Representation of Data

Measure of Central Tendencies and spread

Histograms, Cumulative freq,

Stem & leaf, Box whisker Diagrams.

Mode, median, Mean.

Range, Standard deviation, Variance

Linear Transformation of data.

Probability Distributions and Random Variable

Probability + Conditional

Binomial Distribution

Geometric Distribution

Random Variable

Normal Distribution.

$N \sim (\overset{\text{Mean}}{\mu}, \overset{\text{Variance}}{\sigma^2})$

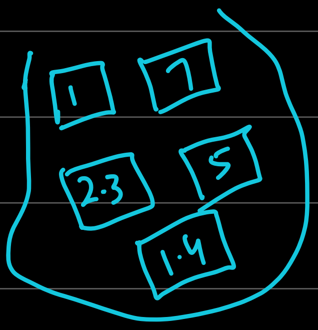
$N \sim (12, 36)$

Mean = 12, Var = 36

SD = $\sqrt{36}$

Arrangements Permutations & Combinations

Binomial
- repeat
- Discrete predetermined → ??
- Success/failure
- n, p, q constant.



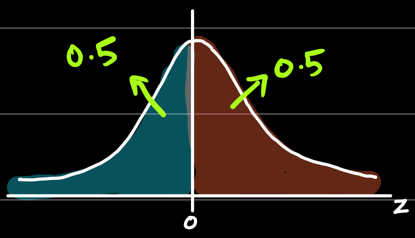
NORMAL DISTRIBUTION

(15 MARKS) (2 QUESTIONS)

Discrete???
↓ Fixed outcomes in experiment!
↳ Binomial Distribution
↳ Geometric Distribution.

CONDITIONS: 1- CONTINOUS DATA (OUTCOMES)

eg: lengths of leaves
height of students in class



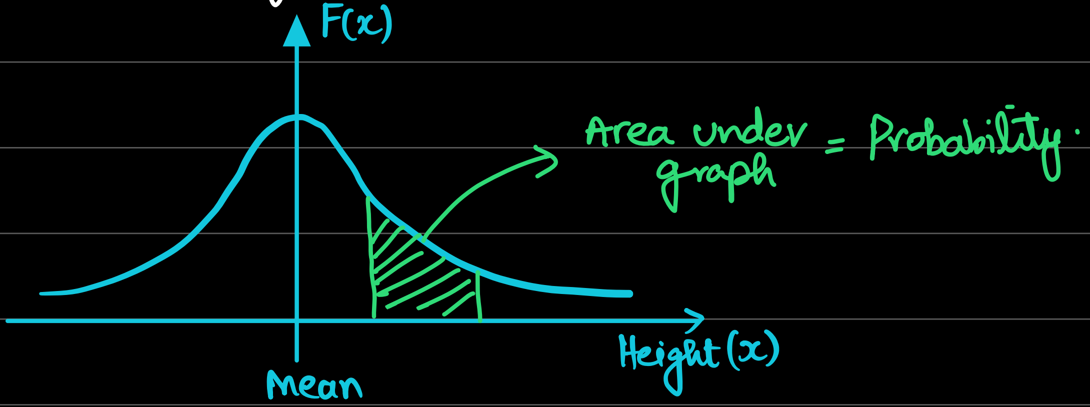
2- Symmetrical data
Data is evenly divided on both sides of Mean.

3- n is large
We will discuss this later in chapter.

IT IS ALWAYS GIVEN WHEN YOU HAVE TO USE THE NORMAL DISTRIBUTION.

Experiment: Heights of students in a class

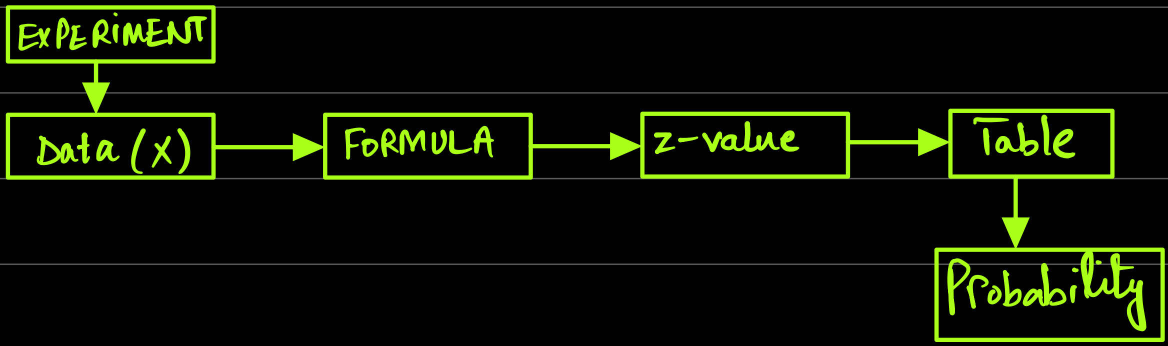
X = heights = Data



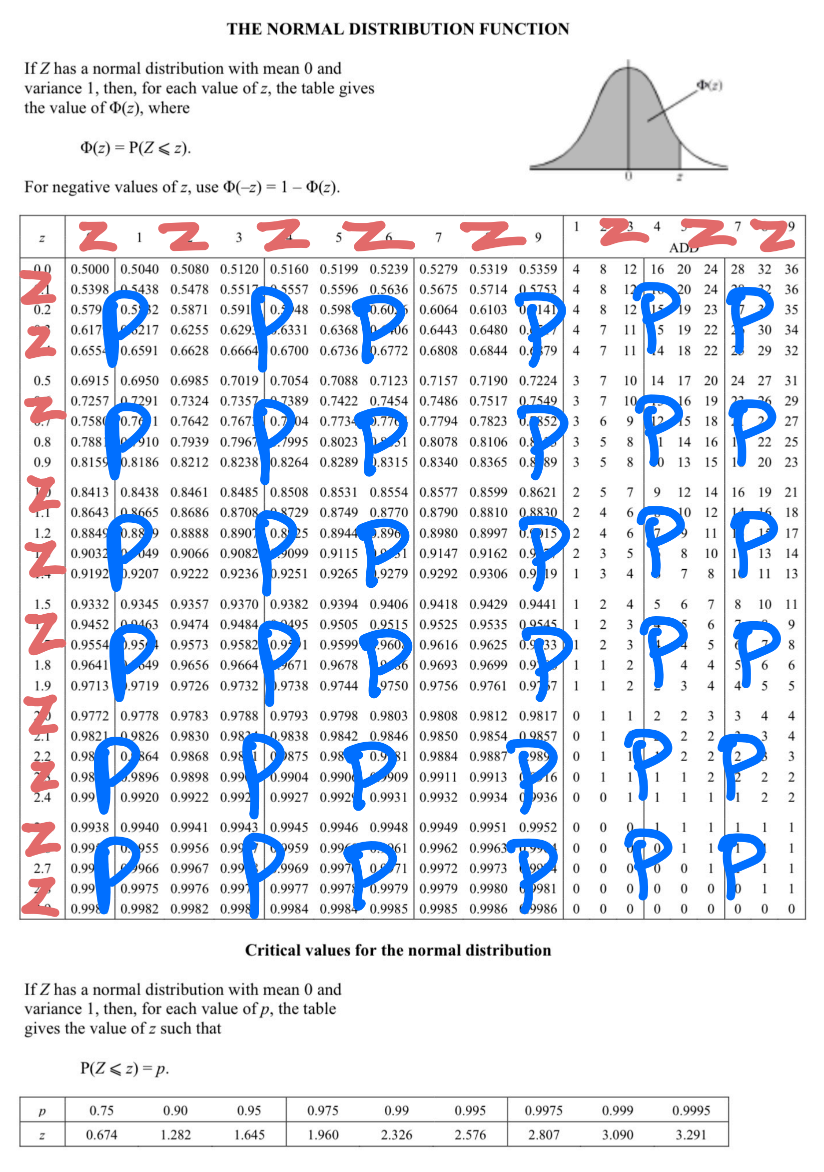
For each new experiment, a new curve is to be made. Finding areas for each new curve gets very complicated.

Maths People did NOT like this at All.

So, we designed a statard graph which can work on any experiment. THAT WAS CALLED A $\boxed{\text{Z-TABLE}}$



This table is given in EXAM.



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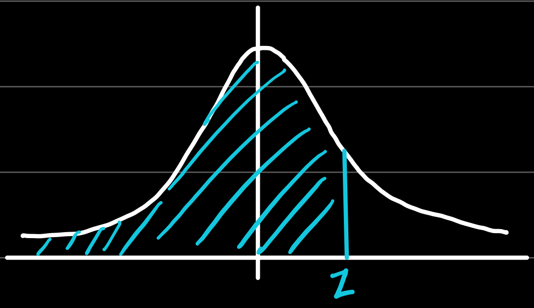
z	[illegible]	1	2	3	4	5	6	7	[illegible]	9	1	2	3	4	5	[illegible]	7	[illegible]	9
											ADD								
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
[illegible]	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	[illegible]	20	24	[illegible]	[illegible]	36
0.2	[illegible]	[illegible]	0.5871	[illegible]	[illegible]	[illegible]	[illegible]	0.6064	0.6103	[illegible]	4	8	12	[illegible]	19	23	[illegible]	[illegible]	35
[illegible]	[illegible]	[illegible]	0.6255	[illegible]	[illegible]	0.6368	[illegible]	0.6443	0.6480	[illegible]	4	7	11	[illegible]	19	22	[illegible]	30	34
[illegible]	[illegible]	0.6591	0.6628	0.6664	0.6700	0.6736	[illegible]	0.6808	0.6844	[illegible]	4	7	11	[illegible]	18	22	[illegible]	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
[illegible]	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	[illegible]	16	19	[illegible]	[illegible]	29
[illegible]	[illegible]	[illegible]	0.7642	[illegible]	[illegible]	[illegible]	[illegible]	0.7794	0.7823	[illegible]	3	6	9	[illegible]	15	18	[illegible]	[illegible]	27
0.8	[illegible]	[illegible]	0.7939	0.7967	[illegible]	0.8023	[illegible]	0.8078	0.8106	[illegible]	3	5	8	[illegible]	14	16	[illegible]	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	[illegible]	0.8340	0.8365	[illegible]	3	5	8	[illegible]	13	15	[illegible]	20	23
[illegible]	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	[illegible]	10	12	[illegible]	[illegible]	18
1.2	0.8849	[illegible]	0.8888	[illegible]	[illegible]	0.8944	[illegible]	0.8980	0.8997	[illegible]	2	4	6	[illegible]	[illegible]	11	[illegible]	[illegible]	17
[illegible]	0.9032	[illegible]	0.9066	0.9082	[illegible]	0.9115	[illegible]	0.9147	0.9162	[illegible]	2	3	5	[illegible]	8	10	[illegible]	13	14
[illegible]	0.9192	[illegible]	0.9222	0.9236	[illegible]	0.9265	[illegible]	0.9292	0.9306	[illegible]	1	3	4	[illegible]	7	8	[illegible]	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
[illegible]	0.9452	0.9463	0.9474	0.9484	[illegible]	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	[illegible]	[illegible]	6	[illegible]	[illegible]	9
[illegible]	0.9554	[illegible]	0.9573	0.9582	[illegible]	0.9599	[illegible]	0.9616	0.9625	[illegible]	1	2	3	[illegible]	[illegible]	5	[illegible]	[illegible]	8
1.8	0.9641	[illegible]	0.9656	0.9664	[illegible]	0.9678	[illegible]	0.9693	0.9699	[illegible]	1	1	2	[illegible]	4	4	5	6	6
1.9	0.9713	[illegible]	0.9726	0.9732	[illegible]	0.9744	[illegible]	0.9756	0.9761	[illegible]	1	1	2	[illegible]	3	4	4	5	5
[illegible]	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	[illegible]	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	[illegible]	[illegible]	2	2	[illegible]	3	4
2.2	[illegible]	[illegible]	0.9868	[illegible]	[illegible]	[illegible]	[illegible]	0.9884	0.9887	[illegible]	0	1	[illegible]	[illegible]	2	2	2	[illegible]	3
[illegible]	[illegible]	[illegible]	0.9898	[illegible]	0.9904	[illegible]	[illegible]	0.9911	0.9913	[illegible]	0	1	1	1	1	2	2	2	2
2.4	[illegible]	0.9920	0.9922	[illegible]	0.9927	[illegible]	0.9931	0.9932	0.9934	[illegible]	0	0	1	1	1	1	[illegible]	2	2
[illegible]	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	[illegible]	1	1	1	1	1
[illegible]	[illegible]	[illegible]	0.9956	[illegible]	[illegible]	[illegible]	[illegible]	0.9962	0.9963	[illegible]	0	0	[illegible]	0	1	1	[illegible]	[illegible]	1
2.7	[illegible]	[illegible]	0.9967	[illegible]	[illegible]	[illegible]	[illegible]	0.9972	0.9973	[illegible]	0	0	0	0	0	1	[illegible]	1	1
[illegible]	[illegible]	0.9975	0.9976	[illegible]	0.9977	[illegible]	0.9979	0.9979	0.9980	[illegible]	0	0	0	0	0	0	0	1	1
[illegible]	[illegible]	0.9982	0.9982	[illegible]	0.9984	[illegible]	0.9985	0.9985	0.9986	[illegible]	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1, then, for each value of p, the table gives the value of z such that

$$P(Z \leqslant z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

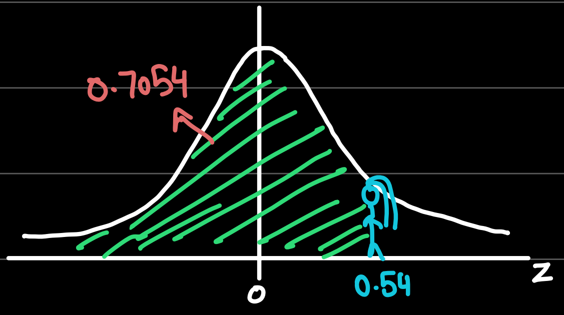


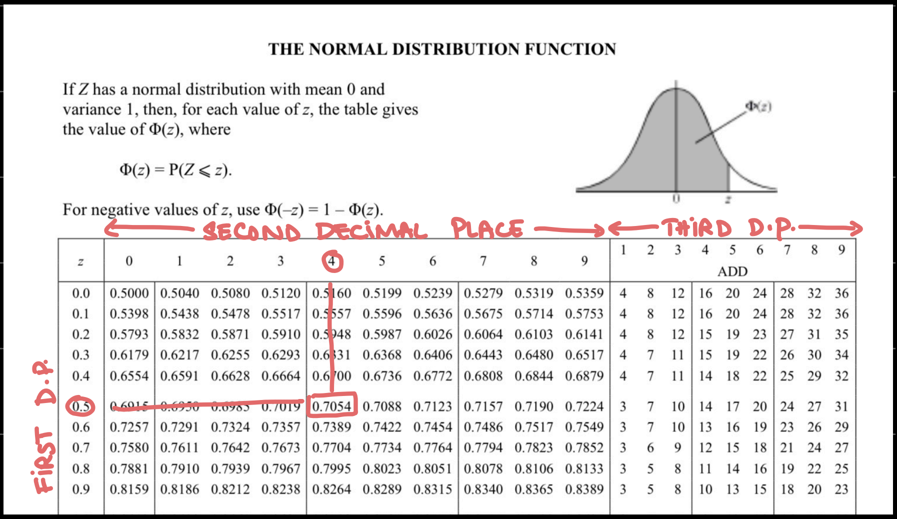
THIS TABLE ALWAYS TELLS AREA TO LEFT SIDE OF ANY z-value.

USING TABLE TO FIND PROBABILITIES

(i) $P(Z < 0.54)$

Left





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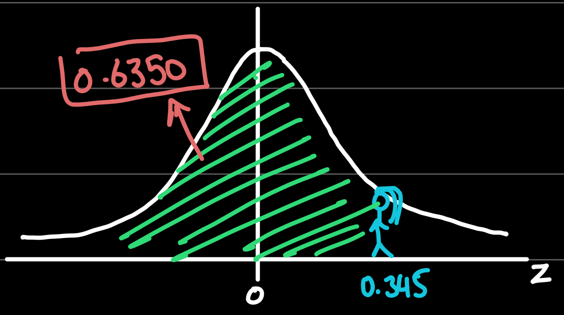
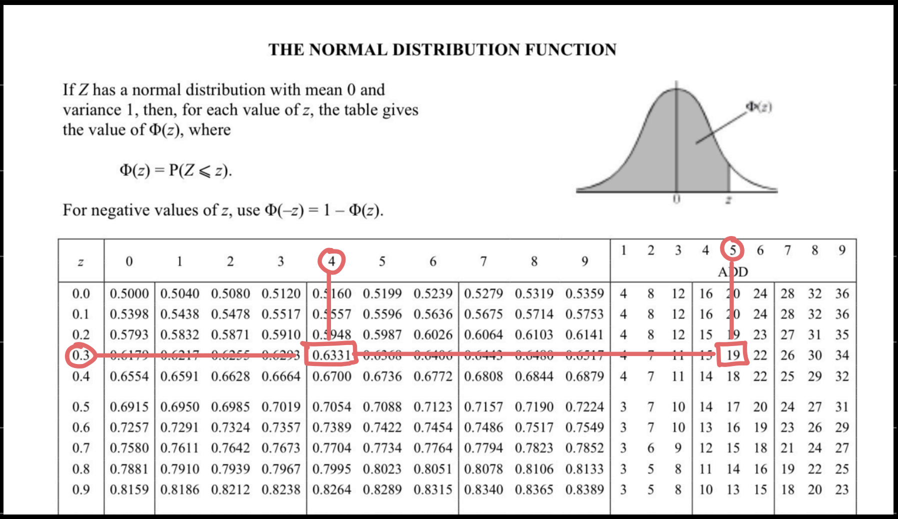
z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

$P(Z < 0.54) = 0.7054$

(ii) $P(Z < 0.345)$

LEFT





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															ADD				
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0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
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0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

Table value of 0.34 → 0.6331

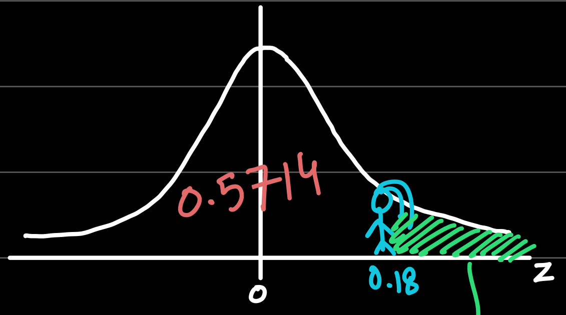
Third decimal $+19$

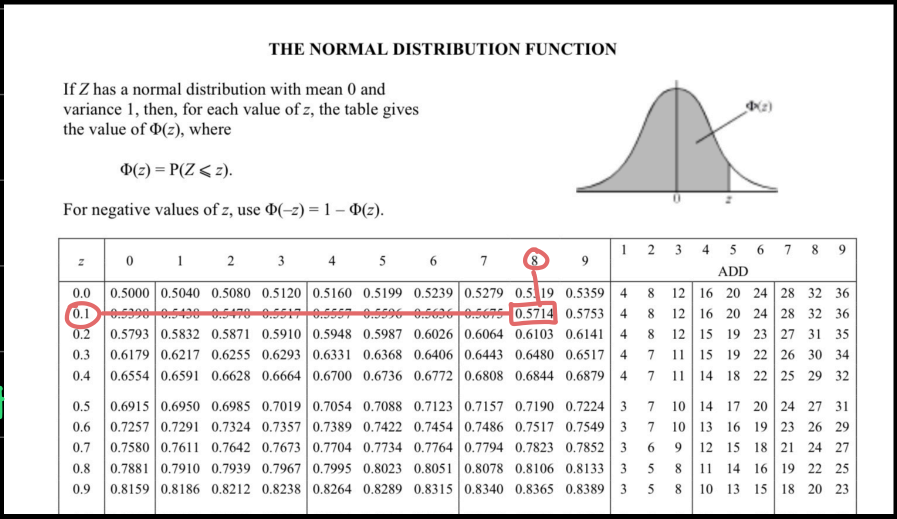
0.6350

$P(Z < 0.345) = 0.6350$

(iii) $P(Z > 0.18)$

RIGHT





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															ADD				
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0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

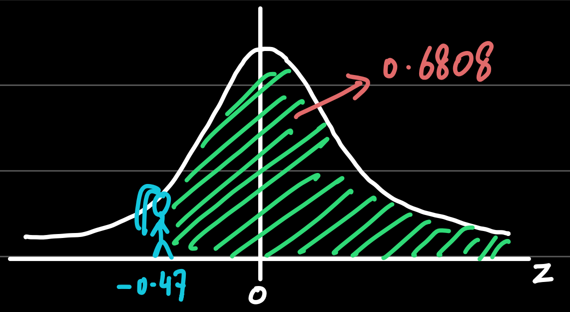
$= 1 - 0.5714$

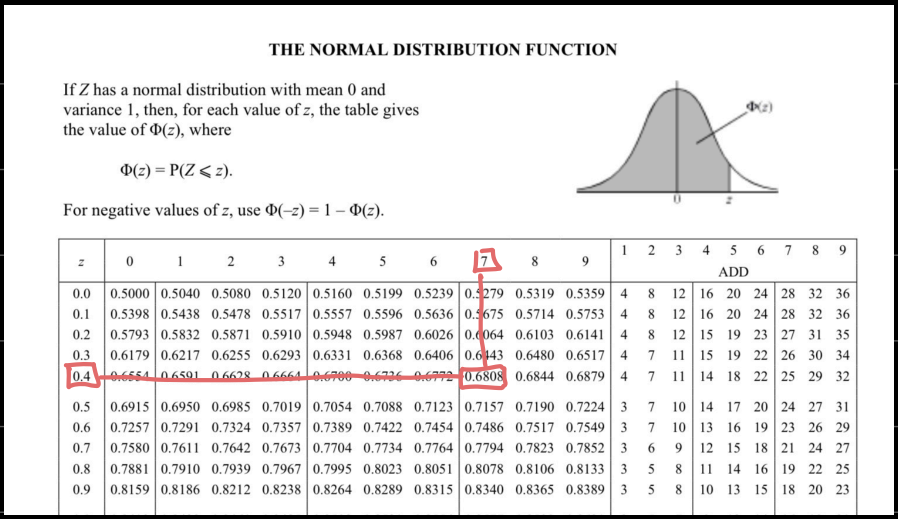
$= 0.4286$

$P(Z > 0.18) = 0.4286$

(iv) $P(Z > -0.47)$

RIGHT





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															ADD				
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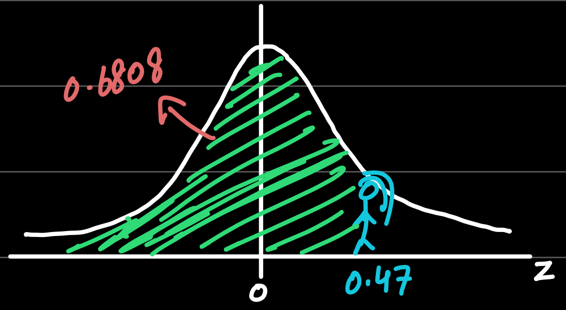
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

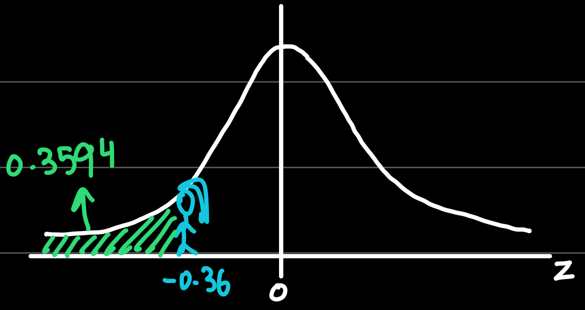
FOR NEGATIVE VALUES, FLIP THE DIAGRAM.

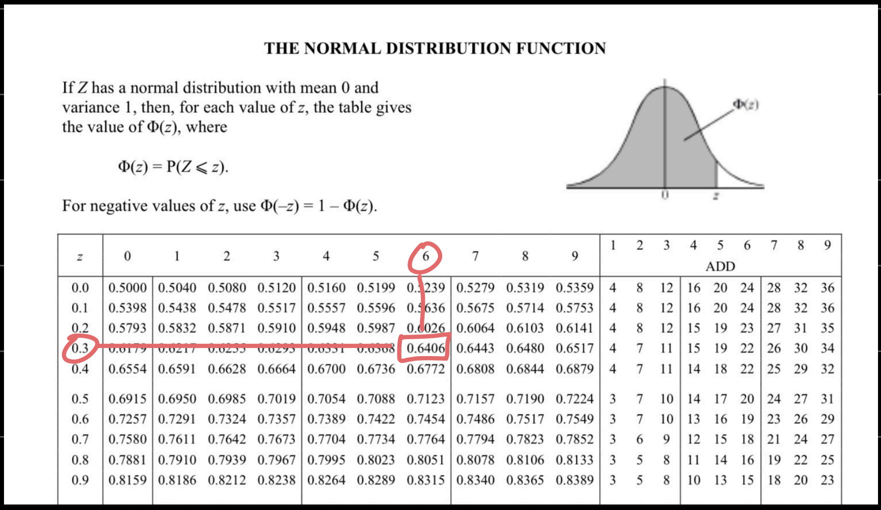


$P(Z > -0.47) = 0.6808$

(v) $P(z < -0.36)$

LEFT





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											1	2	3	4	5	6	7	8	9
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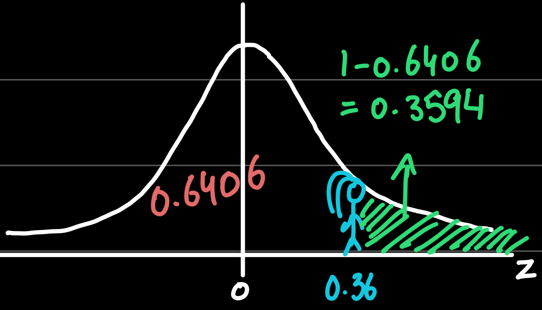


$P(z < -0.36) = 0.3594$

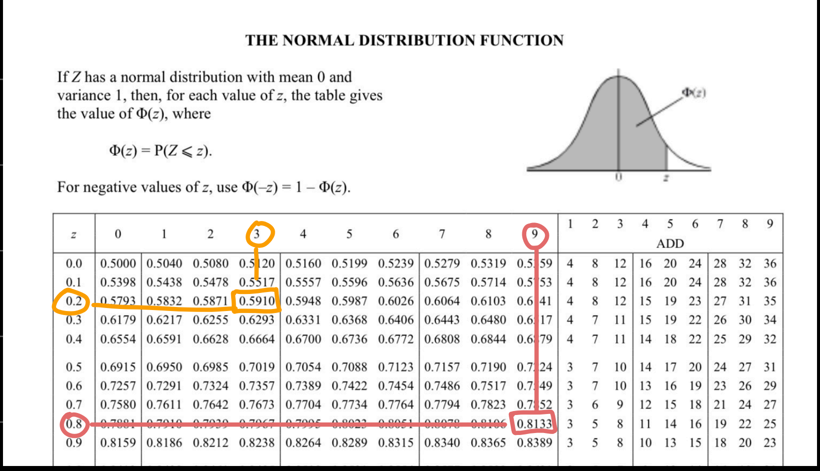
In Normal Distribution

$X < 45$ and $X \leqslant 45$ are exactly same

In Binomial and geometric

$X < 45$ and $X \leqslant 45$ are two separate ideas

(vi) $P(0.23 < z < 0.89)$



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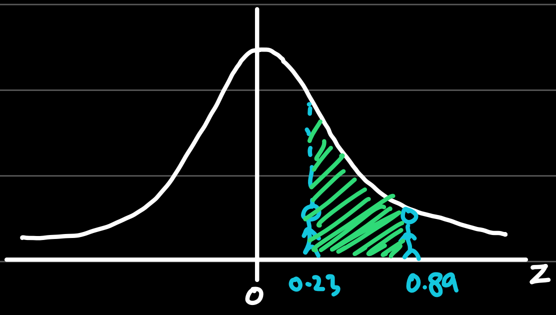


$\phi(0.23) = 0.5910$

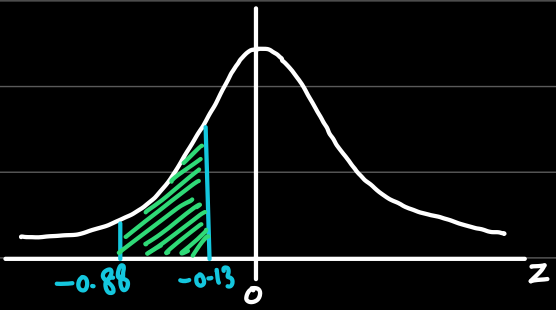
$\phi(0.89) = 0.8133$

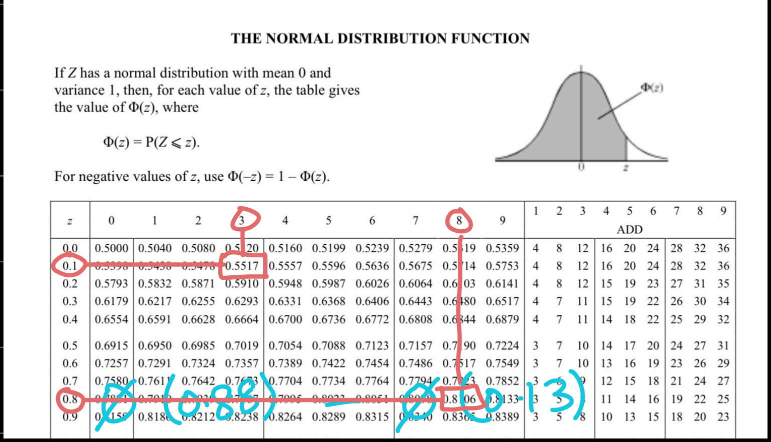
$\phi(z)$ = Table value of z

$\phi(1.2)$ = Table value of 1.2.

$$P(0.23 < z < 0.89) = \phi(0.89) - \phi(0.23)$$
$$= 0.8133 - 0.5910$$
$$= 0.2223.$$

(vii) $P(-0.88 < Z < -0.13)$





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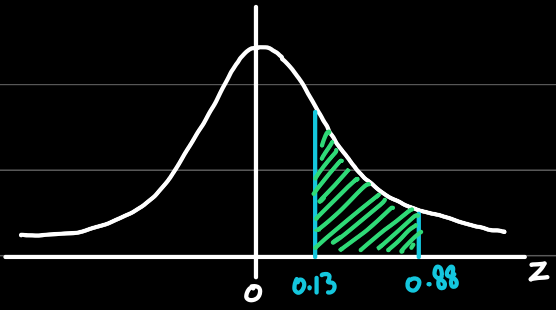
$\phi(0.88) - \phi(0.13)$

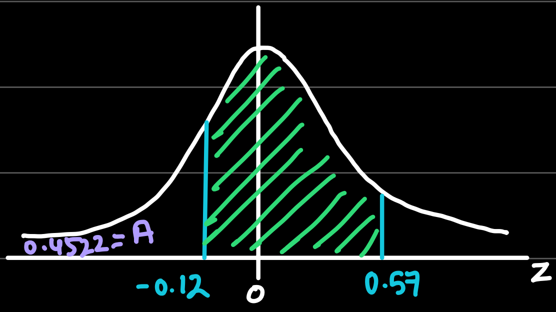


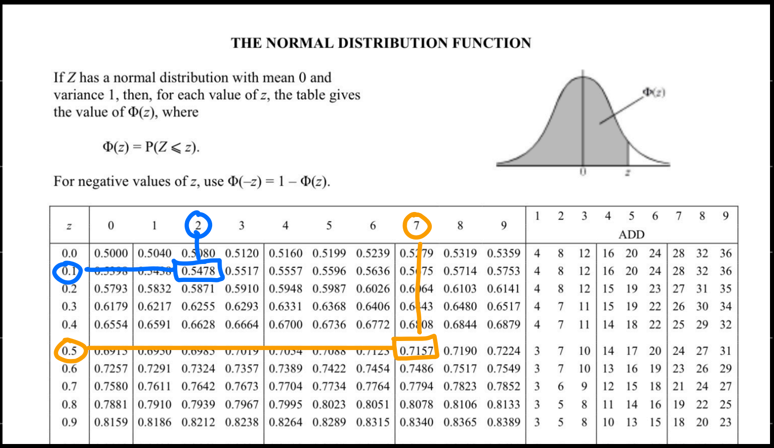
$= 0.8106 - 0.5517$

$= \boxed{0.2589}$

ADVANCED

(viii) $P(-0.12 < Z < 0.57)$





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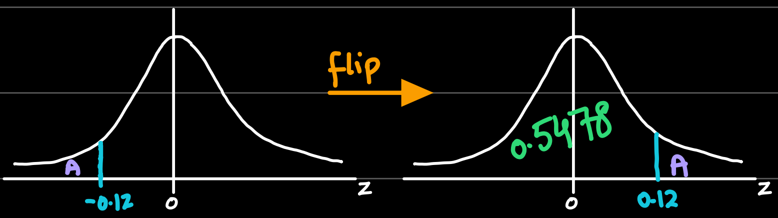
$\phi(0.57) = 0.7157$



$P(-0.12 < Z < 0.57)$

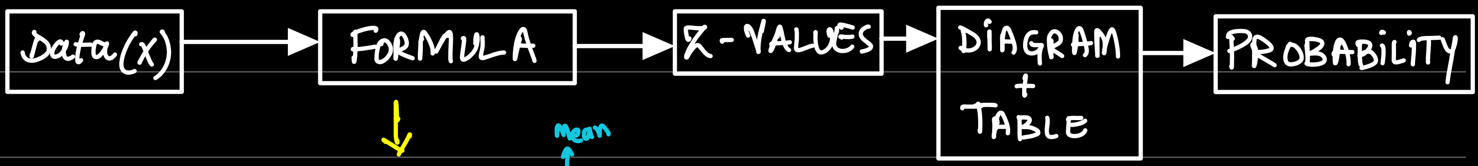
$= 0.7157 - 0.4522$

$= 0.2635$

$A = 1 - 0.5478$

$A = 0.4522$

TYPE 1: FORWARD WORKING


Data(x) → FORMULA → Z-VALUES → DIAGRAM + TABLE → PROBABILITY

$$\boxed{z = \frac{X - \overset{\text{mean}}{\mu}}{\underset{\text{SD}}{\sigma}}}$$

X = Data value (height etc)

μ = Mean

σ = Standard deviation.

Q:) X is normally distributed such that X has mean 36 and SD 12.

$\mu = 36$, $\sigma = 12$

In S1 write all probabilities to 4d.p.

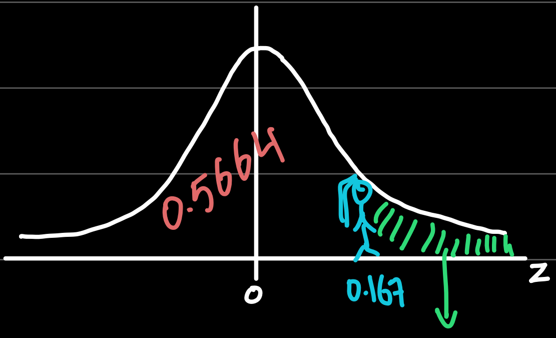
Find all Z-values up to 3 dp

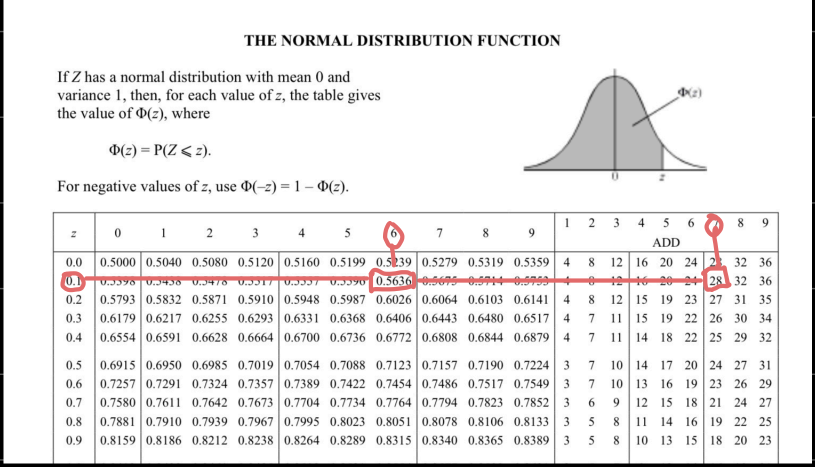
(i) $P(X > 38)$

$$z = \frac{X-\mu}{\sigma} = \frac{38-36}{12} = 0.167$$

$P(z > 0.167)$

right

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$= 1 - 0.5664$

$\phi(0.16) = 0.5636$

Add 3rd dp (7) = +28

0.5664

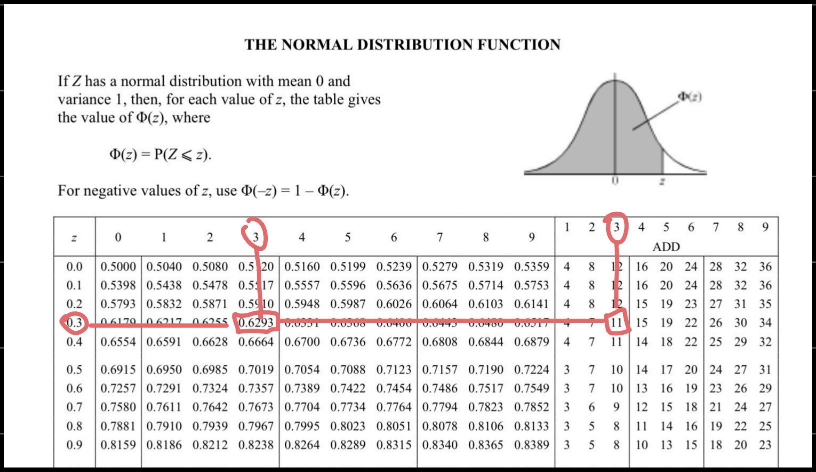
$P(z > 0.167) = 0.4336$

$P(X > 38) = 0.4336$

(ii) $P(X > 32) = 0.6304$

$$z = \frac{X-\mu}{\sigma} = \frac{32-36}{12} = -0.333$$

$P(Z > -0.333) = 0.6304$



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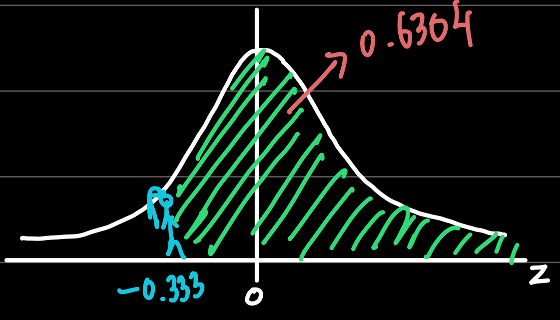
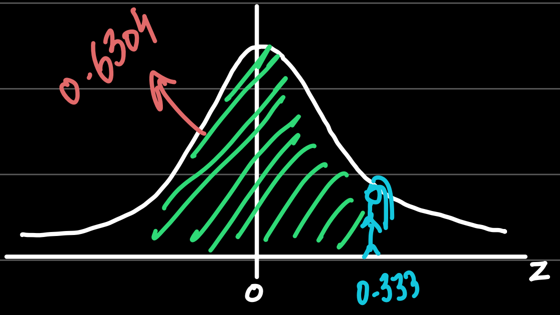


$\phi(0.33) = 0.6293$

$+ 11$

$\phi(0.333) = 0.6304$



Q: X is normally distributed

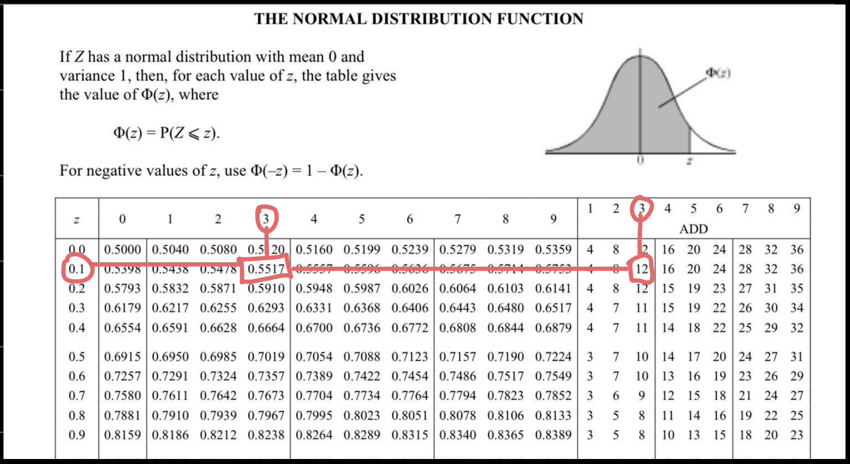
Mean = 30 , SD = 15

(i) $P(X > 32)$

$$z = \frac{X-\mu}{\sigma} = \frac{32-30}{15} = 0.133$$

$P(Z > 0.133)$

right



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															ADD				
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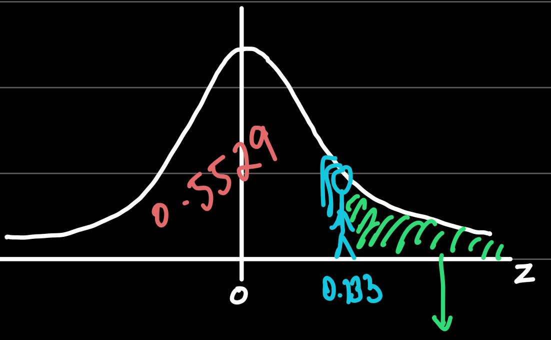
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23



$\phi(0.13) = 0.5517$

$= 1 - 0.5529$

$+ \quad 12$

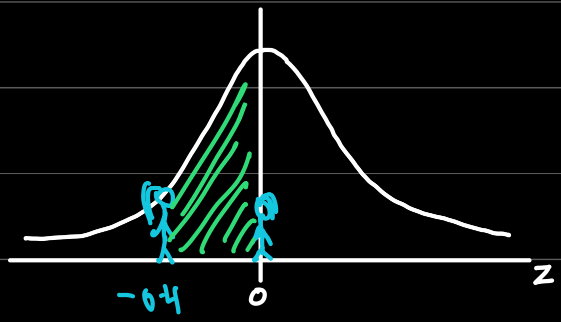
$\Phi(0.133) = 0.5529$

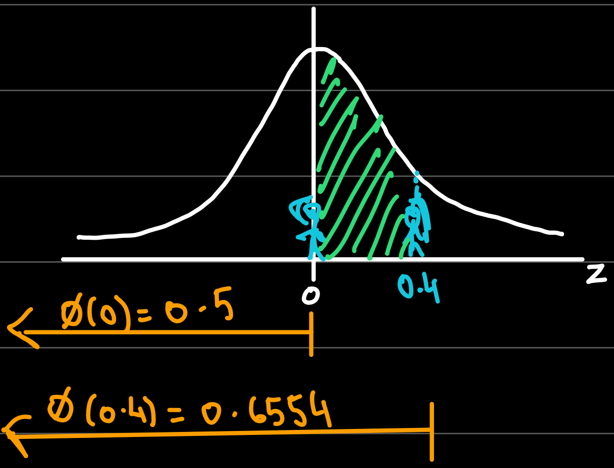
$P(X > 32) = 0.4471$

(ii) $P(24 < X < 30)$

$z = \frac{24-30}{15}$ $\qquad z = \frac{30-30}{15}$

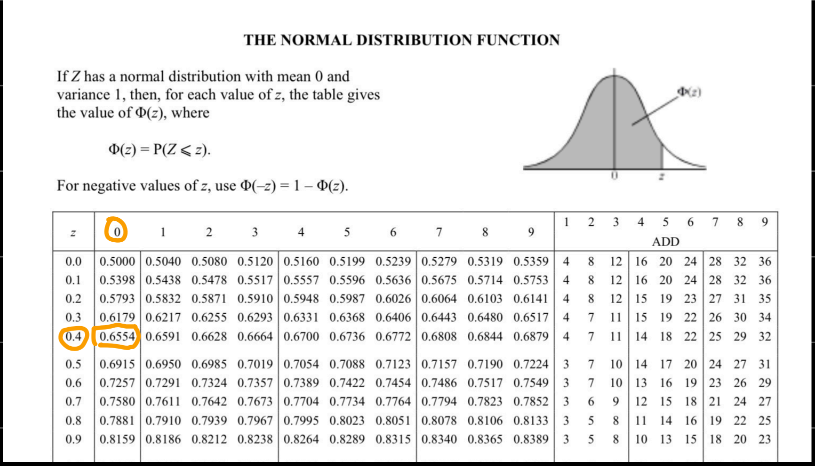
$P(-0.4 < z < 0)$





$\Phi(0) = 0.5$

$\Phi(0.4) = 0.6554$



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0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
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0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
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0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23

$= 0.6554 - 0.5$

$P(24 < x < 30) = 0.1554$

TYPE 2 : REVERSE WORKING

PROBABILITY GIVEN → DIAGRAM & TABLE → Z-value → Formula $z = \frac{x-\mu}{\sigma}$ → Missing values from Data (x, μ, σ)

FIRST STEP IN REVERSE WORKING IS TO DECIDE +/− SIGN OF z. (DRAW MS. S1 FOR HELP.)

1 $P(Z < \boxed{+}) = 0.8531$

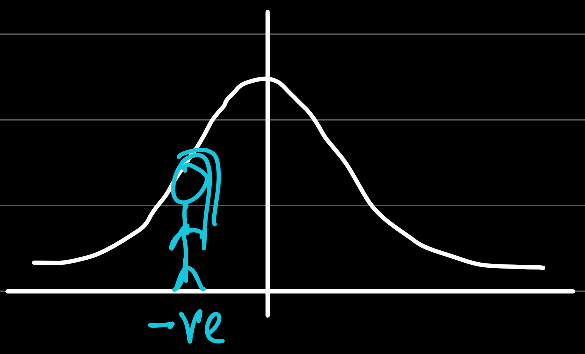
Left

More than half.

2 $P(Z < \boxed{-ve}) = 0.1356$

left

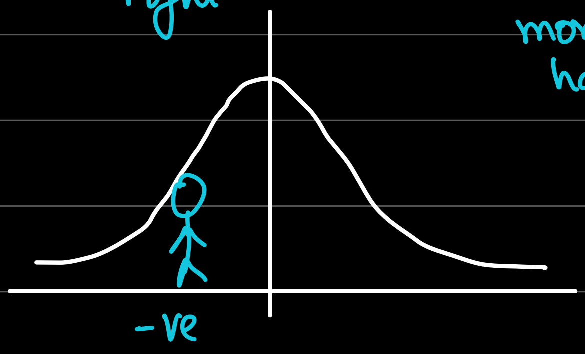
Area is less than half.



$\boxed{3}$ $P(z > \boxed{-ve}) = 0.8713$

right

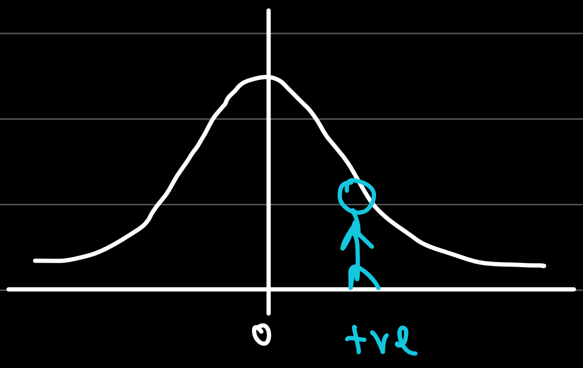
Area is
more than
half .



$\boxed{4}$ $P(z > \boxed{+ve}) = 0.1121$

right

Area is
less than
half

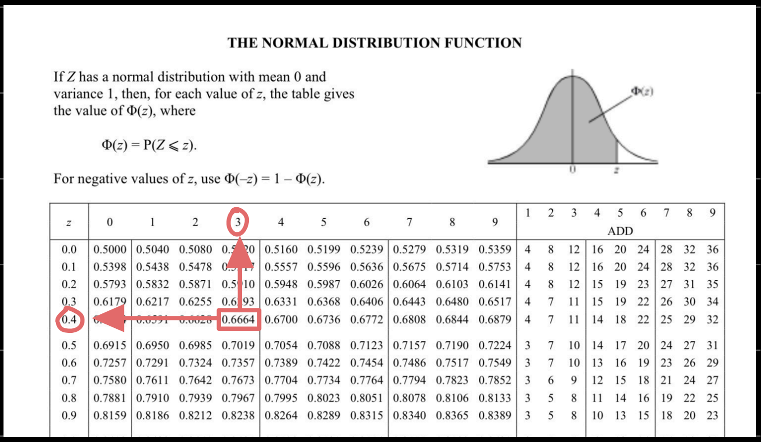


REVERSE WORKING

Q: Find a in all of following:

Mean = 12 SD = 8

(i) $P(X > a) = 0.6664$



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0.4	[illegible]	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
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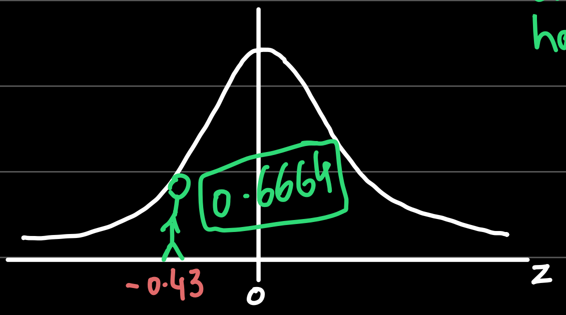
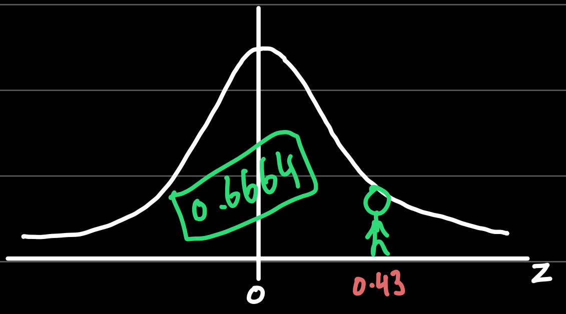
$P(z > \boxed{-0.43}) = 0.6664$

right

area is more than half.



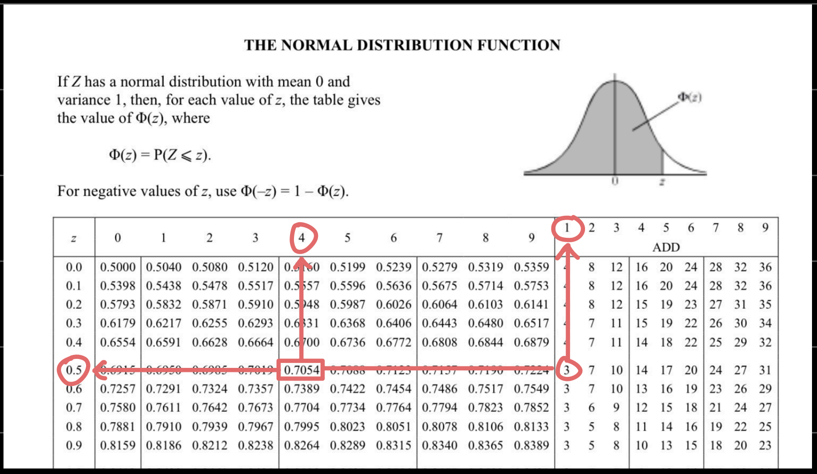


$$z = \frac{X - \mu}{\sigma}$$

$$-0.43 = \frac{a - 12}{8}$$

$$\boxed{a = 8.56}$$

(ii) $P(X < a) = 0.7057$



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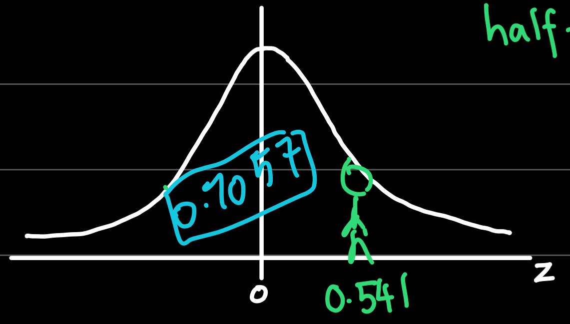
$P(z < \boxed{+0.541}) = 0.7057$

Left

area is more than half.



$\Phi(0.54) = 0.7054$

Third dp: (1) $+ \quad 3$

$\Phi(0.541) = 0.7057$

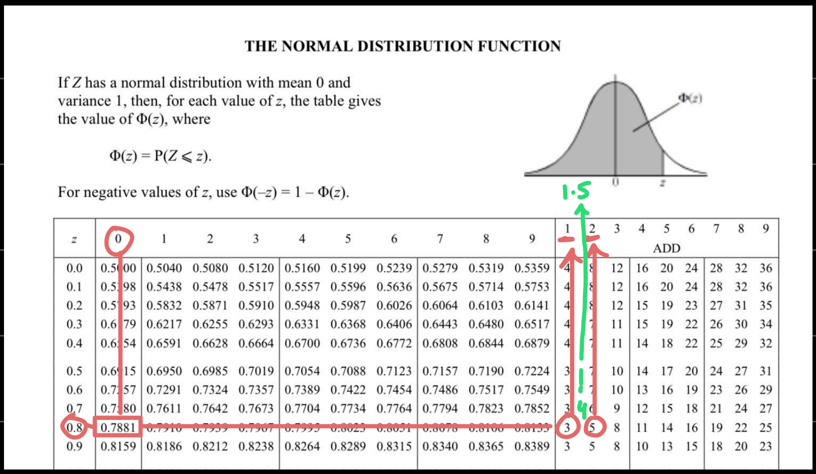
$z = \underline{X - \mu}$

σ

$$+0.541 = \frac{a-12}{8}$$

$$a = \boxed{}$$

(iii) $P(X < a) = 0.7885$



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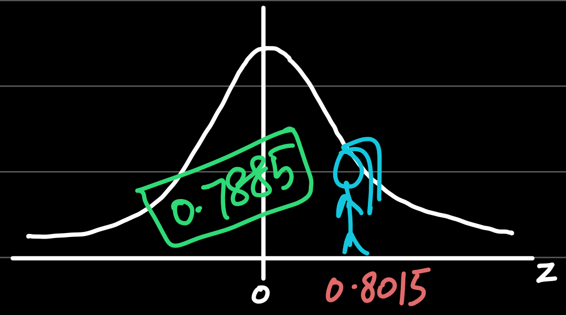
$P(Z < \boxed{+0.8015}) = 0.7885$

Left

area is more than half



$\phi(0.80) = 0.7881$

Third dp: (1.5) $\quad + 4$

$\phi(0.8015) = 0.7885$

$$z = \frac{x-\mu}{\sigma}$$

$$0.8015 = \frac{a-12}{8}$$

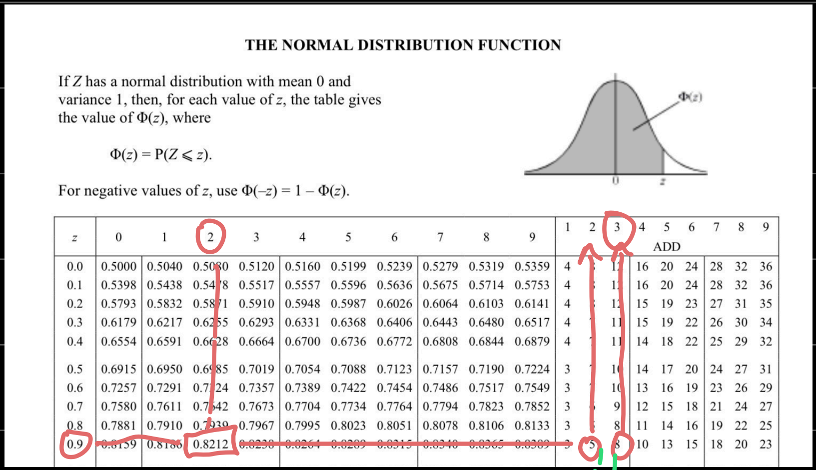
$$a = \boxed{}$$

(iv) $P(X > a) = 0.1781$

$P(Z > \boxed{+0.923}) = 0.1781$

right

area is less than half.



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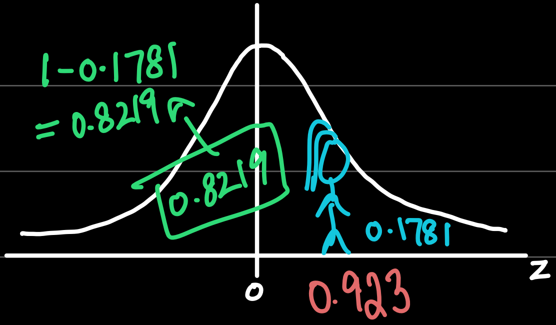
6(7)



If the desired value is not in middle, jump for closest value.

$$z = \frac{x - \mu}{\sigma}$$

$$0.923 = \frac{a - 12}{8}$$

PARAMETERS

SYMBOLS

BINOMIAL	NORMAL.
$X \underset{\text{is distributed}}{\sim} \overset{\text{Binomically}}{B}(\underset{\text{repeats}}{n}, \underset{\text{Prob. of Success}}{p})$	$X \underset{\text{distributed}}{\sim} \overset{\text{Normally}}{N}(\underset{\text{mean}}{\mu}, \underset{\text{variance}}{\sigma^2})$
$X \sim B(12, 0.2)$	$X \sim N(12, 36)$
Binomial Distribution, $n = 12$, $p = 0.2$	Normal Distribution, $\mu = 12$, $\sigma^2 = 36$ $\sigma = 6$ SD = 6

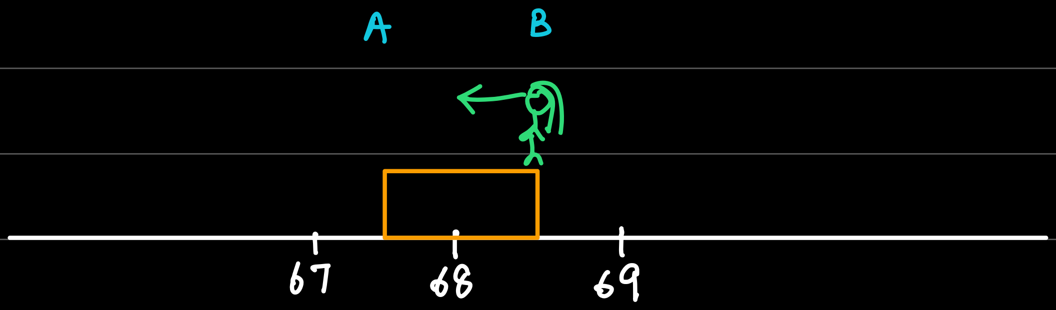
TYPE3 BINOMIAL TO NORMAL APPROXIMATION

(n becomes too large)

BINOMIAL → NORMAL DISTRIBUTION

n, p, q

1. $\mu = np$
2. $SD = \sigma = \sqrt{npq}$
3. Correction of continuation.



BINOMIAL → NORMAL

$$P(X \overset{\text{right}}{>} 68) \longrightarrow P(X > 68.5)$$

$$P(X \overset{\text{right}}{\geq} 68) \longrightarrow P(X > 67.5)$$

$$P(X \overset{\text{left}}{<} 68) \longrightarrow P(X < 67.5)$$

$$P(X \overset{\text{left}}{\leq} 68) \longrightarrow P(X < 68.5)$$

$$P(X = 68) \longrightarrow P(67.5 < X < 68.5)$$

(Discrete) (CONTINUOUS)

IN BINOMIAL, > or ≥ are two different scenarios.

In NORMAL, IT DOES NOT MATTER IF YOU USE > or ≥.

CONDITIONS FOR WHICH THIS APPROXIMATION is JUSTIFIED: $\boxed{np > 5}$ and $\boxed{nq > 5}$

Q. A dice is thrown 140 times. X denotes the random variable for number of times dice lands on a multiple of 3.

$n = 140$, Discrete, $\boxed{p = \frac{2}{6} = \frac{1}{3}}$ $\boxed{q = \frac{4}{6} = \frac{2}{3}}$ Binomial ✓

(i) Find probability that dice lands on multiple of 3 exactly twice.

$$P(X = 2) = {}^{140}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{138} = \boxed{\quad\quad}$$

(ii) Find the probability that dice lands on multiple of 3 more than 68 times.

$$P(X > 68) = P(X=69) + P(X=70) + P(X=71) \ldots\ldots P(X=140)$$

This is where we say 'n has become too large'

Now we use a Normal Approximation for this Question.

$$\boxed{\text{BINOMIAL}} \longrightarrow \boxed{\text{NORMAL}}$$

$n = 140$

$p = 1/3$

$q = 2/3$

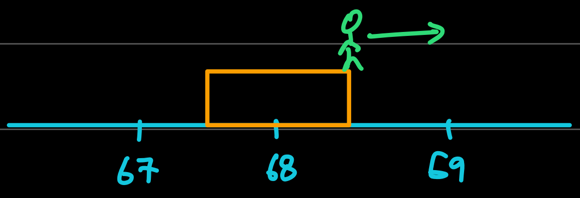
① Mean $= \mu = np = 140\left(\frac{1}{3}\right) = \frac{140}{3}$

② SD $= \sigma = \sqrt{npq} = \sqrt{140 \times \frac{1}{3} \times \frac{2}{3}} = \sqrt{\frac{280}{9}}$

③ Correction of continuation.

$P(X > 68)$

right

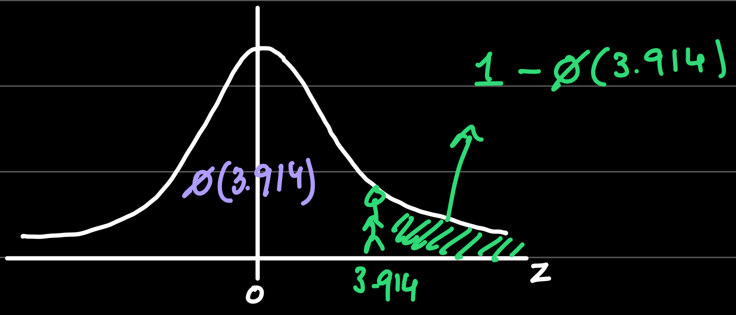


$P(X > 68.5)$

Now this is a simple Forward working (Type 1)

$$Z = \frac{X-\mu}{\sigma} = \frac{68.5 - \frac{140}{3}}{\sqrt{\frac{280}{9}}} = 3.914$$

$$P(Z > 3.914)$$



Go to table and find $\Phi(3.914)$

$$P(X>68) = 1 - \Phi(3.914) =$$

Permutation & Combination (2 classes)

Linear transformation $\Sigma(x+60)\ldots$ }
Box & whisker etc } 1class
Data Representation — OLevel (video) }

4 **(i)** State two conditions which must be satisfied for a situation to be modelled by a binomial distribution. (Repeated)(Discrete)(success/failure)(n, p, q constant). [2]

In a certain village 28% of all cars are made by Ford.

(ii) 14 cars are chosen randomly in this village. Find the probability that fewer than 4 of these cars are made by Ford. [4]

(iii) A random sample of 50 cars in the village is taken. Estimate, using a normal approximation, the probability that more than 18 cars are made by Ford. [4]

$n = 14 \quad , \quad p = 0.28 \qquad q = 0.72$

(ii) $P(X<4) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= {}^{14}C_0 (0.28)^0 (0.72)^{14} + {}^{14}C_1 (0.28)^1 (0.72)^{13} + {}^{14}C_2 (0.28)^2 (0.72)^{12} + {}^{14}C_3 (0.28)^3 (0.72)^{11}$

$= \boxed{\quad\quad}$

(iii) $n = 50 \quad , \quad p = 0.28 \quad , \quad q = 0.72$

$P(X>18) = P(X = 19, 20, 21 \ldots\ldots 50)$

BINOMIAL $\longrightarrow$ NORMAL

$n = 50$

$p = 0.28$

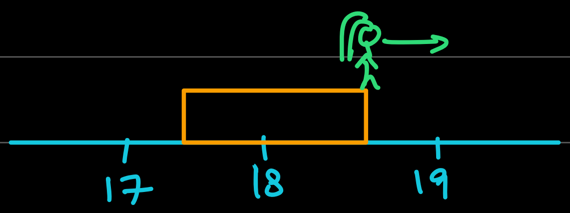
$q = 0.72$

1) Mean $= \mu = np = 50(0.28) = 14$

2) SD $= \sqrt{npq} = \sqrt{50(0.28)(0.72)} = \sqrt{10.08}$

3) Correction of Continuition.

$P(\overset{\text{right}}{X > 18})$



$P(X > 18.5)$

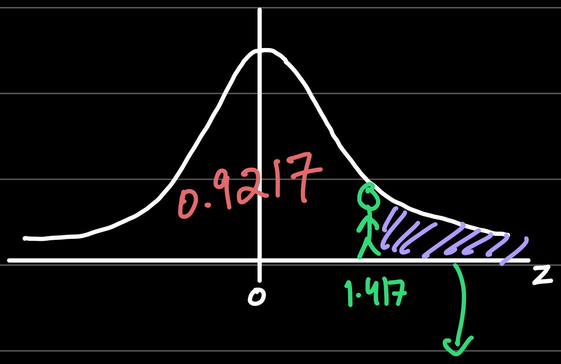
$z = \dfrac{X - \mu}{\sigma} = \dfrac{18.5 - 14}{\sqrt{10.08}} = 1.417$

$P(z > 1.417)$

$\Phi(1.41) = 0.9207$

3dp (Add 10) = $+10$

0.9217



$= 1 - 0.9217$

$= 0.0783$

$P(X > 18) = 0.0783.$