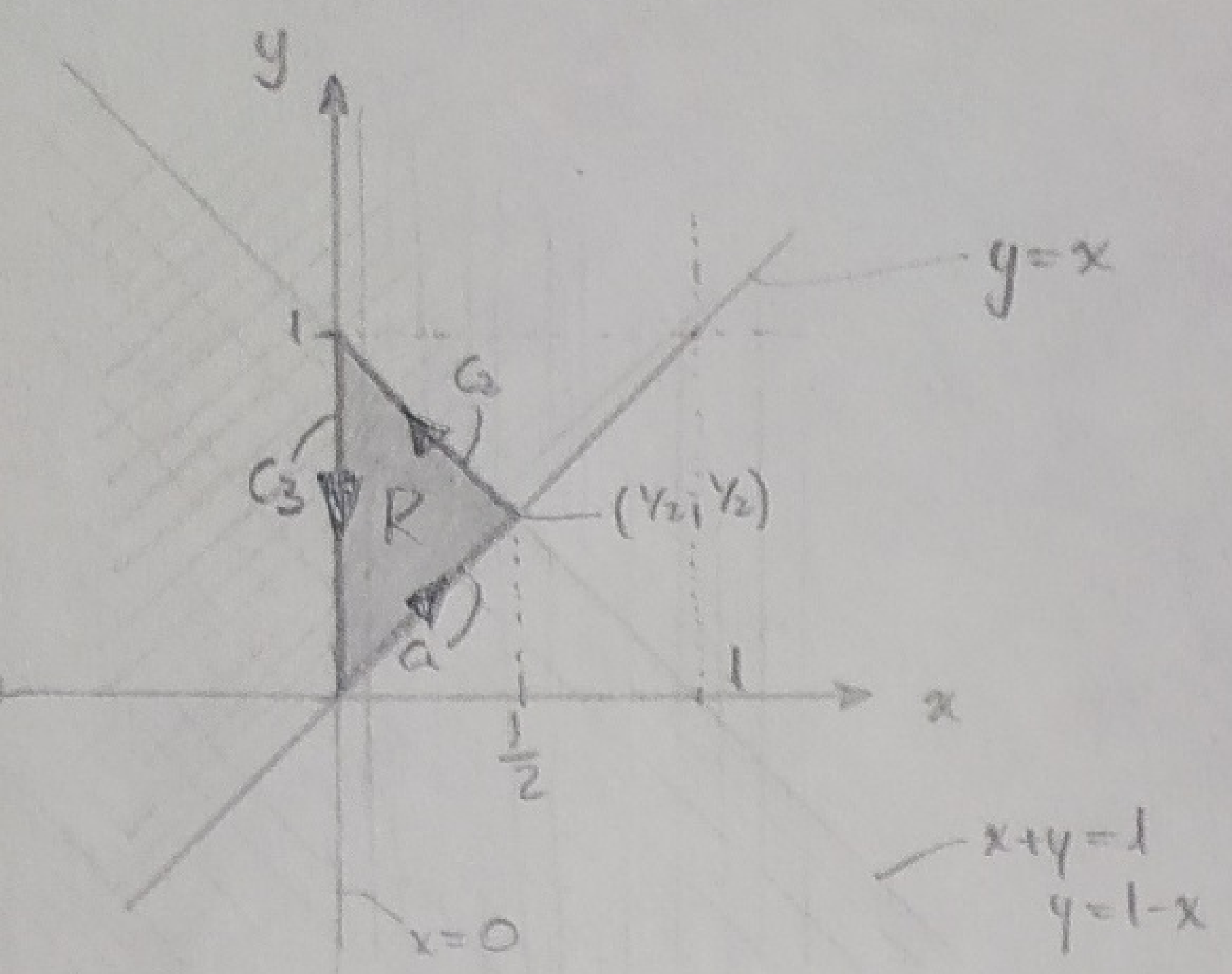
DE PARCIAL → ÁREAS PLANAS (INTEGRAL DE LÍNEA)

1 ③ Mediante integral de línea calcular el área del recinto plano que resulta de: $y \geq x$, $x+y \leq 1$, $x \geq 0$. → $y \leq -x+1$



$\begin{cases} y=x \\ x+y=1 \end{cases} \to x+x=1 \to x=\frac{1}{2}$ $\left(y=\frac{1}{2}\right)$

Planteos $\bar{f}(x,y) = (P_{(x,y)} ; Q_{(x,y)}) = (0, x)$

$P_{(x,y)} = 0 \to P'_y = 0$
$Q_{(x,y)} = x \to Q'_x = 1$
$Q'_x - P'_y = 1 - 0 = 1$ es una constante

$a(R) = \oint \bar{f} \cdot d\bar{s}$

$a(R) = \int_{C_1} \bar{f} \cdot d\bar{s} + \int_{C_2} \bar{f} \cdot d\bar{s} + \int_{C_3} \bar{f} \cdot d\bar{s}$

$C_1: \begin{cases} x=t \\ y=t \end{cases}$ $0 \leq t \leq \frac{1}{2}$ → $\bar{g}_1(t) = (t;t)$ → $\bar{g}'(t) = (1;1)$

$\bar{f}(\bar{g}_1(t)) = \bar{f}(t;t) = (0,t)$

$\int_{C_1} 0dx + xdy = \int_{C_1} \bar{f}(\bar{g}(t)) \cdot \bar{g}(t) \cdot dt = \int_{C_1} (0,t) \cdot (1,1) \cdot dt = \int_0^{1/2} t \cdot dt = \frac{t^2}{2}\Big|_0^{\frac{1}{2}} = \frac{1}{8}$

$C_2: \begin{cases} x=\frac{1}{2}-t \\ y=\frac{1}{2}+t \end{cases}$ $0 \leq t \leq \frac{1}{2}$ → $\bar{g}_2(t) = \left(\frac{1}{2}-t ; \frac{1}{2}+t\right)$ → $\bar{g}'(t) = (-1;1)$

$\bar{f}(\bar{g}_2(t)) = \bar{f}\left(\frac{1}{2}-t;\frac{1}{2}+t\right) = \left(0,\frac{1}{2}-t\right)$

$\int_{C_2} 0dx + xdy = \int_{C_2} \bar{f}(\bar{g}(t)) \cdot \bar{g}'(t) \cdot dt = \int_{C_2} \left(0,\frac{1}{2}-t\right) \cdot (-1;1) \cdot dt = \int_0^{\frac{1}{2}} \left(\frac{1}{2}-t\right) dt = \frac{1}{2}t - \frac{t^2}{2}\Big|_0^{\frac{1}{2}} = \frac{1}{8}$

$C_3: \begin{cases} x=0 \\ y=1-t \end{cases}$ $0 \leq t \leq 1$ → $\bar{g}_3(t) = (0;1-t)$ → $\bar{g}'(t) = (0,-1)$

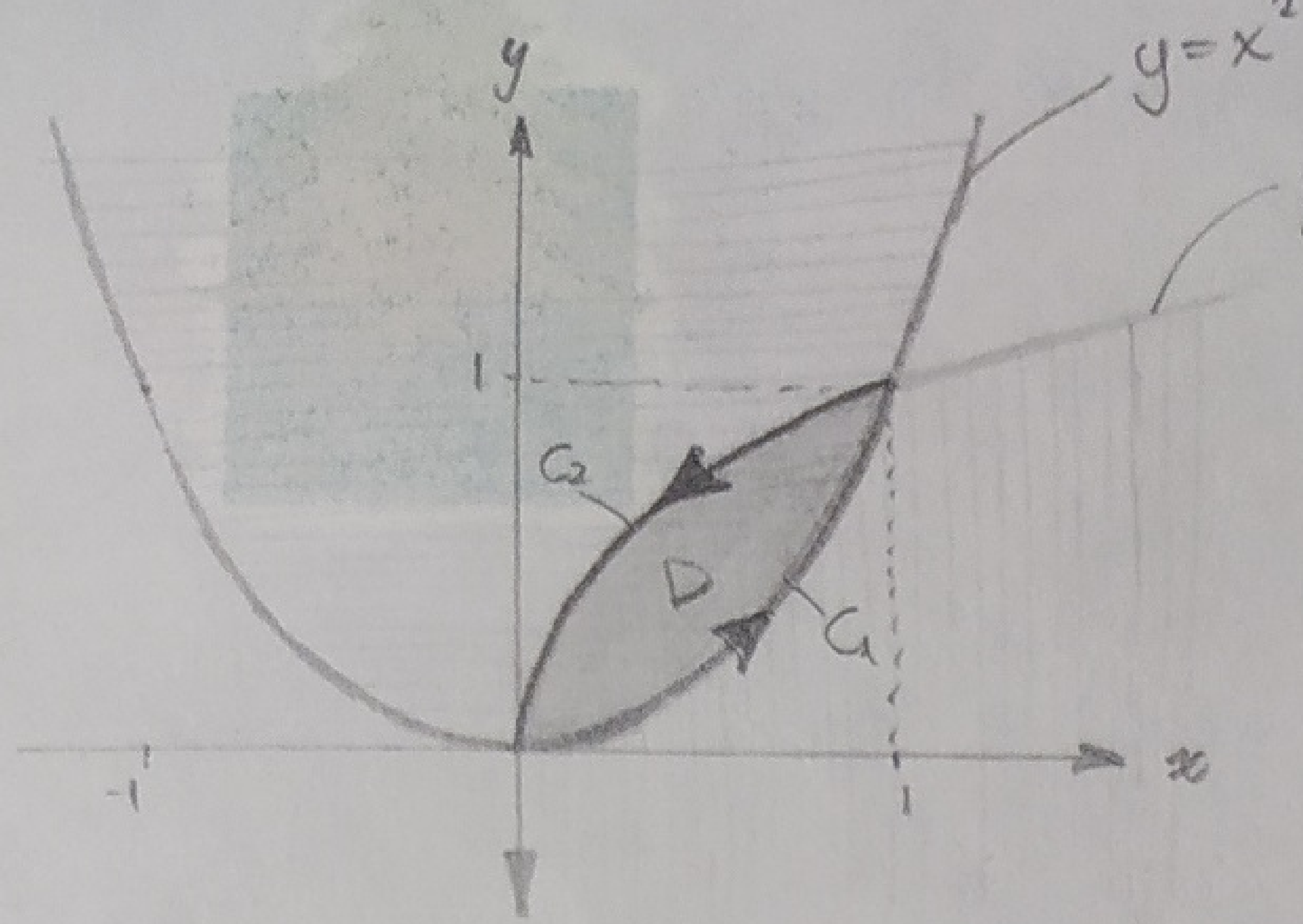
$\bar{f}(\bar{g}_3(t)) = \bar{f}(0,1-t) = (0,0)$

$\int_{C_3} 0dx + xdy = \int_{C_3} \bar{f}(\bar{g}(t)) \cdot \bar{g}'(t) \cdot dt = \int_{C_3} (0,0) \cdot (0,-1)\, dt = \int_0^1 0\, dt = 0$

$\boxed{a(R) = \frac{1}{8} + \frac{1}{8} + 0 = \frac{1}{4}}$

3 ②

Mediante integral de línea calcular el área de recinto plano que resulta de $x^2 \leq y \leq \sqrt{x}$.



$\begin{cases} y=\sqrt{x} \\ y=x^2 \end{cases}$

$(x^2)^2 = (\sqrt{x})^2$

$x^4 = x$

$x^4 - x = 0$

$x(x^3-1) = 0$

$x_1 = 0 \qquad x_2 = 1$

$y_1 = (0)^2 \qquad y_2 = (1)^2$

$y_1 = 0 \qquad y_2 = 1$

Puntos de intersección: $(0,0)$, $(1,1)$

Definimos: $\bar{F}: \mathbb{R}^2 \to \mathbb{R}^2 \,/\, \bar{F}_{(x,y)} = (0, x)$

Si $\bar{F} = (0,x) \begin{cases} P = 0 \longrightarrow P'_y = 0 \\ Q = x \longrightarrow Q'_x = 1 \end{cases} \quad \underbrace{Q'_x}_{1} - \underbrace{P'_y}_{0} = 1$ ⟶ es una constante ✓

$\boxed{\bar{F}_{(x,y)} = (\underset{P}{0}, \underset{Q}{x})}$

$a(R) = \oint_C \bar{F} \cdot d\bar{s}$

$a(R) = \int_{C_1} \bar{F} \cdot d\bar{s} + \int_{C_2} \bar{F}\, ds$

$C_1: \begin{cases} x = t \\ y = t^2 \end{cases} \quad 0 \leq t \leq 1 \qquad \bar{g}_1(t) = (t, t^2) \longrightarrow \bar{g}_1'(t) = (1, 2t)$

$\bar{F}(\bar{g}_1(t)) = \bar{F}(t, t^2) = (0, t)$

$\int_{C_1} \bar{F} \cdot d\bar{s} = \int_{C_1} \bar{F}(\bar{g}_1(t)) \cdot \bar{g}_1'(t)\, dt = \int_{C_1} (0,t)\cdot(1,2t)\, dt = \int_0^1 2t^2 = \frac{2}{3} t^3 \Big|_0^1 = \frac{2}{3}$

$C_2: \begin{cases} x = 1-t \\ y = \sqrt{1-t} \end{cases} \quad 0 \leq t \leq 1 \qquad \bar{g}_2(t) = (1-t; \sqrt{1-t}) \longrightarrow \bar{g}_2'(t) = \left(-1; -\frac{1}{2\sqrt{1-t}}\right)$

$\bar{F}(\bar{g}(t)) = \bar{F}(1-t; \sqrt{1-t}) = (0; 1-t)$

$\boxed{\int_{C_2} \bar{F} \cdot ds} = \int_{C_2} \bar{F}(\bar{g}_2(t)) \cdot \bar{g}_2'(t) \cdot dt = \int_{C_2} (0, 1-t) \cdot \left(-1; -\frac{1}{2\sqrt{1-t}}\right) dt = \int_0^1 -\frac{1-t}{2\sqrt{1-t}}\, dt$

$= -\int_0^1 \frac{\sqrt{1-t}}{2}\, dt = -\left[-\frac{(1-t)^{\frac{3}{2}}}{3}\right]_0^1 = -\left[\underbrace{\cancel{\left(-\frac{(1-1)^{\frac{3}{2}}}{3}\right)}}_{0} - \underbrace{\left(-\frac{(1-0)^{\frac{3}{2}}}{3}\right)}_{(-\frac{1}{3})}\right] = -\frac{1}{3}$

$a(R) = \frac{2}{3} + \left(-\frac{1}{3}\right)$

$\boxed{a(R) = \frac{1}{3}}$ ✓