National University of Computer and Emerging Sciences

Lahore Campus

Operations Research (MT 4031)

Final Exam

Date: 19-12-2025

Total Time	3 hrs
Total Marks:	75
Total Questions:	6

Course Instructor(s)

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Roll No [redacted] Section A

Student Signature [redacted]

Do not write below this line

i) **Attempt all the questions neatly on the answer sheet.**
ii) **Solve all the parts of a question together in order.**
iii) **Don't use a red pen or lead pencil to solve the paper.**

CLO #:1 Recognize the importance of operations research and linear programming by learning the characteristics of different types of decision-making environments, appropriate decision-making approaches, and tools to be used in each type.

Q.1: [15]

Consider the L.P.P

$$Max\ z = 3x_1 + 4x_2$$

Subject to

$$2x_1 + 3x_2 \leq 1200 + M_1 \quad \text{(Machine 1)}$$
$$2x_1 + x_2 \leq 1000 + M_2 \quad \text{(Machine 2)}$$
$$x_2 \leq 200 + M_3 \quad \text{(Machine 3)}$$
$$x_1, x_2 \geq 0$$

The optimal simplex tableau is given below.

Basic	x_1	x_2	s_1	s_2	s_3	Solution	M_1	M_2	M_3
z	0	0	$\frac{5}{4}$	$\frac{1}{4}$	0	A	m_{11}	m_{12}	m_{13}
x_1	1	0	$\frac{-1}{4}$	$\frac{3}{4}$	0	B	m_{21}	m_{22}	m_{23}
s_3	0	0	-2	2	1	C	m_{31}	m_{32}	m_{33}
x_2	0	1	$\frac{1}{2}$	$\frac{-1}{2}$	0	D	m_{41}	m_{42}	m_{43}

i) Find the values of A, B, C, D and $[m_{ij}]$ where $i = 1,2,3,4$ and $j = 1,2,3$.
ii) Determine the dual price for machine 1, machine 2 and machine 3.
iii) Determine the feasibility ranges for the dual prices obtained in (ii)
iv) If $M_1 = 100$, $M_2 = 0$ and $M_3 = 0$ then find the new optimum solution.

CLO #: 2 Solve the Transportation Models, Assignment Models and Network Models.

Q2: [10]

The table gives the optimistic *(a)*, most likely *(m)*, and pessimistic *(b)* time estimates for the activities of a project.

a) Find missing values to complete the table.
b) Construct the **project network diagram** showing all dummy activities.
c) Draw the **time schedule for both critical and non-critical activities**.
d) Find the probability that the project will be completed within **86 days**. Use $F(Z) \approx 0.9505$ for $Z \geq 1.65$.

Activity	(a, m, b)	$D\ (Days)$	σ^2
A–B	(14, 17, 25)	17.83	3.36
B–C	(14, 18, 21)		1.36
B–D	(13, 15, 18)	15.17	
B–H	(16, 19, 28)		
C–D (dummy)	(0, 0, 0)		0
C–E	(15, 18, 27)	19	4
D–F	(13, 17, 21)		1.78
E-G (dummy)	(0, 0, 0)		
E-I	(14,18,21)	17.67	
F-G (dummy)	(0, 0, 0)		0
F-H (dummy)	(0, 0, 0)	0	
G-I	(16,20,41)	22.83	
H-I	(14, 16, 22)		1.78

CLO #: 2

Q3: [8+7]

a- Four workers are available to perform five jobs. The earnings (in $), when a worker is assigned a particular job, are given in the table below.

workers/Jobs	*A*	*B*	*C*	*D*	*E*
1	62	78	50	101	82
2	72	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80

Each worker can be assigned to only one job and each job must be performed by exactly one worker. Find the optimal assignment of jobs to workers that maximizes the earning.

b- In a 3×3 transportation problem, let x_{ij} be the amount shipped from source i to destination j, and let c_{ij} be the corresponding transportation cost per unit. The amounts of supply at sources 1, 2, and 3 are 15, 30, and 85 units, respectively, and the demands at destinations 1, 2, and 3 are 20, 30, and 80 units, respectively. Assume that the starting northwest-corner solution is optimal and that the associated values of the multipliers are given as $u_1 = -2, u_2 = 3, u_3 = 5, v_1 = 2, v_2 = 5, v_3 = 10$

i) Find the associated optimal cost.
ii) Determine the smallest value of c_{ij} for each nonbasic variable that will maintain the optimality of the northwest-corner solution.

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CLO #: 3 Understand the basic methodology for the integer programs, dynamic programs and simulation.

Q4: [3+7+5]

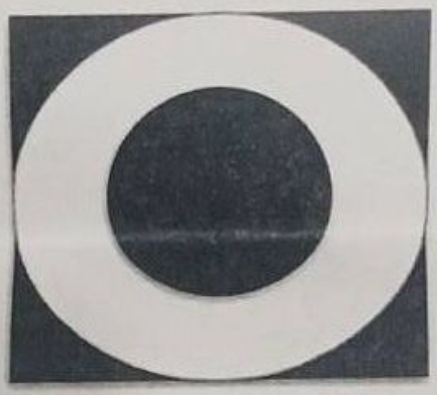
A community center is preparing two types of activity kits, Kit X and Kit Y, for a youth program. Each kit requires three different resources: paper, plastic sheets, and time for assembly. Kit X requires 2 units of paper and Kit Y requires 3 units of paper. The center has at most 48 units of paper available. Kit X requires 1 unit of plastic sheet and Kit Y requires 3 units of plastic sheets. The center has at most 42 units of plastic sheets available. Each kit, whether X or Y, requires 1 hour of assembly time. A total of 21 hours of assembly time is available. The profit earned from two kits is $50 and $100 respectively.

a. Formulate the above problem as a linear programming model.
b. Solve the problem by Dynamic Programming, clearly indicating the stages and states involved.
c. Solve the problem graphically and compare the solution with the one found in part b.

CLO #: 3

Q5: [3+7]

a- The graphical structure of the circle, $(x-4)^2+(y+3)^2=9$ and $(x-4)^2+(y+3)^2=3$ is shown in figure below,



i) Define the corresponding distributions $f(x)$ and $f(y)$, and then show how a sample point (x, y) is determined using the (0,1) random pair (R_1, R_2).

ii) Estimate area of the Shaded part for the following random pairs
R_1: 0.9341, 0.0589, 0.5946, 0.9658
R_2: 0.7644, 0.1709, 0.5426, 0.2231

b- Consider the following definite integral:

$$\int_1^{10} \frac{\ln x}{x}\,dx$$

i) Develop the Monte Carlo experiment to estimate the value of the integral.
ii) Hence, evaluate the integral using the following pairs of random numbers.

R_1	.0589	.3529	.4799	.7676	.6139	.3919	.9341	.5199	.3473	.7472
R_2	.5869	.3455	.2867	.8111	.8261	.4291	.7125	.5954	.3575	.8208

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CLO #: 3

Q6: [6+4]

Consider the following linear model

$$\text{Maximize } z = x_1 + 2x_2$$

Subject to:

$$\begin{aligned} 2x_2 &\leq 7 \\ x_1 + x_2 &\leq 7 \\ 2x_1 &\leq 11 \\ x_1, x_2 &\geq 0 \text{ and integers} \end{aligned}$$

a- Find the continuous solution of the model

b- Apply fractional cut (cutting plane) method by selecting x_1-equation as a source row to obtain the integer solution