

CS 301: Theory of Automata
Quiz 4
November 28, 2019.

Problem

Given the following language SUBSET_SUM. Prove that it is a member of NP class.
 $\text{SUBSET_SUM} = \{ \langle S, t \rangle \mid S \text{ is a set of numbers and there is a subset } m \subseteq S \text{ and the sum of all items in } m \text{ is } t \}$

Solution

There can be two ways of proving this. Any one of them is correct.

Method 1 (via a polynomial time verifier on a deterministic machine)

The certificate is the subset m of S . The verifier would perform the following steps using the certificate:

1. Check that all members of m are also present in S
2. Sum all numbers of m and verify that the sum is t
3. If condition 1 and condition 2 is satisfied then ACCEPT otherwise REJECT.

We can see that if n = number of elements in S then step 1 can be performed in $O(n^2)$ time and step 2 can be performed in $O(n)$ time. Hence the verifier would run in polynomial time on a deterministic Turing machine and the language is a member of NP.

Method 2 (via a polynomial time non-deterministic TM)

Given a non-deterministic TM with $\langle S, t \rangle$ as input. The machine would perform the following steps:

1. Non-deterministically select a subset m of S
2. Sum each element of m . If $\text{sum} == t$ then ACCEPT, otherwise REJECT

We can see that step 2 will take $O(n)$ time and hence the longest branch in the computation tree would not take more than polynomial time. Hence, the language can be decided in polynomial time on a non-deterministic TM. Which satisfies our definition of NP.