DE PARCIAL → ECUACIONES DIFERENCIALES (2^{da} PARTE)

6 (4) Resolver la ED $\left(\frac{1}{x}+3x^2\right)dx + \left(\frac{1}{y}\right)dy = 0$, con $y(1)=1$.

$x=1 \to y=1$

Variables separables:

$$\left(\frac{1}{x}+3x^2\right)dx + \left(\frac{1}{y}\right)dy = 0$$

$$\int\left(\frac{1}{x}+3x^2\right)dx = \int -\frac{1}{y}\,dy$$

$$\underbrace{\ln|x| + x^3 + c = -\ln|y|}_{SG}$$

$$\ln|1| + (1)^3 + c = -\ln|1|$$

$$\underbrace{c=-1}$$

$$\ln|x| + x^3 - 1 = \ln|y|$$

$$\boxed{\ln|x| + x^2 + \ln|y| = 1}$$

Hallar la solución particular de $y'' + y = x^2$ si la gráfica de y tiene RN $y - \frac{x}{2} = 1$ en $(0; y_0)$

$y_0 - \frac{0}{2} = 1$

$\underbrace{y_0 = 1}$, $\to y(0)=1$

$y_N = -\frac{1}{y'}x + x_0$ $\quad$ $y_N = \frac{1}{2}x + 1$

$\underbrace{y' = -2}$ $\to$ $y'(0) = -2$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$\lambda_1 = 0 + 1i$

$\lambda_2 = 0 - 1 \cdot i$

$$y_h(x) = e^{0x}\left(c_1 \cdot \cos(1\cdot x) + c_2 \cdot \operatorname{sen}(1\cdot x)\right)$$

$$\underbrace{y_h(x) = c_1 \cdot \cos x + c_2 \cdot \operatorname{sen} x}$$

$y_p = ax^2 + bx + c$

$y_p' = 2ax + b$

$y_p'' = 2a$

$$y'' + y = x^2$$

$$2a + ax^2 + bx + c = x^2$$

$$x^2(a) + x(b) + (2a+c) = x^2(1)$$

$\underbrace{a=1}$

$\underbrace{b=0}$

$2a + c = 0$

$\underbrace{c=-2}$

$y_p(x) = ax^2 + bx + c$

$\underbrace{y_p(x) = x^2 - 2}$

$$y_G(x) = y_h(x) + y_p(x)$$

SG $\quad y_G(x) = c_1 \cdot \cos x + c_2 \cdot \operatorname{sen} x + x^2 - 2 \quad$ (I)

$y_G'(x) = -c_1 \cdot \operatorname{sen} x + c_2 \cdot \cos x + 2x \quad$ (II)

$$y(0) = c_1 \cos(0) + c_2 \cdot \operatorname{sen}(0) + \cancel{0}^2 - 2$$

$$1 = c_1 - 2$$

$$\underbrace{c_1 = 3}$$

$$y'(0) = -c_1 \cdot \operatorname{sen}(0) + c_2 \cdot \cos(0) + 2(0)$$

$$\underbrace{-2 = c_2}$$

SP $\quad \boxed{y_p = 3 \cdot \cos x - 2 \cdot \operatorname{sen} x + x^2 - 2}$

8 (3) Hallar la SG de la ED $\underbrace{2y\cdot dx}_{P} + \underbrace{(1-\ln(y)-2x)}_{Q}\,dy = 0.$

- $P \in C^1$ ✓
- $Q \in C^1$ ✓
- $P'_y = 2$
- $Q'_x = -2$

$\neq$ NO es una EDTE → debemos multiplicar un factor integrante a cada miembro de la ED.

¿Admite un factor integrante que dependa exclusivamente de x? → para $\mu(x) = e^{\int \frac{P'_y - Q'_x}{Q} dx}$

$\frac{P'_y - Q'_x}{Q} \Rightarrow \frac{2-(-2)}{1-\ln y - 2x} = \frac{4}{1-\ln y - 2x}$ → No depende exclusivamente de x → NO admite factor integrante dependiendo de x.

No hace falta

¿Admite uno, entonces, que dependa exclusivamente de y? → para $\mu(y) = e^{\int \frac{Q'_x - P'_y}{P} dy}$

$\frac{Q'_x - P'_y}{P} \Rightarrow \frac{-2-(2)}{2y} = \frac{-4}{2y} = -\frac{2}{y}$ → Depende exclusivamente de y → Admite un factor integrante que dependa solo de y.

Planteo: $\mu(y) = e^{\int \frac{Q'_x - P'_y}{P} dy} = e^{\int -\frac{2}{y} dy} = e^{\ln\left(\frac{1}{y^2}\right)} = \boxed{\frac{1}{y^2}}$ ✓

Multiplico por $\boxed{\frac{1}{y^2}}$ a cada miembro de la ED: $(2y)\left(\frac{1}{y^2}\right)\cdot dx + (1-\ln(y)-2x)\left(\frac{1}{y^2}\right)dy = 0\cdot\cancel{\left(\frac{1}{y^2}\right)}$

$$\underbrace{\left(\frac{2}{y}\right)}_{P}dx + \underbrace{\left(\frac{1}{y^2} - \frac{\ln(y)}{y^2} - \frac{2x}{y^2}\right)}_{Q}dy = 0 \checkmark$$

¿Es una EDTE ahora?

- $P \in C^1$ ✓
- $Q \in C^1$ ✓
- $P'_y = -\frac{2}{y^2}$
- $Q'_x = -\frac{2}{y^2}$

$=$ Ahora sí es una EDTE. ✓

$U'_x = \frac{2}{y}$ → $u(x,y) = \int \frac{2}{y} dx = \frac{2}{y}x + \alpha(y)$

$\alpha(y) = +\frac{\ln y}{y} + \beta(x)$

$U'_y = \frac{1}{y^2} - \frac{\ln(y)}{y^2} - \frac{2x}{y^2}$ → $u(x,y) = \cancel{-\frac{1}{y}} + \frac{\ln y}{y} + \cancel{\frac{1}{y}} + \frac{2x}{y} + \beta(x)$

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$$u(x,y) = \frac{2x}{y} + \frac{\ln y}{y} + k$$

$$\text{SG: } \boxed{\frac{2x}{y} + \frac{\ln y}{y} = c}$$ ✓

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9 ① Resolver la ED $(y+\ln(x))\,dx = x\,dy$.

$$\underbrace{\left(y+\ln(x)\right)}_{P}dx + \underbrace{(-x)}_{Q}dy = 0$$

$$\left.\begin{array}{l} P'_y = 1 \\ Q'_x = -1 \end{array}\right\} \neq \longrightarrow \text{NO es EDTE}$$

$$\frac{P'_y - Q'_x}{Q} = \frac{1-(-1)}{x} = \frac{2}{x} \xrightarrow{} \text{depende sólo de } x \checkmark \xrightarrow{\text{planteo}} \mu(x) = e^{\int \frac{P'_y - Q'_x}{Q}dx} = e^{\int \frac{2}{x}dx} = e^{\ln(x^2)} = \boxed{x^2}$$

$$\left.\begin{array}{l} P'_y = x^2 \\ Q'_x = -3x^2 \end{array}\right\} \neq \,!$$