National University of Computer and Emerging Sciences, Lahore Campus

Course:	Probability and Statistics	Course Code:	MT-206
Program:	BS(Computer Science)	Semester:	Fall 2016
Duration:	3 Hours	Total Marks:	50
Paper Date:	19-12-2016	Weight	50%
Section:	ALL	Page(s):	2
Exam:	Final	Roll No:	L13-4088

Instruction/Notes: **Attempt all questions.**

Question # 1 The probability that a man will alive in 25 years is 3/5 and the probability that his wife will be alive in 25 years is 2/3. Find the probability that (6)

(a) Both will be alive;
(b) Only the man will be alive;
(c) At least one will be alive.

Question # 2 If the continuous random variable X has a probability distribution function; (6)

$$f(x) = \begin{cases} \frac{3}{4}(3-x)(x-5) & 3 \le x \le 5 \\ 0 & elsewhere \end{cases}$$

(a) Calculate the mean and standard deviation;
(b) Find the cumulative distribution function $F(x)$.

Question # 3 In testing a certain kind of truck tire over a rugged terrain it is found that 20% of the trucks fail to complete the test run without a blowout. Of the next 20 trucks tested find the probability that exactly 4 trucks have blowouts. (4)

Question # 4 Suppose that a random system of police patrol is devised so that a patrol officer may visit a given beat location Y = 0, 1, 2, 3,… times per half-hour period, with each location being visited an average of once per time period. Assume that Y possesses, approximately, a Poisson probability distribution. Calculate the probability that the patrol officer will miss a given location during a half-hour period. What is the probability that it will be visited once? Twice? At least once? (8)

Question # 5 Wires manufactured for use in a computer system are specified to have resistances between 0.12 and 0.14 ohms. The actual measured resistances of the wires produced by company A have a normal probability distribution with mean 0.13 ohm and standard deviation 0.005 ohm.

(a)What is the probability that a randomly selected wire from company A's production will meet the specifications?

(b) If four of these wires are used in each computer system and all are selected from company Ã, what is the probability that all four in a randomly selected system will meet the specifications? (6)

Question # 6 If a certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. What is the probability that a random sample of 36 of these resistors will have a mean resistance;

(a) of more than 40.5 ohms;
(b) At least 39 ohms.

(4)

Question # 7 To reach maximum efficiency in performing an assembly operation in a manufacturing plant, new employees require approximately a 1-month training period. A new method of training was suggested, and a test was conducted to compare the new method with the standard procedure. Two groups of nine new employees each were trained for a period of 3 weeks, one group using the new method and the other following the standard training procedure. The length of time (in minutes) required for each employee to assemble the device was recorded at the end of the 3-week period. The resulting measurements are as shown in Table below.

Procedure	Measurements								
Standard	32	37	35	28	41	44	35	31	34
New	35	31	29	25	34	40	27	32	31

(a) Estimate the true mean difference $(\mu_1 - \mu_2)$ with confidence coefficient .95. Assume that the assembly times are approximately normally distributed, that the variances of the assembly times are approximately equal for the two methods, and that the samples are independent.
(b) Test the hypothesis at the 0.01 level of significance that new procedure will increase the efficiency.

(10)

Question # 8 A machine puts out 16 imperfect articles in a sample of 500. After the machine is overhauled, it puts 3 imperfect articles in a batch of 100. Has the machine been improved? Use 5% level of significance. Use p- value for your conclusion and also find 99% confidence interval for the difference between population proportion.

(6)

National University of Computer and Emerging Sciences, Lahore Campus				
	Course:	**Probability & Stats**	**Course Code:**	**MT205**
	Program:	**BS CS**	**Semester:**	**Spring 20**
	Duration:	**3 hours**	**Total Marks:**	**130**
	Paper Date:	**July 06; 2020**	**Weight**	**50%**
	Section:	**All**	**Page(s):**	**03**
	Exam:	**Final Term**	**Paper Time**	**9:00am - 12:00noon**
Instruction/Notes:	(i) Attempt All Questions. (ii) Solve the following problems and upload your handwritten solutions to Google class room as a single PDF within given time. (iii) Handwritten solutions should be scanned using camscanner app. Ensure the scans are taken on level surface and with proper lighting. The solution PDF file should be named as FirstName-LastName_18L-1234. (iv) Upload your PDF solution file till 12:15pm (15 minutes for scanning and uploading)			

Q1. **(a)** In the manufacture of a certain scientific instrument great importance is attached to the life of particular critical components. This component is obtained in bulk from source ***A*** and in the course of inspection, the lives of 1150 of the components from source ***A*** are determined. The following frequency table is obtained: **Points (10)**

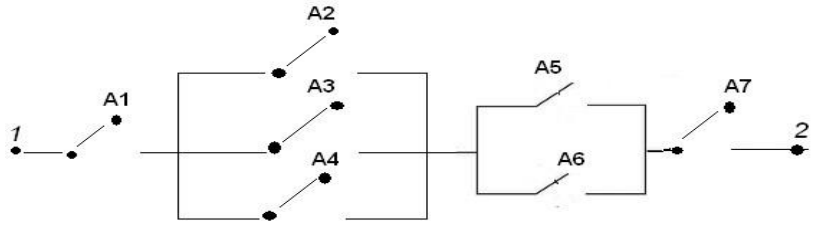
Life in hours	No. of components
1000 - 1020	40
1020 - 1040	96
1040 - 1060	364
1060 - 1080	372
1080 - 1100	85
1100 - 1120	76
1120 - 1140	65
1140 - 1160	52

Find Arithmetic Mean, Median and Mode.

(b) Use method of least square to fit an equation of the form $y = ab^x$ to the following data in which y represents the number of bacteria per unit volume existing in a culture at the end of x hours. **Points (10)**

x	0	1	2	3	4	5
y	73	91	112	131	162	250

Q2. **(a)** In the circuit, given below, the switches open and close independently and randomly. The probability that the particular switch is closed is "0.65" which is same for all the switches. Find the probability of finding closed path from 1 to 2. **Points (10)**



Hint: Use addition law for two and three events. Also use the definition of independent events.

(b) In an experiment, *A, B, C* and *D* are events with probabilities **Points (10)**
$P(A \cup B) = \frac{5}{8}$, $P(A) = \frac{3}{8}$, $P(C \cap D) = \frac{1}{3}$ and $P(C) = \frac{1}{2}$. Furthermore, *A* and *B* are disjoint while *C* and *D* are independent. Answer the following questions with proper mathematical justification:

(i) What is $P(B)$? (ii) What is $P(A \cap B^c)$? (iii) What is $P(A \cup B^c)$?
(iv) Are A and B independent? (v) What is $P(D)$? (vi) What is $P(C \cup D)$?
(vii) What is $P(C|D)$? (viii) What is $P(C \cap D^c)$? (ix) What is $P(C \cup D^c)$?
(x) What is $P(C^c \cap D^c)$?

(c) A machine produces photo detectors in pairs. Test show that the first photo detector is acceptable with probability $\frac{2}{5}$. When the first photo detector is acceptable, the second photo detector is acceptable with probability $\frac{4}{5}$. Otherwise, if the first photo detector is defective, the second photo detector is acceptable with probability $\frac{3}{5}$. **Points (05)**

(i) Draw a tree diagram for the experiment.
(ii) What is the probability that exactly one photo detector of a pair is acceptable?
(iii) What is the probability that both photo detectors in a pair are defective?

(d) Dr. James has been teaching basic statistics for many years. He knows that 90% of the students will completed the assigned problems. He has also determined that among those who do their assignments, 95% will pass the course. Among those students who do not do their assignments, 60% will pass. Peter took statistics in last semester with Dr. James and received a passing grade. What is the probability that he completed the assignments? **Points (05)**

Q**3.** **(a)** Each time a modem transmits one bit, the receiving modem analyzes the signal that arrives and decides whether the transmitted bit is 0 or 1. It makes an error with probability *p,* independent of whether any other bit is received correctly. Answer the following questions with justification (reason). **Points (10)**

(i) If the transmission continuous until the receiving modem makes its first error, what is the Probability Mass Function (*PMF*) of *X*, the number of bits transmitted?
(ii) If the probability of error is $p = 0.1$, what is the probability that $X \geq 6$?
(iii) If the modem transmits 100 bits, what is the *PMF* of *Y*, the number of errors?
(iv) If the probability of error is $p = 0.01$ and the modem transmits 200 bits, what is the probability that $Y \leq 3$?

(b) The painted light bulbs produced by a company are 60% white, 25% blue and 15% green. In a sample of 10 bulbs, find the probability that 5 are white, 2 are green and 3 are blue. **Points (5)**

(c) Telephone calls are being placed through a certain exchange at random times on the average of ten per minute. Assuming a Poisson random variable, determine the probability that in a 12 second interval, there are 3 or more calls. **Point (5)**

Q4. **(a)** The density function of a random variable *X* is given by **Points (10)**

$$f(x) = \begin{cases} a + bx^2, & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

If $E[X] = \frac{3}{5}$, find *a* and *b*?

(b) Find moment generating function $M_X(t)$ of the random variables having following probability functions: **Points (10)**

$$P_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(c) The random vector (X, Y) has a joint PDF **Points (10)**

$$f(x,y) = \begin{pmatrix} 2e^{-x}e^{-2y}, & x>0,\ y>0 \\ 0, & otherwise \end{pmatrix}$$

Find the following:

(i) Marginal Probability Density Function (PDF) of *X*.

(ii) Find Expected value of *X* and *XY*.

(d) Let *X* have zero mean and unit variance, and put *Y = 3X*. Find the following: **Points (10)**

(i) Correlation between *X* and *Y*. i.e. $E(XY)$

(ii) Covariance $Cov(X,Y) = \sigma_{XY}$

(iii) Coefficient of correlation ρ_{XY}

(iv) Make conclusions on the basis of obtained results.

Q5. **(a)** Let $Y = 5X - 10$, and *X* be normally distributed with mean 10 and variance 25. Find $P(Y \geq 68)$. **Points (10)**

Hint: Use basic properties of mean and variance to proceed further.

(b) A process is in control when the average amount of instant coffee that is packed in a jar, is 6 oz. The standard deviation is 0.2 oz. A sample of 100 jars is selected at random and the sample average is found to be 6.1 oz. Is the process out of control? **Points (05)**

(c) A random sample of size 20, from a normal population, has mean 182 and standard deviation is 2.3. Test the following hypothesis at 5% significance level. **Points (05)**

H_0: $\mu \leq 181$

H_1: $\mu > 181$

National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Probability & Statistics	Course Code:	MT2005/2009
Degree Program:	BS CS/SE	Semester:	Spring 2022
Exam Duration:	3 Hours	Total Marks:	100
Paper Date:	16-06-2022	Weight	50
Section:	ALL	Page(s):	6
Exam Type:	Final Term	Time:	1:00pm - 4:00pm

Student : Name: _____ Roll No. _____ Section: _____

Instruction/Notes:
1. Attempt all the questions on the answer book and show proper working.
2. Use of Scientific calculator is allowed but Exchange of calculators or use of programmable calculators is not allowed.
3. Students are not allowed to write anything on the question paper except roll number.
4. If you have any ambiguity in the data then do not ask anything from invigilator, just make an assumption and continue your paper.

Q 1.

(a) **Points (05)**

The following distribution shows Kilowatt-Hours of Electricity used in one month by 62 residential consumers in a certain locality of Lahore:

Consumption in KWH	5-24	25-44	45-64	65-84	85-104
No. of consumers	4	6	11	22	14

Estimate Median consumption in KWH.

(b) **Points (10)**

Two candidates X and Y in BS exam obtained the following marks in six subjects. By using coefficient of variation, check that which candidate perform more consistently?

Paper	I	II	III	IV	V	VI
X	58	49	76	80	47	72
Y	39	38	86	72	75	69

(c) **Points (10)**

In an experiment to measure the stiffness of a spring, the length of the spring under different loads was measured as follows:

X = Load (lb)	3	5	6	9	10	12
Y = Length (in)	10	12	15	17	19	22

Find the least square regression equation for predicting the weight (load), given the length of the spring.

Q 2.

(a) **Points (10)**

Due to an Internet configuration error, packets sent from New York to Los Angeles are routed through El Paso with probability 3/4. Given that a packet is routed through El Paso, suppose it has conditional probability 1/3 of being dropped. Given that a packet is not routed through El Paso, suppose it has conditional probability 1/4 of being dropped. Find the following probabilities:

(i) The probability that a packet is dropped.

(ii) Find the conditional probability that a packet is routed through El Paso given that it is not dropped.

(b) **Points (05)**

Is it possible for A and B to be independent events yet satisfy $A = B$? Justify your answer.
Hint: Apply the definition of independence of two events to get the required path.

(a) Points

The number of hits at a Web site in any time interval is a Poisson random variable. A particular sit has on average $\lambda = 2$ hits per second. What is the probability that there are no hits in an interval of 0.25 seconds? What is the probability that there are no more than two hits in an interval of one second?

(b) Points (5)

Let N be the number of times a computer polls a terminal until the terminal has a message ready for transmission. If we suppose that the terminal produces messages according to a sequence of independent Bernoulli trials, the N has a geometric distribution. Find the mean of N.

(c) Points (10)

Continuous random variable X has $E[X] = 3$ and $Var[X] = 9$. Find probability density function (PDF) $f_X(x)$, if X has a uniform PDF.

Q 4.

(a) Points (10)

For a random variable X, let $Y = aX + b$. Show that if $a > 0$ then $\rho_{X,Y} = 1$. Also show that if $a < 0$, then $\rho_{X,Y} = -1$.

Hint: (i) Use basic properties of expectation to get the required result.

(ii) $\rho_{X,Y} = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$

(b) Points (10)

Random variable X and Y have the joint PMF

$$P_{X,Y}(x,y) = \begin{cases} cxy & x = 1, 2, 4; \quad y = 1, 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(b) What is $P[Y < X]$?

Q 5.

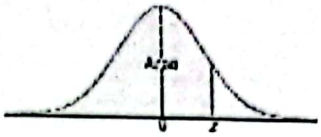
(a) Points (10)

A publishing company has just published a new college textbook. Before the company decides the price at which to sell this textbook, it wants to know the average price of all such textbooks in the market. The research department at the company took a sample of 25 comparable textbooks and collected information on their prices. This information produced a mean price of \$145 and standard deviation is \$35 for this sample. It is known that the population of such prices is normal. Is it reasonable to conclude that mean price of all such college textbooks is at most \$150? Draw your conclusions at 1% level of significance.

(b) Points (10)

Suppose we want to compare the average yearly income in Providence and Boston, two neighboring cities in New England. It is known from experience that the variance of yearly incomes in Providence is \$40000 and the variance for yearly incomes in Boston is \$90000. A random sample of 40 families was taken in Providence, yielding a mean yearly income of \$47000, while a random sample of 50 families was taken in Boston, yielding a mean yearly income of \$52000. At the $\alpha = .01$ significance level, test whether or not there is a significant different in average yearly income between the two cities.

Table A.3 Areas under the Normal Curve



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Probability and Statistics	Course Code:	MT2005
Program:	BSE/BSCS/BDS	Semester:	Fall 2023
Duration:	180 Minutes	Total Marks:	100
Paper Date:	30-12-2023	Weight	50%
Section:	ALL SECTIONS	Page(s):	8
Exam Type:	FINAL	Moderator	Ms. Sarah Ahmad

Student : Name: Ahmed Abdullah Roll No. 22L-7503 Section: BDS-3 A

Instruction/Notes:

1. **All the questions are compulsory. Use answer book to solve the questions.**
2. Exchange of calculators is not allowed. You can only **use your own scientific calculator (programmable calculators are not allowed).**
3. **Statistical Tables are attached with the question paper. You are not allowed to bring it.**
4. Don't get panic. If you found any ambiguity in the data then do not ask anything to the invigilator, just make assumption and continue solving your paper.
5. **Pencil work would not be marked and if you attempt any question with all possibilities then only first solution will be considered for marking.**
6. Believe in yourself & do not waste your time by looking in answer sheets of your fellows and copying them.
7. If you are thinking that it's a revenge. No, it is not. It is just an exam. We want you to be a most successful person in life. All the Best!

Don't Hurry. Don't Worry. Do your Best and Let it rest.

Question 1:

Baysian

[CLO-2, Marks: 07]

Computer assembling company receives 24% of parts from supplier X, 36% of parts from supplier Y, and the remaining 40% of parts from supplier Z. Five percent of parts supplied by X, ten percent of parts supplied by Y, and six percent of parts supplied by Z are defective. If an assembled computer has a defective part in it, what is the probability that this part was received from supplier Z?

Question 2:

[CLO-5, Marks: 5+5=10]

In a tech company, an algorithm that models the data processing time (in minutes), say X, for a computer system tasked with analyzing and organizing information, operates in two distinct phases based on the amount of data received.

$$f(x) = \begin{cases} \frac{x}{25} & 0 < x < 5 \\ \frac{10 - x}{25} & 5 < x < 10 \end{cases}$$

pdf → cdf approach

a) Verify the given function of data processing time as a proper pdf.
b) Compute the probability that data processing time is less than 7 minutes.

0076

Question 3: **[CLO-4, Marks: 3+6+3=12]**

According to a June 2022 poll conducted by the Massachusetts Health Benchmarks project, approximately 55 percent of residents answered "serious problem" to the question: "Some people think that childhood obesity is a national health problem. What do you think? Is it a very serious problem, somewhat of a problem, not much of a problem, or not a problem at all?" Assuming that the probability of giving this answer to the question is 0.55 for any Massachusetts resident, find the probability that if 12 residents are chosen at random

a) Exactly seven will answer "serious problem."
b) Two or fewer households will answer "serious problem."
c) No one will answer "serious problem."

Question 4: **[CLO-8 &1, Marks: 5+5+(5+3)+(7+5)=30]**

In a software development firm, the management team is keen on understanding how different programming aptitude levels might influence the efficiency of code compilation, measured in time taken (in seconds). To optimize coding processes and identifying proficient programmers based on their aptitude scores, the team has collected aptitude test scores of 8 programmers alongside their corresponding code compilation times, and they aim to explore the following:

Programmer No.	Aptitude Test Scores	Code Compilation Time (seconds)
1	57	67
2	58	68
3	59	65
4	59	68
5	60	72
6	61	72
7	62	69
8	64	71

a) Whether there is a relationship between code compilation time and aptitude test scores? Calculate and Interpret.
b) Fit a linear regression model to predict code compilation time based on programmers' aptitude test scores.
c) What is the sum of squared residuals? Also assess the goodness of fit of the obtained regression model and interpret.
d) Show the five number summary and Draw a box & whisker plot to pinpoint any potential outliers within the dataset of aptitude test scores.

0076

Question 5:

[CLO-5, Marks: 6+3=9]

The finished inside diameter of a piston ring is normally distributed with a mean of 10 centimeters and a standard deviation of 0.03 centimeters.

a) What is the probability that a piston ring will not have an inside diameter between 9.97 and 10.03 centimeters?

b) Below what value of inside diameter will 15% of the piston rings fall?

Question 6:

[CLO-6, Marks: 15]

THE VIDEO GAME SATISFACTION RATING CASE STUDY: A company that produces and markets video game systems wishes to assess its customers' level of satisfaction with a relatively new model, the XYZ-Box. In the six months since the introduction of the model, the company has received many warranty registrations from purchasers. The company will select a random sample of 12 of these registrations and will conduct telephone interviews with the purchasers. Specifically, each purchaser will be asked to state his or her level of agreement with each of the seven statements listed on the survey instrument. The level of agreement for each statement is measured on a 7-point Likert scale.

The Video Game Satisfaction Survey Instrument

Statement	Strongly disagree						Strongly agree
The game console of the XYZ-Box is well designed.	1	2	3	4	5	6	7
The game controller of the XYZ-Box is easy to handle.	1	2	3	4	5	6	7
The XYZ-Box has high quality graphics capabilities.	1	2	3	4	5	6	7
The XYZ-Box has high quality audio capabilities.	1	2	3	4	5	6	7
The XYZ-Box serves as a complete entertainment center.	1	2	3	4	5	6	7
There is a large selection of XYZ-Box games to choose from.	1	2	3	4	5	6	7
I am totally satisfied with my XYZ-Box game system.	1	2	3	4	5	6	7

Purchaser satisfaction will be measured by adding the purchaser's responses to the seven statements stated above. Experience has shown that a purchaser of a video game system is "very satisfied" if his or her composite score is at least 42. The composite scores for the sample of 12 customers are given in the table below:

Composite Scores	39	38	42	38	46	40	45	42	41	42	44	39

Let μ denote the mean of all possible customer satisfaction ratings based composite scores for the XYZ-Box video game system.

a) Test at 5% significance level that mean of all possible customer satisfaction ratings based composite scores is at least 42.

Fractional Distribution

Question 7: **[CLO-2 & 4, Marks: 4+5+8=17]**

The response time is the speed of page downloads and it is critical for a mobile Web site. As the response time increases, customers become more frustrated and potentially abandon the site for a competitive one. Let X denote the number of bars of service, and let Y denote the response time (to the nearest second) for a particular user and site. The Joint Probability distribution of X and Y is given below.

	X: No. of Bars of Signal Strength		
Y: Response time, (nearest second)	1	2	3
4	0.15	0.1	0.05
3	0.02	0.1	0.05
2	0.04	0.03	0.2
1	0.01	0.02	0.25

a) Obtain the conditional probability of 2 bars of signal strength if the response time is 3 seconds.
b) For a particular user, what is the probability of 4 seconds response time or 1 bar of signal strength?
c) Calculate covariance i.e. Cov (X,Y).

X: 1-3

FORMULA SHEET FINAL EXAM FALL 2023

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

$$\bar{x} \pm t_{(\alpha/2,\nu)}\frac{s}{\sqrt{n}}; \quad t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

$$b_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$E(X) = \sum xP(x)$

Lower limit = Q1-1.5(IQR)

Upper limit = Q3+1.5(IQR)

$Var(X) = \frac{\sum fx^2}{\sum f} - (\frac{\sum fx}{\sum f})^2$

$s^2 = \frac{\sum(X-\bar{X})^2}{n-1}$

$P(A \cap B \cap C) = P(A)P(B)P(C)$

$P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$

$P(AUB) = P(A)+P(B)-P(A \cap B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$E(X^2) = \sum x^2 f(x)$

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$

$i = \frac{p}{100} * n$

$$F = \frac{s_1^2}{s_2^2} \text{ If } s_1^2 > s_2^2$$

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_i^k P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_i^k P(B_i)P(A|B_i)} \quad , r = 1,2,,k$$

$$b(x;n,p) = \binom{n}{x}p^x q^{n-x}, x = 0,1,\dots n\,, q = 1-p\,,$$

$$f(x_1,x_2,\dots,x_k;p_1,p_2,\dots,p_k;n) = \binom{n}{x_1,x_2,\dots,x_k}p_1^{x_1}p_2^{x_2},\dots,p_k^{x_k}$$

$$h(x;N,n,k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$g(x;p) = pq^{x-1}, \qquad x = 1,2,3,\dots$$

$$p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}, \qquad x = 0,1,2,\dots, \qquad \mu = \lambda t$$

$$f(x) = \frac{1}{n}$$

$$z = \frac{x-np}{\sqrt{npq}}$$

$R^2 = 1 - \frac{SSE}{SST}$ or $R^2 = \frac{SSR}{SST}$

$e = Y - \hat{y}$

$SST = \sum(Y-\bar{Y})^2 = \sum Y^2 - (\sum y)^2/n$

$SSE = \sum(Y-\hat{y})^2 = \sum Y^2 - b_0\sum y - b_1\sum XY$

$SSR = \sum(\hat{y}-\bar{Y})^2$

$\mu = E(X)$

$\sigma^2 = E(X^2) - \mu^2$

$$t = \frac{\bar{d}-d_0}{s_d/\sqrt{n}}$$

$$\bar{d} \pm t_{(\alpha/2,\nu)}\, s_d/\sqrt{n}$$

$$z = \frac{x-\mu}{\sigma}$$

0076

University of Computer and Emerging Sciences
(FAST)

Final Paper **Fall-2013**

Course: Probability and Statistics
Time Allowed: 3 hours Maximum marks: 50

Q.1: Suppose that the 4 inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John who stamps 20% of the packages, fails to stamp the expiration date twice in every 100 packages; Tom who stamps 60% of the packages, fails to the stamp once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 150 packages. If a customer complains that his package of film does not show the expiration date, what is the probability that it was inspected by Tom? (5)

Q.2: Consider the density function: (8)

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

(a) Evaluate k (b) Find Mean and Variance of X.
(c) Find $F(x)$ and use it to evaluate $P(0.2 < x < 0.6)$.

Q.3: Suppose that X and Y are independent random variables having the joint probability distribution: (6)

f(x,y)		x	
		2	4
y	1	0.10	0.15
	3	0.20	0.30
	5	0.10	0.15

Find (a) E(2X – 3Y) ; (b) E(XY)

Q.4: A manufacturing company uses an acceptance scheme on items from a production line before they are shipped. The plan is a two-stage one. Boxes of 25 items are readied for shipment, and a sample of 3 items is tested for defectives. If any defectives are found, the entire box is sent back for 100% screening. If no defectives are found, the box is shipped.
(a) What is the probability that a box containing 3 defectives will be shipped?
(b) What is the probability that a box containing only 1 defective will be sent back for screening? (5)

Q.5: Suppose the probability that any given person will believe a tale about the transgressions of a famous actress is 0.7. What is the probability that:
(a) The fifth person to hear this tale is the third one to believe it.
(b) The fourth person to hear this tale is the first one to believe it. (4)

P.T.O.

1/6

Q.6: The average life of a certain type of small motor is 10 years with a S.D. of 2 years. The manufacturer replaces free all motors that fail while under guarantee. If he is willing to replace only 4% of the motors that fail, how long a guarantee should be offered? Assume that the lifetime of a motor follows a normal distribution. (5)

Q.7: The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution:

x	4	5	6	7
P(X=x)	0.2	0.4	0.3	0.1

(a)Find the mean μ and the variance σ^2.

(b)Find the mean $\mu_{\bar{x}}$ and the variance $\sigma^2_{\bar{x}}$ of the mean $\bar{x}$ for random samples of 36 cherry puffs.

(c) Find the probability that the average number of cherries in 36 cherry puffs will be less than 5.1 or greater than 5.6. (6)

Q.8: A random sample of 12 shearing pins is taken in a study of the Rockwell hardness of the pin head. Measurements on the Rockwell hardness are made for each of the 12, yielding an average value of 48.50 with a sample S.D. of 1.5. Assuming the measurements to be normally distributed:

(a) Construct 95% confidence interval for the mean Rockwell hardness.

(b) What will be the 95% confidence interval if population variance is 4? (6)

Q.9: A manufacturer claims that the average tensile strength of thread A exceeds the tensile strength of thread B by at least 10 kg. To test this claim, 60 pieces of each type of thread were tested under similar conditions. Type A thread had an average tensile strength of 87 kg with a S.D. of 6 kg, while thread B had an average tensile strength of 78 with a S.D. of 5 kg. Test the manufacturer's claim using a 0.05 level of significance. (5)

National University of Computer and Emerging Sciences
(FAST)

Final Exam FALL-2014

Course: Probability and Statistics (for CS)
Time Allowed: 3 hours Maximum marks: 50

Note: Attempt all questions.

Q1: Write short answers to the following questions. (8)

i. Differentiate between Sample and Population.
ii. Define Statistical Inference.
iii. Write any two properties of Normal Distribution.
iv. What is the difference between one-tailed test and two-tailed test?

Q2: In a high school graduating class of 110 students, 58 studied Mathematics, 72 studied Statistics and 40 studied both Statistics and Mathematics. If one of these students is selected at random, find the probability that :
(a) the student took Mathematics or Statistics;
(b) the student took Statistics but not Mathematics. (4)

Q3: Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as:

f(x,y)		x		
		1	2	3
y	1	0.05	0.05	0.1
	2	0.05	0.1	0.35
	3	0	0.2	0.1

(a) Find P(Y=3/X=2),
(b) Find the covariance of the random variables X and Y,
(c) Show whether X and Y are independent or not. (6)

Q4: Suppose the probability is 0.7 that any given person will believe a tale about the transgressions of a famous actress. What is the probability that:
(a) the fifth person to hear this tale is the third one to believe it?
(b) the fourth person to hear this tale is the first one to believe it? (4)

National University of Computer and Emerging Sciences, Lahore Campus

Course:	Probability &Statistics	Course Code:	MT206
Program:	BS Computer Science	Semester:	Fall-17
Duration:	3 hour	Total Marks:	50
Paper Date:	December 22, 2017	Weight	50%
Section:	All	Page(s):	02
		Roll No:	[redacted]
Exam:	Final	Section:	[redacted]

Instruction/Notes: Attempt all questions.
Formula Sheet is allowed.
Exchange of calculators and stationary is strictly prohibited. Attempt parts of same question together. If you think some information is missing or wrong make assumptions and clearly state them.

Question 1: (6 marks) In FAST University in the class of 100 CS students, 54 studied Sociology, 69 studied Image processing, and 35 studied both Sociology and Image Processing. If one of these students is selected at random, find the probability that

(a) The student took sociology or image processing. $S \cup IP$

(b) The student did not take either of these subjects. $S' \cup IP'$, $(S \cap IP)'$

(c) The student took image processing but not sociology. $IP \cup S'$

Question 2: (5 marks) Arrivals to a bank automated teller machine (ATM) are distributed according to a Poisson distribution with a mean equal to three per 15 minutes.

a) Determine the probability that in a given 15-minute segment no customers will arrive at the ATM.

b) What is the probability that fewer than four customers will arrive in a 30-minute segment?

Question 3: (6 marks) A rocket motor is manufactured by bonding an igniter propellant and a sustainer propellant together inside a metal housing. The shear strength of the bond between the two types of propellant is an important quality characteristic. It is suspected that the shear strength is related to the age in weeks of the batch of sustainer propellant. Eight observations on shear strength and age are shown in the table below

$\hat{y} = b_0 + b_1 x$

Shear Strength (psi) y	2158.7	1678.2	2316.0	2061.3	2207.5	1708.3	1784.7	2575.0
Age of Propellant (weeks) x	15.5	23.8	8.0	17.0	5.5	19.0	24.0	2.5

a) Estimate the linear regression line. $e = y - \hat{y}$

b) Predict the shear strength (y) when age of propellant = 6.00 (x) and calculate the error.

Question 4: (5 marks) It is known that screws produced by a certain company will be defective with probability 0.01, independently of each other. The company sells the screws in packages of 10 and offers a money-back guarantee that at most 1 of the 10 screw is defective. What proportion of packages sold must the company replace? $X < 1$

$n = 10$

$p = 0.01$

$\hat{p} = \frac{p}{n}$

np

$p = P(\text{Success})$

p

$\hat{p} =$ 1/7

Question 5: (6 marks) Entry to Fast University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100.

a) If Maida wants to be admitted to this university and she knows that she must score better than at least 70% of the students who took the test. Maida takes the test and scores 585. Will she be admitted to this university?

b) Maida is told that 10% of the students taking the test have higher scores than she does? What was Maida's score?

c) What percentage of students taking this test will have scores below 300?

Question 6: (8 marks) Samples of two types of cord have breaking strengths in pounds as shown.

Type-1:	28	26	20	24	22
Type-2:	22	18	16	14	20

a) On the basis of the 01% level, test if is type-1 is significantly better than type-2? Assuming the population variances are equal.

b) Construct the 99% confidence interval for the mean difference, assuming the population variances are equal.

Question 7: (6 marks) A university has found over the years that out of all the students who are offered admission, the proportion who accept is .70. After a new director of admissions is hired, the university wants to check if the proportion of students accepting has increased significantly. Suppose they offer admission to 1200 students and 888 accept. Is this evidence at the 5% level of significance that there has been a significant increase in proportion of students accepting admission? Use P.value in your conclusion.

Question 8: (8 marks) Suppose that a radioactive particle is randomly located in a square with sides of unit length. That is, if two regions within the unit square and of equal area are considered, the particle is equally likely to be in either region. Let Y_1 and Y_2 denote the coordinates of the particle's location. A reasonable model for the relative frequency histogram for Y_1 and Y_2 is the bivariate analogue of the univariate uniform density function

$$f(y_1, y_2) = 1, \qquad 0 \le y_1 \le 1, 0 \le y_2 \le 1$$

a) Find (Cummulative Density Function) $F(0.2, 0.4)$

b) Find $P(0.1 \le Y_1 \le 0.3, 0 \le Y_2 \le 0.5)$

c) Find $Var(Y_1 + 2Y_2)$

Do not write below this line

Attempt all the questions on Answer Book.

Statistical tables & Formula sheet are attached. You are not allowed to use your own tables.

Recreate tables/graphs (if any) on the answer book.

Don't write anything on the question paper and Do Not Attach it.

Lead Pencil work wouldn't marked or claimed for rechecking. Use permanent ink pen.

If you found any ambiguity in the data then do not ask anything to the invigilator, just make assumption and continue solving your paper.

Show the calculation/procedure of each Question/Sub-Parts properly.

Must write the final answer of each question up to 4 significant figures.

CLO 2: Use of basic counting principles & all laws of probability to analyze probabilistic experiment.

[marks: 7+8]

Q1:

An individual has 3 different email accounts. Most of her emails, in fact 70%, come into account no. 1, whereas 20% come into account no. 2 and the remaining 10% into account no. 3. It is known from past experience that 1%, 2% and 5% emails from account no 1, account no 2 and account no 3 respectively are spam.

a) What is the probability that a randomly selected email is spam?

b) If the observed email is identified as spam, to which account it is most likely associated?

CLO 3: Identify and analyze the type of random variables and its probability *distributions.*

Q2: **[marks: 6+3+11]**

Let X denote the number of software updates a company's server undergoes in a month: 1, 2, or 3 updates. Let Y denote the number of times the server experiences downtime. Their joint probability distribution is given as:

$f(x,y)$		x		
		1	2	3
y	1	0.05	0.05	0.10
	3	0.05	0.10	0.35
	5	0.00	0.20	0.10

a) Evaluate the marginal distribution of X and Y.
b) Find $P(Y = 3 \mid X = 2)$.
c) Verify whether $E(XY) = E(X)\,E(Y)$ or not?

CLO 4 & 5: Determine the type of discrete & continuous distribution and evaluate its probability distribution.

Q3: **[marks: 10+5]**

a) Each time a modem transmits one bit, the receiving modem analyzes the signal that arrives and decides whether the transmitted bit is 0 or 1. It makes an error with probability p, independent of whether any other bit is received correctly. If the probability of error is $p = 0.01$ and the modem transmits 200 bits, what is the probability that at most 3 errors will be observed?

b) A software development company releases updates to its mobile application every evening. Once the updates are released, they must be tested on various devices to ensure compatibility and functionality. The time required to test each update follows a normal distribution, with a mean of 120 minutes and a standard deviation of 20 minutes. If a manager must be present until 90% of the updates are tested, how long will the manager need to be there? Also interpret the result.

CLO 1 & 6: Compute and interpret various measures of location and variation. Apply classical hypothesis testing/confidence interval for single population.

Q4: **[marks: 10]**

The operations manager of a plant making cellular telephones has proposed rearranging the production process to be more efficient. He wants to estimate the average time to assemble the telephone using the new arrangement. For this purpose, a sample of 15 time points used to assemble the cellular telephone under the new arrangement is provided below:

Assembling Time (in minutes)														
8	8	8	8	8	10	14	15	13	8	11	13	11	10	7

What is a reasonable range of values for the population mean assembling time? Also, interpret the results.

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CLO 6 & 8: **Apply classical hypothesis testing/confidence intervals for single population and to compare two populations and draw inferences.** ***Methodologies of regression analysis for the future predictions. Able to check goodness of fit and strength of relationship between two variables.***

Q5: **[marks: 10+ (10+10) +10]**

PCW World rated four component characteristics for 10 ultraportable laptop computers: features, performance, design, and price. Each characteristic was rated using a 0-100 point scale. An overall rating, referred to as the PCW World Rating, was then developed for each laptop. The following table shows the features rating and the PCW World Rating for the 10 laptop computers.

Model	Features Rating	PCW World rating
Thinkpad X200	87	83
VGN-Z598U	85	82
U6V	80	81
Elitebook 2530p	75	78
X360	80	78
Thinkpad X360	76	78
Ideapad U110	81	77
Micro Express JFT250	73	75
Toughbook W7	79	73
HP Envy 133	68	72

a) Derive an estimated regression equation to predict PCW world ranking based on features rating.

b) Compute explained and unexplained variation? Complete the table below and comment on the overall significance of the regression model.

Source of Variation	Degrees of Freedom	Sum of Square	Mean Square	F-Ratio	p-value
Regression	$k-1$	SSR	SSR/k-1		0.006
Error	$k\ n-k$	SSE	SSE/n-k		
Total	$n-1$				
Decision/Conclusion:					

Note: *Recreate the above ANOVA table in the same sequence of entries. Use a full page in landscape orientation in your answer book for the table. Write the final answers in the table. Show the calculations on a separate page, either before or after the table.*

:) Is it reasonable to conclude that in a population, there is a positive relationship between the PCW world ranking and features rating? Test at 5% level of significance using correlation coefficient.

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$$\bar{x} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$$

$$\bar{x} \pm t_{(\alpha/2,\nu)}\frac{s}{\sqrt{n}};$$

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}}$$

$$b_1 = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$b_0 = \bar{y} - b_1\bar{x}$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$$t = \frac{b_1-\beta_1}{s_{yx}/\sqrt{S_{xx}}} \text{ or } \frac{b_1-\beta_1}{s_{b1}}$$

$$s_{yx} = \sqrt{\frac{\sum(Y-\hat{y})^2}{n-2}}$$

$$S_{xx} = \sum(X-\bar{X})^2$$

$$\text{Var(X)} = \text{E}(X^2) - [E(X)]^2$$

$$\text{Var(X)} = \frac{\sum fx^2}{\sum f} - (\frac{\sum fx}{\sum f})^2$$

$$\text{P(A} \cap B \cap C) = P(A)P(B)P(C)$$

$$\text{P(A} \cap B \cap C) = P(A)P(B/A)P(C/A \cap B)$$

$$\text{P(AUB)} = \text{P(A)+P(B)-P(A} \cap B)$$

$$F = \frac{s_1^2}{s_2^2} \text{ If } s_1^2 > s_2^2, \quad F = \frac{MSR}{MSE}$$

$$P(B_r/A) = \frac{P(B_r \cap A)}{\sum_l^k P(B_l \cap A)} = \frac{P(B_r)P(A/B_r)}{\sum_l^k P(B_l)P(A/B_l)} \quad ,r = 1,2,,k$$

$$b(x;n,p) = \binom{n}{x}p^x q^{n-x}, x = 0,1,\dots n, q = 1-p,$$

$$f(x_1,x_2,\dots,x_k;p_1,p_2,\dots,p_k;n) = \binom{n}{x_1,x_2,\dots,x_k}p_1^{x_1}p_2^{x_2},\dots,p_k^{x_k}$$

$$h(x;N,n,k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$p(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}, \quad x = 0,1,2,\dots, \quad \mu = \lambda t$$

$$R^2 = 1 - \frac{SSE}{SST} \text{ or } \frac{SSR}{SST}$$

$$SSR = \sum(\hat{y}-\bar{Y})^2$$ explained

$$SST = \sum(Y-\bar{Y})^2 = \sum Y^2 - \frac{(\sum Y)^2}{n}$$ unexplained

$$SSE = \sum(Y-\hat{y})^2 = \sum Y^2 - b_0\sum y - b_1\sum XY$$

$$\mu = E(X)$$

$$\text{E(X)} = \sum xf(x)$$

$$t = \frac{\bar{d}-d_0}{s_d/\sqrt{n}};$$

$$\bar{d} \pm t_{(\alpha/2,\nu)}\, s_d/\sqrt{n}$$

$$z = \frac{x-\mu}{\sigma}$$

Lower limit = Q1-1.5(IQR)

Upper limit = Q3+1.5(IQR)

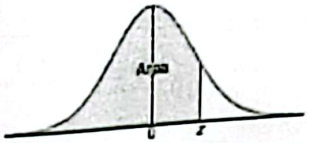

Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998