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Numerical Computing

Assignment 3

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BCS-5D

Q No 1

$y(1.1) = ?$

$y(1) = 1$

$$\frac{d^2y}{dx^2} + y^2\frac{dy}{dx} = x^3$$

$y'(1) = 1$

Using Taylor Series

$x_0 = 1$ $\quad y'_0 = 1$

$y_0 = 1$ $\quad x = 1.1$

$\Rightarrow y'' + y^2y' = x^3$

$y'' = x^3 - y^2y' \rightarrow (1)$

$h = x - x_0 = 1.1 - 1$

$\boxed{h = 0.1}$

$$\Rightarrow y(x) = y_1 = y_0 + hy'_0 + \frac{h^2}{2!}y''_0 + \frac{h^3}{3!}y'''_0 + \frac{h^4}{4!}y^{iv}_0 + \ldots$$

for (1) $\quad y'' = x^3 - y^2y'$

$y''_0 = x_0^3 - y_0^2y'_0$

$= (1)^3 - (1)^2(1)$

$\boxed{y''_0 = 0}$

diff (1) w.r.t x and get

$\Rightarrow y''' = 3x^2 - y^2y_0 - 2yy'^2 \rightarrow (2)$

②

Now $y_0''' = 3x_0^2 - y_0^2 y_0'' - 2y_0 y_0'^2$

$\Rightarrow y_0''' = 3(1)^2 - 0 - 2(1)(1)^2$

$= 3 - 2$

$\boxed{y_0''' = 1}$

Again diff ② w.r.t x

$\Rightarrow y^{iv} = 6x - [y^2 y''' - 2yy'(y'')] - 2[y'^2 y' - y(2(y')^2 + yy'')]$

$\Rightarrow y^{iv} = 6x - y^2 y''' + 2yy'y'' - 2y'^3 + 2y[2y'^2 + 2yy'']$

$= 6x - y^2 y''' + 2yy'y'' - 2y'^3 + 4yy'^2 + 4y^2 y''$

$y^{iv} = 6x - 3y^2 y'' + 2y'[yy'' - y'^2 + 2yy']$

Now

$y_0^{iv} = 6x_0 - 3y_0^2 y_0'' + 2y_0'[y_0 y_0'' - y_0'^2 + 2y_0 y_0']$

$= 6(1) - 3(1)^2(0) + 2(1)[(1)(0) - (1)^2 + 2(1)(1)]$

$= 6 - 0 + 2[0 - 1 + 2]$

$= 6 + 2(1)$

$= 8$

Put in Taylor Series

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \frac{h^4}{4!}y_0^{(iv)}$$

$$= 1 + (0.1)(1) + \frac{(0.1)^2(0)}{2} + \frac{(0.1)^3(1)}{6} + \frac{(0.1)^4(8)}{24}$$

$$= 1 + 0.1 + 0 + \overset{0.00067}{\cancel{+.6667}} + 0.000334$$

$$\boxed{y_1 = 1.160279}$$

Similarly we get finally

$$y(1.1) = 1.160279$$

Q No 2

$y(0.8) = ?$ $\qquad y(0) = 1$

given $\frac{dy}{dx} = x + y$ $\qquad h = 0.2$

Use Adam's Method

first of all we need atleast four values of y

So

$$x_0 = 0, \; y_0 = 1, \; h = 0.2$$

We have

$$y' = x + y \rightarrow (1)$$

$$y_0' = x_0 + y_0 = 0 + 1 = 1 \Rightarrow \boxed{y_0' = 1}$$

diff 1 w.r.t x

$$y'' = 1 + y' \rightarrow 2$$

$$y_0'' = 1 + y_0'$$

$$= 1 + 1$$

$$\boxed{y_0'' = 2}$$

again diff 2 w.r.t x

$$y''' = 0 + y''$$

$$y_0''' = 0 + y_0''$$

$$= 0 + 2$$

$$\boxed{y_0''' = 2}$$

Now by formula

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \cdots$$

$$= 1 + (0.2)(1) + \frac{(0.2)^2}{2}(2) + \frac{(0.2)^3}{3.2}(2) + \cdots$$

$$\boxed{y_1 = 1.2428}$$

Now $x_1 = 0.2$ $y_1 = 1.2428$

$x_2 = 0.4$ $y_2 \longrightarrow$ find it

as $y' = x + y$

$y_1' = x_1 + y_1$

$= 0.2 + 1.2428$

$y_1' = 1.4428$

$\Rightarrow y_1'' = 1 + y_1'$

$= 1 + 1.4428$

$\boxed{y_1'' = 2.4428}$

also $y_1''' = 0 + y_1'' = 2.4428$

So

$$y_2 = y_1 + hy_1' + \frac{h^2}{2!}y_1'' + \frac{h^3}{3!}y_1''' + \cdots$$

$$= 1.2428 + (0.2)(1.4428) + \frac{(0.2)^2}{2}(2.4428) + \frac{(0.2)^3}{3.2}\cdot(2.4428)$$

$+ \cdots$

$\Rightarrow \boxed{y_2 = 1.5836}$

∴ for next value of y

$x_2 = 0.4 \qquad y_2 = 1.5836$

$x_3 = 0.6$

$\Rightarrow \quad y_2' = x_2 + y_2 = 1.9836$

$y_2'' \Rightarrow 2.9836$ and $y_2''' = 2.9836$

$y_2^{iv} = 2.9836$

Now

$$y_3 \Rightarrow 1.5836 + (0.2)(1.9836) + \frac{(0.2)^2}{2}(2.9836) + \frac{(0.2)^3}{6}(2.9836)$$

$+ \cdots$

$$\boxed{y_3 = 2.04420}$$

Now we can apply Adam's predictor formula

$$y_{n+1,p} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

for $n=3$

$$y_{4,p} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right] \rightarrow 3$$

As $\Rightarrow y_3' = x_3 + y_3 = 0.6 + 2.04420 = 2.6442$

Put all values in (3)

$$y_{4,p} = 2.0442 + \frac{0.2}{24}\left[55(2.6442) - 59(1.9836) + 37(1.4428) - 9(1)\right]$$

$$\boxed{y_{4,p} = 2.6507}$$

Applying Adam's Corrector formula

$$y_{n+1,c} = y_n + \frac{h}{24}\left[9y'_{n+1} + 19y'_n - 5y'_{n} + y'_{n-2}\right]$$

for $n = 3$

$$y_4' = x_4 + y_4$$
$$= 0.8 + 2.6507$$
$$\boxed{y_4' = 3.4507}$$

$$y_{4,c} = 2.0442 + \frac{0.2}{24}\left[9(3.4507) + 19(2.6442) - 5(1.9836) + 1.4428\right]$$

$$\Rightarrow \boxed{y_{4,c} = 2.6510}$$

Q No 3

$$\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = 0 \qquad y(0) = 2$$

$$a = 0, \; b = 1$$

$$y'' - 3y' + 2y = 0 \longrightarrow (1) \qquad n = 4$$

As $\quad b - a = nh$

$$\Rightarrow h = \frac{b-a}{n} = \frac{1}{4}$$

$$y''_c = \frac{y_{i-1} + y_{i+1} - 2y_i}{h^2}$$

replace in (1)

$$\frac{y_{i-1} + y_{i+1} - 2y_i}{h^2} - 3\left[\frac{y_{i+1} - y_{i-1}}{2h}\right] + 2y_i = 0$$

Put $h = 1/4$

$$\Rightarrow 16(y_{i-1} + y_{i+1} - 2y_i) - 6(y_{i+1} - y_{i-1}) + 2y_i = 0$$

$$16y_{i-1} + 16y_{i+1} - 32y_i - 6y_{i+1} + 6y_{i-1} + 2y_i = 0$$

$$22y_{i-1} + 10y_{i+1} - 30y_i = 0, \text{ for } i = 1, 2, 3, \ldots$$

for $i = 1$

$$22y_0 + 10y_2 - 30y_1 = 0$$

$$10y_2 - 30y_1 + 22y_0 = 0 \quad \rightarrow (1)$$

for $i=2$

$$22y_1 + 10y_3 - 30y_2 = 0 \quad \rightarrow (2)$$

for $i=3$

$$22y_2 + 10y_4 - 30y_3 = 0 \quad \rightarrow (3)$$

from (3) put values

$$30y_3 = 22y_2 + 10y_4$$

$$y_3 = \frac{22(4.3) + 10(10.1)}{30}$$

$$\boxed{y_3 = 6.52}$$

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for 1

$$30y_1 = 22y_0 + 10y_2$$

$$y_1 = \frac{22(2) + 10(4.3)}{30}$$

$$\boxed{y_1 = 2.9}$$

finally

$x_0 = 0$	$y_0 = 2$
$x_1 = 0.25$	$y_1 = 2.9$
$x_2 = 0.5$	$y_2 = 4.3$ (found using Taylor Series)
$x_3 = 0.75$	$y_3 = 6.52$
$x_4 = 1$	$y_4 = 10.1$