Lecture #3

4 Bits code

BCD code range from 0 to 9. {Total $= 2^{4-1} = 16$}

$6 = 0110$ $\quad$ $6 = 0110$

$10 = 0001\ 0000$ $\quad$ $+10 =$

6 code words in BCD are invalid. Ranging from $10 \to 15$.

BCD addition:

$$
\begin{array}{r}
7 = 0111 \\
+\ 5 = 0101 \\
\hline
1100 > 9
\end{array}
$$

Now we will add six to level 2 Byte

$$
\begin{array}{r}
+6 = 0110 \\
\hline
0001\ 0010 = 12
\end{array}
$$

$$
\begin{array}{rrrrr}
1897 & = 0001 & 1000 & 1001 & 0111 \\
+\ 2905 & = 0010 & 1001 & 0000 & 0101 \\
\hline
 & 0100 & 0001 & 1010 & 1100 \\
 & & & & +6\ \ 0110 \\
\hline
 & 0100 & 0001 & 1010 & 0010
\end{array}
$$

ASCII 7 bits. $2^7 - 1 = 128$

It contains 98 graphic printing characters and 34 non printing characters.

$B_7\ B_6\ B_5\ B_4\ B_3\ B_2\ B_1$

It represents column $\qquad$ Rows

Parity Bit. :- It identify the odd number of means. 0101 → If it changes to 0001 detected. ~~Back~~

It counts the no of 1's, if no of 1 changed is odd Then it can be detected

---

1's complement. with sign bit

7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0

Sign bit

-10

1 0 0 0 1 0 1 0

→ 1 0 1 1 1 0 1 0 1

+ 1

2's

~~1 1 1 1 1 2~~

1 0 1 1 1 1 1 1 0 0

1 1

1

Lecture #4

(i) Subtraction by 1's complement.

$X - Y = 1010100 - 1000011$

$\downarrow$ 1's complem

```
  1010100
+ 0111100
 10010000
```

9th ↑ 8 bits.

```
      +1
```

$X - Y = 0010001$

steps:

① Take 1's complement of Y then add if carry exist (>8 bit) then add that carry.

② If there's no end carry then Take 1's complement of the ~~carry~~ ~~carry~~ answer and then insert neg sign.

$Y - X = 1000011$

```
  1000011
+ 0101011
  1101110
```

No end carry so Take 1's com

Sum $= -0010001$

Subtraction by 2's complement.

ⓐ X − Y

Take 2's complement of Y
Then add.

X = 1010100
Y = 1000011

2's Y = 0111101

$$\begin{array}{r} 1010100 \\ 0111101 \\ \hline 10010001 \end{array}$$

Discard end carry then answer is
X − Y = 0010001

ⓑ Y − X

$$\begin{array}{lr} Y = & 1000011 \\ 2's \to X & 0101100 \\ \hline & -1101111 \end{array}$$

Then put -ve sign in answer.

In Bolean Algebra.

1+1 = 1

1 represent on
0 represent off.

logic gates

(i)  And $X \cdot Y$

(2)  OR $X + Y$

(3)  Z $Z = \bar{X}$

wave form.

(i) And

X 0 0 1 1 + Y 0 1 0 1 = 0 0 0 1

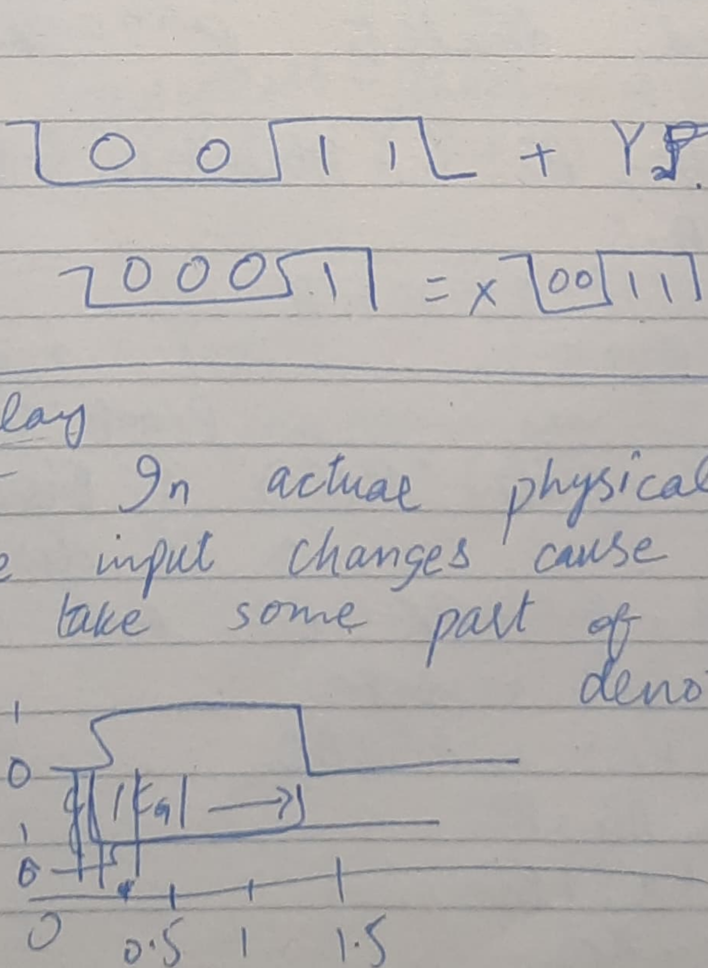
(2) OR

0 0 0 1 1 = x 0 0 1 1 + 0 1 0 1

Logic Gate Delay

In actual physical gates, if one or more input changes cause the output to change it take some part of second.

denoted by t_{gd}

output 

0.5 - 0.2
= 0.3 ns

→ The order of evaluation in a Bolean algebre. enpression.

1) Paranthesis
2) NOT ~
3) AND ·
4) OR +

→ Dual and Duality Principale.

Dual → obtained by changing and to OR or OR to And

Duality If the output no change occurs then It's called duality principale.

(1) $A + A \cdot B = A$ Absorption Theoram.

L.H.S $= A \cdot 1 + A \cdot B$

$= A(1 + B) = A(1) = A$

L.H.S = R.H.S

(2) $A + BC = (A + B)(A + C)$ Proof too. Distributive la

OR distributive over And.

R.HS

$= A \cdot A + A \cdot C + AB + BC$

$= A + AC + AB + BC$

$= A(1 + C) + AB + BC$

$= A + AB + BC$

$= A(1 + B) + BC$

$= A + BC$

1) Consensus Theorem

$$XY + \bar{X}Z + YZ = XY + \bar{X}Z$$

Both XY and $\bar{X}Z$ have similarity in the face of X where YZ doesn't so. It can be eliminated.

$$= XY + \bar{X}Z + YZ.1$$
$$= XY + \bar{X}Z + YZ(X+\bar{X})$$
$$= XY + XYZ + \bar{X}Z + \bar{X}YZ$$
$$= XY(1+Z) + \bar{X}Z(1+Y)$$
$$= XY + \bar{X}Z$$

L.H.S = R.H.S

Dual of consensus Theorem is.

$$(X+Y)(\bar{X}+Z)(Y+Z) = (X+Y)(\bar{X}+Z)$$

Minimization.

$$X.Y + \bar{X}.Y = Y \qquad (x+\bar{y})(\bar{x}+y) = y$$

Demorgan's law.

$$\overline{x+y} = \bar{x}.y \qquad \overline{x.y} = \bar{x}+\bar{y}$$

lecture #6

1) Conanical Forms.

Bolean functions expressed as sum of minterms or product of max term are called conanical forms.

2) It's useful to specify Bolean function in the form that
   1) allows comparison for equality
   2) has a correspondence to truth table

③ Usage
   (i) SOP aka sum of product.
   (2) POS aka product of sums

④ Minterm

And term with every variable present in orignal or complemented form called minterms.
Formula = 2^n where n is total counted literals.

Such

$X = 1$, $\bar{X} = 0$, $Y = 1$, $\bar{Y} = 0$

$XY = 1$ $X\bar{Y} = (1)(0) = 0$

⑤ Maxterms

OR-terms with every variable in either original or complemented way.
formula = 2^n

here = $[X = 0]$ $\bar{X} = 1$, $Y = 0$, $\bar{Y} = 1$

$X + Y = 00$

$\boxed{X + \bar{Y} = 1}$

⑥ Standard Order.

(i) Order of minterms and maxterms matters.

② Both of terms are designated with a subscript. It's a number expressed as binary number which shows that the literal is represented in the orignal format or complemented one.

-> Minterm.

x, Y 0 = complemented

1 = Not "

-> Max term =

X, Y 1 = complemented

0 = N.C

⑦ Question Practice

(i) minterm of 6 Represented as Σm

Binary of 6 = $2|6$, $2|3-0$, $1-1$

1 1 0

$= X Y \bar{Z}$

(2) Maxterm of 6 Πm

= 1 1 0

$= \bar{x} + \bar{y} + z$

⑧ Relationship b/w Min and Max term.

<u>By Demorgan's</u>

i) $\overline{X \cdot Y} = \bar{X} + \bar{Y}$

(2) $\overline{X + Y} = \bar{x} \cdot \bar{y}$

They are complement of each other.

<u>steps</u>

i) Take dual then inverse

⑨ Observations.

(i) In Minterm table. Then in a row only one 1 is present in 2^n terms. All the other terms are 0.

(2) In Maxterm table. There will be only one zero in a row. All others are 1.

③ SOP or POS are used for stating Bolean function.

⑩ SOP Example.

$F = A + \bar{B}C$

$= A(B+\bar{B}) + (A+\bar{A})\bar{B}C$

$= AB + A\bar{B} + A\bar{B}C + \bar{A}\bar{B}C$

$= AB(C+\bar{C}) + A\bar{B}(C+\bar{C}) + A\bar{B}C + \bar{A}\bar{B}C$

$= \underset{m_7}{ABC} + \underset{m_6}{AB\bar{C}} + \underset{m_5}{A\bar{B}C} + \underset{m_4}{A\bar{B}\bar{C}} + A\bar{B}C + \underset{m_1}{\bar{A}\bar{B}C}$

same

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C$

<u>Expressed as SOP</u>

$F = m_7 + m_6 + m_5 + m_4 + m_1 = m_1 + m_4 + m_5 + m_6 + m_7$

Representation in Shorthand SOP Form

$$F(A,B,C) = \sum m(\overset{m_1}{1}, \overset{m_4}{4}, \overset{m_5}{5}, \overset{m_6}{6}, \overset{m_7}{7})$$

represents the minterm

POS

$$f(x,y,z) = x + \bar{x}y$$
$$= (x+\bar{x})(x+\bar{y}) = 1\cdot(x+\bar{y})$$
$$= (x+\bar{y}+z\bar{z})$$
$$= (x+\bar{y}+z)(x+\bar{y}+\bar{z})$$

$$f = M_2 . M_3$$

Convert SOP to POS.

$$f(A,B,C) = A\bar{C} + BC + \bar{A}\bar{B}$$

Distributive law $\quad x + yz = (x+y).(x+z)$

let $\quad A\bar{C} + BC = x \quad , \quad \bar{A}\bar{B} = yz$

Then

$$= (A\bar{C} + BC + \underbrace{\bar{A}}_{y})(A\bar{C} + BC + \underbrace{\bar{B}}_{z})$$
$$= [\bar{A} + A\bar{C} + BC][\bar{B} + BC + A\bar{C}]$$
$$= (\bar{C} + BC + \bar{A})(A\bar{C} + C + \bar{B})$$
$$= (C + B + \bar{A})(A + C + \bar{B})$$

Rearrange

$$f = (\bar{A}+B+C)(A+\bar{B}+C) \text{ to give } f = M_5 . M_2$$

Conanical terms includes all the literal and are under minterm, maxterm.

lecture # 7

Functions Complement.

(1)

→ The complement of a function expressed as a SOP is constructed by selecting the minterms missing in The SOP, conanical form

→ Alternatively, The complement of a function expressed by a SOP is simply the POS with the same indices.

→ اگر کسی فنکشن کا میکس ٹرم نکالنی ہو تو وہ ٹرم لکھو جو اس
جو کے فنکشن میں نہ ہوں، مثال

$m_i = \overline{M_i}$

$M_i = \overline{m_i}$

$$F(x, y, z) = \Sigma_m(1, 3, 5, 7)$$

$$\overline{F}(x, y, z) = \Sigma_m(0, 2, 4, 6)$$

The maxterm of " is $\Pi_m(1, 3, 5, 7)$

(2) Conversion b/w forms.

(i) $F(x, y, z) = \Sigma_m(1, 3, 5, 7)$

$$\overline{F}(x, y, z) = \Sigma_m(0, 2, 4, 6)$$

But now here double complement will result in

$$\overline{\overline{F}}(x, y, z) = \Pi_m(0, 2, 4, 6)$$

$$F(x, y, z) = \Pi_m(0, 2, 4, 6)$$

Not obeyed by conanoical form.

Standard Forms

(i) Standard SOP form :-

Function written in the form of 'OR' of 'And' terms. such as $ABC + \bar{A}\bar{B}C + B$

(ii) Standard POS form.

Functions written in the form of 'And' of 'OR'.

such as $(A+B).(A+\bar{B}+\bar{C}).C$

These mixed forms are neither SOP nor POS

(i) $(AB+C).(A+C)$ (ii) $ABC + AC(A+B)$

Standard SOP :-

A sum of minterms form for n variables can be written down directly from a truth table.

(i) Implementation of this form is a two level network of gates such that :

→ The first level consists of n-input And gates.

→ The second level is a single OR gate (with fewer 2^n input)

$$\begin{array}{r|l} 2 & 4 \\ \hline 2 & 2 - 0 \\ \hline & 1 - 0 \end{array}$$

(ii) This form often can be simplified so that the corresponding circuit is simpler.

The example of the simplification :-

$F(A, B, C) = \Sigma_m (1, 4, 5, 6, 7)$

$F = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$

Simplified

$F = \bar{A}\bar{B}C + A\bar{B}(\bar{C}+C) + AB(\bar{C}+C)$

$= \bar{A}\bar{B}C + A\bar{B} + AB$

$= \bar{A}\bar{B}C + A(\bar{B}+B)$

$= A + \bar{A}\bar{B}C$

$= (A+\bar{A})(A+\bar{B}C)$

$= A+\bar{B}C$

It contains three literals as compared to 15 literals.

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SOP and POS observations.

(i) Canonical forms (SOPs, POS, or SSOP, SPOS) differs in complexity.
(ii) Bolean algebra can be used to manipulate into simpler form.
(iii) Simpler equations leads to simpler solution.

Properties of

9

Literal Cost :

(i) Literal – a variable or it's complement.
(2) Literal cost = No of literals appeared in a ~~literal~~ Bolean expression – corresponding to logic Circuit Diagram.

→ Gate Input cost.

The number of (gates) inputs to the gates in the implementation corresponding exactly to the given equation
(G – inverter not canted, GN – inverter counted)

* For POS, or SOP – It can be found from the equations by finding the sum of
  (i) All literal appearences
  (2) The number of terms excluding the single literals G.
  (3) Optionally. The number of distinct literal count. GN.

Example

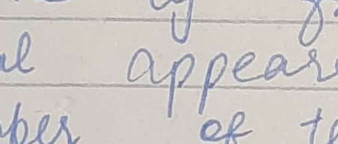
inputs 2 — inputs 3 — input 3 — combined literal = 3

(i) $F = BD + A\bar{B}C + A\bar{C}\bar{D}$

$1 + 2 + 3 \Rightarrow 3$

Literals = 8 | 8 + 3 = G (Inputs) $\underline{G = 11}$, GN = 14

↓



(ii) $F = BD + A\bar{B}C + A\overline{BD} + AB\bar{C}$

Literals = 11

G = Literals + 4 = 15

GN = Literals + 3 = 18 — distinct inverted complements Such as $\bar{B}, \bar{D}, \bar{C}$

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Bolean Function Optimization.

(i) ~~Definition~~

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