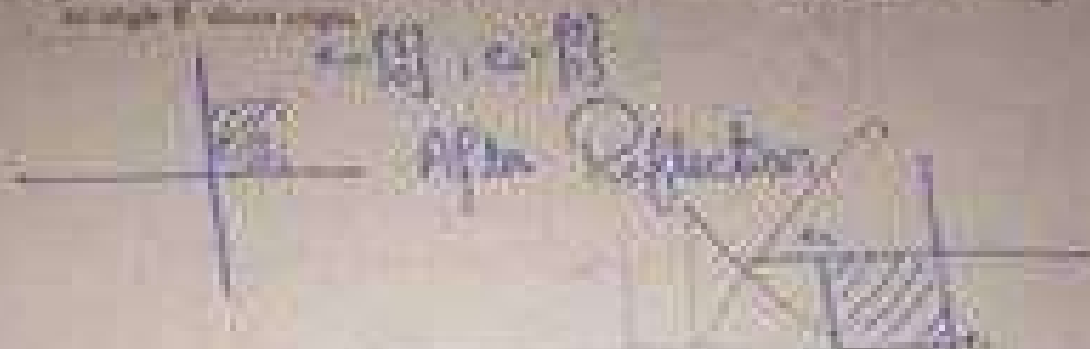


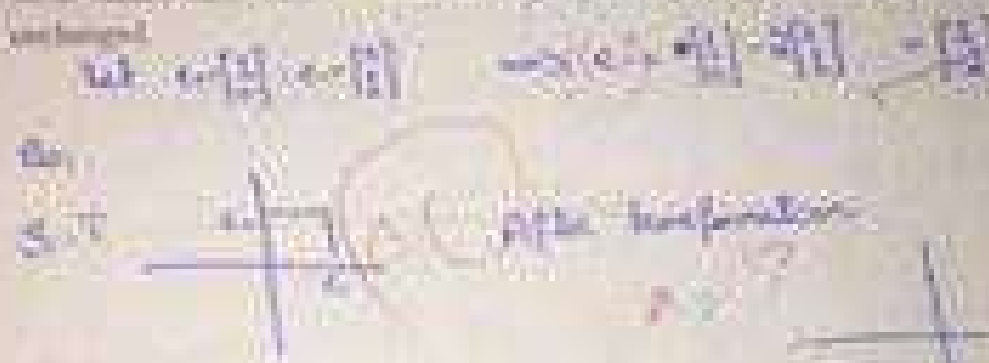
Question # _____ Page # _____

Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a transformation. Find the image of each of the

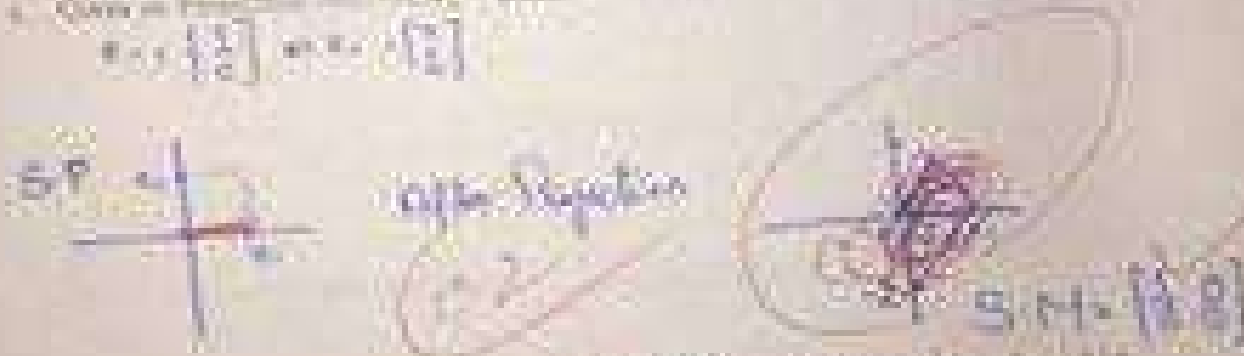
- a. Rotate each vector that belongs to the domain in counter clockwise direction through an angle θ about origin



- b. Given the vectors \vec{u} and \vec{v} in \mathbb{R}^2 , find $T(\vec{u})$ and $T(\vec{v})$ if T is a linear transformation that maps \vec{u} to \vec{v} and \vec{v} to \vec{u} .



- c. Given an ellipse E in \mathbb{R}^2 , find $T(E)$ if T is a linear transformation that maps \vec{u} to \vec{v} and \vec{v} to \vec{u} .



- d. Find vectors \vec{u} and \vec{v} in \mathbb{R}^2 such that $T(\vec{u}) = \vec{v}$ and $T(\vec{v}) = \vec{u}$ if T is a linear transformation that maps \vec{u} to \vec{v} and \vec{v} to \vec{u} .



Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation such that $T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

a) Determine whether the transformation is linear or not.

$$\begin{aligned} & T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = T\left(x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) \\ &= \left[x_1 T\begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 T\begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] \\ &= \left[x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] \\ &= \begin{pmatrix} x_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ x_2 \end{pmatrix} \\ &= \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\ &= \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\ &= CT(x) + dT(x) = CT(x) \end{aligned}$$

So, transformation is linear.

b) Find matrix of transformation.

$$\begin{bmatrix} x_1 & x_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let T be a tree with n vertices.

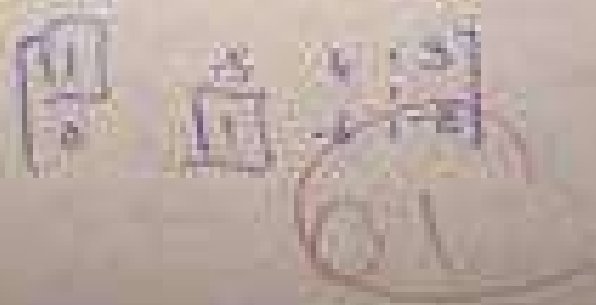
As all the $P_{n-1}(v, w)$ are
 paths and there are not any cycles in
 a tree, T is a path and is
 P_n .

Let T be a tree with n vertices.

As T is a tree, it has no cycles and
 only P_n is a path.

Find a matrix A whose image is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Page 407, Ex 1



PTO

1. Find the range of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5x_1 + 4x_2 \\ 0 \\ x_1 - 6x_2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - 10x_1 + 4x_2 \\ 0 \\ 0 + 2x_1 - 18x_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 - 10 + 4 \\ 0 \\ 2 - 18 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -5 \\ 0 \\ -16 \end{bmatrix}$$

2. Determine which the columns of A are linearly dependent or independent

As there are 3 columns and 2 rows and last column is 0 so it is linearly dependent.

0

3. Do the column vectors of A span \mathbb{R}^2

As there are 3 columns and 2 rows hence they do not span \mathbb{R}^2

Q1. Find a matrix A such that $A^2 = A$ and $A \neq 0$.
 (Hint: Consider the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and check if it satisfies the conditions.)



Question 11

Answer 11

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Check whether the matrix A satisfies the conditions of the question.

Properties of A :

- (1) System A is invertible.
- (2) Solution of A is found.
- (3) System is linearly dependent.
- (4) System is not in row echelon form.
- (5) System is not in row echelon form.

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Let $T: \mathbb{R}^4 \rightarrow \mathbb{R}^4$ be a linear transformation. The matrix of T with respect to the standard basis of \mathbb{R}^4 is given by

As given

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

As we can see that T maps \mathbb{R}^4 into \mathbb{R}^4 and there are all parts positive. So transformation must be one to one.



Find the standard matrix A that gives a reflection through the plane $x_1 = 0$.

