Question 2

a

From the definition of the Rademacher Complexity we get:

$$R(\hat{F} \circ S) = \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[sup_{\hat{f} \in \hat{F}} \sum_{i=1}^{m} \sigma_i \hat{f}(z_i)]$$

$$= \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[sup_{f \in F} \sum_{i=1}^{m} \sigma_i c \cdot f(z_i)]$$

case 1 $c \geq 0$:

$$^1 = \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[c \cdot sup_{f \in F} \sum_{i=1}^{m} \sigma_i f(z_i)]$$

$$^2 = c \cdot \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[sup_{f \in F} \sum_{i=1}^{m} \sigma_i f(z_i)]$$

$$= c \cdot R(F \circ S) = |c| \cdot R(F \circ S)$$

case 2 $c < 0$:

$$^3 = \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[|c| \cdot sup_{f \in F} \sum_{i=1}^{m} -\sigma_i f(z_i)]$$

$$^4 = |c| \cdot \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[sup_{f \in F} \sum_{i=1}^{m} -\sigma_i f(z_i)]$$

$$= |c| \cdot R(F \circ S)$$

[1] Linearity of sup for positive integers.

[2] Linearity of Expectation.

[3] Linearity of sup for positive numbers. $|c|$

[4] Since $\sigma \in \{\pm 1\}$ multiplying the σ by -1 doesn't change the distribution and so doesn't change the value of the Expectation

b

Denote $\hat{F}_1$ to be the $\hat{F}$ defined in part a
From the definition of the Rademacher Complexity we get:

$$R(\hat{F} \circ S) = \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[sup_{\hat{f} \in \hat{F}} \sum_{i=1}^{m} \sigma_i \hat{f}(z_i)]$$

$$^1 = \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[sup_{f \in F} \sum_{i=1}^{m} \sigma_i (c \cdot f(z_i) + b)]$$

$$^2 = \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[sup_{f \in F} \sum_{i=1}^{m} \sigma_i c \cdot f(z_i) + \sum_{i=1}^{m} \sigma_i b]$$

$$^3 = \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[sup_{f \in F} \sum_{i=1}^{m} \sigma_i c \cdot f(z_i)] + \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[\sum_{i=1}^{m} \sigma_i b]$$

$$^4 = R(\hat{F}_1 \circ S) + b \sum_{i=1}^{m} \frac{1}{m} E_{\sigma \sim \{\pm 1\}^m}[\sigma_i]$$

$$^5 = R(\hat{F}_1 \circ S)$$

$$^6 = |c| \cdot R(F \circ S)$$

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[1] substitute $\hat{f}$ by c$\cdot f + b$

[2] $\sum_{i=1}^{m} \sigma_i b$ isn't dependant on f so can be taken out of the sup

[3] Linearity of Expectation

[4] Definition of $R(\hat{F}_1 \circ S)$ and [3]

[5] E$[\sigma_i] = 0, \forall i$

[6] from a