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DE PARCIAL - IMPLÍCITAS/COMPUESTAS

1 ① Aplicando diferencial total, hallar aproximadamente $z = h(\overset{u=1}{0{,}98};\ \overset{v=2}{2{,}01})$ si $z = h(u,v)$ resulta de la composición de $z = f(x,y)$ definida implícitamente por la ecuación $z^2 + \sqrt{z} = e^{x-2} + e^{y-2}$ con $\begin{cases} x = 2u \\ y = g(v)^2 \end{cases}$ donde $w = g(v)$ queda definida por la ecuación $w^2 + \ln(w^2 - 1) = v$.

$$h(0{,}98;2{,}01) \simeq h(1,2) + h'_x(1,2)\cdot(0{,}98-1) + h'_y(1,2)\cdot(2{,}01-2)$$

$h(1,2)$ — $\underline{u=1}$, $\underline{v=2}$

$x = 2(1)$

$\underline{x=2}$

$y = g(2)^2$

$y = 1^2$

$\underline{y=1}$

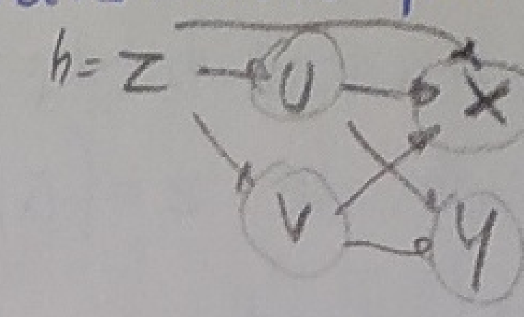
$w^2 + \ln(w^2-1) = 2$

por tanteo: $\underline{w=1}$

$z^2 + \sqrt{z} = e^{2-2} + e^{1-2}$

$z^2 + \sqrt{z} = 1 + \frac{1}{e}$

3 ① Hallar valor y dirección de derivada direccional máxima en el punto $(1,4)$ de la función compuesta $z=h(x,y)$ que resulta de $z=u\cdot x\cdot v^2$ donde $u=x\sqrt{y}$; $v(x,y)$ definida implícitamente por $2v+e^{v-2x}-\frac{y}{x}=1$



$$h'((1,4),\breve{u})_{máx}=\left\|\left(h'_x(1,4);\,h'_y(1,4)\right)\right\|=\sqrt{\left(h'_x(1,4)\right)^2+\left(h'_y(1,4)\right)^2}$$

$$h'_x=z'_u\cdot u'_x+z'_v\cdot v'_x+z'_x=\overbrace{4\cdot 2}^{8}+\overbrace{8\cdot 2}^{16}+8=\boxed{32}$$

$$h'_y=z'_u\,u'_y+z'_v\cdot v'_y=\overbrace{4\cdot\frac{1}{2}}^{2}+\overbrace{8\cdot\frac{1}{3}}^{8/3}=\boxed{\frac{14}{3}}$$

$$z'_u=x\cdot v^2\Big|_{\substack{x=1\\v=2}}=\boxed{4}\qquad u'_x=\sqrt{y}\,\Big|_{y=4}=\boxed{2}$$

$$z'_v=2uxv\Big|_{\substack{x=1\\v=2\\u=2}}=\boxed{8}\qquad u'_y=u\cdot\frac{1}{2\sqrt{y}}\Big|_{\substack{u=2\\y=4}}=\boxed{\frac{1}{2}}$$

$$z'_x=uv^2\Big|_{\substack{u=2\\v=2}}=\boxed{8}$$

$$F(x,y,v)=2v+e^{v-2x}-\frac{y}{x}-1$$

$$F'_x=e^{v-2x}(-2)-\frac{y}{x^2}\Big|_{\substack{x=1\\y=4\\v=2}}=-6$$

$$F'_y=-\frac{1}{x}\Big|_{x=1}=-1$$

$$F'_v=2+e^{v-2x}\Big|_{\substack{x=1\\v=2}}=3$$

$$v'_x=-\frac{F'_x}{F'_v}=-\frac{-6}{3}=\boxed{2}$$

$$v'_y=-\frac{F'_y}{F'_v}=-\frac{-1}{3}=\boxed{\frac{1}{3}}$$

$x=1$ $\quad$ $y=4$

$u=1\sqrt{4}$

$u=2$

$$2v+e^{v-2x}-\frac{y}{x}=1$$

$$2v+e^{v-2}-\frac{4}{1}$$

$$2v+e^{v-2}=5$$

$v=2$

$$\boxed{h'((1,4);\breve{u}_{máx})}=\sqrt{32^2+\left(\frac{14}{3}\right)^2}=\boxed{32{,}33}$$

$$\boxed{\breve{u}_{máx}}=\frac{(32;\,14/3)}{\|(32;\,14/3)\|}=\boxed{(0{,}9895;\;0{,}1443)}$$

Creo que está bien...

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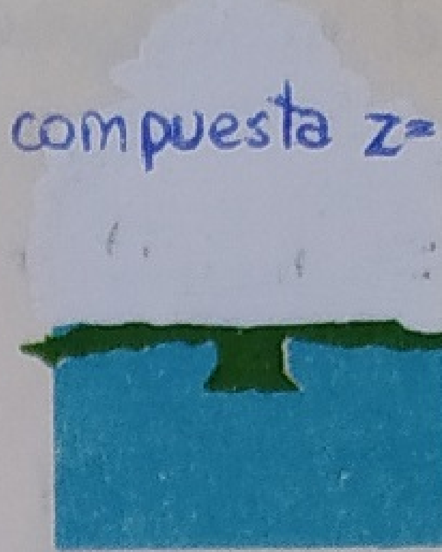
5 ① Hallar dirección y valor de derivada direccional máxima de la función compuesta $z=h(u,v)$ en el $(1,1)$ definida por $z=x^2 \cdot e^{xy}$ con $\begin{cases} x=\sqrt{uv} \\ y=\ln v \end{cases}$.



$$h : z \begin{cases} \to x \to u, v \\ \to y \to v \end{cases}$$

$$h'((1,1);\breve{u}_{máx}) = \|\nabla h'(1,1)\| = \|(h'_u(1,1);h'_v(1,1))\| = \|(1,2)\|$$

$$\underbrace{u=1}_{} \quad \underbrace{v=1}_{}$$

$x=\sqrt{uv}$, $x=\sqrt{1\cdot 1}$, $\underbrace{x=1}$

$y=\ln v$, $y=\ln 1$, $\underbrace{y=0}$

$z=x^2\cdot e^{xy}$, $z=1^2\cdot e^{1\cdot 0}$, $\underbrace{z=1}$

$$h'_u = z'_x \cdot x'_u = 2\cdot\frac{1}{2} = \boxed{1}$$

$$h'_v = z'_x \cdot x'_v + z'_y \cdot y'_v = 2\cdot\frac{1}{2} + 1\cdot 1 = \boxed{2}$$

$$\underbrace{z'_x} = 2x\cdot e^{xy} + x^2\cdot e^{xy}\cdot y \Big|_{\substack{x=1\\y=0}} = \underbrace{2}$$

$$\underbrace{z'_y} = x^2\cdot x\cdot e^{xy} \Big|_{\substack{x=1\\y=0}} = \underbrace{1}$$

$$\underbrace{x'_u} = \frac{1}{2\sqrt{uv}}\cdot v \Big|_{\substack{u=1\\v=1}} = \underbrace{\frac{1}{2}}$$

$$\underbrace{x'_v} = \frac{1}{2\sqrt{uv}}\cdot u \Big|_{\substack{u=1\\v=1}} = \underbrace{\frac{1}{2}}$$

$$\underbrace{y'_v} = \frac{1}{v}\Big|_{v=1} = \underbrace{1}$$

$$\boxed{h'((1,1);\breve{u}_{máx})} = \|(h'_u(1,1);h'_v(1,1))\| = \sqrt{[h'_u(1,1)]^2+[h'_v(1,1)]^2} = \sqrt{1^2+2^2} = \boxed{\sqrt{5}}$$

$$\boxed{\breve{u}_{máx}} = \frac{(1;2)}{\sqrt{5}} = \boxed{\left(\frac{1}{\sqrt{5}};\frac{2}{\sqrt{5}}\right)}$$

8 ① Dada $h(x,y) = g(x,y) + x^2 + y^2$ donde $w = g(x,y)$ viene definida implícitamente por $xy + \cos(wx)$

Hallar el valor aproximado de $h(0{,}91;\ 1{,}02)$ usando diferencial total.

$$h(0{,}91;\ 1{,}02) \simeq h(1,1) + h'_x(1,1)\cdot(0{,}91-1) + h'_y(1,1)\cdot(1{,}02-1)$$

$x=1$ $\quad$ $y=1$

$$xy + \cos(wx) = yw$$
$$1 + \cos(w) = w$$
$$w = 1{,}99 \simeq 2$$

$$F = xy + \cos(wx) - yw$$

$$F'_x = y + (-\operatorname{sen}(wx)\cdot w)\Big|_{\substack{x=1\\y=1\\w=1{,}999}} = 0{,}93$$

$$F'_y = x - w\Big|_{\substack{x=1\\w=1{,}999}} = -0{,}999$$

$$F'_w = -\operatorname{sen}(wx)\cdot x - y\Big|_{\substack{x=1\\y=1\\w=1{,}999}} = -1{,}0349$$

$h'_x =$

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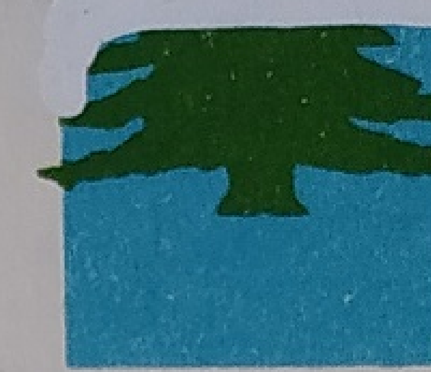
10 Hallar las direcciones de derivada direccional nula de la función $z = h(x,y)$ siendo $z = u \cdot \ln v$ con

$$\left\{ \begin{array}{l} u = \frac{x^2}{v} \\ v = y^2 \end{array} \right\} \text{ en el punto } (2, 1).$$



$$\begin{tikzcd} & u \arrow[r] \arrow[d] & x \\ z \arrow[ur] \arrow[r] & v \arrow[r] & y \end{tikzcd}$$

$\check{r}_{nula} \perp \nabla h(2,1) = \check{r}_{nula} \perp \left(h'_x(2,1) ; h'_y(2,1)\right)$

$x = 2$ $\quad$ $y = 1$

$u = \frac{x^2}{v}$, $u = \frac{2^2}{1}$, $u = 4$

$v = y^2$, $v = 1^2$, $v = 1$

$z = u \cdot \ln v$, $z = 4 \ln 1$, $z = 0$

$h'_x = z'_u \cdot u'_x$

$h'_x = \cancel{0 \cdot 4}$

$\underline{h'_x = 0}$

$h'_y = z'_u \cdot u'_v \cdot v'_y + z'_v \cdot v'_y$

$h'_y = \cancel{0 \cdot 1 \cdot 2} + 4 \cdot 2$

$\underline{h'_y = 8}$

$z'_u = \ln v \,\Big|_{v=1} = 0$

$z'_v = u \cdot \frac{1}{v} \,\Big|_{\substack{u=4 \\ v=1}} = 4$

$u'_x = \frac{1}{v} \cdot 2x \,\Big|_{\substack{x=2 \\ v=1}} = 4$

$u'_v = -x \cdot \frac{1}{v^2} \,\Big|_{\substack{x=2 \\ v=1}} = 1$

$v'_y = 2y \,\Big|_{y=1} = 2$

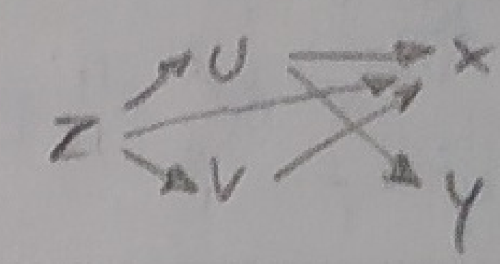
$\check{r}_{nula} \perp (0 ; 8)$

$\check{r}_{nula_1} = (8 ; 0)$

$\check{r}_{nula_2} = (-8 ; 0)$

$\boxed{12}$ ① Calcular aproximadamente el valor de la función $z = h(x,y)$ en $h(\overset{x}{0{,}98};\ \overset{y}{1{,}01})$ siendo $h(x,y)$ la composición de $z = f(u,v,x) = ux + \ln v^2$ donde $\left\{\begin{array}{l} u = 2\cdot\sqrt{xy} \\ v = g(x) \end{array}\right\}$ y $g(x)$ queda definida implícitamente por $v^2 + \ln v = \sqrt{x}$.

$$h(0{,}98;\ 1{,}01) \simeq h(1,1) + h'_x(1,1)\cdot(0{,}98-1) + h'_y(1,1)\cdot(1{,}01-1)$$



$\underline{x=1}$ $\quad \underline{y=1}$

$u = 2\sqrt{xy}$
$u = 2\sqrt{1\cdot 1}$
$\underline{u=2}$

$v^2 + \ln v = \sqrt{x}$
$v^2 + \ln v = 1$
$\underline{v=1}$

$z'_u = x\big|_{x=1} = 1$

$z'_v = \frac{1}{v^2}\cdot 2v\big|_{v=1} = 2$

$z'_x = u\big|_{u=2} = 2$

$u'_x = 2\,\frac{1}{2\sqrt{xy}}\cdot y\Big|_{\substack{x=1\\y=1}} = 1$

$u'_y = 2\cdot\frac{1}{2\sqrt{xy}}\,x\Big|_{\substack{x=1\\y=1}} = 1$

$\underline{h(1,1)} = f(\underset{2}{u},\underset{1}{v},\underset{1}{x}) = ux + \ln v^2$
$= 2\cdot 1 + \ln 1^2$
$= 2$

$h'_x = z'_u\cdot u'_x + z'_v\cdot v'_x + z'_x$
$= 1\cdot 1 + 2\left(\frac{1}{6}\right) + 2$
$= \frac{10}{3}$

$\boxed{h'_y} = z'_u\cdot u'_y$
$= 1\cdot 1$
$= 1$

$v = g(x) \Rightarrow v^2 + \ln v = \sqrt{x}$
$\underbrace{v^2 + \ln v - \sqrt{x}}_{F} = 0$

$F = v^2 + \ln v - \sqrt{x}$

$F'_x = -\frac{1}{2\sqrt{x}}\Big|_{x=1} = -\frac{1}{2}$

$F'_v = 2v + \frac{1}{v}\Big|_{v=1} = 3$

$v'_x = -\frac{F'_x}{F'_v} = -\frac{-\frac{1}{2}}{3} = \frac{1}{6}$

$$h(0{,}98;\ 1{,}01) \simeq 2 + \frac{10}{3}(-0{,}02) + 1\cdot 0{,}01$$

$$\boxed{h(0{,}98;\ 1{,}01) \simeq 1{,}94\overline{3}}$$