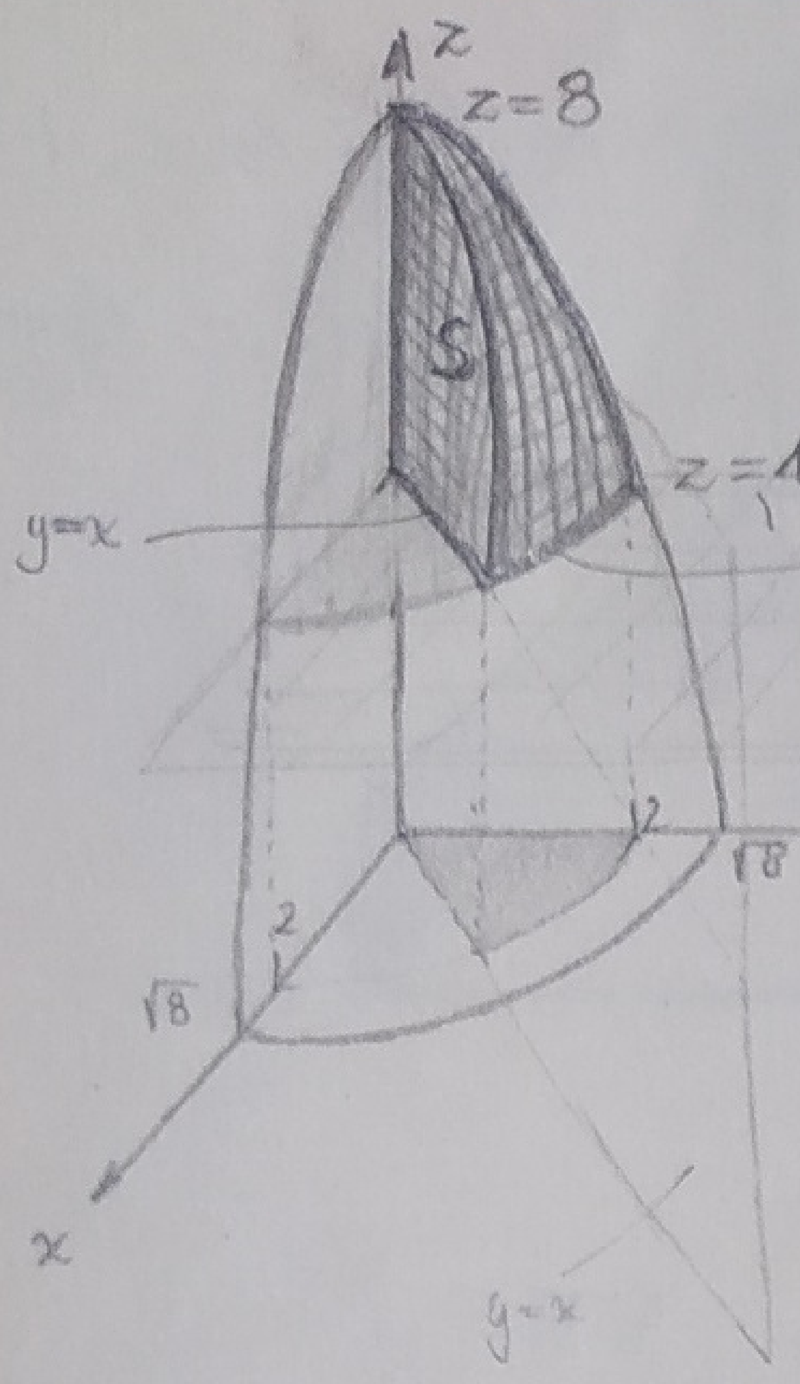
DE PARCIAL → INTEGRALES TRIPLES

$\boxed{2}$ ② Calcular el volumen del sólido que resulta de: $z \leq 8-x^2-y^2$ → paraboloide

$z \geq 4.$

$x \geq 0$

$y \geq x.$



$4 = 8-x^2-y^2$

$x^2+y^2=4$

$y^2 = 4-x^2$

$y = \sqrt{4-x^2}$

$0 \leq x \leq 2$

$x \leq y \leq \sqrt{4-x^2}$

$4 \leq z \leq 8-x^2-y^2$

COORDENADAS CARTESIANAS → COORDENADAS CILÍNDRICAS

$$V(S) = \int_0^2 dx \int_x^{\sqrt{4-x^2}} dy \int_4^{8-x^2-y^2} dz = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_0^2 \rho \, d\rho \int_4^{8-\rho^2} dz$$

$$z\Big|_4^{8-\rho^2} = 8-\rho^2-4 = 4-\rho^2$$

$0 \leq \rho \leq 2$

$\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

$4 \leq z \leq 8-x^2-y^2$

$8-(x^2+y^2)$

$4 \leq z \leq 8-\rho^2$

$\rho^2 = x^2+y^2$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_0^2 \rho \cdot (4-\rho^2) \, d\rho = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_0^2 (4\rho - \rho^3) \, d\rho$$

$$2\rho^2 - \frac{1}{3}\rho^3 \Big|_0^2 = 2 \cdot 2^2 - \frac{1}{3} \cdot 2^3 = \frac{16}{3}$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{16}{3} \, d\theta = \frac{16}{3}\theta \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \frac{16}{3}\frac{\pi}{2} - \frac{16}{3}\frac{\pi}{4} = \boxed{\frac{4}{3}\pi}$$

7 (2) Calcular el volumen del cuerpo definido por: $x^2+3y^2+z \leq 2z$.

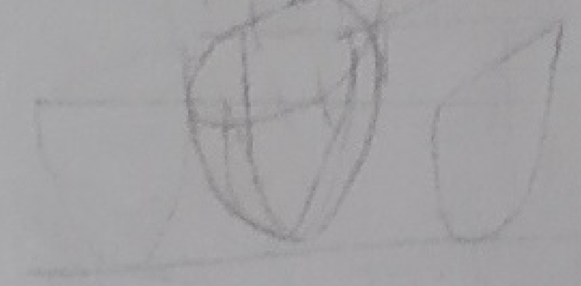
$z \geq 2x^2$. — paraboloide cilíndrico

$x^2+3y^2 = z$, paraboloide elíptico

$\begin{matrix} x=0 \\ y=0 \end{matrix} \rightarrow z=0$

$\begin{matrix} x=0 \\ z=0 \end{matrix} \rightarrow 3y$

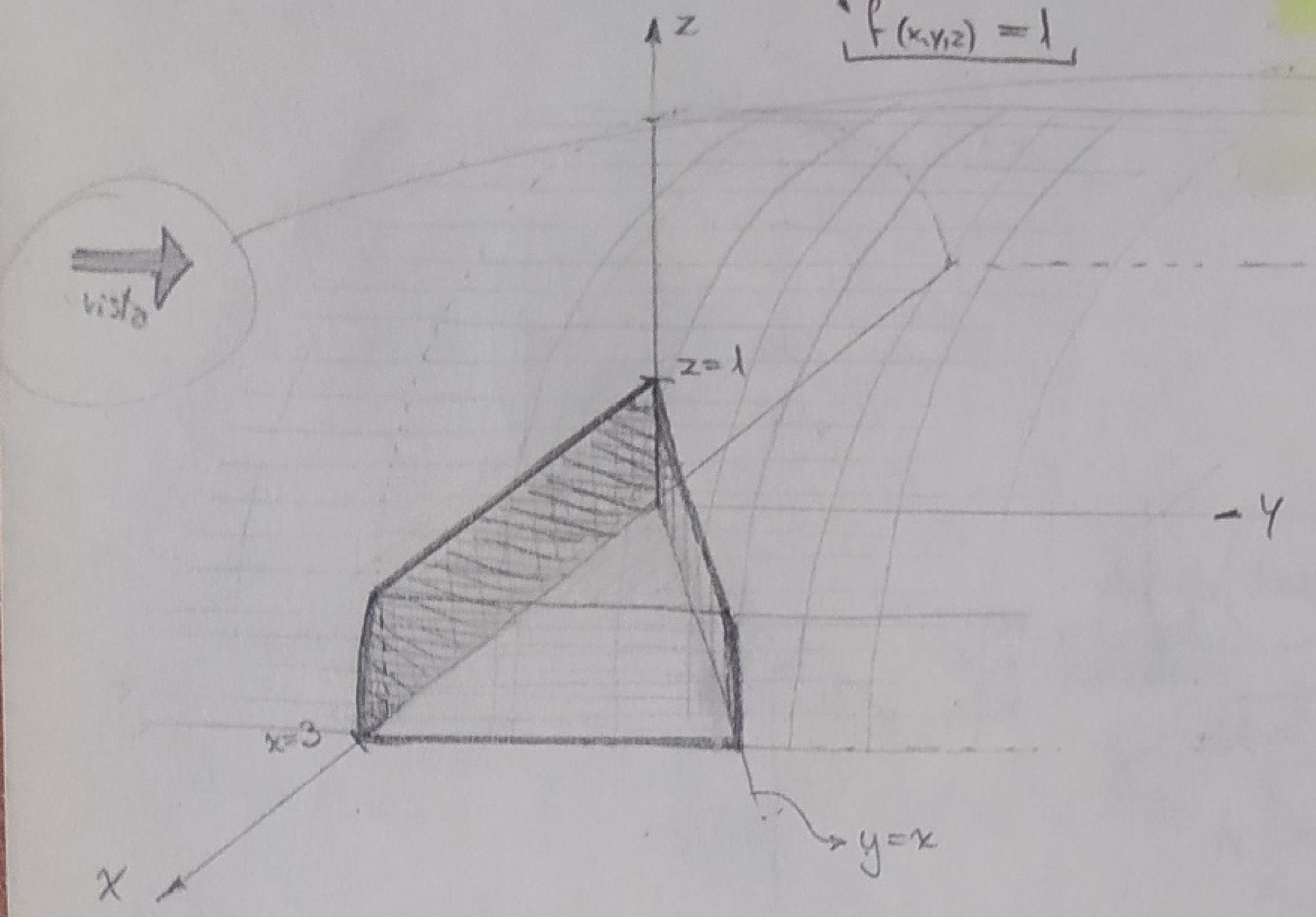
Queda un sólido infinito... — no tiene techo!

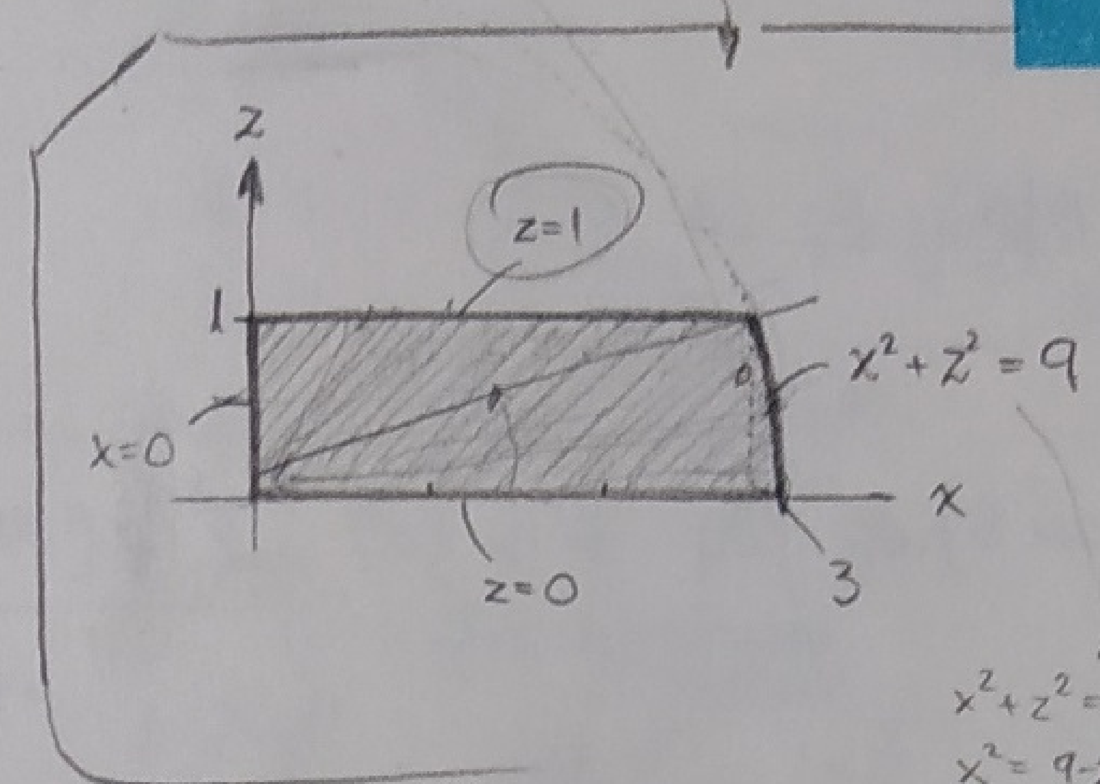


DE PARCIAL → INTEGRALES TRIPLES

♦ Calcular el volumen del sólido que resulta de: $x^2+z^2 \leq 9$, $y \leq x$, $y \geq 0$, $0 \leq z \leq 1$.

$f_{(x,y,z)} = 1$





$x^2+z^2=9$
$x^2=9-z^2$
$x=\sqrt{9-z^2}$

$$\begin{cases} x = \rho \cdot \cos\theta \\ y = y \\ z = \rho \cdot \operatorname{sen}\theta \end{cases}$$

$0 \leq x \leq \sqrt{9-z^2}$
$0 \leq z \leq 1$
$0 \leq y \leq x$

$$V(S) = \int_0^1 dz \int_0^{\sqrt{9-z^2}} dx \int_0^x dy = \int_0^1 dz \int_0^{\sqrt{9-z^2}} dx \Big[\underbrace{y\Big|_0^x}_{x}\Big] = \int_0^1 dz \int_0^{\sqrt{9-z^2}} x\,dx = \int_0^1 dz \Big[\underbrace{\frac{x^2}{2}\Big|_0^{\sqrt{9-z^2}}}_{\frac{9-x^2}{2}}\Big]$$

$$= \int_0^1 \left(\frac{9}{2} - \frac{1}{2}z^2\right) dz = \frac{9}{2}z - \frac{1}{6}z^3\Big|_0^1 = \frac{9}{2} - \frac{1}{6} = \boxed{\frac{13}{3}}$$

No conviene plantearlo coordenadas cilíndricas porque la proyección en xz NO es simétrica

Si hubiera sido algo así , ahí sí.

1 ① Hallar el momento de inercia respecto del eje y del sólido que resulta de $\sqrt{x^2+z^2} \le y \le 1$ si la densidad en cada punto es proporcional a la distancia del punto del sólido al plano yz.

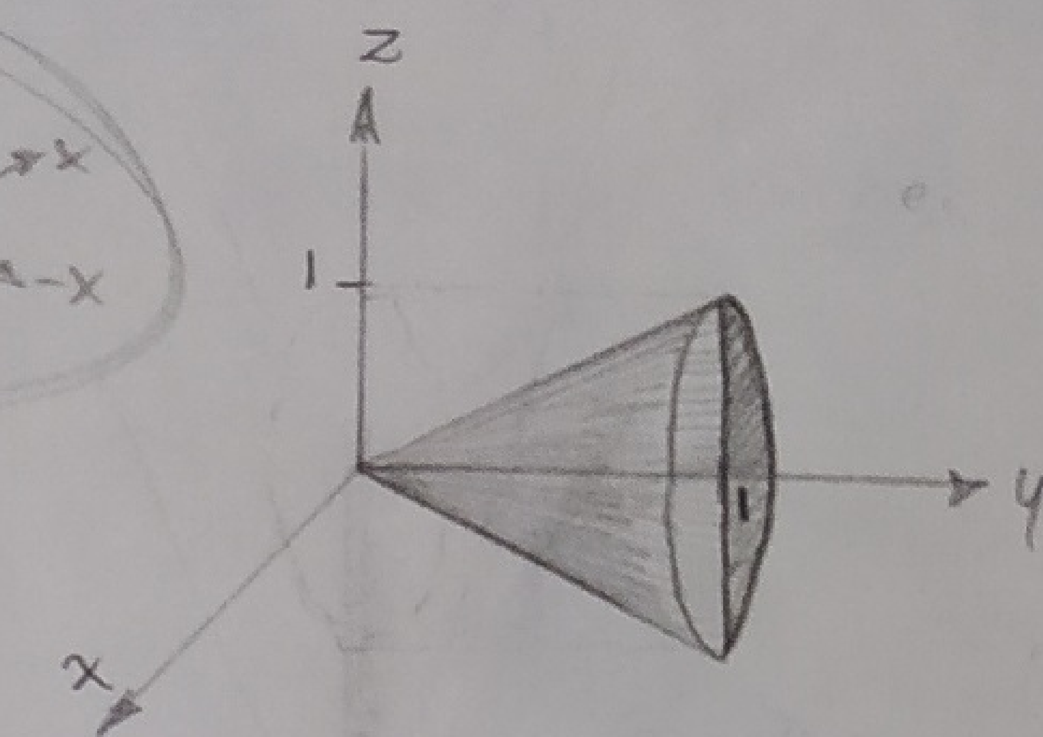
la parte de arriba

$\delta = k|x|$

$$I_y = \iiint \underbrace{(x^2+z^2)}_{\rho^2} \cdot \underbrace{\overbrace{k|x|}^{\delta}}_{|\rho\cdot\cos\theta|} \, dx\,dy\,dz$$

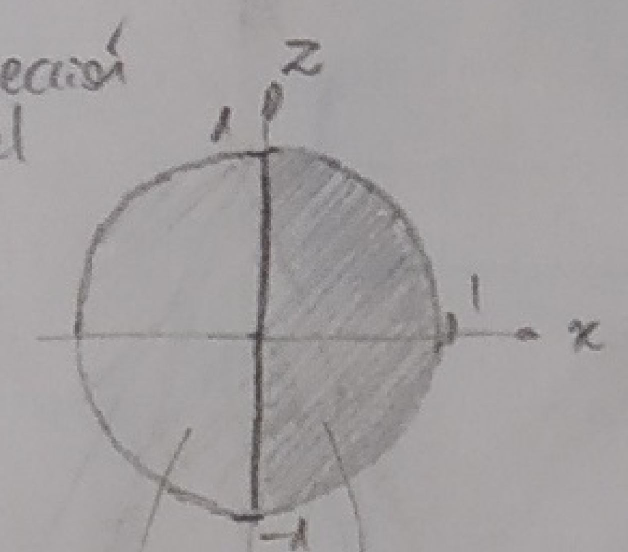
hay que abrir $|x|$ en $+x$, $-x$

$$I_y = k \iiint \rho^2 \underbrace{|\rho\cdot\cos\theta|}_{\rho|\cos\theta|} \overbrace{\rho}^{\text{jacobiano}} \, dy\, d\rho\, d\theta + k$$



$$I_y = k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^1 \rho^3(\rho\cdot\cos\theta)\,d\rho \int_\rho^1 dy + k\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\theta \int_0^1 \rho^3(-\rho\cdot\cos\theta)\,d\rho\int_\rho^1 dy$$

proyección en el xz



$$\begin{cases} x = \rho\cdot\cos\theta \\ y = y \\ z = \rho\cdot\operatorname{sen}\theta \end{cases}$$

$x^2+z^2 = \rho^2$

$|J| = \rho$

$$I_y = k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\cdot d\rho \cdot \int_0^1 \rho^4(1-\rho)\,d\rho - k\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi}\cos\theta\, d\theta\cdot\int_0^1\rho^4(1-\rho)\,d\rho$$

$$I_y = k\cdot 2\cdot\frac{1}{30} - k\cdot(-2)\cdot\frac{1}{30}$$

$$I_y = \frac{1}{15}k + \frac{1}{15}k$$

$$\boxed{I_y = \frac{2}{15}k}$$

$0 \le \rho \le 1 \;=\; 0 \le \rho \le 1$

$\rho \le y \le 1 \;=\; \rho \le y \le 1$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \;\neq\; \frac{\pi}{2} \le \theta \le \frac{3}{2}\pi$

$$\int_\rho^1 dy = y\Big|_\rho^1 = 1-\rho$$

$$\int_0^1 \rho^4(1-\rho)\,d\rho = \int_0^1 \rho^4-\rho^5\,d\rho = \frac{\rho^5}{5}-\frac{\rho^6}{6}\Big|_0^1 = \frac{1}{30}$$

$$\int\cos\theta\, d\theta = \operatorname{sen}\theta \rightarrow \operatorname{sen}\theta\Big|_{-\pi/2}^{\pi/2} = 2$$

$$\operatorname{sen}\theta\Big|_{\frac{\pi}{2}}^{\frac{3}{2}\pi} = -2$$