DS 501: STATISTICAL AND MATHEMATICAL METHODS FOR DATA SCIENCE

Quiz 2

October 18, 2018

PROBLEM

Find the singular value decomposition of:

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

SOLUTION

$$X = A^T A = \begin{pmatrix} 4 & 0 \\ 0 & 9 \end{pmatrix}$$

Since X is a diagonal matrix, its eigen values are the values on the diagonals ({4,9}) and the eigen vectors are parallel to the coordinate axis. Giving us the following:

$$\Sigma = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$

(note we write the higher diagonal value first)

$$V = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$\mathbf{u}_1 = X\mathbf{v}_1 = [\,0\ 2]^T$
$\mathbf{u}_2 = X\mathbf{v}_2 = [3\ 0]^T$

Normalizing the above vectors we get:

$$U = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Hence

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^T$$