CS 301: Theory of Automata
Quiz 2
October 08, 2019.

Problem
Use the pumping lemma to prove that the following language is not regular:
$L = \{0^m1^n \mid m>n \text{ and } n\geq 0\}$

Solution

We prove that L is not regular using a proof by contradiction technique.

Let us assume that L is regular, which means that there is a DFA with p states that can represent L. The pumping length for L is then taken as p.

Let us take the following string: $s = 0^{p+1}1^p$. As $s\in L$, then according to pumping lemma, we should be able to pump s if L is regular.

According to pumping lemmas let us split s into three strings: s = xyz such that |xy| <= p

as |xy| <= p, hence x and y both consist of a string of zeros. Here x can be the empty string or a string of zeros. Also, y would be a string with at least one zero as required by the pumping lemma. The maximum possible length for y is 0^p and minimum possible length is 0^a

We can see that $xy^iz \in L$ for i>0, however $xz \notin L$, as this string will have equal or less number of zeros than ones in both cases when we take y as 0^p or when we take y as 0. As any possible split of the string with |xy|<=p will lead to a string which cannot be pumped for i=0, we have a contradiction. Hence L is not regular.