Discrete Structures

(exercise question with solutions)

Text book: Kenneth H. Rosen, Discrete Mathematics and Its Applications

Exercise 6.2

9. What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

$$\left\lceil \frac{N}{50} \right\rceil \geq 100$$

$$\Rightarrow N \geq 99 \cdot 50 + 1 = 4951$$

Exercise 6.2

15. How many numbers must be selected from the set $\{1, 2, 3, 4, 5, 6\}$ to guarantee that at least one pair of these numbers add up to 7?

$\{1, 6\}, \{2, 5\}, \{3, 4\}$

we have to select 4 numbers to have at least one pair of these numbers add up to 7.

Exercise 6.2

19. Suppose that every student in a discrete mathematics class of 25 students is a freshman, a sophomore, or a junior.

a) Show that there are at least nine freshmen, at least nine sophomores, or at least nine juniors in the class.

b) Show that there are either at least three freshmen, at least 19 sophomores, or at least five juniors in the class.

(a) we assume otherwise, that this is not the case, then every group has at most 8 students which is not possible as total students are 25.

(b) again if the statement were not true then would be at most 2 freshman, at most 18 sophomores & at most 4 juniors. which makes a total of 24 students. Contradiction.

Exercise 6.2

***23.** Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.

No. of Seats = 50

1st seat is adjacent to seat 50

no. of odd numbered seats = 25

even " " = 25

let us assume that no more than 12 boys occupied the odd numbered seats

then at least 13 boys occupy even numbered seats. and vice versa

assume that at least 13 boys occupy the 25 odd numbered seats.

Then two of those boys must be consecutive in odd numbered seats and the person sitting those two boys always have boys as right & left neighbour

Exercise 6.2

39. Find the least number of cables required to connect 100 computers to 20 printers to guarantee that 2every subset of 20 computers can directly access 20 different printers. (Here, the assumptions about cables and computers are the same as in Example 9.) Justify your answer.

Do Yourself.

hint is given.

Exercise 6.5

7. How many ways are there to select three unordered elements from a set with five elements when repetition is allowed?

$$x_1 + x_2 + x_3 + x_4 + x_5 = 3$$

$$C(5+3-1, 3) = C(7,3) = 35$$

Exercise 6.5

21. How many ways are there to distribute six indistinguishable balls into nine distinguishable bins?

Using Theorem 2

$$C(9+6-1, 6) = C(14, 6)$$

$$= 3003$$

Exercise 6.5

20. How many solutions are there to the inequality

$$x_1 + x_2 + x_3 \leq 11,$$

where x_1, x_2, and x_3 are nonnegative integers? [*Hint:* Introduce an auxiliary variable x_4 such that $x_1 + x_2 + x_3 + x_4 = 11$.]

$$x_1 + x_2 + x_3 \leq 11$$

$$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 = 11 \quad , \quad x_4 \geq 0$$

$$C(11+4-1, 11) = C(14, 11)$$

$$\frac{14 \times 13 \times 12}{3!} = 28 \times 13$$

$$= 364$$

Exercise 6.5

31. How many different strings can be made from the letters in *ABRACADABRA*, using all the letters?

$$\frac{n!}{n_1!\, n_2!\, n_3! \cdots n_k!}$$

Using Theorem 3

Here $n = 11$

$n_1 = 5$ [5 As]

$n_2 = 2$ [2 Bs]

$n_3 = 1$ [1 C]

$n_4 = 1$ [1 D]

$n_5 = 2$ [2 Rs]

Answer:

$$\frac{11!}{5!\, 2!\, 1!\, 1!\, 2!}$$

$= 83160$