National University of Computer and Emerging Sciences

Lahore Campus

Discrete Structures (CS1005)

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Course Instructors

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Sessional-I Exam

Total Time: 1 Hour

Total Marks: 30

Total Questions: 03

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Roll No Section Student Signature

CLO #1: Express statements in terms of predicates, quantifiers and logical connectives. Apply formal logic proofs, logical reasoning to practical problems related to offered program.

Q. No 1:

i) Write the following English sentences in symbolic form. [5]

a) To qualify the programming contest, it is necessary to submit code in Python or Java, but not both.	$r \rightarrow (p \oplus q)$
b) The room is dark whenever the lights are not on,	$\sim p \rightarrow q$
c) The team played well but they lost the match.	$p \wedge \sim q$
d) If the passenger shows neither a passport nor an ID card, then entry is denied.	$\sim(p \vee q) \rightarrow r$ Or $(\sim p \wedge \sim q) \rightarrow r$
e) A triangle is equilateral only if all its sides are equal, and conversely	$p \leftrightarrow q$

ii) Use Laws of equivalence to prove that $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg p$ is tautology. [5]

L.H.S $\equiv (\neg p \wedge (p \rightarrow q)) \rightarrow \neg p$	
$\equiv \neg(\neg p \wedge (\neg p \vee q)) \vee \neg p$	*Conditional Disjunction*
$\equiv (p \vee \neg(\neg p \vee q)) \vee \neg p$	*De Morgan's Law*
$\equiv p \vee (p \wedge \neg q) \vee \neg p$	*De Morgan's Law and Double Negation*
$\equiv (p \vee \neg p) \vee (p \wedge \neg q)$	*Associative Law*
$\equiv T \vee (p \wedge \neg q)$	*Negation Law*
$\equiv (T \vee p) \wedge (T \vee \neg q)$	*Distributive Law*
$\equiv T \wedge T$	*Domination Law*
$\equiv T$	*Idempotent Law*
$\equiv \boldsymbol{R.H.S}$	

Q. No 2:

i) Write the following statement in the form of if p then q. Also, write its contraposition.

"Having a valid passport is necessary but not sufficient for international travel." [2]

If p then q: If a person can travel internationally, then they have a valid passport.	
Contrapositive: If a person does not have a valid passport, then they cannot travel internationally.	

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ii) Use quantifiers and predicates with more than one variable to express these statements. [2]
Let *C* (*x*, *y*) be the statement "*x* and *y* have chatted over the Internet," where the domain for the variables *x* and *y* consists of all students in your class**.**
a) There is a student in your class who has chatted with everyone in your class over the Internet.

Solution: $\exists x \forall y\, C(x, y)$

b) There are two students in your class who have not chatted with each other over the Internet.

Solution: $\exists x \exists y((x \neq y) \wedge \sim C(x, y))$

iii) Answer the following questions. [6]

a) Determine the truth value of $\exists x \forall y(y \neq 0 \rightarrow xy = 1)$ if the domain of each variable consists of all real numbers. Give a counter example if it is false.

Solution: This is false, since the reciprocal of y depends on y - there is not one x that works for all y.

b) Translate the $\forall x \forall y\left(((x < 0) \wedge (y < 0)) \rightarrow (xy > 0)\right)$ into an English. The domain in each case consists of all real numbers.

Solution: For all real numbers x and for all real numbers y, if both x and y are negative, then their product is positive. **"Product of two negative real numbers gives a positive real number."**

c) Express the negations of "No one has climbed every mountain in the Himalayas." using quantifiers, and in English.

Solution: Logically it means " $\neg \exists x \forall m C(x, m)$"

Using De Morgan's $\neg\left(\neg \exists x \forall m C(x, m)\right) \equiv \forall x \exists m \neg C(x, m)$

In English "Every person has at least one Himalayan Mountain they haven't climbed."

Q. No. 3

i) Use rules of interference to determine if the following argument is valid, [5]

"If Lois will not mow her lawn, then there is either a chance of rain, or her red headband is missing." "Whenever the temperature is over 80°F, there is no chance for rain.", "Today the temperature is 85°F and Lois red headband is not missing". Therefore (sometime today) Lois will mow her lawn.

Solution: Let:

- p: Lois will mow her lawn.
- q: There is a chance of rain.
- r: Her red headband is missing.
- s: The temperature is over 80°F

Premises:

If Lois will not mow her lawn, then there is either a chance of rain, or her red headband is missing.

P_1 $\sim p \rightarrow (q \vee r)$

Whenever the temperature is over 80°F, there is no chance of rain.

P_2 $s \longrightarrow \sim q$

Today the temperature is 85°F and Lois red headband is not missing.

P_3 $s \wedge \sim r$

Lois will mow her lawn.

$\therefore$ p

Steps	Reasons
1) $s \wedge \sim r$	Premise 3
2) s	Simplification 1
3) $s \longrightarrow \sim q$	Premise 2
4) $\sim q$	Modus Ponens on 2 and 3
5) $\sim r$	Simplification on 1
6) $\sim q \wedge \sim r$	Conjunction on 4 and 5
7) $\sim (q \vee r)$	Demorgan's law on 6
8) $\sim p \rightarrow (q \vee r)$	Premise 2
9) $\sim(\sim p)$	Modus Tollens on 7 and 8
10) p	Double Negation on 9
$\Longrightarrow$ ***Argument is Valid***	

ii) Let $n \in \mathbb{Z}$. Use a direct proof to show that if n is odd, then $n^2 - 1$ is divisible by 8. [5]

Solution: Let's be an odd integer,

So $n = 2k + 1$, where $k \in Z$

Squaring both sides we have

$$n^2 = (2k+1)^2$$
$$n^2 = 4k^2 + 4k + 1$$

Subtracting 1 from both sides, we have

$$n^2 - 1 = 4k^2 + 4k$$
$$n^2 - 1 = 4k(k+1)$$

Since $k(k+1)$ represents the product of two consecutive terms.

We know that the "Product of two consecutive terms is even"

$$\text{So } k(k+1) = 2m, \quad where\ m \in Z$$

Substituting in equation above, we get

$$n^2 - 1 = 4(2l) = 8m$$

This implies that $n^2 - 1$ is divisible by 8.