^4\right] = \frac{512}{3}\operatorname{sen}\theta$

$\int_0^4 -\frac{\rho^4}{3}\, d\rho = -\frac{1}{3}\frac{\rho^5}{5}\Big|_0^4 = \frac{-\rho^5}{15}\Big|_0^4 = -\frac{1024}{15}$

$\int_0^4 -2\rho^3 \operatorname{sen}\theta\, d\rho = -2\frac{\rho^4}{4}\operatorname{sen}\theta\Big|_0^4 = \operatorname{sen}\theta\left[-2\rho^4\Big|_0^4\right] = -128 \cdot \operatorname{sen}\theta$

$\frac{512}{3} - \frac{1024}{15} + \frac{512}{3}\operatorname{sen}\theta - 128\operatorname{sen}\theta$

$\frac{512}{5} - \frac{128}{3}\operatorname{sen}\theta$

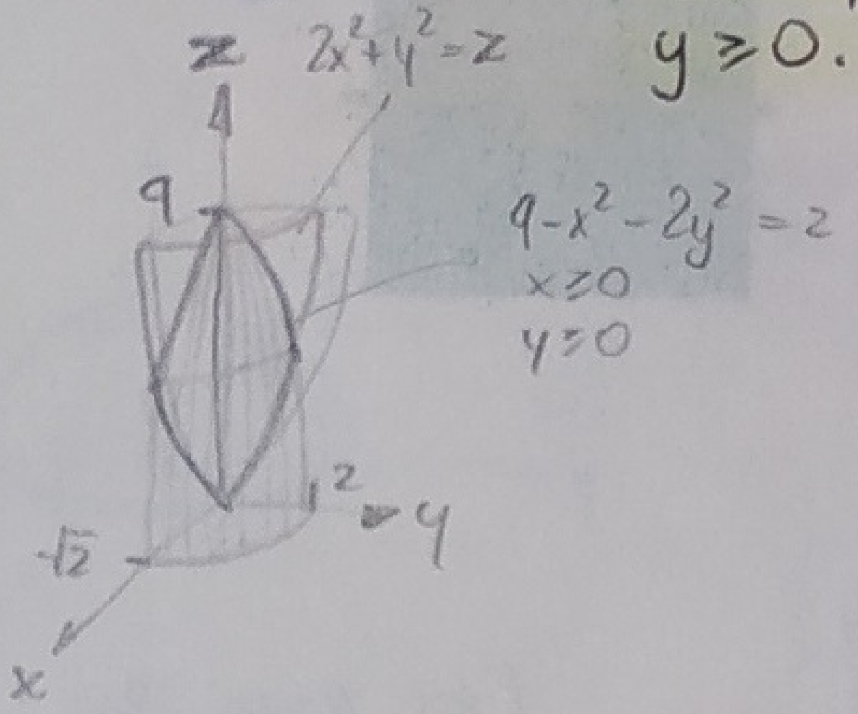
$$= \int_0^{2\pi} \left(\frac{512}{5} - \frac{128}{3}\operatorname{sen}\theta\right) d\theta$$

$$= \frac{512}{5}\theta + \frac{128}{3}\cos\theta \Big|_0^{2\pi}$$

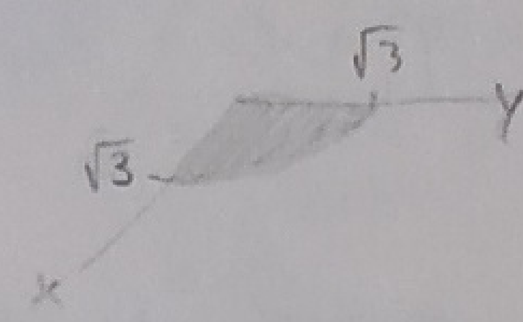
$$= \boxed{\frac{1024}{5}\pi}$$

4 ① Calcular el flujo del campo $\bar{F}(x,y,z) = (x^2y;\ -xy^2;\ x^2z)$ a través de la superficie frontera del sólido $2x^2+y^2 \le z \le 9-x^2-2y^2$, $x \ge 0$, $y \ge 0$.

$$\operatorname{div} \bar{F} = \cancel{2xy} - \cancel{2xy} + x^2 = x^2$$



$$\begin{cases} z = 2x^2+y^2 \\ z = 9-x^2-2y^2 \end{cases}$$
$$\downarrow$$
$$2x^2+y^2 = 9-x^2-2y^2$$
$$3x^2+3y^2 = 9$$
$$x^2+y^2 = 3$$



$$\begin{cases} x = \rho\cos\theta \\ y = \rho \operatorname{sen}\theta \\ z = z \end{cases} \qquad 0 \le \rho \le \sqrt{3} \qquad 0 \le \theta \le \frac{\pi}{2}$$

$$x^2+y^2 = \rho^2 \qquad |J| = \rho$$

$$\boxed{\iint_S \bar{F}\cdot\bar{n}\cdot ds} = \iiint_M \operatorname{div}\bar{F}\cdot dx\,dy\,dz$$

$$x^2 = \rho^2\cos^2\theta$$

$$\underbrace{2x^2+y^2}_{x^2+x^2+y^2} \le z \le 9-\underbrace{(x^2+2y^2)}_{(x^2+y^2+y^2)}$$

$$\rho^2\cos^2\theta+\rho^2 \le z \le 9-\rho^2-\rho^2\operatorname{sen}^2\theta$$

$$= \iiint_M x^2\,dx\,dy\,dz$$

$$= \int_0^{\frac{\pi}{2}} d\theta \cdot \int_0^{\sqrt{3}} d\rho \cdot \int_{\rho^2+\rho^2\cos^2\theta}^{9-\rho^2-\rho^2\operatorname{sen}^2\theta} \rho\cdot\rho^2\cdot\cos^2\theta\cdot dz$$

$$\int dz = z\Big|_{\rho^2+\rho^2\cos^2\theta}^{9-\rho^2-\rho^2\operatorname{sen}^2\theta} = 9-\rho^2-\rho^2\operatorname{sen}^2\theta-\rho^2-\rho^2\cos^2\theta$$
$$= 9-\rho^2(1+\operatorname{sen}^2\theta+1+\cos^2\theta)$$
$$= 9-3\rho^2$$

$$= \int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta \cdot \int_0^{\sqrt{3}} \rho^3\,d\rho \cdot \int_{\rho^2+\rho^2\cos^2\theta}^{9-\rho^2-\rho^2\operatorname{sen}^2\theta} dz$$

$$= \int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta \cdot \int_0^{\sqrt{3}} \rho^3(9-3\rho^2)\,d\rho$$

$$9\rho^3-3\rho^5$$

$$= 3\cdot\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta \cdot \int_0^{\sqrt{3}} 3\rho^3-\rho^5\,d\rho$$

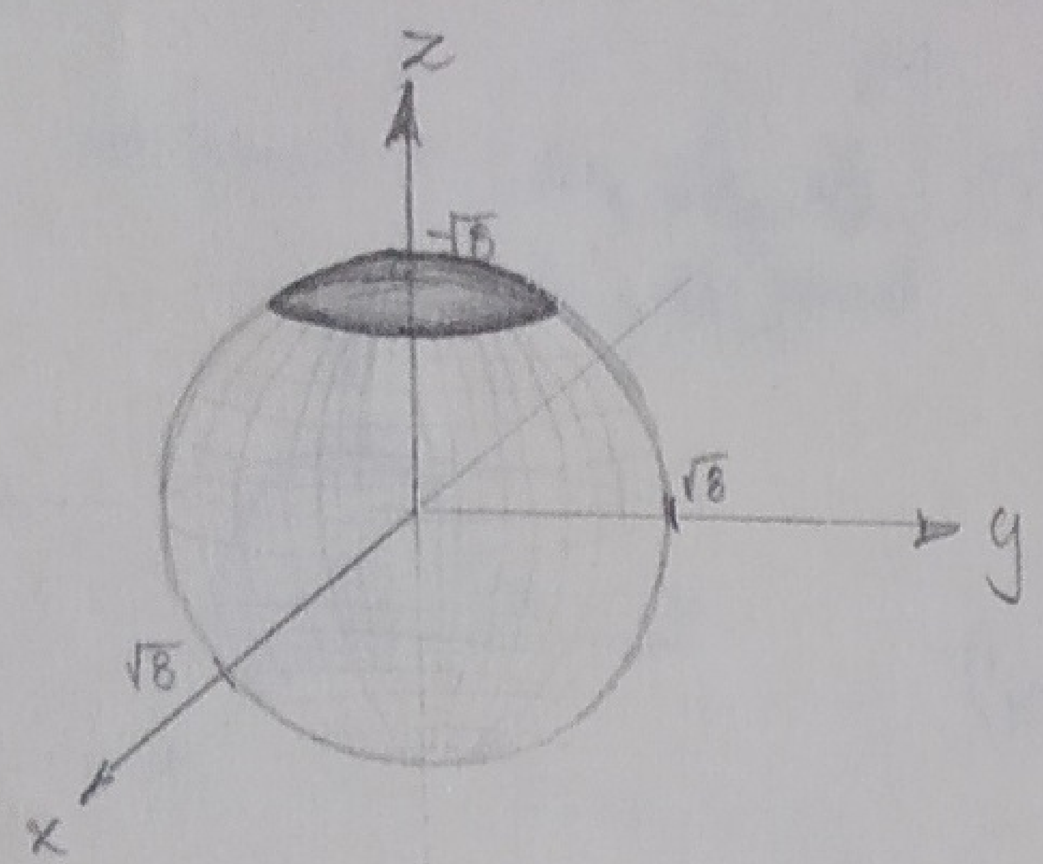
$$\int_0^{\sqrt{3}} 3\rho^3\,d\rho - \int_0^{\sqrt{3}} \rho^5\,d\rho = \frac{3}{4}\rho^4 - \frac{1}{6}\rho^6\Big|_0^{\sqrt{3}} = \frac{27}{4}-\frac{9}{2} = \frac{9}{4}$$

$$= 3\cdot\frac{\pi}{4}\cdot\frac{9}{4}$$

$$\boxed{= \frac{27}{16}\pi}$$

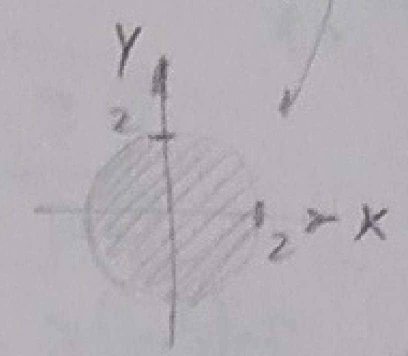
5 ① Calcular el flujo del campo vectorial $\bar{F}(x,y,z) = \left(\frac{x^3}{3}; y^2 z; -yz^2\right)$ a través de la superfice frontera del sólido $x^2+y^2+z^2 \leq 8 \ \wedge \ z \geq 2$.

esfera de radio $\sqrt{8}$ — plano $z=2$



$z=2 \to x^2+y^2+2^2=8$

$x^2+y^2=4$



$\operatorname{div}\bar{F} = x^2 + 2yz - 2yz = x^2$

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\cdot\sin\theta \\ z = z \end{cases}$$

$x^2+y^2=\rho^2$

$J = \rho$

$0 \leq \theta \leq 2\pi$

$0 \leq \rho \leq 2$

$2 \leq z \leq \sqrt{8-x^2-y^2} = \sqrt{8-\rho^2}$

$$\boxed{\iint_S \bar{F}\cdot\bar{n}\,ds} = \iiint_M \operatorname{div}\bar{F}\cdot dx\,dy\,dz = \iiint_M x^2\,dx\,dy\,dz$$

$x^2 \to \rho^2\cos^2\theta$

$$= \int_0^{2\pi} d\theta \cdot \int_0^2 d\rho \int_2^{\sqrt{8-\rho^2}} \rho\cdot\rho^2\cdot\cos^2\theta\,dz$$

$$= \underbrace{\int_0^{2\pi}\cos^2\theta\,d\theta}_{=\pi} \cdot \int_0^2 \rho^3 d\rho \cdot \int_2^{\sqrt{8-\rho^2}} dz$$

$$\int_2^{\sqrt{8-\rho^2}} dz = z\Big|_2^{\sqrt{8-\rho^2}} = \sqrt{8-\rho^2}-2$$

$$= \pi\cdot\int_0^2 \rho^3\cdot\left(\sqrt{8-\rho^2}-2\right)d\rho$$

$$= \pi\left[\int_0^2 \rho^3\sqrt{8-\rho^2}\,d\rho - \underbrace{\int_0^2 \rho^3\,d\rho}_{=8}\right]$$

$$\int_0^2 \rho^3\sqrt{8-\rho^2}\,d\rho = \frac{\sqrt{(8-\rho^2)^5}}{5} - \frac{8\sqrt{(8-\rho^2)^3}}{3}\Bigg|_0^2 = 9{,}2026$$

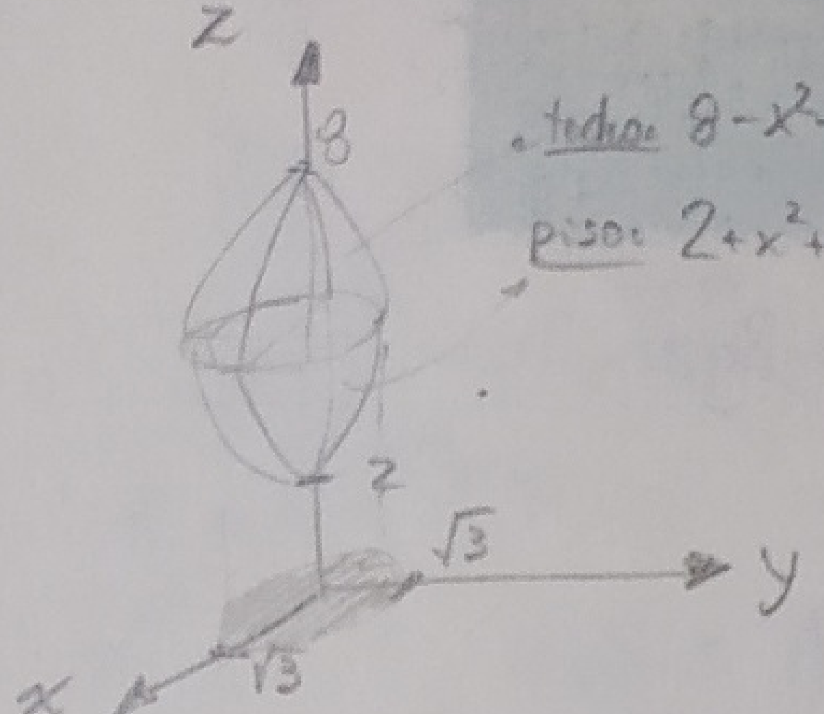
$$= \pi\,(9{,}2026 - 8)$$

$$\boxed{= 1{,}2026\cdot\pi}$$

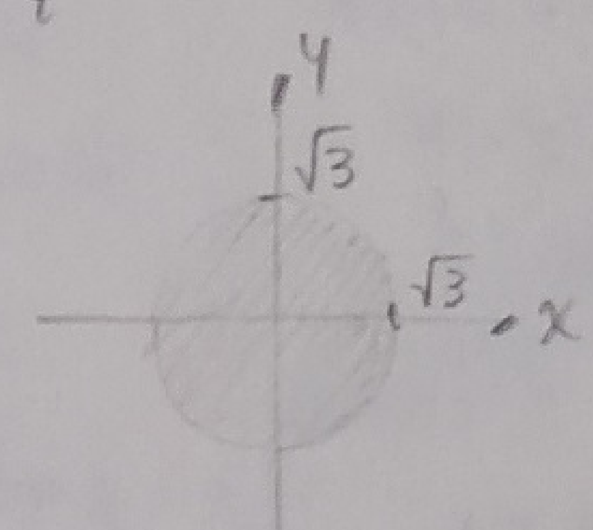
6 ① Calcular el flujo de $f(x,y) = (x-y;\ y-z;\ z-x)$ a través de la superficie frontera del cuerpo definido por $2+x^2+y^2 \le z \le 8-x^2-y^2$.

$$2+x^2+y^2 = 8-x^2-y^2$$
$$2x^2+2y^2 = 6$$
$$x^2+y^2 = 3$$

$$\text{div } \bar{f} = 1+1+1 = 3$$



- techo: $8-x^2-y^2 = z$
- piso: $2+x^2+y^2 = z$



$$\begin{cases} x = \rho\cdot\cos\theta \\ y = \rho\cdot\sin\theta \end{cases} \longrightarrow x^2+y^2 = \rho^2 \qquad |J| = \rho$$

$$0 \le \theta \le 2\pi$$
$$0 \le \rho \le \sqrt{3}$$
$$2+\underbrace{(x^2+y^2)}_{\rho^2} \le z \le 8-(x^2+y^2)$$
$$2+\rho^2 \le z \le 8-\rho^2$$

$$\boxed{\iint_S \bar{f}\cdot\bar{n}\, ds} = \iiint_M \text{div } \bar{f}\cdot dx\,dy\,dz$$

$$= \iiint_M 3\cdot dx\,dy\,dz$$

$$= 3\int_0^{2\pi} d\theta \cdot \int_0^{\sqrt{3}} \rho\, d\rho \int_{2+\rho^2}^{8-\rho^2} dz$$

$$z\Big|_{2+\rho^2}^{8-\rho^2} = 8-\rho^2-(2+\rho^2)$$
$$8-\rho^2-2-\rho^2$$
$$6-2\rho^2$$

$$= 3\int_0^{2\pi} d\theta \int_0^{\sqrt{3}} \rho\cdot(6-2\rho^2)\, d\rho = 3\int_0^{2\pi} d\theta \int_0^{\sqrt{3}} (6\rho-2\rho^3)\, d\rho$$

$$\theta\Big|_0^{2\pi} = 2\pi$$

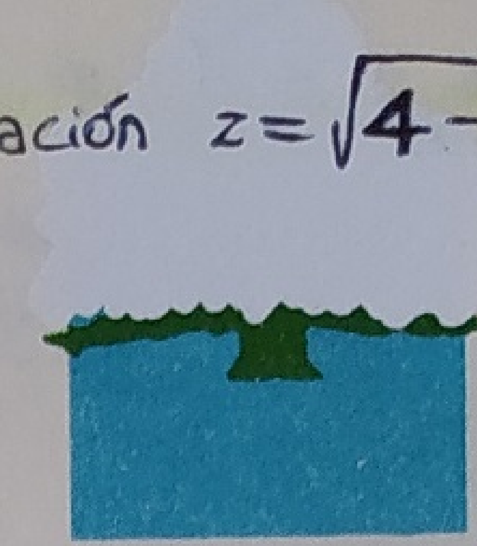
$$3\rho^2-\frac{1}{2}\rho^4\Big|_0^{\sqrt{3}} = \frac{9}{2}$$

$$= 3\cdot \cancel{2}\pi\cdot\frac{9}{\cancel{2}}$$

$$\boxed{= 27\pi}$$

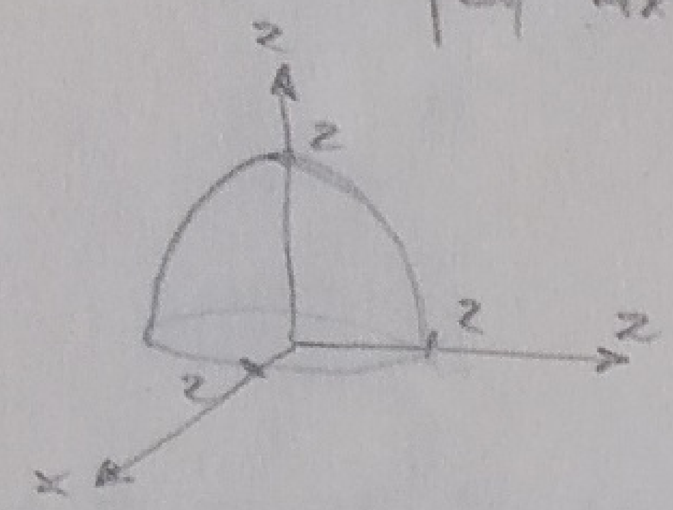
DE PARCIAL → INTEGRAL DE SUPERFICIE – FLUJO 4/5

7 ① Calcule el flujo del rotor de $\vec{F}$ a través de la superficie S de ecuación $z=\sqrt{4-x^2-y^2}$ con $z \geq 0$ siendo $\vec{f}(x,y,z) = (2y;\ 4x;\ g(x,y,z))$ con $g \in C^2$.

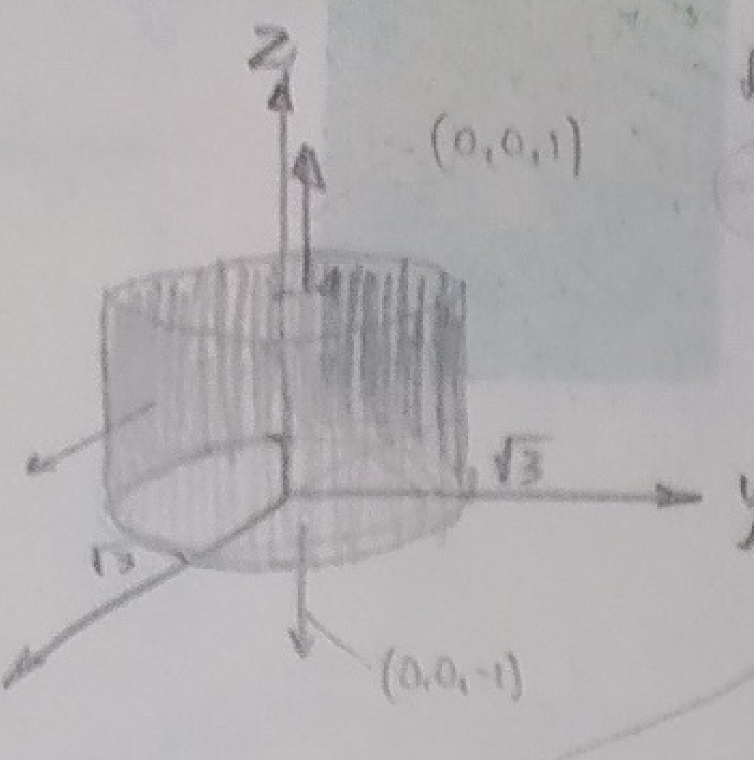


semiesfera

$$\text{rot}(f(x,y,z)) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y & 4x & g(x,y,z) \end{vmatrix} = (g'_y - 0;\ -(g'_x - 0);\ 4-2) = (g'_y;\ -g'_x;\ 2)$$



8 ② Calcular el flujo de $\vec F \in C^1$ a través de la porción de cilindro $x^2+y^2=3$ limitado por $0 \le z \le 4$ si $\operatorname{div}\vec F = x^2+1$, $\vec F(x,y,0) = (y^2; x^2y; 0)$; $F(x,y,4) = (y^2; x^2y; 1)$.



$$\underbrace{S^*}_{\text{todo}} = \underbrace{S}_{\text{cilindro}} \cup \underbrace{S_1}_{\text{tapa abajo}} \cup \underbrace{S_2}_{\text{tapa arriba}}$$

"frontera" → superficie cerrada

$$\iiint_V \operatorname{div}\vec F\,dx\,dy\,dz = \int_0^{2\pi} d\theta \cdot \int_0^{\sqrt3} \rho\,d\rho \int_0^4 (\rho^2\cos^2\theta + 1)\,dz = 4\int_0^{2\pi} d\theta \cdot \int_0^{\sqrt3} (\rho^3\cos^2\theta + \rho)\,d\rho = 4\int_0^{2\pi}\left(\frac{9}{4}\cos^2\theta + \frac{3}{2}\right)d\theta = 21\pi$$

$\frac{9}{4}\pi + 3\pi = \frac{21}{4}\pi$

piso (z=0)

$$\iint_{S_1} \vec F\,ds = \iint_{D_{xy}} (y^2; x^2y; 0)\cdot(0,0,-1)\,dx\,dy = \iint_{D_{xy}} 0 = 0$$

tapa (z=4)

$$\iint_{S_2} \vec F\,ds = \iint_{D_{xy}} (y^2; x^2y; 1)\cdot(0,0,1)\,dx\,dy = \iint_{D_{xy}} dx\,dy = \pi\cdot(\sqrt3)^2 = 3\pi$$

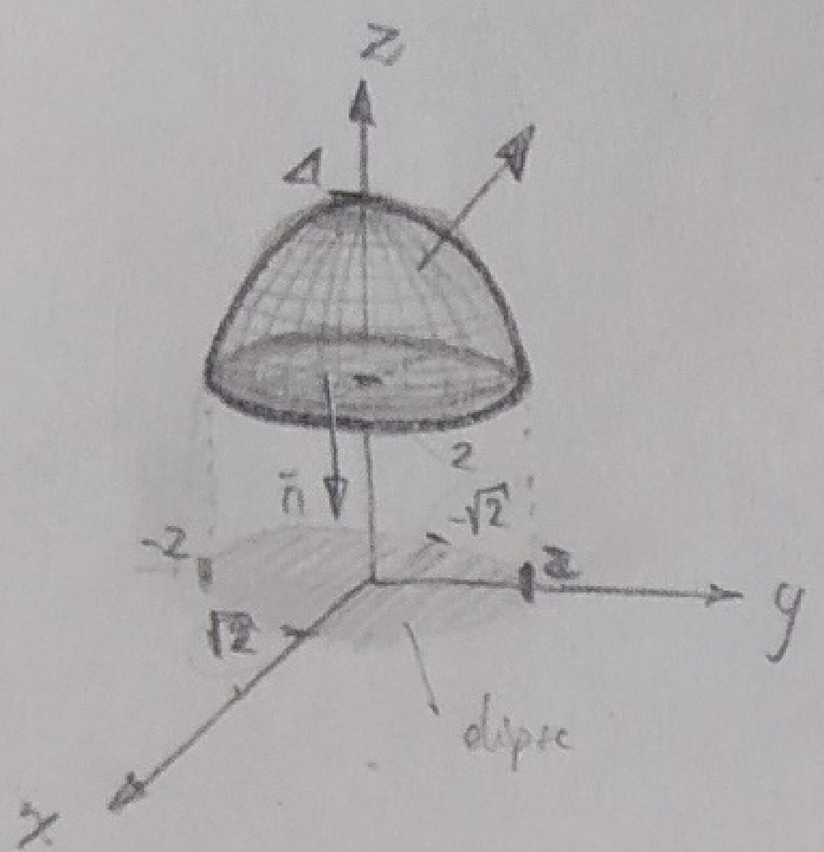
$$\iint_{S_{cilindro}} \vec F\,ds = 21\pi - 3\pi = \boxed{18\pi}$$

$$\underbrace{\iint_{S^*} \vec F\cdot\vec n\,ds}_{21\pi} = \iint_S \vec F\cdot\vec n\,ds + \underbrace{\iint_{S_1} \vec F\cdot\vec n\,ds}_{0} + \underbrace{\iint_{S_2} \vec F\cdot\vec n\,ds}_{3\pi}$$

$$\iint_S \vec F\cdot\vec n\,ds = 18\pi$$

9 ③ Mediante una conveniente aplicación del teorema de la divergencia, calcular el flujo de $\bar{F}(x,y,z) = (y; x; y^2)$ sobre la porción de superficie $z = 4 - 2x^2 - y^2$ con $z \geq 2$.

Indicar el sentido en que calcula el flujo.



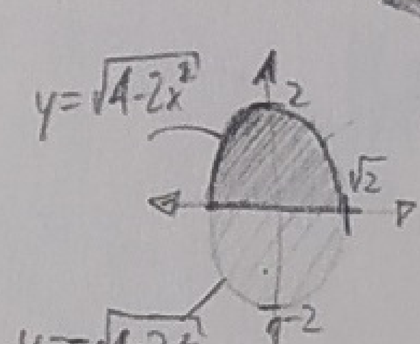
$\left.\begin{matrix} x=0 \\ z=0 \end{matrix}\right\}$ $z = 4 - 2x^2 - y^2$
$0 = 4 - y^2$
$y = -2 \wedge y = 2$

$\left.\begin{matrix} y=0 \\ z=0 \end{matrix}\right\}$ $z = 4 - 2x^2 - y^2$
$0 = 4 - 2x^2$
$x = -\sqrt{2} \wedge x = \sqrt{2}$

$$\operatorname{div} \bar{F} = \nabla \cdot \bar{F} = \frac{\partial (y)}{\partial x} + \frac{\partial (x)}{\partial y} + \frac{\partial (y^2)}{\partial z} = 0$$

$$\iint_{S^*} \bar{F} \cdot \bar{n}\, ds = \iiint_M \operatorname{div} \bar{F} \cdot dx\,dy\,dz = \iiint_M 0\, dx\,dy\,dz = 0$$

$$\iint_{S^*} \bar{F} \cdot \bar{n} \cdot ds = \iint_S \bar{F} \cdot \bar{n} \cdot ds + \iint_{S_1} \bar{F} \cdot \bar{n}\, ds$$



$$0 = \iint_S \bar{F} \cdot \bar{n} \cdot ds - 8{,}8857$$

$$\boxed{\iint_S \bar{F} \cdot \bar{n} \cdot ds = 8{,}8857}$$

$\bar{n} = (0, 0, -1)$

$$\iint_{S_1} \bar{F} \cdot \bar{n}\, ds = \iint_{D_{xy}} (y, x, y^2) \cdot (0, 0, -1)\, dx\, dy$$

$$= \iint_{D_{xy}} -y^2 = \int_{-\sqrt{2}}^{\sqrt{2}} dx \int_0^{\sqrt{4-2x^2}} -y^2 dy + \int_{-\sqrt{2}}^{\sqrt{2}} dx \int_{-\sqrt{4-2x^2}}^{0} -y^2 dy$$

$$-\frac{y^3}{3}\Big|_0^{\sqrt{4-2x^2}}$$

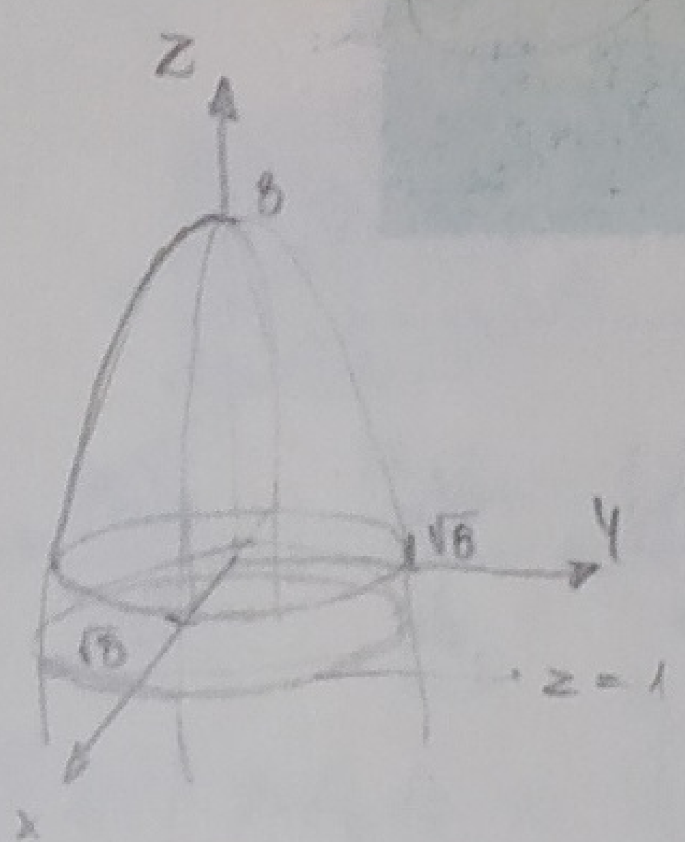
$$= -\frac{1}{3} \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{(4-2x^2)}^3\, dx - \frac{1}{3} \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{(4-2x^2)}^3\, dx$$

$$= -\frac{2\sqrt{2}}{3} \int_{-\sqrt{2}}^{\sqrt{2}} \left(\sqrt{2-x^2}\right)^3 dx$$

$$= -\frac{2\sqrt{2}}{3} \left[\frac{x\sqrt{(2-x^2)^3}}{4} + \frac{6x\sqrt{2-x^2}}{8} + \frac{3 \cdot 4}{8} \operatorname{arcsen}\left(\frac{x}{\sqrt{2}}\right) \right]_{-\sqrt{2}}^{\sqrt{2}}$$

$$= -8{,}8857$$

10 ② Mediante una conveniente aplicación del teorema de la divergencia, calcular el flujo de $\bar{F}(x,y,z) = (e^x z; \cos(xy); -x^2)$ a través de la porción de paraboloide $z = 8 - y^2 - x^2$ con $z \leq -1$.



¿z menor a -1?

$$\operatorname{div} \bar{F} = \nabla \cdot \bar{F} = \underbrace{\frac{\partial(e^x \cdot z)}{\partial x}}_{z \cdot e^x} + \underbrace{\frac{\partial(\cos(xy))}{\partial y}}_{-x \cdot \operatorname{sen} x} + \underbrace{\frac{\partial(-x^2)}{\partial z}}_{0}$$

$$= z \cdot e^x - x \cdot \operatorname{sen}(xy)$$

$$\iint \bar{F} \cdot \bar{n}\, ds = \iiint_M \operatorname{div} \bar{F}\, dx\, dy\, dz = \int dx \int dy \int (ze^x - x \operatorname{sen}(xy))\, dz$$