

Q1

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -4 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{bmatrix}$$

(a) $x_1 + 3x_2 = 2$

$$-2x_1 - 4x_2 + 7x_3 = 4$$

$$3x_1 + 5x_2 + 2x_3 = 1$$

$$x_1 - x_2 + 2x_3 = -3$$

(1 mark)

(b) $\begin{bmatrix} 1 & 3 & 0 & 2 \\ -2 & -4 & 7 & 4 \\ 3 & 5 & 2 & 1 \\ 1 & -1 & 2 & -3 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ 2 & 7 & 8 & 8 \\ -4 & 2 & -5 & -5 \\ 0 & -4 & 2 & -5 \end{bmatrix} \begin{array}{l} R_2 \leftarrow R_2 + 2R_1 \\ R_3 \leftarrow R_3 - 3R_1 \\ R_4 \leftarrow R_4 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7/2 & 4 \\ 0 & -4 & 2 & -5 \\ 0 & -4 & 2 & -5 \end{bmatrix} \quad R_2 \leftarrow \frac{1}{2} R_2$$

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7/2 & 4 \\ 0 & 0 & 16 & 11 \\ 0 & 0 & 16 & 11 \end{bmatrix} \begin{array}{l} R_3 \leftarrow R_3 + 4R_2 \\ R_4 \leftarrow R_4 + 4R_2 \end{array}$$

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7/2 & 4 \\ 0 & 0 & 1 & 11/16 \\ 0 & 0 & 16 & 11 \end{bmatrix} \quad R_3 \leftarrow \frac{1}{16} R_3$$

$$\begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 7/2 & 4 \\ 0 & 0 & 1 & 11/16 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 \leftarrow R_4 - 16R_3$$

$x_1 + 3x_2 = 2$ (5 marks)

$$x_2 + 7/2 x_3 = 4$$

$$x_3 = 11/16$$

$$x_2 + 7/2 \cdot 11/16 = 4$$

$$x_2 + \frac{77}{32} = 4$$

$$x_2 = 4 - \frac{77}{32}$$

$$x_2 = \frac{128 - 77}{32} = \frac{51}{32}$$

$$x_1 + 3\left(\frac{51}{32}\right) = 2$$

$$x_1 = 2 - \frac{153}{32}$$

$$x_1 = \frac{64 - 153}{32}$$

$$x_1 = -\frac{89}{32}$$

$$(x_1, x_2, x_3) = \left(-\frac{89}{32}, \frac{51}{32}, \frac{11}{16}\right)$$

(1 mark)

Q2

$$\begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{bmatrix}$$

(a) Minor $M_{21} :- M_{21} = \begin{vmatrix} 1 & 2 \\ 7 & 2 \end{vmatrix} = 2 - 14 = -12$ (2 marks)

Cofactor $C_{21} :- C_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-12) = -12$ (3 marks)

(b)

$$\begin{vmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{vmatrix} = (-1)(0 + 35) - 1(6 + 5) + 2(21 - 0)$$

$$= -35 - 11 + 42$$

$$= -46 + 42$$

$$= -4$$
 (5 marks)

Q3 formula for

(a) The eq. of plane is :-

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \quad \rightarrow (1 \text{ mark})$$

$$1(x-1) + 9(y-1) + 8(z-4) = 0$$

$$x - 1 + 9y - 9 + 8z - 32 = 0$$

$$x + 9y + 8z - 1 - 9 - 32 = 0$$

$$x + 9y + 8z - 42 = 0$$

$$\boxed{x + 9y + 8z = 42}$$

\rightarrow (4 marks)

(b) Distance formula is :-

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} \quad \rightarrow (1 \text{ mark})$$

$$= \frac{|1(-1) + 9(-1) + 8(2) - 42|}{\sqrt{1^2 + 9^2 + 8^2}}$$

$$= \frac{|-1 - 9 + 16 - 42|}{\sqrt{1 + 81 + 64}}$$

$$= \frac{|-36|}{\sqrt{146}} = \frac{36}{\sqrt{146}}$$

$$= \frac{36}{\sqrt{146}}$$

\rightarrow (4 marks)

Q4

$$\text{Proj}_{\vec{a}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{a}}{|\vec{a}|^2} \right) \vec{a} \rightarrow (1 \text{ mark})$$

$$= \left(\frac{(2, 1, 1, 2) \cdot (4, -4, 2, -2)}{(\sqrt{4^2 + 4^2 + 2^2 + 2^2})^2} \right) (4, -4, 2, -2)$$

$$= \left(\frac{8 - 4 + 2 - 4}{(\sqrt{40})^2} \right) (4, -4, 2, -2)$$

$$= \left(\frac{\frac{2}{40}}{20} \right) (4, -4, 2, -2)$$

$$= \left(\frac{4}{20}, \frac{-4}{20}, \frac{2}{20}, \frac{-2}{20} \right)$$

$$= \left(\frac{1}{5}, -\frac{1}{5}, \frac{1}{10}, -\frac{1}{10} \right) \rightarrow (4 \text{ mark})$$

$$\vec{u} - \text{Proj}_{\vec{a}} \vec{u} = (2, 1, 1, 2) - \left(\frac{1}{5}, -\frac{1}{5}, \frac{1}{10}, -\frac{1}{10} \right) (1 \text{ mark})$$

$$= (2 - \frac{1}{5}, 1 + \frac{1}{5}, 1 - \frac{1}{10}, 2 + \frac{1}{10})$$

$$= \left(\frac{9}{5}, \frac{6}{5}, \frac{9}{10}, \frac{21}{10} \right)$$

$$\rightarrow (4 \text{ marks})$$

