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18L - 1135

Numerical Computing

CS - SD

Assignment 4

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⑥ No 1

1

Picard's method

$$\frac{dy}{dx} = 2x - y^2 \quad \text{where } y=0 \text{ at } x=0$$

As $y=y_0$ and $x=x_0$ $\Rightarrow y_0=0$ and $x_0=0$

also : $y_n = y_0 + \int_{x_0}^x f(x, y_{n-1}) dx$

$$\Rightarrow y_n = 0 + \int_0^x (-y_{n-1}^2 + 2x) dx$$

when $n=1$ // 1st iteration

$$y_1 = 0 + \int_0^x (-y_0^2 + 2x) dx$$

Put $y_0=0$

$$\Rightarrow y_1 = \int_0^x (2x) dx$$

So

$$y_1 = x^2$$

for $n=2$ // Second Iteration (2)

$$y_2 = 0 + \int_0^x (-y_1^2 + 2x) dx$$

Put $y_1 = x^2$

$$y_2 = \int_0^x (-x^4 + 2x) dx$$

$$\Rightarrow y_2 = x^2 - \frac{x^5}{5} \Rightarrow y_2 = x^2 - \frac{x^5}{5}$$

when $n=3$ // 3rd iteration

$$y_3 = \int_0^x \left[\left(x^2 - \frac{x^5}{5} \right)^2 + 2x \right] dx$$

$$\Rightarrow y_3 = x^2 - \frac{x^5}{5} + \frac{x^8}{20} - \frac{x^{11}}{275}$$

Q No 2

(3)

$$y' = xy + y^2 \quad y(0) = 1$$

find y for $x = 0.1, 0.2$

Now

$$x_0 = 0 \text{ and } y_0 = 1$$

$$h = 0.1$$

Using R-K method for fourth order

$$k_1 = h(x_0 + y_0^2)$$

we have

$$k_1 = h(x_0 y_0 + y_0^2)$$

Put values and get

$$k_1 = 0.1$$

$$k_2 = h f((x_0 + h/2), (y_0 + k_1/2))$$

$$\text{so } k_2 = h \left((x_0 + h/2) \times (y_0 + k_1/2) + (y_0 + k_1/2)^2 \right)$$

after putting values

$$k_2 = 0.1155$$

Similarly $k_3 = hf(x_0 + h/2, y_0 + k_2/2)$

(4)

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

we get $\Rightarrow k_3 = 0.117172$

$$k_4 = 0.135979$$

$$\Delta y = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} \Rightarrow 0.116887$$

$$y_1 = y_0 + \Delta y \Rightarrow 1.11689 \leftarrow y(0.1)$$

for y_2

$$x_1 = 0.1$$

$$y_1 = 1.11689$$

just like previous formulas replace y_0 with y_1
and get

$$k_1 = 0.135913$$

$$k_2 = 0.158159$$

$$k_3 = 0.160974$$

$$k_4 = 0.187241$$

(5)

again $y_2 = y_1 + \Delta y$

$$y_2 = y_1 + \left[\frac{k_1 + 2k_2 + 2k_3 + k_4}{6} \right]$$

$$= 1.11689 + 0.160237$$

$$\boxed{y_2 = 1.22713} \leftarrow y(0.2)$$