Zaghum Sarwar

18L-1135

Numerical Computing

CS-5D

Assignment 2

Mr. Shoaib

Q No 1

let's suppose our transcendental eq. as

$$x^3 - x - 1 = 0$$

(1) Bisection method

$f(0) = -1$ -ve

$f(1) = -1$ -ve

$f(2) = 5$ +ve

so $a = 1$ $b = 2$

$$x_0 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$$

$f(x_0) = f(1.5) = 0.875$ +ve

root lies b/w 1 and 1.5

$$x_1 = \frac{1+1.5}{2} = 1.25$$

$f(x_1) = f(1.25) = -0.2968$ -ve

so root's b/w 1.5 and 1.25

$$x_2 = \frac{1.25+1.5}{2} = 1.375$$

$f(1.375) = 0.2246$ +ve

root b/w 1.25 and 1.375 $\Rightarrow x_3 = 1.3125$

also $f(x_3) = -0.0515$

But given error tolerance 10^{-3} or 0.001

so $\frac{b-a}{2} < 0.0001$

At first step when
$a = 1$
$b = 2$

$$\frac{2-1}{2} < 0.0001$$

$$0.5 \not< 0.0001$$

So we stop at that step, so maximum iterations we can do is '1'.
So, We ignore remaining iterations.

2 Regula-falsi method

as already $f(1) = -1$ −ve
$f(2) = 5$ +ve

so $a = 1$, $b = 2$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{5 - 2(-1)}{5+1}$$

$\longrightarrow x_1 = 1.1667$

$\Rightarrow f(x_1) = -0.5786$ −ve

So $a = 1$
$b = 1.1667$

$$x_2 \Rightarrow \frac{-0.5786 - (-1.1667)}{1.1667 - 1}$$

$\longrightarrow x_2 = 1.0562$

$f(x_2) = -0.8779$ −ve

so root's b/w

$a = 1.1667$

$b = 1.0562$

-0.8779

$$x_3 = \frac{(1.1667)(-0.8779) - (1.0562)(-[illegible])}{-0.8779 - (-0.5786)}$$

$\longrightarrow x_3 = 1.0903$

$f(x_3) = -0.7942$ -ve

so root's between

$a = 1.1667$

$b = 1.0903$

$$x_4 = \frac{(1.1667)(-0.7942) - (1.0903)(1.9632)}{-0.7942 - 1.9632}$$

$\longrightarrow x_4 = 1.1123$

$f(x_4) = -0.7361$ -ve

so

root lies b/w

$a = 1.1667$

$b = 1.1123$

$\Rightarrow x_5 = 1.1271$

$f(x_5) = -0.6952$

so

$a = 1.1667$

$b = 1.1271$

so $x_6 = 1.1374$

$f(x_6) = -0.6944$ -ve

so $a = 1.1667$
$b = 1.1270$

$x_7 = 1.1376$

at x_6, x_7, values are correct upto three decimal places
so we stop here
root is x_7

③ Iteration Method

as $f(x) = x^3 - x - 1 = 0$

$f(x) \Rightarrow x^3 = x + 1$
$x = (1+x)^{1/3}$

$f'(x) = \frac{1}{3}(1+x)^{-2/3}$

also $|f'(1)| < 1$ exists

and $|f'(2)| < 1$ exists

So we proceed

starting from $x_0 = 1.3$

$x_1 = F(x_0)$
$= F(1.3)$

$\Rightarrow (1+1.3)^{1/3}$

$\longrightarrow x_1 = 1.3200$

$x_2 = F(x_1)$
$= (1+1.3200)^{1/3}$

$\longrightarrow x_2 = 1.3238$

$x_3 = F(x_2)$
$= (1+1.3238)^{1/3}$

$\longrightarrow x_3 = 1.3245$

$x_4 = (1+1.3245)^{1/3}$

$\longrightarrow x_4 = 1.3247$

$x_5 = (1+1.3247)^{1/3}$

$\longrightarrow x_5 = 1.3247$

Since root is correct to three decimal places at x_3, x_4 So we stop.
root is x_4

4 Secant Method

$x_0 = 1$
$x_1 = 2$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = 2 - \frac{5(1)}{5+1}$$

$\longrightarrow x_2 = 1.1667$

$f(x_2) = -0.5786$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= 1.1667 - \frac{(-0.5786)(-0.8333)}{-0.5786 - 5}$$

$\longrightarrow x_3 = 1.2531$

$f(x_3) = -0.2854$

$$x_4 = x_3 - \frac{f(x_3)(x_3 - x_2)}{f(x_3) - f(x_2)}$$

$$= 1.2531 - \frac{(-0.2854)(0.0864)}{-0.2854 - 1.1667}$$

$\longrightarrow x_4 = 1.0832$

$f(x_4) = -0.8122$

$$x_5 \Rightarrow 1.0832 - \frac{(-0.8122)(-0.1699)}{-0.8122+0.2854}$$

$$\longrightarrow x_5 = 1.0854$$

$$f(x_5) = -0.8066$$

$$x_6 = 1.0854 - \frac{(-0.8066)(-0.1384)}{-0.8066+0.8122}$$

$$x_6 = 1.08593$$

at x_5, x_6 values correct to 3 decimal place so we stop

root is x_6

⑤ Newton-rhapson method

as $f(1) = -1$ $\boxed{f'(x) = 3x^2-1}$
$f(2) = 5$

let's put $x_0 = 1.5$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.5 - \frac{0.875}{5.75}$$

$$\longrightarrow x_1 = 1.3478$$

$$f(x_1) = 0.1005$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.3478 - \frac{0.1005}{4.4496}$$

$$\longrightarrow x_2 = 1.3252$$

$$f(x_2) = 0.0020$$

$$x_3 = 1.3252 - \frac{0.0020}{4.2684}$$

$$\longrightarrow x_3 = 1.3247$$

$$f(x_3) = -0.00007$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.3247 + \frac{0.00007}{4.2644}$$

$$\longrightarrow x_4 = 1.3246$$

at x_3, x_4 is correct to 3 decimal places

so root is x_4

→ Q # 2

let's have system of equations

$$x_1 + x_2 + x_3 = 5$$
$$x_1 + 2x_2 + 2x_3 = 6$$
$$x_1 + 2x_2 + 3x_3 = 8$$

① Dolittle's method

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$$

$$A = LU$$

for ease let's have entries of L and U as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \begin{bmatrix} d & e & f \\ 0 & g & h \\ 0 & 0 & i \end{bmatrix}$$

from here we derive values as

$d = 1 \qquad ad = 1 \quad \Rightarrow \quad a = 1$
$e = 1 \qquad ae + g = 2 \quad \Rightarrow \quad g = 1$
$f = 1 \qquad af + h = 2 \quad \Rightarrow \quad h = 1$

also $\qquad bd = 1 \quad \Rightarrow \quad b = 1$
$be + cg = 2 \quad \Rightarrow \quad c = 1$
$bf + ch + i = 3 \quad \Rightarrow \quad i = 1$

let $Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$$

$y_1 = 5$

$y_1 + y_2 = 6 \Rightarrow y_2 = 1$

$y_1 + y_2 + y_3 = 8 \Rightarrow y_3 = 2$

let $Ux = y$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$$

$x_1 + x_2 + x_3 = 5 \Rightarrow x_1 = 4$

$x_2 + x_3 = 1 \Rightarrow x_2 = -1$

$x_3 = 2$

so result is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 2\cancel{8} \end{bmatrix}$$

② Crout's method

here $A = LU$

$$\Rightarrow \begin{array}{ccc} A & & L & U \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} & = & \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} & \begin{bmatrix} 1 & g & h \\ 0 & 1 & i \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$\Rightarrow$ $a = 1$ $\quad ag = 1 \Rightarrow g = 1$

$b = 1$ $\quad bg + c = 2 \Rightarrow c = 1$

$d = 1$ $\quad dg + e = 2 \Rightarrow e = 1$

$ah = 1 \Rightarrow h = 1$

$bh + ci = 2 \Rightarrow i = 1$

$f = 1$

$Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$$

$y_1 = 5$

$y_1 + y_2 = 6 \Rightarrow y_2 = 1$

$y_1 + y_2 + y_3 = 8 \Rightarrow y_3 = 2$

$Ux = y$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$$

$$x_1 + x_2 + x_3 = 5 \quad \Rightarrow x_1 = 4$$
$$x_2 + x_3 = 1 \quad \Rightarrow x_2 = -1$$
$$x_3 = 2$$

so $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

③ Gauss-Seidel Method

Let's change variable names from previous equations.

$$\left.\begin{aligned} x + y + z &= 5 \\ x + 2y + 2z &= 6 \\ x + 2y + 3z &= 8 \end{aligned}\right\}$$ eq's are same as before

$x = 5 - y - z$ ①

$y = \frac{1}{2}(6 - x - 2z)$ ②

$z = \frac{1}{3}(8 - x - 2y)$ ③

Let $x_0 = y_0 = z_0 = 0$, put in 1,2,3

$x^{(1)} = 5$

$y^{(1)} = 3$

$z^{(1)} = 8/3$

2^{nd} iteration.

$$x^{(2)} = 5 - y^{(1)} - z^{(1)}$$

$$\longrightarrow x^{(2)} = 7 - \frac{2}{3}$$

$$y^{(2)} = \frac{1}{2}\left(6 - x^{(1)} - 2z^{(1)}\right)$$

$$\longrightarrow y^{(2)} \Rightarrow \frac{19}{6}$$

$$z^{(2)} = \frac{1}{3}\left(8 - x^{(1)} - 2y^{(1)}\right)$$

$$\longrightarrow z^{(2)} \Rightarrow 3$$

3^{rd} iteration

$$x^{(3)} = 5 - y^{(2)} - z^{(2)}$$

$$\longrightarrow x^{(3)} = \frac{29}{6}$$

$$y^{(3)} = \frac{1}{2}\left(6 - x^{(2)} - 2z^{(2)}\right)$$

$$\longrightarrow y^{(3)} = \frac{1}{3}$$

$$z^{(3)} = \frac{1}{2}\left(6 - x^{(2)} - 2z^{(2)}\right)$$

$$\longrightarrow z^{(3)} = \frac{7}{9}$$