/

DE PARCIAL — CURVAS ⟷ RT y PN.
SUPERFICIES ⟷ PT y RN.

$\boxed{3}$ (4) Con $f \in C^1$, mediante diferencial total, calcular aproximadamente $f(0{,}98;0{,}01)$ sabiendo que el plano tangente en $(1;0;f(1,0))$ es normal a la recta tangente a la curva definida por $\begin{cases} x^2 - z^2 = y \\ z = x^2 \end{cases}$ en dicho punto.

curva $\begin{cases} x^2 - z^2 = y \to \overbrace{x^2 - y - z^2 = 0}^{F} \longrightarrow \bar{\nabla} F = (2x, -1, -2z)\big|_{(1,0,1)} = (2,-1,-2) \\ z = x^2 \to \underbrace{-x^2 + z^2 = 0}_{G} \longrightarrow \nabla G = (-2x, 0, 2z)\big|_{(1,0,1)} = (-2, 0, 2) \end{cases}$

punto $(1, 0, z_0)$

$\begin{cases} 1^2 - z^2 = 0 \\ z = 1^2 \end{cases} \begin{matrix} \to z^2 = 1 \\ \to z = 1 \end{matrix}$

punto: $(x=1, y=0, z=1)$.

$$\vec{U}_{RT} = \nabla F(1,0,1) \otimes \nabla G(1,0,1) = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 2 & -1 & -2 \\ -2 & 0 & 2 \end{vmatrix} = (-2;0;-2)$$

RT de la curva: $(x,y,z) = (1,0,1) + \lambda(-2,0,-2),\ \lambda \in \mathbb{R}$

$(1, 0, 1)$

PT en $(1,0,1)$ $\perp$ RT de la curva.

$\vec{n}_p \parallel \vec{U}_{RT\,curva} \longrightarrow \vec{n}_p = (1,0,1) = k\,(-f'_x(1,0);\ -f'_y(1,0);\ 1) \longrightarrow \begin{cases} 1 = k\cdot(-f'_x(1,0)) \to f'_x(1,0) = -1 \\ 0 = k\cdot(-f'_y(1,0)) \to f'_y(1,0) = 0 \\ 1 = k \end{cases}$

$$\begin{aligned} f(0{,}98;0{,}01) &\simeq f(1,0) \pm f'_x(1,0)\cdot(x-1) + f'_y(1,0)\cdot(y-0) \\ &\simeq 1 + (-1)(0{,}98-1) + \cancel{0\cdot(0{,}01-0)} \\ &\simeq 1 + (-1)(-0{,}02) \\ &\simeq 1 + 0{,}02 \end{aligned}$$

$\boxed{f(0{,}98;0{,}01) \simeq 1{,}02}$

5 ③ Dada la curva C como intersección de $\begin{cases} x^2+y^2+z^2=8 \\ y=\sqrt{x^2+z^2} \end{cases}$ $F=x^2+y^2+z^2-8$, $G=\sqrt{x^2+z^2}-y$

a. Verificar si $(2,0,2)$ es un punto regular de la curva.

b. Verificar si C es una curva plana.

$\frac{1}{2\sqrt{x^2+z^2}}\ 2x$

$$\nabla F = (2x;\ 2y;\ 2z)\Big|_{(2,0,2)} = (4;\ 0;\ 4)$$

$$\nabla G = \left(\frac{x}{\sqrt{x^2+z^2}};\ -1;\ \frac{z}{\sqrt{x^2+z^2}}\right)\Big|_{(2,0,2)} = \left(\frac{\sqrt{2}}{2};\ -1;\ \frac{\sqrt{2}}{2}\right)$$

$$\bar{U}_c = \nabla F_{(2,0,2)} \otimes \nabla G_{(2,0,2)} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 4 & 0 & 4 \\ \frac{\sqrt{2}}{2} & -1 & \frac{\sqrt{2}}{2} \end{vmatrix} = (4;\ 0;\ -4) \neq \bar{0}$$

$\Downarrow$

$(2,0,2)$ es un punto regular de la curva

¿C es una curva plana $\Leftrightarrow$ está incluida en un plano?

Hay que parametrizar? $\begin{cases} x^2+y^2+z^2=8 \\ y=\sqrt{x^2+y^2} \end{cases}$

$x^2+x^2+z^2+z^2=8$

$2x^2+2z^2=8$

$x^2+z^2=4$

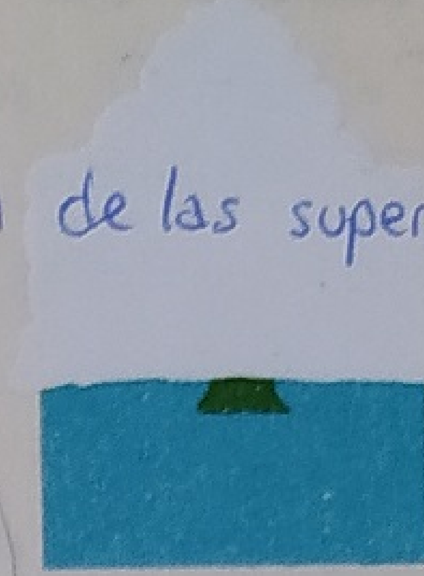
$y=2$

DE PARCIAL → CURVAS → RT y PN.
→ SUPERFICIES → PT y RN.

6 ② Hallar la ecuación vectorial de la RT a la curva dada como intersección de las superficies $\begin{cases} y = x^3 \\ x + 2z = 5 \end{cases}$ en el punto $(1, 1, 2)$.

$2z = 5 - x$

$z = -\frac{1}{2}x + \frac{5}{2}$



$x = t$

$y = t^3$

$z = -\frac{1}{2}t + \frac{5}{2}$

$g(t) = (t; t^3; -\frac{1}{2}t + \frac{5}{2}),\ t \in \mathbb{R}$

$\left.\begin{cases} t = 1 \\ t^3 = 1 \quad \checkmark \\ -\frac{1}{2}t + \frac{5}{2} = 2 \longrightarrow -\frac{1}{2}(1) + \frac{5}{2} = 2 \checkmark \end{cases}\right\} g(t=1) = (1; 1; 2)$

$g'(t) = (1, 3t^2, -\frac{1}{2})$

$\hookrightarrow g'(1) = (1, 3, -\frac{1}{2})$ — vector director de la RT en $(1,1,2)$.

$\boxed{\underline{RT:}\ (x, y, z) = (1; 1; 2) + \lambda \cdot (1; 3; -\frac{1}{2}),\ \lambda \in \mathbb{R}}$

Se puede hacer así...

$y = x^3 \longrightarrow \overbrace{x^3 - y}^{F} = 0 \longrightarrow \nabla F = (3x, -1, 0)\Big|_{(1,1,2)} = (3, -1, 0)$

$x + 2z = 5 \longrightarrow \underbrace{x + 2z - 5}_{G} = 0 \longrightarrow \nabla G = (1, 0, 2)\Big|_{(1,1,2)} = (1, 0, 2)$

$\vec{u} = \nabla F_{(1,1,2)} \otimes \nabla G_{(1,1,2)} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 0 \\ 1 & 0 & 2 \end{vmatrix} = (-2; -6; 1)$

$\boxed{RT:\ (x, y, z) = (1, 1, 2) + \alpha(-2, -6, 1),\ \alpha \in \mathbb{R}}$

... o así

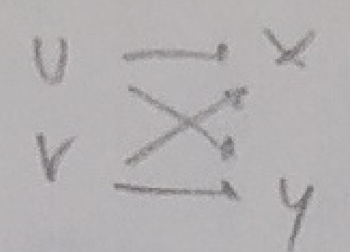
$(1, 3, -\frac{1}{2})$ y $(-2, -6, 1)$ son paralelas... ✓

7 ① Hallar la ecuación del PT en el punto $(1,1,1)$ de la función definida implícitamente por $F \circ \bar{g} = 0$ siendo $w = F(u,v) = \ln\left(\frac{v}{u}\right)$

$F(\bar{g}(x,y,z)) = 0$ $\qquad \bar{g}(x,y,z) = (x^2 y ; z^2 x)$. $\qquad [(x,y,z) - (1,1,1)] \cdot \bar{\nabla} F(1,1,1) = 0$

$F(x^2 y ; z^2 x) \Rightarrow$

$\ln\left(\frac{z^2 x}{x^2 y}\right) \Rightarrow \ln\left(\frac{z^2}{xy}\right)$



$(x-1 ; y-1 ; z-1) \cdot ($

8 ③ Hallar la ecuación de la RT a la curva dada por $\begin{cases} z = x^2 + y^2 \\ z = x + y \end{cases}$ en el punto $(1, 1, z_0)$.

$z = 1^2 + 1^2 \rightarrow z = 2$ ✓

$z = 1 + 1 \rightarrow z = 2$ ✓

punto: $(1, 1, 2)$

$F = x^2 + y^2 - z \longrightarrow \bar{\nabla} F = (2x ; 2y ; -1) \Big|_{(1,1,2)} = (2, 2, -1)$

$G = x + y - z \longrightarrow \bar{\nabla} G = (1 ; 1 ; -1) \Big|_{(1,1,2)} = (1, 1, -1)$

$$\bar{u}_{RT} = \bar{\nabla} F(1,1,2) \otimes \bar{\nabla} G(1,1,2) = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 2 & 2 & -1 \\ 1 & 1 & -1 \end{vmatrix} = (-2+1 ; -(-2+1) ; 2-2) = (-1 ; 1 ; 0)$$

$$\boxed{RT: (x, y, z) = (1, 1, 2) + \lambda \cdot (-1 ; 1 ; 0), \quad \lambda \in \mathbb{R}}$$

DE PARCIAL → CURVAS → RT y PN.
→ SUPERFICIES → PT y RN.

9 ③ Hallar la ecuación del PN a la curva C dada por $\left\{\begin{array}{l} z = x^2+y^2 \\ x^2+y^2+z^2=6 \end{array}\right\}$ en el punto $(1, 1, z_0)$.
Analizar si C es regular en dicho punto.

$z = 1^2+1^2 \Rightarrow \underline{z=2}$

$1^2+1^2+2^2=6$

$6=6$ ✓

punto: $(1, 1, 2)$

$F = x^2+y^2-z \longrightarrow \bar{\nabla} F = (2x, 2y, -1)\Big|_{(1,1,2)} = (2; 2; -1)$

$G = x^2+y^2+z^2-6 \longrightarrow \bar{\nabla} G = (2x, 2y, 2z)\Big|_{(1,1,2)} = (2; 2; 4)$

$\vec{n}_\pi = \bar{\nabla} F_{(1,1,2)} \otimes \bar{\nabla} G_{(1,1,2)} = \begin{vmatrix} i & j & k \\ 2 & 2 & -1 \\ 2 & 2 & 4 \end{vmatrix} = (10; -10; 0) \neq \bar{0} \Rightarrow \boxed{C \text{ es regular en } (1,1,2)}$

$/\!/ (1, -1, 0)$

$\left[(x, y, z)-(1,1,2)\right] \cdot (1; -1; 0) = 0$

$(x-1; y-1; z-2) \cdot (1; -1; 0) = 0$

$x-1 \qquad x-1+(y-1)\cdot(-1) = 0$

$x\cancel{-1}-y\cancel{+1} = 0$

$\boxed{x = y}$ PN a la curva C en $(1,1,2)$.

12 ② Dada la curva C definida por $\left\{\begin{array}{l} x+z=2 \\ y = x^2 \end{array}\right\}$, verificar que C es una curva regular en $\bar{A}(2,4,0)$ y hallar la ecuación ~~cartesiana y vectorial~~ del PN.

$\left.\begin{array}{l} x+z=2 \longrightarrow \underline{z=2-x} \\ \underline{y=x^2} \end{array}\right\} \quad \bar{g}(t) = (t; t^2; 2-t)$

$\left\{\begin{array}{l} t = 2 \longrightarrow t=2 \\ t^2 = 4 \longrightarrow 2^2=4 \Rightarrow 4=4 ✓ \\ 2-t = 0 \longrightarrow 2-2=0 \Rightarrow 0=0 ✓ \end{array}\right.$

$\bar{g}'(t) = (1; 2t; -1)$

$g(2) = (2, 4, 0)$

$\bar{g}'(t=2) = (1; 4; -1) \neq \bar{0} \Rightarrow \boxed{C \text{ es una curva regular en } \bar{A}(2,4,0)}$

PN: $\left[(x, y, z)-(2, 4, 0)\right] \cdot (1; 4; -1) = 0$

$(x-2; y-4; z) \cdot (1; 4; -1) = 0$

$x-2+4(y-4)+(-1)z = 0$

$x-2+4y-16-z = 0$

$\boxed{x+4y-z = 18}$

10 (3) Hallar en qué puntos la RT a la curva C en el punto $(1, 2, z_0)$ corta a la superficie $z + y = 4$ cuando la curva viene definida por $\begin{cases} x^2 + y^2 = z^2 + 4 \\ z + 2 = x + y \end{cases}$.

- $1^2 + 2^2 = z^2 + 4$
  $1 + 2 = z^2 + 4$
  $z^2 = 1$ — $\cancel{z=-1}$, $z = 1$
- $z + \cancel{2} = 1 + \cancel{2}$
  $z = 1$

$F = x^2 + y^2 - z^2 - 4$ — $\bar{\nabla} F = (2x; 2y; -2z)\big|_{(1,2,1)} = (2, 4, -2)$

$G = x + y - z - 2$ — $\bar{\nabla} G = (1; 1; -1)\big|_{(1,2,1)} = (1, 1, -1)$

$$\bar{U}_{RT} = \bar{\nabla} F_{(1,2,1)} \otimes \bar{\nabla} G_{(1,2,1)} = \begin{vmatrix} \breve{i} & \breve{j} & \breve{k} \\ 2 & 4 & -2 \\ 1 & 1 & -1 \end{vmatrix} = (-4+2; -(-2+2); 2-4) = (-2; 0; -2)$$

$\parallel$ a $(1, 0, 1)$

RT: $(x, y, z) = (1, 2, 1) + \lambda(1, 0, 1)$, $\lambda \in \mathbb{R}$ / RT: $\begin{cases} x = 1 + \lambda \\ y = 2 \\ z = 1 + \lambda \end{cases}$, $\lambda \in \mathbb{R}$

$z + y = 4$
$(1 + \lambda) + (2) = 4$
$\lambda + 3 = 4$
$\lambda = 1$

$x = 1 + 1 = 2$
$y = 2$
$z = 1 + 1 = 2$

→ $\boxed{\text{punto } (2, 2, 2)}$