Information Security
CS 3002

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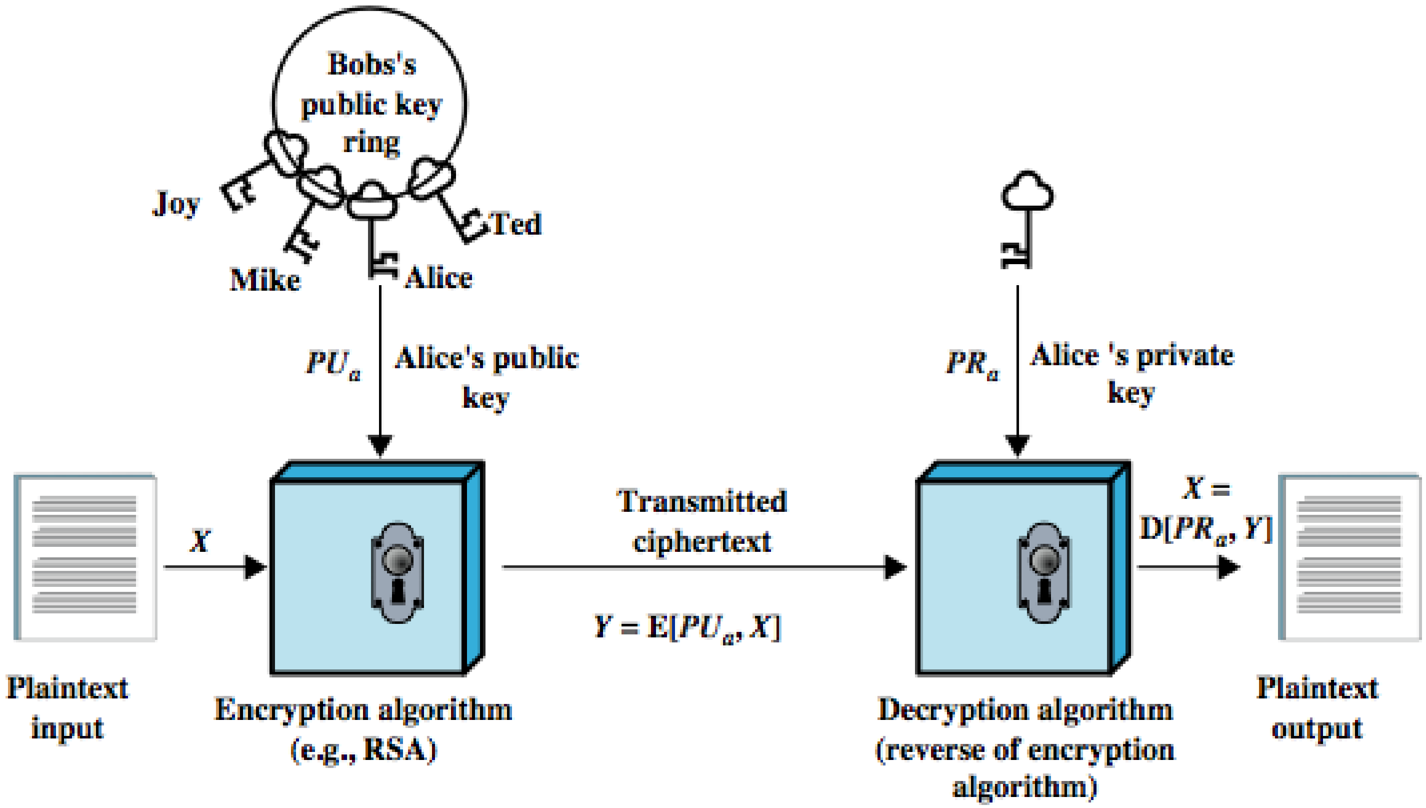
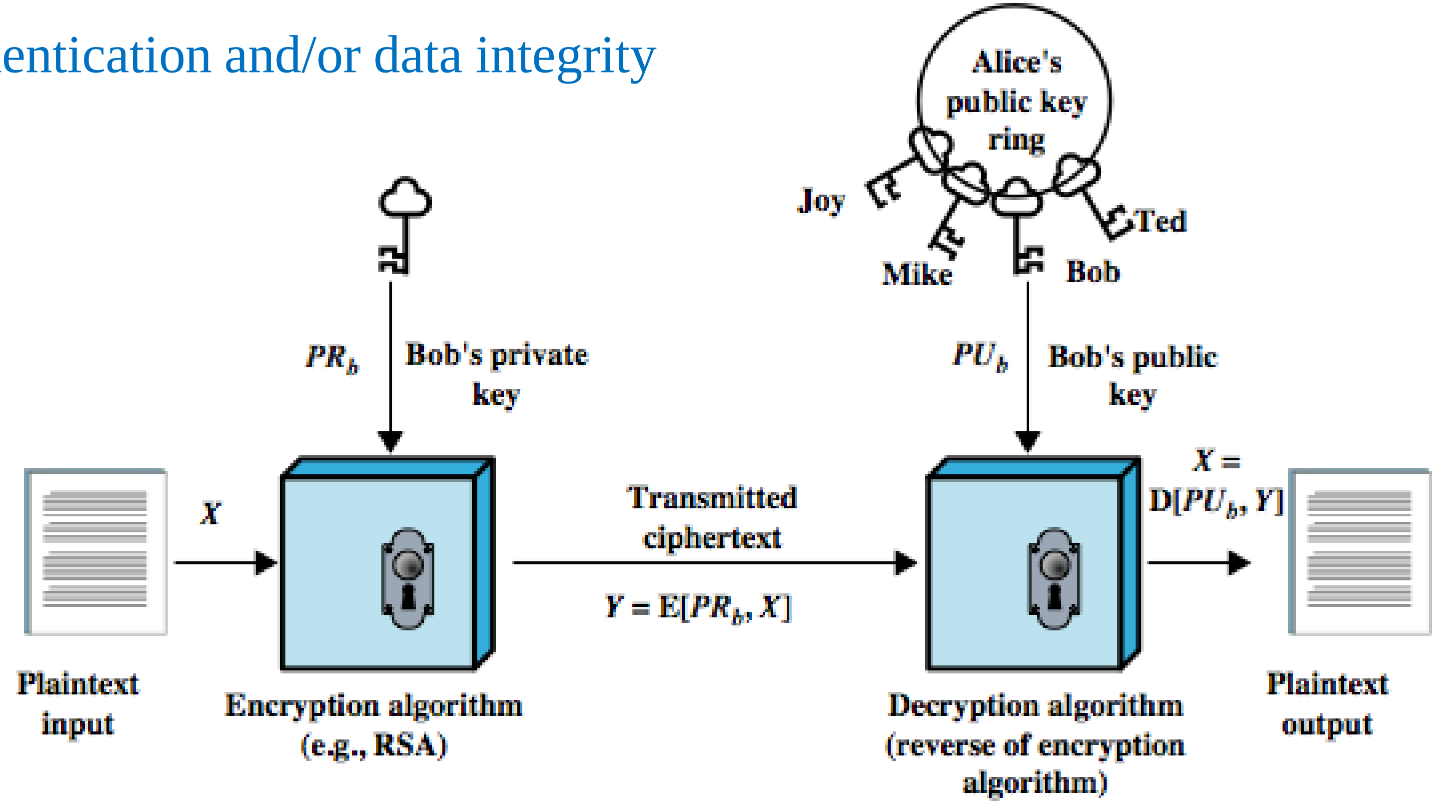
Public Key Encryption



(a) Confidentiality

Public Key Authentication

Authentication and/or data integrity



(b) Authentication

Public Key Requirements

1. computationally easy to create key pairs
2. computationally easy for sender knowing public key to encrypt messages
3. computationally easy for receiver knowing private key to decrypt ciphertext
4. computationally infeasible for opponent to determine private key from public key
5. computationally infeasible for opponent to otherwise recover original message
6. useful if either key can be used for each role

Public Key Algorithms

- RSA (Rivest, Shamir, Adleman)
  - developed in 1977
  - only widely accepted public-key encryption alg
  - given tech advances need 1024+ bit keys
- Diffie-Hellman key exchange algorithm
  - only allows exchange of a secret key
- Digital Signature Standard (DSS)
  - provides only a digital signature function with SHA-1
- Elliptic curve cryptography (ECC)
  - new, security like RSA, but with much smaller keys

Diffie-Hellman

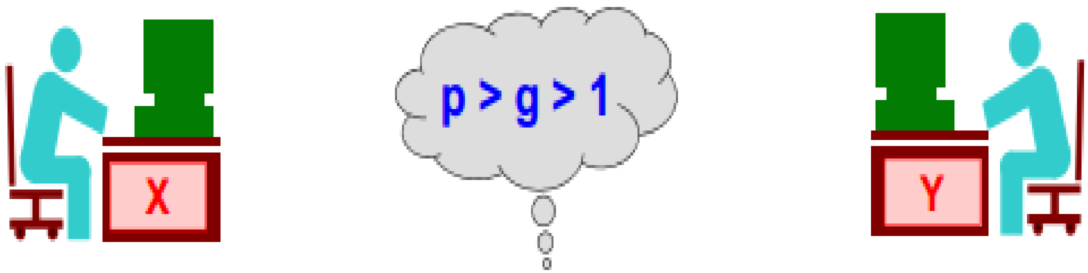
- First public key algorithm invented
- Published in 1976
- Specific method for securely exchanging cryptographic keys over a public channel
- Concept given by Ralph Merkle
- Named after Whitfield Diffie and Martin Hellman

- Public key exchange algorithm, neither encryption nor signature

Diffie-Hellman

- Uses modular arithmetic also called the clock arithmetic:
  - g mod p. where g is the generator and p is the prime modulus
  - $1 < g < p$
- g is a primitive root of p
- Consider two numbers g & p shared publically between A & B
- A computes $X = g^x \bmod p$ (x is the secret from Alice)
- B computes $Y = g^y \bmod p$ (y is the secret from Bob)
- A & B exchanged X & Y
- A computes $K_{AB} = Y^x \bmod p$ and B computes $K_{BA} = X^y \bmod p$
- $K_{AB} = K_{BA} = g^{xy} \bmod p$

Diffie-Hellman



$A = g^X \bmod p$ $\qquad\qquad$ $B = g^Y \bmod p$

$$\xrightarrow{\quad\quad\quad A \quad\quad\quad}$$

$$\xleftarrow{\quad\quad\quad B \quad\quad\quad}$$

$K_A = B^X \bmod p$ $\qquad\qquad$ $K_B = A^Y \bmod p$

$$\boxed{K_A = K_B = g^{XY} \bmod p = K_{AB}}$$

Diffie-Hellman: example

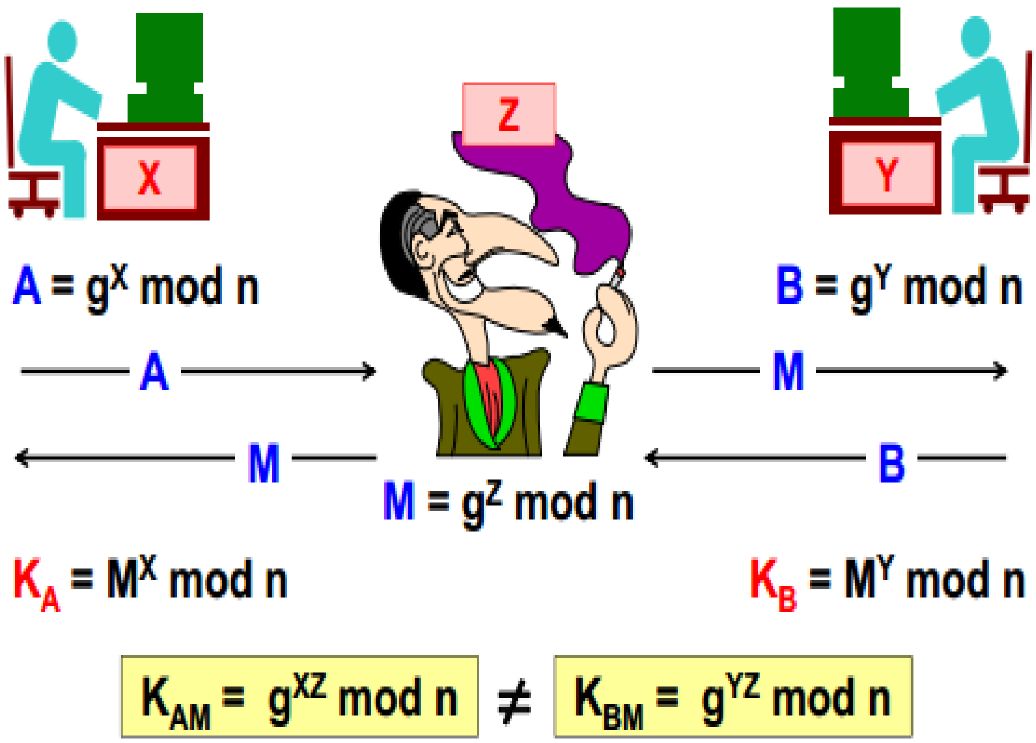
- E.g: $g = 3$, $p = 353$

- Alice computes secret => $x = 97$

  $g^x \bmod p = 3^{97} \bmod 353 = 40$
- Bob computes secret => $y = 233$
- $g^y \bmod p = 3^{233} \bmod 353 = 248$

- Alice gets 248 from Bob =>
- Alice computes => $248^{97} \bmod 353 = 160$
- Bob gets 40 from Alice =>
- Bob computes => $40^{233} \bmod 353 = 160$

Man in the middle attack on DH



RSA

- Invented by Ron Rivest, Adi Shamir, and Len Adleman at MIT 1978
- Block size can be variable
- Key length can be variable
- Plaintext must be smaller than the key length
- Ciphertext block will be the length of the key
- product of prime numbers, factoring of result

- Applications: secrecy(secret exchange of keys) and digital signature

Co-prime or relatively prime

- **Two numbers are co-prime if the greatest common divisor (GCD) between them is only 1.**

- **A and B are coprime iff gcd(A,B) = 1.**
- **E.g: 6 and 11. gcd between 6 & 11 is 1 only**

- **It isn’t necessary that the two numbers are prime! They have to be prime to each other!**

Euler Totient function

- If n is a positive integer, ϕ(phi) function counts all the positive integers less than n that are relatively prime to n.
- $\phi(n)$ = Totient(n) = numbers of integers < n that are coprime to n
- $\phi(10) = 4$
  - (1,3,7,9) are all coprime to 10
- Generally, $\phi(p) = (p-1)$ where p is a prime number

Multiplicative Inverse

- **Multiplicative Inverse of a number X is Y iff Y multiplied with X yields 1.**

- **i.e. X * Y = 1, Both X and Y are the multiplicative inverses of each other.**

PKC – How it works?

- **Plain text M and ciphertext C are integers between 0 and n-1(n is product of two prime numbers).**
- **$Key_1 = \{e, n\}$, $Key_2 = \{d, n\}$**
  - **Where e and d are two secret number**
  - **$C = M^e \bmod n$**
  - **$M = C^d \bmod n$**

RSA - Key Construction

- Select two large primes: $p, q, p \neq q$
- $n = p \times q$
- Calculate Euler's Totient Function, $\phi(n) = (p-1)(q-1)$
- Select **e** relatively prime to ϕ => $gcd(\phi, e) = 1$; $0 < e < \phi$
- Calculate **d** = inverse of **e** mod ϕ => **d** * **e** mod $\phi = 1$

- public key = (**e**, n)
- private key = (**d**, n)
- The roles of **e** & **d** are interchangeable. i.e. $(x^d)^e \bmod n = (x^e)^d \bmod n$

RSA Key Construction: Example

- Select two large primes: $p, q, p \neq q$
- $p = 17, q = 11$
- $n = p \times q = 17 \times 11 = 187$
- Calculate $\phi = (p-1)(q-1) = 16 \text{x} 10 = 160$
- Select e, such that $\gcd(\phi, e) = 1; 0 < e < \phi$ say, $e = 7$
- Calculate d such that $\rightarrow$ $de \bmod \phi = 1$
- Use Euclid's algorithm to find $d=e^{-1} \bmod \phi$
  - $160k+1 = 161, 321, 481, 641$
  - Check which of these is divisible by 7
  - 161 is divisible by 7 giving $d = 161/7 = 23$
- Key_1 = Public key = {7, 187}, Key_2 = Private key = {23, 187}

RSA – encryption: Example

- **Messages to encrypt:**
- $m_1 = 2$ & $m_2 = 3$
- Key_1 = Public key = {7, 187}, Key_2 = Private key = {23, 187}
- $c_1 = 2^7 \bmod 187 = 128 \bmod 187 = 128$
- $c_2 = 3^7 \bmod 187 = 2187 \bmod 187 = 130$
- $m_1 = 128^{23} \bmod 187 = 2$
- $m_2 = 130^{23} \bmod 187 = 3$