FINAL EXAM SOLUTIONS
fall 2018

QUESTION 1 **(Marks: 5+2+5+2)**

Given the following predictions and their corresponding labels:

prediction	0.9	-0.2	0.82	0.01	0.97	0.2	-0.7	0.6	-0.95	0.8
label	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1

Suppose predictedLabel = +1 if prediction ≥ threshold, otherwise predictedLabel = -1.

a. Find 3 points on the ROC curve for thresholds 0.85, 0.5, 0. Write the points and plot them (working not required).

Solution: The three points are:

1. threshold = 0.85, (fpr,tpr) = (1/6,1/4)
2. threshold = 0. 5, (fpr,tpr) = (1/2,1/2)
3. threshold = 0, (fpr,tpr) = (2/3,3/4)

b. Make the confusion matrix for threshold = 0

TP = 3, FP =4, FN = 1, TN = 2

c. Find for threshold = 0

i. balanced error rate = __11/24__ ii. sensitivity = __3/4__

iii. specificity = _____1/3_________ iv. precision = ___3/7______ v. recall = ___3/4__

d. For what maximum value of threshold do you get maximum recall? __-0.2__

QUESTION 2 **(Marks: 2+2+2+2+2)**

Given the following transition matrix for a Markov chain:

1 0 0
0
0.9 0 0.1
0

a. Is it an absorbing Markov chain? x yes • no

b. $P(S_2$ at time $t=3|S_1$ at time $t=2)$ = _______0_____________

c. $P(S_3$ at time $t=4|S_2$ at time $t=2)$ =______0.05_____________

d. $P(S_4$ at time $t=5|S_2$ at time $t=2)$ =______0.025+0.05=0.075___________

e. Probability of the Sequence $S_2S_3S_3S_4$ =______0.025________

QUESTION 3 **(Marks: 6+2)**

a. Find the steady state vector of the following transition matrix P of a regular Markov chain. Show all working.

P = 1/5 4/5
3/10

Solution

v = [3/11 8/11]^T

b. What is the value of P^{1000}?

Solution

$$\begin{pmatrix} 3/11 & 8/11 \\ 3/11 & 8/11 \end{pmatrix}$$

QUESTION 4 (Marks: 3+3+3+3+4)

a. Do the following set of vectors form a linearly independent set?

i. [1 10 20]^T [3 10 5]^T [4 20 25]^T • yes **• no**

ii. [1 10]^T [3 31]^T [2 22]^T • yes **• no**

iii. [1 10 20 1]^T [3 11 15 2]^T [14 20 26 4]^T [19 90 66 4]^T **• yes** • no

b. What is the span of [1 2]^T and [5 8]^T

The two dimensional space R^2 of all vectors

c. What is the span of [1 2]^T and [2 4]^T

Subspace of all vectors in R^1 in the direction of the unit vector [1/sqrt(5) 2/sqrt(5)]^T

d. Find the projection of (2,1,0,1) onto **w** = (0,1,2,-1).

zero vector

e. Find the Mahalonobis distance between the point (1,2,1) and (0,0,0) when given the following covariance matrix. Show working/formula:

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Solution

Not possible as the covariance matrix should be a 3x3 matrix as the points also lie in the three dimensional space.

QUESTION 5 (Marks: 6+2+2)

Given the following parameters after training an SVM machine.

x1	1	2	1	3	2	3
x2	1	2	3	4	5	6
target	+1	+1	+1	-1	-1	-1
α	0	1	3	2	2	0

a. Find the weights of the SVM machine and the equation of the separating boundary.

Solution

w = [-5 -7]^T

w_0 = 25 (for the first support vector (2, 2)

b. Suppose the weights found by the SVM machine are: w_0 = 1, **w** = [1 2]^T. Find the classification of the points: (1,4) and (0,6)

Solution

(1,4) classified as positive point (y=10)
(0,6) classified as positive point (y=13)
c. Draw the decision boundary for weights $w_0 = -1$, $\mathbf{w} = [2\ \ 1]^T$

QUESTION 6 (Marks: 6)

Diagonalize the following matrix. Show working and write the final answer.

$$\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

Solution

$$\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1/sqrt(2) \\ 1/sqrt(2) \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1/sqrt(2) \\ -1/sqrt(2) \end{pmatrix}$$

QUESTION 7 (Marks: 6+6+6+6)

a. Given the following values of the standard Normal variable:
$z_{0.10} = 1.282$, $z_{0.05} = 1.645$, $z_{0.025} = 1.960$, $z_{0.01} = 2.326$, $z_{0.005} = 2.576$.
Find the 95% confidence interval for the errors made by a machine learning algorithm, which was tested on 10 OCR samples and committed two errors.

Solution

Interval: [-0.0479   0.4479]

b. Suppose a security system is used to authorize people to enter a building. In 70% cases the system allows a person to enter. Suppose the probability of an un-authorized person entering the building is 0.1. The system only allows 60% of authorized persons to enter the building. Suppose the system allows a person to enter then what are the chances that the person is authorized? Show all working/formula.

Solution:

0.6*0.9/0.7 = 27/35

c. Suppose a lab technician observed a total of 240 people. 80 people had cancer. 25% people with cancer were exposed to high nitrogen levels. Out of 240 people observed, 60 people were exposed to high nitrogen levels. Using probability theory can we conclude from this data that having cancer depends on being exposed to high nitrogen levels or are the two independent? How do you make your conclusion? Show your justification/formulas clearly.

Solution

Yes, the data shows that they are independent as
P(nitrogen exposure| cancer) = P(nitrogen exposure) = 1/4

d. Given the following data for 3 attributes x_1,x_2,x_3 and their corresponding labels

x_1	0	1	0	1	0
x_2	0	1	1	1	1
x_3	1	0	1	0	1
label	+1	+1	+1	-1	-1

Give the classification of the point (0,1,0) when a naive Baye's classifier is used. Show all working.

Solution

Let $\mathbf{x}$ = (0,1,0)
P(C=+1|$\mathbf{x}$) = 4/45*1/P($\mathbf{x}$)
P(C=-1|$\mathbf{x}$) = 1/10*1/P($\mathbf{x}$)

Predicted label = -1

QUESTION 8 **(Marks: 1+6)**

Draw the feasible region and find the stationary points of the given function using the method of Lagrange multipliers. Show working:

$f(\boldsymbol{x}) = 2x_1^2+x_2$

subject to $x_1 = -x_2$

Solution

feasible region is all points on the line $x_1 = -x_2$

stationary point = (1/4, -1/4)