National University of Computer and Emerging Sciences, Lahore Campus CS1005 Course Code: Discrete Structures Course: Spring 22 Semester: BSE Program: Total Marks: 30 60 mins Duration: 15 Weightage 09-05-2022 Paper Date: Page(s): 2A,4B,4A Section: Roll No: Mid-II Exam: Attempt All Questions(Each question carry five marks) Instruction/Notes:

- 1. Show that $2^{2n}-1$ is divisible by 3 by using mathematical induction where n is positive integer.
 - a) What is the statement P(1)?
 - b) Show that P(1) is true, completing the basis step of the proof.
 - c) What is the inductive hypothesis?
 - d) What do you need to prove in the inductive step?
 - e) Complete the inductive step.

2. Prove that √2 is irrational number. Contradiction.

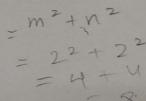
3. Let m be an integer with m > 1. Show that the relation $R = \{(a, b) \mid a \equiv b \pmod{m}\}$ is an equivalence relation on the set of

integers.

4. Let R and S be relations on a set A represented by the matrices

$$M_{R} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} , M_{S} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find a) $R \circ S$ b) $R \cup S$



5. Determine whether the function $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is bijective if

$$\mathbf{a}) f(m, n) = m + n$$

b)
$$f(m, n) = m^2 + n^2$$

6. Given the matrix representing a relation on a finite set, Write code fragment to determine whether the relation is symmetric and/or antisymmetric.

GOOD LUCK