National University of Computer and Emerging Sciences, Lahore Campus

Course:	Numerical Computing	Course Code:	CS2008
Program:	BSCS/BSR/BSSE	Semester:	Fall 2023
Duration:	3 hours	Total Marks:	100
Paper Date:	December 22, 2023	Weight	50%
Section:	All	Page(s):	02
Exam:	Final Term	Roll No:	

Instruction/Notes:	Attempt all questions on the answer book. Don't write anything on a question paper except your name and roll number.

Q1. Let $P_2(x)$ be the interpolating polynomials for the data $(0.5, y), (1, 3),$ and $(2, 2)$. The coefficient of x^2 in $P_2(x)$ is 6. Find y.   **Points (10)**

Q2. In a circuit with impressed voltage $v(t)$ and inductance L, Kirchhoff's first law gives the relationship   **Points (10)**

$$v(t) = L\frac{di}{dt} + R\,i,$$

where R is the resistance in the circuit and i is the current. Suppose we measure the current for several values of t and obtain

T	1.00	1.01	1.02	1.03
i	3.10	3.12	3.14	3.18

Where t is measured in seconds, i is in amperes, the inductance L is a constant 0.98 henries, and the resistance is 0.142 ohms. Approximate the voltage $v(t)$ when $t = 1.03$.
Hint: Choose numerical differentiation formula based on time t when $v(t)$ is required.

Q3. The solid of revolution obtained by rotating the region under the curve $y = g(x)$ over the interval $a \le x \le b$ about the axis has surface area is given by the following formula: **Points (10)**

$$Surface\ Area = \int_a^b 2\pi\, g(x)\, \sqrt{1 + [g'(x)]^2}\ dx$$

1.88663

Approximate the surface area if $g(x) = e^{-x}$ for $0 \le x \le 1$, using composite Trapezoidal rule with $h = 0.2$.
Note: Throughout the computations, use at least five decimal approximations.

Q4. Using Newton Raphson's method, establish the formula $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)$ to calculate the cube root of N. Hence find cube root of 15 correct to three decimals places.   **Points (10)**

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Q5. Solve the following linear system using LU decomposition algorithm with $l_{ii} = 1$ for all i. Points (10)

$$4x + y - z = 5$$
$$-x + 3y + z = -4$$
$$2x + 2y + 5z = 1$$

x_1 $1.45955,$
y_2 $0.8485.$
3_2 $-0.0103,$

Q6. The concentration of salt y in a homemade soap maker is given as a function of time t by

$$\frac{dy}{dt} = 1 - 0.5\,y - 0.1\,y\,t, \qquad 4.0386$$

at the initial time $t = 0$, the salt concentration in the tank is 5g/L. Use Picard method to find the thid approximate solution. Also, find residual function in this case. Moreover, compute salt concentration and corresponding residual errors at
$t = 0.25, 0.5$ and 0.75?   Points (15)

Q7. Use obtained solutions from Q6 evaluate $y(1)$ through Milne's Predictor-Corrector scheme. Points (10)
where

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n) \qquad 3.61537$$

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1})$$

Q8. Transform the following second order IVP   Points (15)

$$y'' - 2y' + 2y - e^{2x}\,Sinx, \quad with\ y(0) = -0.4,\ y'(0) = -0.6$$

in to a system of first order IVPs and solve the obtain system at $x = 0.2$ using Runge-Kutta method of order 4 (RK4).
$-1.157331,$

Q9. Use the finite difference algorithm, solve the following BVP   Points (10)

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0, \quad with\ y(0) = 0,\ y(1) = 1.$$

Divide the domain interval in to five subintervals.