# DE PARCIAL ⟶ POLINOMIOS DE TAYLOR

$\boxed{2}$ ① Si $f \in C^3 \ \forall (x,y) \in \mathbb{R}^2$ determinar el Polinomio de Taylor de $2^{do}$ grado en $(3,1)$; sabiendo que: ① el plano tangente de la función en el punto $(3,1,5)$ es paralelo al plano $x+y=2z$ y ② el Hessiano en $(3,1)$ es $H = \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix}$.

$x_0 = 3$

$y_0 = 1$

$f(3,1) = 5$

$$f(x,y) = f(3,1) + f'_x(3,1)\cdot(x-3) + f'_y(3,1)\cdot(y-1) + \frac{1}{2}\left[f''_{xx}(3,1)\cdot(x-3)^2 + 2\cdot f''_{xy}(3,1)\cdot(x-3)(y-1) + f''_{yy}(3,1)\cdot(y-1)^2\right]$$

$$H(x,y) = \begin{vmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{vmatrix}_{(x_0,y_0)} \Rightarrow H(3,1) = \begin{vmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{vmatrix}_{(3,1)} = \begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix}$$

$$\begin{cases} f''_{xx}(3,1) = 2 \\ f''_{xy}(3,1) = f''_{yx}(3,1) = 1 \\ f''_{yy}(3,1) = 4 \end{cases}$$

$\underset{\alpha}{\text{PT a } f \text{ en } (3,1,5)}$ // plano $\overset{\beta}{x+y=2z}$ $\Rightarrow$ tienen normales proporcionales $\vec{n}_\beta = (1,1,-2)$

$$z - z_0 = f'_x(x_0,y_0)\cdot(x-x_0) + f'_y(x_0,y_0)\cdot(y-y_0)$$

$$z - 5 = f'_x\cdot(x-3) + f'_y(y-1)$$

$$0 = f'_x x - 3f'_x + f'_y y - f'_y - z + 5$$

$$\vec{n}_\alpha = (f'_x; f'_y; -1)$$

$$\begin{cases} f'_x = k\cdot(1) \\ f'_y = k\cdot(1) \\ -1 = k\cdot(-2) \end{cases} \quad k = \frac{1}{2} \quad f'_x = \frac{1}{2} \quad f'_y = \frac{1}{2}$$

$$f(x,y) = 5 + \frac{1}{2}(x-3) + \frac{1}{2}(y-1) + \frac{1}{2}\left[2\cdot(x-3)^2 + 2\cdot 1\cdot(x-3)(y-1) + 4\cdot(y-1)^2\right]$$

$$\boxed{f(x,y) = 5 + \frac{1}{2}(x-3) + \frac{1}{2}(y-1) + (x-3)^2 + (x-3)(y-1) + 2(y-1)^2}$$

3 ② Si $f \in C^3 \ \forall (x,y) \in \mathbb{R}^2$ con Polinomio de Taylor de 2do grado en un entorno del $(1,-2)$

$p(\bar{x}) = 5 + 4x + 2y + 2xy + y^2$.

a. Hallar la ecuación del plano tangente en $(1;-2;f(1,-2))$.

b. Analizar si $f(1;-2)$ es un extremo.

$$p(1,-2) = f(1,-2) + f'_x(1,-2)\cdot(x-1) + f'_y(1,-2)\cdot(y+2) + \frac{1}{2}\left[f''_{xx}(1,-2)\cdot(x-1)^2 + 2\cdot f''_{xy}(1,-2)\cdot(x-1)(y+2) + f''_{yy}(1,-2)\,(y-2)^2\right]$$

$$= f(1,-2) + f'_x\cdot x - f'_x + f'_y\cdot y + 2\cdot f'_y + \frac{1}{2}\cdot f''_{xx}\cdot(x^2-2x+1) + f''_{xy}\cdot(xy+2x-y-2) + \frac{1}{2}\,f''_{yy}\cdot(y^2-4y+4)$$

$$= f(1,-2) + (f'_x)\cdot x - f'_x + (f'_y)\cdot y + 2\cdot f'_y + \frac{1}{2}f''_{xx}\cdot x^2 - f''_{xx}\cdot x + \frac{1}{2}f''_{xx} + f''_{xy}\cdot xy + 2\cdot f''_{xy}\cdot x - f''_{xy}\cdot y - 2f''_{xy} + \frac{1}{2}f''_{yy}\cdot y^2 - 2f''_{yy}\cdot y + 2f''_{yy}$$

$$= \underbrace{\left(f(1,-2) - f'_x + 2\cdot f'_y + \frac{1}{2}f''_{xx} - 2f''_{xy} + 2\cdot f''_{yy}\right)}_{=5} + \underbrace{(f'_x - f''_{xx} + 2\cdot f''_{xy})}_{=4}\cdot x + \underbrace{(f'_y - f''_{xy} - 2\cdot f''_{yy})}_{=2}\cdot y + \underbrace{\left(\frac{1}{2}f''_{xx}\right)}_{=0}x^2 + \underbrace{(f''_{xy})}_{=2}xy + \underbrace{\left(\frac{1}{2}f''_{yy}\right)}_{=1}y^2$$

$$\begin{cases} f(1,-2) - f'_x + 2\cdot f'_y + \frac{1}{2}\cdot f''_{xx} - 2\cdot f''_{xy} + 2\cdot f''_{yy} = 5 \\ f'_x - f''_{xx} + 2\cdot f''_{xy} = 4 \\ f'_y - f''_{xy} - 2\cdot f''_{yy} = 2 \\ \frac{1}{2}\cdot f''_{xx} = 0 \\ f''_{xy} = 2 \\ \frac{1}{2}\cdot f''_{yy} = 1 \end{cases}$$

$f(1,-2) \cancel{-0} + \overset{16}{2\cdot 8} + \cancel{\tfrac{1}{2}\cdot 0} \overset{-4}{-2\cdot 2} \overset{+4}{+2\cdot 2} = 5$

$\boxed{f(1,-2) = 5-16 = -11}$

$f'_x - 0 + 2\cdot 2 = 4 \Rightarrow \boxed{f'_x = 0}$

$f'_y - 2 - 2\cdot 2 = 2 \longrightarrow f'_y - 6 = 2 \Rightarrow \boxed{f'_y = 8}$

$\boxed{f''_{xx} = 0}$

$\boxed{f''_{yy} = 2}$

ⓐ PT: $\overset{f(1,-2)}{z - z_0} = \overset{f'_x(1,-2)}{f'_x(x_0,y_0)}\,(x-x_0) - \overset{f'_y(1,-2)}{f'_y(x_0,y_0)}\cdot(y-y_0)$

$z - (-11) = \cancel{0\,(x-1)} - 8\,(y+2)$

$z + 11 = -8y - 16$

$\boxed{8y + z = -27}$ ecuación del plano tangente en $(1,-2,-11)$.

ⓑ ¿Es un extremo relativo $(1,-2)$? $=$ ¿$\nabla f(1,-2) = (0,0)$?

$(f'_x(1,-2);\ f'_y(1,-2)) \overset{?}{=} (0,0)$

$(0,8) \neq (0,0)$. $\Rightarrow$ NO es extremo relativo.