15th IEEE INTERNATIONAL CONFERENCE
ON EMERGING TECHNOLOGIES 2019
National University Of Computer & Emerging Sciences
Peshawar Campus

① Differentiation by I.F

Numerical Differenciation

Newton forward I. Formula

x_0	y_0
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4

$$y(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \cdots$$

$x = x_i$, $y'(x_i)$, $y''(x_i)$

where

$$P = \frac{x - x_0}{h} \Rightarrow x = x_0 + Ph$$

→ Equal interval
→ Unequal interval.

$$\Downarrow$$

$$\Rightarrow y(x_0 + Ph) = y_0 + P\Delta y_0 + \left(\frac{P^2 - P}{2!}\right)\Delta^2 y_0 + \frac{(P^3 - 3P^2 + 2P)}{3!}\Delta^3 y_0 + \cdots$$

Differentiating w.r.t 'P'

$$\Rightarrow \frac{dy}{dx} \cdot h = y'(x) = \Delta y_0 + \frac{(2P-1)}{2!}\Delta^2 y_0 + \frac{(3P^2 - 6P + 2)}{3!}\Delta^3 y_0 + \cdots$$

$$\boxed{\Rightarrow y'(x) = \frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2P-1)}{2!}\Delta^2 y_0 + \frac{(3P^2 - 6P + 2)}{3!}\Delta^3 y_0 + \cdots\right]}$$

This is the numerical differential formula derived from Newton forward I. Formula.

Differentiating again

$$\boxed{\Rightarrow \frac{d^2 y}{dy^2} = y''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + (P-1)\Delta^3 y_0 + \cdots\right]}$$

Question:

Find $\frac{dy}{dx}$ at $x = 1.5$ from following table.

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.0	13.625	24	38.875	59

Sol

The forward difference table for given data.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 1.5$	3.375				
		$\Delta y_0 = 3.625$			
2.0	7.0		$\Delta^2 y_0 = 3.0$		
		6.625		$\Delta^3 y_0 = 0.75$	
2.5	13.625		3.75		0
		10.375		0.75	
3.0	24		4.5		0
		14.875		0.75	
3.5	38.875		2.25		
		20.125			
4.0	59				

Given: $x_0 = 1.5$, $x = 1.5$, $h = 0.5$, $P = 0$

since

$$\Rightarrow \frac{dy}{dx} = y'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{2P-1}{2!}\Delta^2 y_0 + \frac{3P^2-6P+2}{3!}\Delta^3 y_0 + \dots\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1.5} = \frac{1}{0.5}\left[3.625 + \left(-\frac{1}{2}\right)(3) + \frac{2}{6}(0.75)\right]$$

$$= 4.75 \quad \underline{\text{Ans}}$$

For unequal interval.

First, we will find $f(x)$ then we differentiate $f(x)$ for $f'(x)$.

At $x = x_0 \Rightarrow p = 0$

$$\left.\frac{dy}{dx}\right]_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + \cdots\right]$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \cdots\right]$$

Ex

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	
2.00	0.6932					
		0.0953				
2.20	0.7885		-0.0083			
		0.0870		0.0013		
2.40	0.8755		-0.0070		0.0002	
		0.0800		0.0011		0.0001
2.60	0.9555		0.0059		0.0003	
		0.0741		0.0008		
2.80	1.0296		-0.0051			
		0.0690				
3.00	1.0986					

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + \Delta^3 y_0\right]$$

Here $x_0 = 2.00$; $P = 0$; $h = 0.20$

$$y'(2.00) = \frac{1}{0.2}\left(0.0953 - \frac{1}{2}(-0.0083) + \frac{1}{3}(0.0013)\right)$$

$$= 0.4994.$$

&

$$y''(2.00) = \frac{1}{(0.2)^2}\left(-0.0083 - 0.0013\right)$$

$$= -0.24.$$

Newton Backward

$$y(x) = y_0 + P\nabla y_0 + \frac{P(P+1)}{2!}\nabla^2 y_0 + \cdots\cdots$$

$$P = \frac{x - x_0}{h} \quad \text{or} \quad x = x_0 + Ph$$

$$\frac{dP}{dx} = \frac{1}{h}$$

$$\frac{dy}{dx} = \frac{dy}{dP}\cdot\frac{dP}{dx} = \frac{1}{h}\frac{dy}{dP}$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\frac{d^2y}{dP^2}$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_0 + \frac{2P+1}{2}\nabla^2 y_0 + \cdots\cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_0 + (P+1)\nabla^3 y_0 + \cdots\cdots\right]$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_0 + \frac{2P+1}{2}\nabla^2 y_0 + \frac{3P^2+6P+2}{6}\nabla^3 y_0 + \frac{2P^3+9P^2+11P+3}{12}\nabla^4 y_0 + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_0 + (P+1)\nabla^3 y_0 + \frac{6P^2+18P+11}{12}\nabla^4 y_0 + \cdots\right]$$

At $x = x_0 \Rightarrow P = 0$
the formula reduces to,

$$\frac{dy}{dx} = \frac{1}{h}\left(\nabla y_0 + \frac{1}{2}\nabla^2 y_0 + \frac{1}{3}\nabla^3 y_0 + \frac{1}{4}\nabla^4 y_0 + \cdots\right)$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left(\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0 + \cdots\right)$$

Ex From the following data, find y' & y'' at $x = 2.25$ using up to 3rd difference only

x	1.00	1.25	1.50	1.75	2.00	2.25
y	2.7183	3.4903	4.4817	5.7546	7.3891	9.4877