

**Discrete Structures (CS1005)**

Date: November 4<sup>th</sup> 2024

**Course Instructors**

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**Sessional-II Exam**

**Total Time: 1 Hour**

**Total Marks: 30**

**Total Questions: 03**

Roll No

Section

Student Signature

**Attempt all questions. Simple scientific calculators are allowed. Attach question paper to the answer sheet.**

**CLO #3:** Apply fundamental concepts of number theory, such as divisibility, greatest common divisors, modular arithmetic, prime numbers, and congruences.

**Q. No 1:**

[10 = 3 + 4 + 3]

- If  $a$ ,  $b$  and  $c$  are integers, where  $a \neq 0$  such that  $a \mid b$  and  $a \mid c$ , then prove that  $a \mid mb + nc$  whenever  $m$  and  $n$  are integers.
- Find all solutions of the congruence  $5x \equiv 3 \pmod{7}$ .
- Find the remainder when  $5^{201}$  is divided by 13.

**CLO #4:** Apply mathematical induction to prove properties of sequences.

**Q. No 2:**

[5 = 2 + 3]

- Use mathematical induction to prove that 21 divides  $4^{n+1} + 5^{2n-1}$  whenever  $n$  is a positive integer.
- Use strong induction to prove that every integer  $n \geq 2$  can be written as a product of prime numbers.

**CLO #5:** Apply fundamental counting principles to solve combinatorial problems.

**Q. No 3:**

[5 + (10 = 2 + 2 + 2 + 2 + 2)]

- Show that the set of all positive rational numbers is countable.
- In a class, 28 students know C++, 26 know Python, and 32 know Java. Additionally, 14 know both C++ and Python, 12 know both Python and Java, and 10 know both C++ and Java. If 6 students know all three languages, how many students know at least one of these languages?

- (b) What is the least number of area codes needed to guarantee that the 24 million phones in a state can be assigned distinct 10-digit telephone numbers? (Assume that telephone numbers are of the form  $NXX-NXX-XXXX$ , where the first three digits from the area code,  $N$  represents a digit from 2 to 9 inclusive, and  $X$  represents any digit.)
- (c) How many bit strings of length 10 contain at least three 1s and at least three 0s?
- (d) Use the binomial theorem to find the coefficient of  $x^{12}y^6$  in the expansion of  $(2x^3 - 4y^2)^7$ .
- (e) How many different strings can be made by reordering the letters of the word "ARTHROSCOPICALLY".

$$4^{k+1} + 5^{2k+1} - 1$$

$$4^k \cdot 4 \quad \cancel{5^{2k+1}} \quad 5^2 + 5^k$$

$$4^k \cdot 4 - 25 + 5^k$$

$$4^k \cdot 4 - (25 + 4) + 5^k$$

$$4(4^k - 21 + 5^k)$$

$$2 \cdot 5^{2k+1}$$

$$25^k \cdot 25$$

$$25^k (21 - 4)$$