

(1)

Q#1

$$x_0 = 0, x_1 = 10, x_2 = 15, x_3 = 20, x_4 = 35, x_5 = 50$$

$$y_0 = 0, y_1 = 227.04, y_2 = 362.78, y_3 = 517.35, y_4 = 602.97, y_5 = 901.67$$

$$\begin{matrix} 0 & D_{00} \\ 0 & 0 \end{matrix}$$

$$10 \quad 227.04 \quad 22.704$$

$$15 \quad 362.78 \quad 27.148 \quad D_{22}$$

$$20 \quad 517.35 \quad 30.914 \quad 0.3766 \quad D_{33}$$

$$35 \quad 602.97 \quad 5.708 \quad -1.2603 \quad -0.065476 \quad -0.001985505$$

$$50 \quad 901.67 \quad 19.913 \quad 0.47351111 \quad 0.04953746 \quad 0.00287534 \quad 0.000097217$$

 D_{44} D_{55}

$$y = D_{00} + (x - x_0) D_{11} + (x - x_0)(x - x_1) D_{22} + (x - x_0)(x - x_1)(x - x_2) D_{33} + (x - x_0)(x - x_1)(x - x_2)(x - x_3) D_{44} + (x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4) D_{55}$$

$$y = 0 + (60 - 0) 22.704 + (60 - 0)(60 - 10) 0.29626667 + (60 - 0)(60 - 10)(60 - 15) 0.004016667 + (60 - 0)(60 - 10)(60 - 15)(60 - 20) (-0.001985505) + (60 - 0)(60 - 10)(60 - 15)(60 - 20)(60 - 35) 0.000097217$$

$$y = 1362.24 + 888.80001 + 542.250045 - 10721.721 + 13124.295$$

$$= 5195.864055$$

(2)

$$\begin{aligned}
 y = & 22.704x + (x^2 - 10x) 0.29626667 \\
 & + (x^3 - 25x^2 + 150x) 0.004016667 \\
 & + (x^4 - 45x^3 + 650x^2 - 3000x) (-0.001985505) \\
 & + (x^5 - 80x^4 + 2225x^3 - 25750x^2 + 105000x) 0.000097217
 \end{aligned}$$

$$\begin{aligned}
 \frac{dy}{dx} = & 22.704 + (2x - 10) 0.29626667 \\
 & + (3x^2 - 50x + 150) 0.004016667 \\
 & + (4x^3 - 135x^2 + 1300x - 3000) (-0.001985505) \\
 & + (5x^4 - 320x^3 + 6675x^2 - 51500x + 105000) 0.000097217
 \end{aligned}$$

$$\left. \frac{dy}{dx} \right|_{x=15} = 22.704 + 5.925333 + 0.301250025 + 0.744564375 + 0.7291275$$

$$= 30.4042753$$

Q#2

(3)

$$I = \int_0^1 x^3 \sqrt{1+9x^4}$$

$$R_{11} = \frac{h}{3} [f(x_0) + 4f(x_1) + f(x_2)]$$

$$= \frac{0.5}{3} [f(0) + 4f(0.5) + f(1)]$$

$$= \frac{0.5}{3} [0 + 4 * 0.15625 + 3.16227766]$$

$$= 0.6312129433$$

$$A = 2\pi * R_{11} = 3.966027891$$

$$R_{21} = \frac{h}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

$$= \frac{0.25}{3} [f(0) + 4f(0.25) + 2f(0.5) + 4f(0.75) + f(1)]$$

$$= \frac{0.25}{3} [0 + 4 * 0.01589728574 + 2 * 0.15625 + 4 * 0.8275265241 + 3.16227766]$$

$$= 0.5707060749$$

$$A = 2\pi * R_{21} = 3.585852025$$

$$R_{22} = R_{21} + \frac{(R_{21} - R_{11})}{4 - 1}$$

$$= 0.5707060749 + \frac{(0.5707060749 - 0.6312129433)}{3}$$

$$= 0.5505371188$$

Q#3

(4)

$$f(\omega) = \tan\left(\frac{\omega * 0.5}{4}\right) - \omega * 0.01 * 4 = 0$$

$$f(\omega) = \tan\left(\frac{\omega}{8}\right) - \frac{\omega}{25} - 1 = 0$$

$$f(7) = -0.08257837077$$

$$f(8) = 0.2374077247$$

Interval is $[7, 8]$

Let $P_0 = 7.5$

$$f'(\omega) = \sec^2\left(\frac{\omega}{8}\right) \cdot \frac{1}{8} - \frac{1}{25}$$

$$P_i = P_{i-1} - \frac{f(P_{i-1})}{f'(P_{i-1})}$$

$$P_i = P_{i-1} - \frac{\tan(0.125P_{i-1}) - 0.04P_{i-1} - 1}{0.125 \sec^2(0.125P_{i-1}) - 0.04}$$

$$\begin{aligned} i=1 & \quad P_1 = 7.5 - \frac{\tan(0.125 * 7.5) - 0.04 * 7.5}{0.125 \sec^2(0.125 * 7.5) - 0.04} \\ & \quad P_1 = 7.304130655 \end{aligned}$$

$$i=2 \quad P_2 = 7.296512831$$

$$i=3 \quad P_3 = 7.296502152$$

(5)

Q#4

$$|A_1| = 1 > 0$$

$$|A_2| = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1 > 0$$

$$|A_3| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$\begin{aligned} &= 1[4-1] - 1[2+1] + 1[-1-2] \\ &= 3 - 3 - 3 \\ &= -3 < 0 \end{aligned}$$

A is not positive definite.

System can not be solved using Cholesky Method.

(6)

STEP 1: Find L and L^T from $A = L * L^T$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} l_{00} & 0 & 0 \\ l_{10} & l_{11} & 0 \\ l_{20} & l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} l_{00} & l_{10} & l_{20} \\ 0 & l_{11} & l_{21} \\ 0 & 0 & l_{22} \end{bmatrix}$$

$$= \begin{bmatrix} l_{00}^2 & l_{00}l_{10} & l_{00}l_{20} \\ l_{10}l_{00} & l_{10}^2 + l_{11}^2 & l_{10}l_{20} + l_{11}l_{21} \\ l_{20}l_{00} & l_{20}l_{10} + l_{21}l_{11} & l_{20}^2 + l_{21}^2 + l_{22}^2 \end{bmatrix}$$

$$l_{00}^2 = 1$$

$$l_{00} = \pm 1$$

$$l_{10}l_{00} = 1$$

$$l_{10} = \frac{1}{l_{00}}$$

$$= \pm 1$$

$$l_{10}^2 + l_{11}^2 = 2$$

$$l_{11} = \sqrt{2 - l_{10}^2}$$

$$= \sqrt{2 - 1}$$

$$= \pm 1$$

$$l_{20}l_{00} = 1$$

$$l_{20} = \frac{1}{l_{00}}$$

$$= 1$$

$$l_{20}l_{10} + l_{21}l_{11} = -1$$

$$l_{21} = \frac{-1 - l_{20}l_{10}}{l_{11}}$$

$$l_{21} = \frac{-1 - 1 \times 1}{1}$$

$$= -2$$

$$l_{20}^2 + l_{21}^2 + l_{22}^2 = 2$$

$$l_{22} = \sqrt{2 - l_{20}^2 - l_{21}^2}$$

$$= \sqrt{2 - 1 - 4}$$

$$= \sqrt{-3}$$

Q#5

(7)

$$\frac{dP}{dt} = 0.02P + 0.1 \quad , \quad P(0) = 10$$

$$P_i = P_0 + \int_{t_0}^t f(t, P_{i-1}) dt$$

$$P_i = 10 + \int_{t_0}^t f(t, P_{i-1}) dt$$

$$P_i = 10 + \int_0^t f(t, P_0) dt$$

$$= 10 + \int_0^t (0.02 P_0 + 0.1) dt$$

$$= 10 + \int_0^t (0.02 \times 10 + 0.1) dt$$

$$= 10 + 0.3t$$

$$P_2 = 10 + \int_0^t f(t, P_1) dt$$

$$= 10 + \int_0^t (0.02 P_1 + 0.1) dt$$

$$= 10 + \int_0^t (0.02 (10 + 0.3t) + 0.1) dt$$

$$= 10 + \int_0^t (0.2 + 0.006t + 0.1) dt$$

$$= 10 + \int_0^t (0.3 + 0.006t) dt$$

$$P_2 = 10 + 0.3t + 0.003t^2$$

(8)

i=3

$$P_3 = 10 + \int_0^t f(t, P_2) dt$$

$$= 10 + \int_0^t [0.02(10 + 0.3t + 0.003t^2) + 0.1] dt$$

$$= 10 + \int_0^t (0.3 + 0.006t + 0.00006t^2) dt$$

$$= 10 + 0.3t + 0.003t^2 + 0.00002t^3$$

i=4

$$P_4 = 10 + \int_0^t f(t, P_3) dt$$

$$= 10 + \int_0^t [0.02(10 + 0.3t + 0.003t^2 + 0.00002t^3) + 0.1] dt$$

$$= 10 + \int_0^t [0.3 + 0.006t + 0.00006t^2 + 0.0000004t^3] dt$$

$$= 10 + 0.3t + 0.003t^2 + 0.00002t^3 + 0.0000001t^4$$

i=5

$$P_5 = 10 + \int_0^t [0.02(10 + 0.3t + 0.003t^2 + 0.00002t^3 + 0.0000001t^4) + 0.1] dt$$

$$= 10 + \int_0^t [0.3 + 0.006t + 0.00006t^2 + 0.0000004t^3 + 0.00000002t^4] dt$$

$$= 10 + 0.3t + 0.003t^2 + 0.00002t^3 + 0.0000001t^4 + 0.000000004t^5$$

at $t = 3$

(9)

at $t=5$

$$P_1 = 11.5$$

$$P_2 = 11.575$$

$$P_3 = 11.5775$$

$$P_4 = 11.5775625$$

$$P_5 = 11.57756375$$

(10)

Q#6

$$\frac{dI}{dt} = (0.4IU - 0.1I), \quad I(0) = 0$$

$$\frac{dU}{dt} = -0.4IU \quad U(0) = 9$$

RK1: $h=0.5$

$$\begin{cases} f(t, I, U) = 0.4IU - 0.1I \\ g(t, I, U) = -0.4IU \end{cases}$$

$$\begin{cases} t_{i+1} = t_i + h \\ I_{i+1} = I_i + h f(t_i, I_i, U_i) \\ U_{i+1} = U_i + h g(t_i, I_i, U_i) \end{cases}$$

$$\begin{aligned} t_0 &= 0, I_0 = 0, U_0 = 9 \\ t_1 &= 0.5, I_1 = 0, U_1 = 9 \\ t_2 &= 1, I_2 = 0, U_2 = 9 \end{aligned}$$

$$t_1 = t_0 + h = 0 + 0.5 = 0.5$$

$$\begin{aligned} I_1 &= I_0 + h f(t_0, I_0, U_0) \\ &= 0 + 0.5 f(0, 0, 9) \\ &= 0.5 [0.4(0)(9) - 0.1(0)] \\ &= 0 \end{aligned}$$

$$\begin{aligned} U_1 &= U_0 + 0.5 g(t_0, I_0, U_0) \\ &= 9 + 0.5 [-0.4(0)(9)] \\ &= 9 \end{aligned}$$

(11)

$$t_2 = t_1 + h = 0.5 + 0.5 = 1$$

$$\underline{i=1} \quad I_2 = I_1 + h f(t_1, I_1, U_1)$$

$$= 0 + 0.5 f(0.5, 0, 9)$$

$$= 0.5 [0.4(0)(9) - 0.1(0)]$$

$$= 0$$

$$U_2 = U_1 + h g(t_1, I_1, U_1)$$

$$= 9 + 0.5 g(0.5, 0, 9)$$

$$= 9 + 0.5 [-0.4(0)(9)]$$

$$= 9$$

RK2

$$for \quad i=0, 1$$

$$t_{i+1} = t_i + h$$

$$k_1 = h f(t_i, I_i, U_i), \quad l_1 = h g(t_i, I_i, U_i)$$

$$k_2 = h f(t_i + h, I_i + k_1, U_i + l_1), \quad I_2 = h g(t_i + h, I_i + k_1, U_i + l_1)$$

$$\rightarrow I_{i+1} = I_i + 0.5 [k_1 + k_2], \quad U_{i+1} = U_i + 0.5 [l_1 + l_2]$$

$$\underline{i=0} \quad t_1 = t_0 + h = 0 + 0.5 = 0.5$$

$$\begin{aligned} K_1 &= h f(t_0, I_0, U_0) \\ &= 0.5 f(0, 0, 9) \\ &= 0.5 [0.4(0)(0) - 0.1(0)] \\ &= 0 \end{aligned}$$

$$\begin{aligned} \ell_1 &= h g(t_0, I_0, U_0) \\ &= 0.5 g(0, 0, 9) \\ &= 0.5 [-0.4(0)(9)] \\ &= 0 \end{aligned}$$

$$\begin{aligned} K_2 &= h f(t_0 + h, I_0 + K_1, U_0 + \ell_1) \\ &= 0.5 f(0 + 0.5, 0 + 0, 9 + 0) \\ &= 0.5 f(0.5, 0, 9) \\ &= 0.5 [0.4(0)9 - 0.1(0)] \\ &= 0 \end{aligned}$$

$$\begin{aligned} \ell_2 &= h g(t_0 + h, I_0 + K_1, U_0 + \ell_1) \\ &= h(0.5, 0, 9) \\ &= -0.4(0)(9) \\ &= 0 \end{aligned}$$

$$\begin{aligned} I_1 &= I_0 + 0.5 [K_1 + K_2], \quad U_1 = U_0 + 0.5 [\ell_1 + \ell_2] \\ &= 0 \quad \quad \quad = 9 \end{aligned}$$

$$\underline{i=1} \quad t_2 = t_1 + h = 0.5 + 0.5 = 1$$

$$\begin{aligned}
 K_1 &= h f(t_1, I_1, U_1) \\
 &= 0.5 f(0.5, 0, 9) \\
 &= 0.5 [0.4(0)9 - 0.1(0)] \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 L_1 &= h g(0.5, 0, 9) \\
 &= 0.5 [-0.4(0)(9)] \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 K_2 &= h f(t_1 + h, I_1 + K_1, U_1 + L_1) \\
 &= 0.5 f(0.5 + 0.5, 0 + 0, 9 + 0) \\
 &= 0.5 f(1, 0, 9) \\
 &= 0.5 [-0.4(0)(9) - 0.1(0)] \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 L_2 &= h g(1, 0, 9) \\
 &= 0.5 [-0.4(0)(9)] \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 I_2 &= I_1 + 0.5 * [K_1 + K_2], \quad U_2 = U_1 + 0.5 [L_1 + L_2] \\
 &= 0 \qquad \qquad \qquad = 9
 \end{aligned}$$

(14)

Q4.7

$$\frac{d^2u}{dx^2} + \frac{2}{\gamma} \left(\frac{du}{dx} \right) = 0 \quad u(1) = 10 \\ u(5) = 0$$

$$\gamma_0 = 1, \gamma_n = 5 \quad n = 4 \quad u_0 = 10, \quad u_n = 0$$

$$h = \frac{\gamma_4 - \gamma_0}{n} = \frac{5 - 1}{4} = 1$$

$\gamma_0 = 1$	$u_0 = 10$
$\gamma_1 = 2$	$u_1 = ?$
$\gamma_2 = 3$	$u_2 = ?$
$\gamma_3 = 4$	$u_3 = ?$
$\gamma_4 = 5$	$u_4 = 0$

$$\frac{w_{i-1} - 2w_i + w_{i+1}}{h^2} + \frac{2}{\gamma_i} \left(\frac{w_{i+1} - w_{i-1}}{2h} \right) = 0$$

$$\frac{w_{i-1}}{h^2} - \frac{2}{h^2} w_i + \frac{w_{i+1}}{h^2} + \frac{1}{\gamma_i h} w_{i+1} - \frac{1}{\gamma_i h} w_{i-1} = 0$$

$$\left(\frac{1}{h^2} - \frac{1}{\gamma_i h} \right) w_{i-1} - \frac{2}{h^2} w_i + \left(\frac{1}{h^2} + \frac{1}{\gamma_i h} \right) w_{i+1} = 0$$

$$\left(1 - \frac{1}{\gamma_i} \right) w_{i-1} - 2 w_i + \left(1 + \frac{1}{\gamma_i} \right) w_{i+1} = 0$$

i=1

$$\left(1 - \frac{1}{\gamma_1}\right)w_0 - 2w_1 + \left(1 + \frac{1}{\gamma_1}\right)w_2 = 0$$

$$\gamma_1 = 2 \text{ and } w_0 = 10$$

$$5 - 2w_1 + 1.5w_2 = 0$$

$$-2w_1 + 1.5w_2 = -5 \quad - \textcircled{1}$$

i=2

$$\left(1 - \frac{1}{\gamma_2}\right)w_1 - 2w_2 + \left(1 + \frac{1}{\gamma_2}\right)w_3 = 0$$

$$\gamma_2 = 3$$

$$\frac{2}{3}w_1 - 2w_2 + \frac{4}{3}w_3 = 0 \quad - \textcircled{2}$$

$$\rightarrow 2w_1 - 6w_2 + 4w_3 = 0 \quad - \textcircled{2}$$

$$\left(1 - \frac{1}{\gamma_3}\right)w_2 - 2w_3 + \left(1 + \frac{1}{\gamma_3}\right)w_4 = 0$$

$$\gamma_3 = 4, w_4 = 0$$

$$0.75w_2 - 2w_3 + 1.25(0) = 0$$

$$0.75w_2 - 2w_3 = 0 \quad - \textcircled{3}$$

(16)

Equations ①, ② and ③ constitute a tridiagonal system.

$$-2w_1 + 1.5w_2 = -5$$

$$2w_1 - 6w_2 + 4w_3 = 0$$

$$0.75w_2 - 2w_3 = 0$$

$$\left[\begin{array}{ccc|c} a_0 = -2 & d_0 = 1.5 & & b_0 = -5 \\ c_0 = 2 & a_1 = -6 & d_1 = 4 & b_1 = 0 \\ c_1 = 0.75 & a_2 = -2 & & b_2 = 0 \end{array} \right]$$

$$\begin{aligned} l_0 &= a_0 \\ &= -2 \end{aligned}$$

$$i=1 \quad l_1 = a_1 - \frac{c_0 d_0}{l_0} = -6 - \frac{-2 * 1.5}{-2} = -4.5$$

$$i=2 \quad l_2 = a_2 - \frac{c_1 d_1}{l_1} = -2 - \frac{0.75 * 4}{-4.5} = -\frac{4}{3}$$

$$y_0 = b_0/l_0 = \frac{-5}{-2} = 2.5$$

$$i=1 \quad y_1 = \frac{b_1 - c_0 y_0}{l_1} = \frac{0 - 2 * 2.5}{-4.5} = \frac{10}{9}$$

$$y_2 = \frac{b_2 - c_1 y_1}{l_2} = \frac{0 - 0.75 * \frac{10}{9}}{-\frac{4}{3}} = \frac{15}{8} = 0.625$$

(17)

$$x_2 = y_2$$

$$= 0.625$$

$$\stackrel{i=1}{=} x_1 = y_1 - \frac{d_1 x_2}{\ell_1} = \frac{10}{9} - \frac{4 * 0.625}{-4.5} = \frac{5}{3}$$

$$\stackrel{i=0}{=} x_0 = y_0 - \frac{d_0 x_1}{\ell_0} = 2.5 - \frac{1.5 * \frac{5}{3}}{-2} = \frac{15}{4} = 3.75$$

$$\gamma_0 = 1 \quad u_0 = 10$$

$$\gamma_1 = 2 \quad u_1 = 15/4$$

$$\gamma_2 = 3 \quad u_2 = 5/3$$

$$\gamma_3 = 4 \quad u_3 = 5/8$$

$$\gamma_4 = 5 \quad u_4 = 0$$