Parallel and Distributed Computing
CS3006 (BCS-6C/6D)
Lecture 21

Instructor: Dr. Syed Mohammad Irteza

Assistant Professor, Department of Computer Science, FAST

13 April, 2023

Previous Lecture

- MPI
  - MPI_Probe and MPI_Get_count
  - MPI_Barrier
  - MPI_Bcast
  - MPI_Reduce (different operations)
  - MPI_Allreduce
  - MPI_Scan (different operations)
  - MPI_Gather and MPI_Scatter
  - SPMD Model
  - MPI_Alltoall

Sorting in the Parallel Era…..

- Can we efficiently apply a Bubble-Sort type of sorting algorithm when the individual values are dispersed across different machines (processes)?

Sorting - Overview

- One of the most commonly used and well-studied Algorithms.
- Sorting can be *comparison-based* or *non-comparison-based*.
- The fundamental operation of comparison-based sorting is *compare-exchange*.
- The lower bound on any comparison-based sort of *n* numbers is $\Theta(n\log n)$.
- Let’s explore a comparison-based sorting algorithm.

Sorting – Basics

What is a parallel sorted sequence?

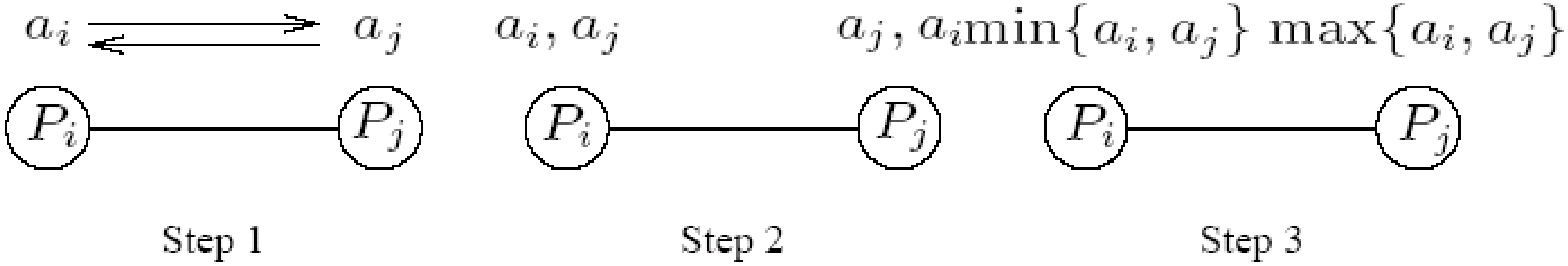
→ Where are the input and output lists stored?

Answers:

- We assume that the input and output lists are distributed.
- The sorted list is partitioned with the property that each partitioned list is sorted and each element in processor P_i's list is less than that in P_j's list if $i < j$.

Sorting: Parallel Compare Exchange Operation

- A parallel compare-exchange operation. Processes P_i and P_j send their elements to each other. Process P_i keeps $\min\{a_i, a_j\}$, and P_j keeps $\max\{a_i, a_j\}$.

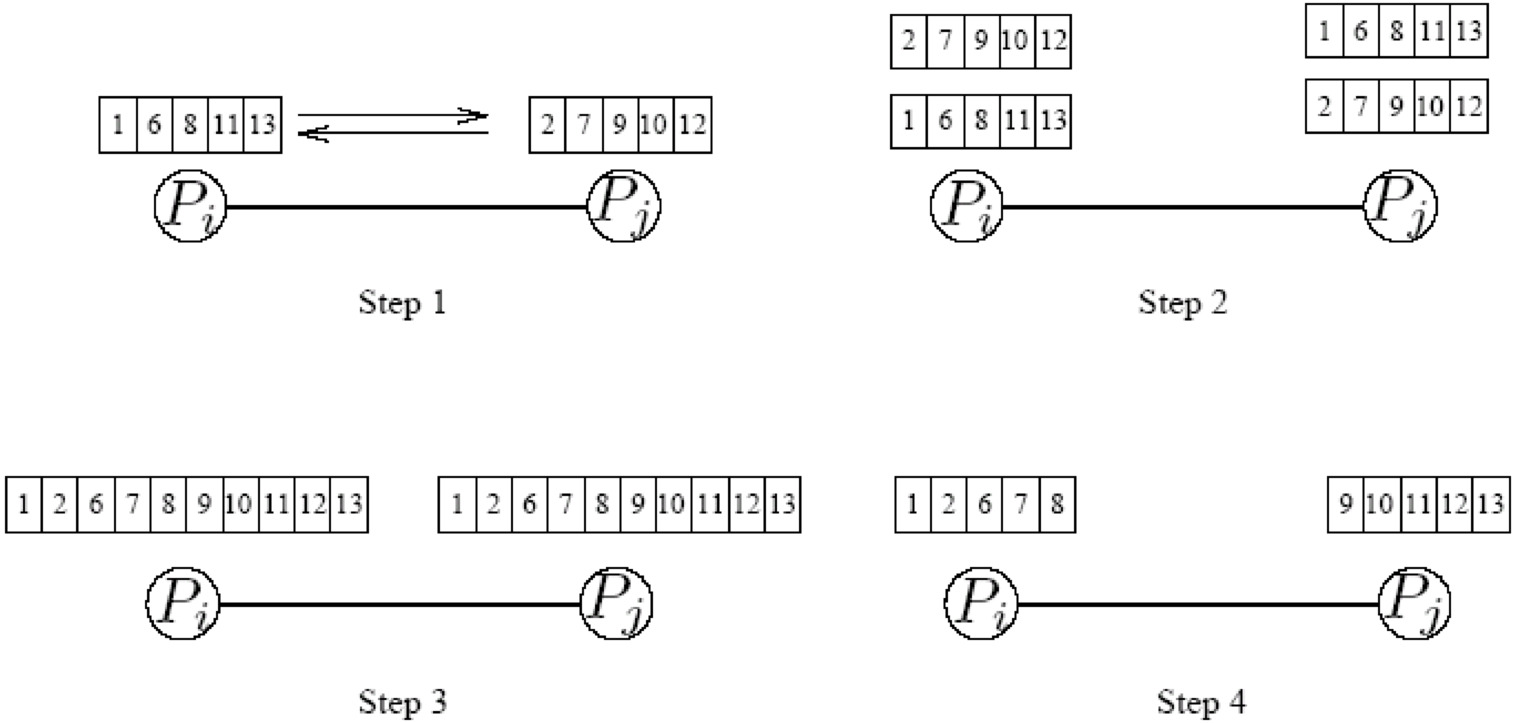


Sorting: Parallel Compare Exchange Operation [cost estimation]

- If each processor has one element, the compare exchange operation stores the smaller element at the processor with smaller id. This can be done in $t_s + t_w$ time.
- If we have more than one element per processor, we call this operation a compare split. Assume each of two processors have n/p elements.
- After the compare-split operation, the smaller n/p elements are at processor P_i and the larger n/p elements at P_j, where $i < j$.
- The time for a compare-split operation is $(t_s + t_w n/p)$, assuming that the two partial lists were initially sorted.
  - Note that this time is only accounting communication costs. Computation and memory complexities are separate things.

Sorting: Parallel Compare Exchange

- A compare-split operation. Each process sends its block of size *n/p* to the other process.
- Each process merges the received block with its own block and retains only the appropriate half of the merged block.
- In this example, process P_i retains the smaller elements and process P_j retains the larger elements.

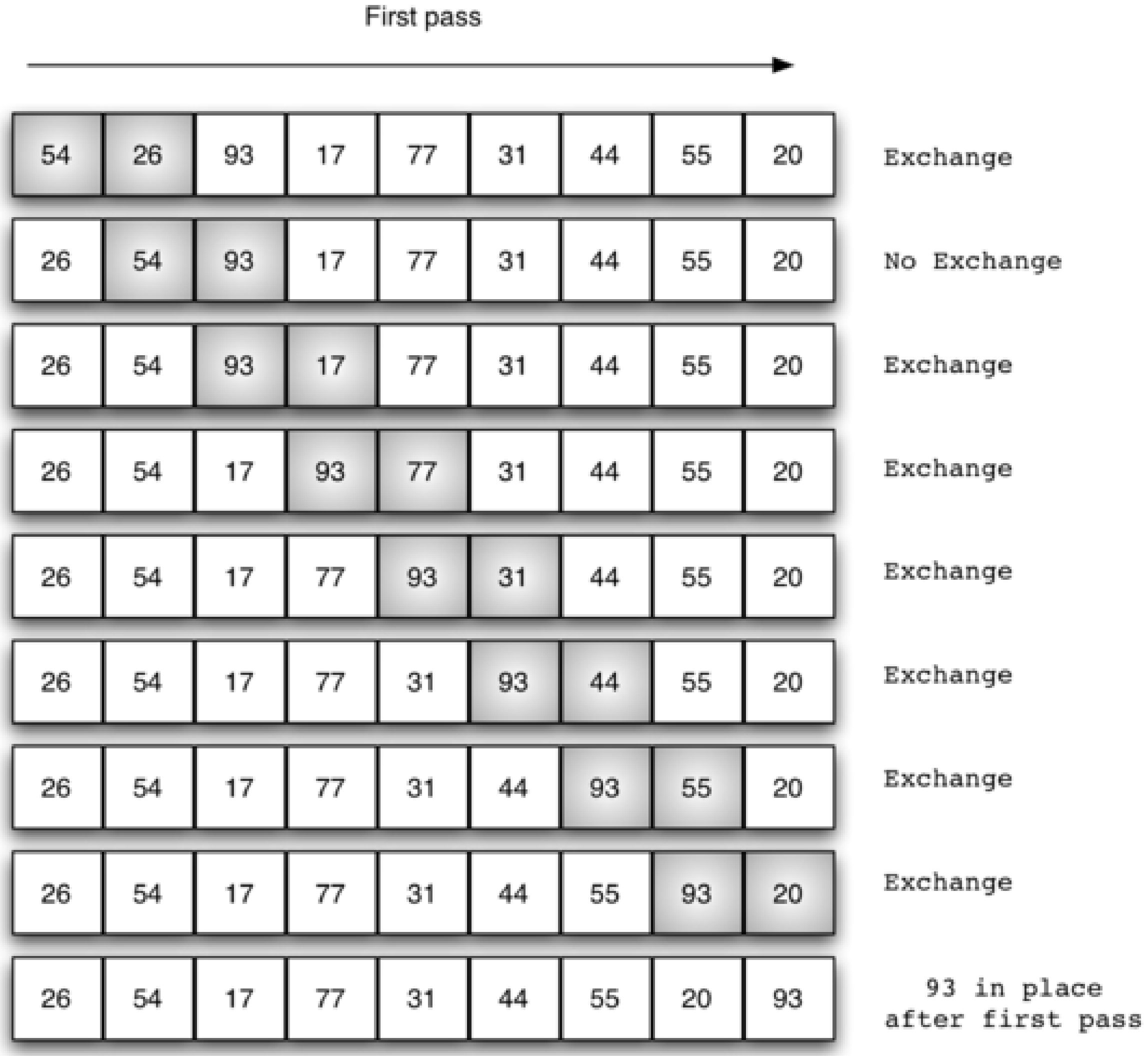


Bubble Sort and its Variant

- The sequential bubble sort algorithm compares and exchanges adjacent elements in the sequence to be sorted:

procedure BUBBLE_SORT(n)
begin
    for $i := n - 1$ **downto** 1 **do**
        for $j := 1$ **to** i **do**
            compare-exchange(a_j, a_{j+1});
end BUBBLE_SORT

Visualization (first pass, seq. BubbleSort)

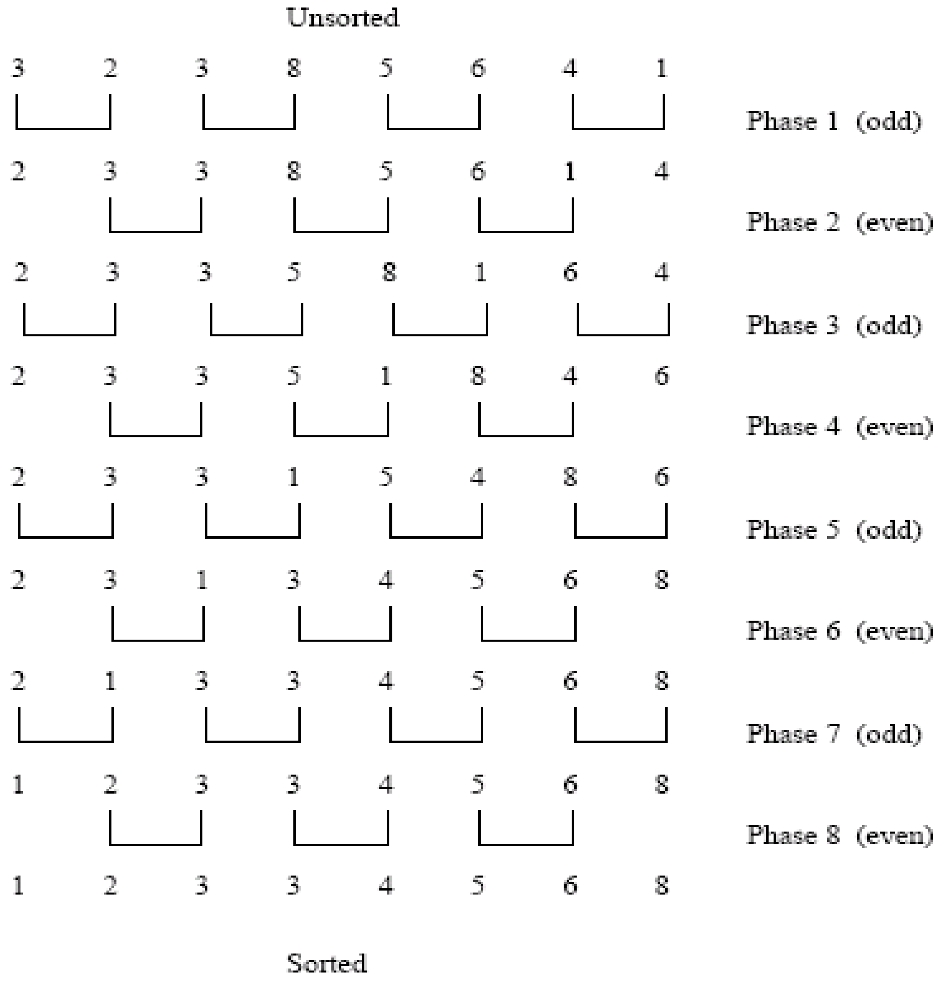


Bubble Sort and its Variant

- The complexity of bubble sort is $\Theta(n^2)$.
- Bubble sort is difficult to parallelize since the algorithm has no concurrency.
- A simple variant, though, uncovers the possible concurrency.

Bubble Sort [Odd-Even Transposition]

- Sorting n = 8 elements, using the odd-even transposition sort algorithm.
- During each phase, at most 8 elements are compared.
- **[This is according to the sequential algorithm]**



Bubble Sort [Odd-Even Transposition]

- Sequential odd-even sort algorithm

```
procedure ODD-EVEN(n)
begin
    for i := 1 to n do
    begin
        if i is odd then
            for j := 0 to n/2 - 1 do
                compare-exchange(a_{2j+1}, a_{2j+2});
        if i is even then
            for j := 1 to n/2 - 1 do
                compare-exchange(a_{2j}, a_{2j+1});
    end for
end ODD-EVEN
```

Odd-Even Sort (Seq. Complexity)

- After n phases of odd-even exchanges, the sequence is sorted.
- Each phase of the algorithm (either odd or even) requires $\Theta(n)$ comparisons.
- Serial complexity is $\Theta(n^2)$.

Parallel Odd-Even Sort

Time / Step	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
0	4	2	7	8	5	1	3	6
1	2	4	7	8	1	5	3	6
2	2	4	7	1	8	3	5	6
3	2	4	1	7	3	8	5	6
4	2	1	4	3	7	5	8	6
5	1	2	3	4	5	7	6	8
6	1	2	3	4	5	6	7	8
7	1	2	3	4	5	6	7	8

Parallel time complexity: $T_{par} = O(n)$ (for P=n)

Source: https://www.slideshare.net/richakumari37266/parallel-sorting-algorithm

Parallel Odd-Even Sort

- **Algorithm Through Observations:**

1. There are total **P** phases/steps. Where P is number of processes
2. **For even phases**
   i. If 'myrank' is even → Communication partner is ('myrank'+1)
   ii. If 'myrank' is odd → Communication partner is ('myrank' - 1)

3. **For odd phases:**
   i. If 'myrank' is even → Communication partner is ('myrank' - 1)
   ii. If 'myrank' is odd → Communication partner is ('myrank'+1)
4. Communication partners remain constant
5. If 'myrank' is less-than the partner, then keep lower values in compare-split-operation

Parallel Odd-Even Sort

Complexity when *n* = P

- Consider the one item per processor case.
- There are P iterations, in each iteration, each processor does one compare-exchange.
- The parallel run time of this formulation is **Θ(*n*)**.
- Parallel run time means computation performed by each of the processors in parallel.

Parallel Odd-Even Sort

Complexity when *n* > P

- Consider a block of *n/p* elements per processor.
- The first step is a local sort.
- In each subsequent step, the compare exchange operation is replaced by the compare split operation.
- The parallel run time of the formulation is:

$$T_P = \overbrace{\Theta\left(\frac{n}{p}\log\frac{n}{p}\right)}^{\text{local sort}} + \overbrace{\Theta(n)}^{\text{comparisons}} + \overbrace{\Theta(n)}^{\text{communication}}.$$

comm. steps for a single process

```
#include <stdlib.h>
#include <mpi.h> /* Include MPI's header file */

main(int argc, char *argv[])
{
  int n;          /* The total number of elements to be sorted */
  int npes;       /* The total number of processes */
  int myrank;     /* The rank of the calling process */
  int nlocal;     /* The local number of elements, and the array that stores them */
  int *elmnts;    /* The array that stores the local elements */
  int *relmnts;   /* The array that stores the received elements */
  int oddrank;    /* The rank of the process during odd-phase communication */
  int evenrank;   /* The rank of the process during even-phase communication */
  int *wspace;    /* Working space during the compare-split operation */
```

```
int i;
MPI_Status status;

/* Initialize MPI and get system information */
MPI_Init(&argc, &argv);
MPI_Comm_size(MPI_COMM_WORLD, &npes);
MPI_Comm_rank(MPI_COMM_WORLD, &myrank);

n = atoi(argv[1]);
nlocal = n/npes; /* Compute the number of elements to be stored locally. */

/* Allocate memory for the various arrays */
elmnts  = (int *)malloc(nlocal*sizeof(int));
relmnts = (int *)malloc(nlocal*sizeof(int));
wspace  = (int *)malloc(nlocal*sizeof(int));
```

```
/* Fill-in the elmnts array with random elements */
srandom(myrank);
for (i=0; i<nlocal; i++)
  elmnts[i] = random();

/* Sort the local elements using the built-in quicksort routine */
qsort(elmnts, nlocal, sizeof(int), IncOrder);

/* Determine the rank of the processors that myrank needs to communicate during */
/* the odd and even phases of the algorithm */
if (myrank%2 == 0) {
  oddrank  = myrank-1;
  evenrank = myrank+1;
}
else {
  oddrank  = myrank+1;
  evenrank = myrank-1;
}
```

```
  /* Set the ranks of the processors at the end of the linear */
  if (oddrank == -1 || oddrank == npes)
    oddrank = MPI_PROC_NULL;
  if (evenrank == -1 || evenrank == npes)
    evenrank = MPI_PROC_NULL;

  /* Get into the main loop of the odd-even sorting algorithm */
  for (i=0; i<npes-1; i++) {
    if (i%2 == 1) /* Odd phase */
      MPI_Sendrecv(elmnts, nlocal, MPI_INT, oddrank, 1, relmnts,
          nlocal, MPI_INT, oddrank, 1, MPI_COMM_WORLD, &status);
    else /* Even phase */
      MPI_Sendrecv(elmnts, nlocal, MPI_INT, evenrank, 1, relmnts,
          nlocal, MPI_INT, evenrank, 1, MPI_COMM_WORLD, &status);

    CompareSplit(nlocal, elmnts, relmnts, wspace,
                 myrank < status.MPI_SOURCE);
  }

  free(elmnts); free(relmnts); free(wspace);
  MPI_Finalize();
}
```

```c
/* This is the CompareSplit function */
CompareSplit(int nlocal, int *elmnts, int *relmnts, int *wspace,
             int keepsmall)
{
  int i, j, k;

  for (i=0; i<nlocal; i++)
    wspace[i] = elmnts[i]; /* Copy the elmnts array into the wspace array */

  if (keepsmall) { /* Keep the nlocal smaller elements */
    for (i=j=k=0; k<nlocal; k++) {
      if (j == nlocal || (i < nlocal && wspace[i] < relmnts[j]))
        elmnts[k] = wspace[i++];
      else
        elmnts[k] = relmnts[j++];
    }
  }
  else { /* Keep the nlocal larger elements */
    for (i=k=nlocal-1, j=nlocal-1; k>=0; k--) {
      if (j == 0 || (i >= 0 && wspace[i] >= relmnts[j]))
        elmnts[k] = wspace[i--];
      else
        elmnts[k] = relmnts[j--];
    }
  }
}
```

```
/* The IncOrder function that is called by qsort is defined as follows */
int IncOrder(const void *e1, const void *e2)
{
  return (*((int *)e1) - *((int *)e2));
}
```

Failure Handling

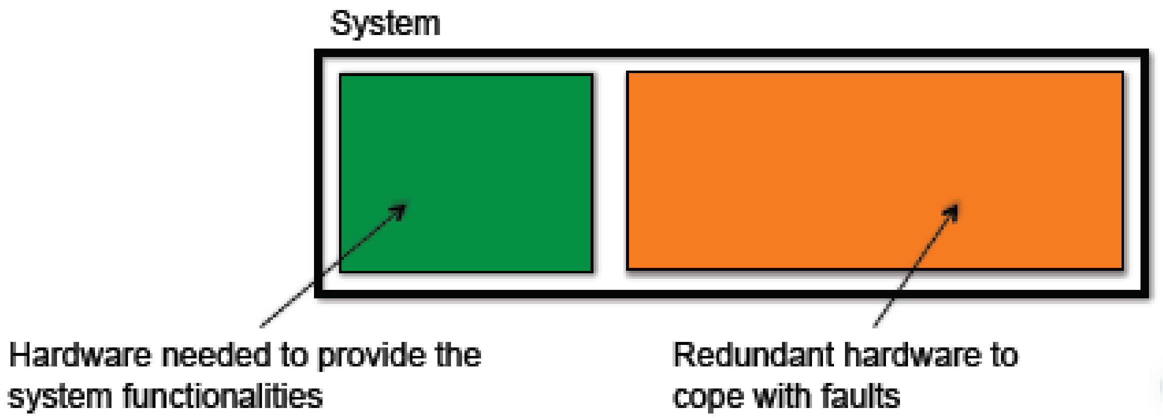
- As the number of components increases, the fault rate also increases
- How to detect failures and make corrections?
- Fault Tolerance
  - fault tolerance or graceful degradation is the property that enables a system to continue operating properly in the event of the failure of some of its components
  - a system continues its intended operation, possibly at a reduced level, rather than failing completely, when some part of the system fails
- Redundancy
  - redundancy is the duplication of critical components or functions of a system with the intention of increasing reliability of the system

Redundancy design techniques

- Hardware redundancy
- Information redundancy
- Time redundancy

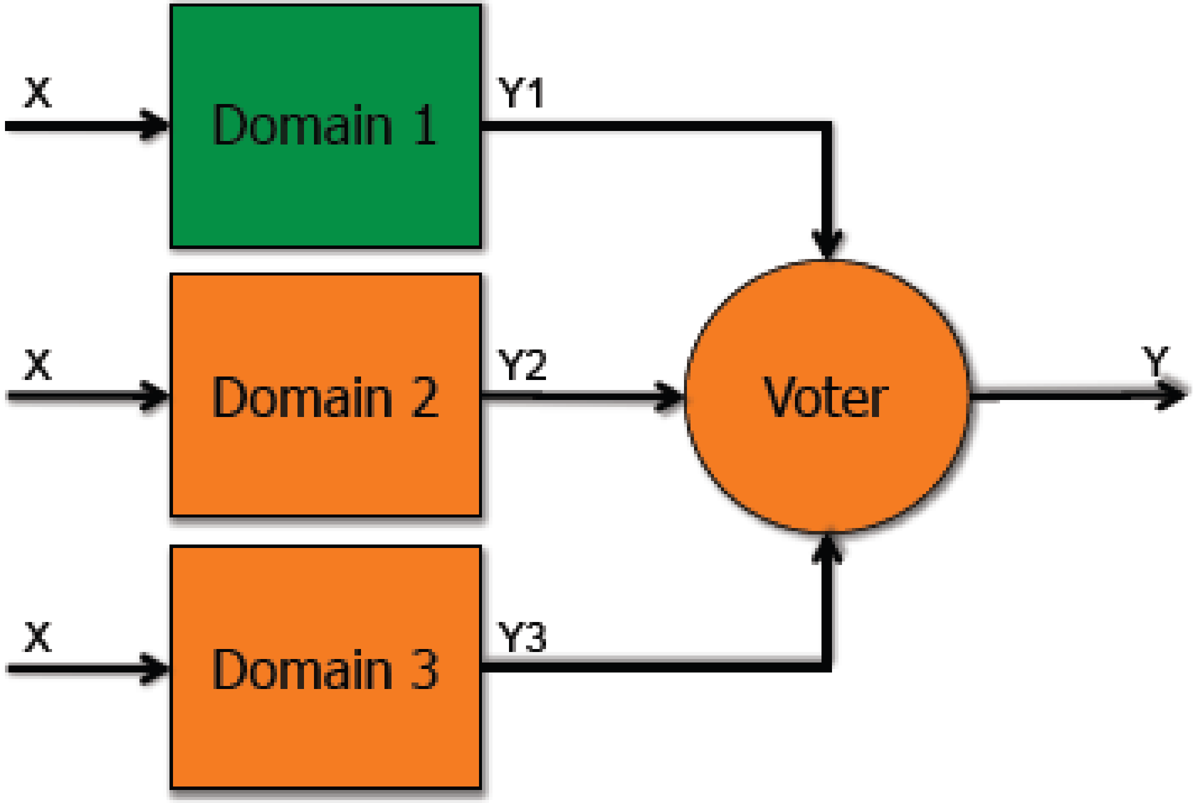
Hardware redundancy

- The system is implemented using *more hardware than that which is needed* for implementing the system functionalities
- The redundant hardware is used for *dealing with faults*



- Types of Hardware redundancy
  - Passive redundancy
  - Active redundancy
  - Hybrid redundancy

Passive Redundancy



- The HW needed to implement the system is replicated 3 times:
  - Triple Module Redundancy (TMR)

Passive Redundancy

- A *majority voter* decides the output to be committed to the user on the basis of the outputs coming from the three domains
- Voter implements the following functionality

```
P_VOTER: process( Y1, Y2, Y3 )
    Begin
      if Y1=Y2 then
            Y = Y1;
      else if Y1=Y3 then
            Y = Y1;
      else if Y2=Y3 then
            Y = Y2;
      else “display error”;
      end if;
    end process;
```

Passive Redundancy

Used to achieve fault tolerance:

- The voter *stops the propagation of faults* that never reach the outputs: error masking

- In case of *permanent fault of a domain no corrective actions are taken*: no error correction

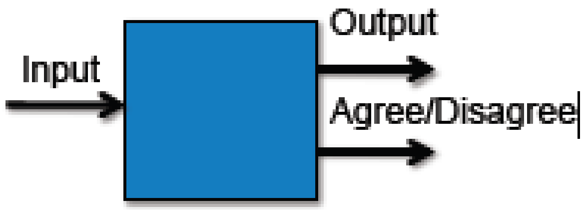
Cost:

- Area: >3x
- Time: negligible

Active Redundancy

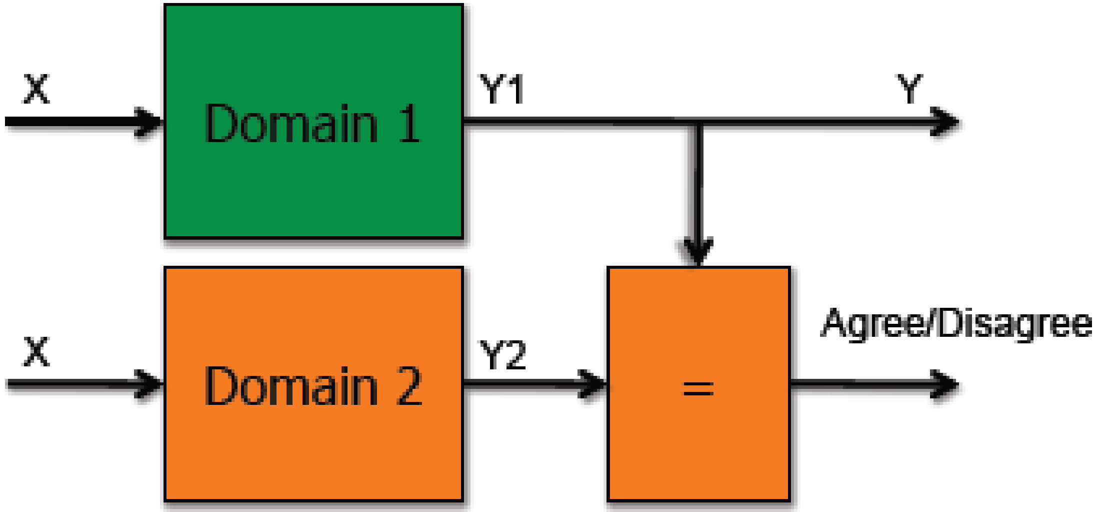
Alternative to passive redundancy

- It implements fault detection
- It may also possibly implement:
  - Error localization
  - Error containment
  - Error recovery



- The purpose of fault detection is to assert a *signal (Agree/Disagree)* every time the system output *differs from the expected* for the given input

Active Redundancy

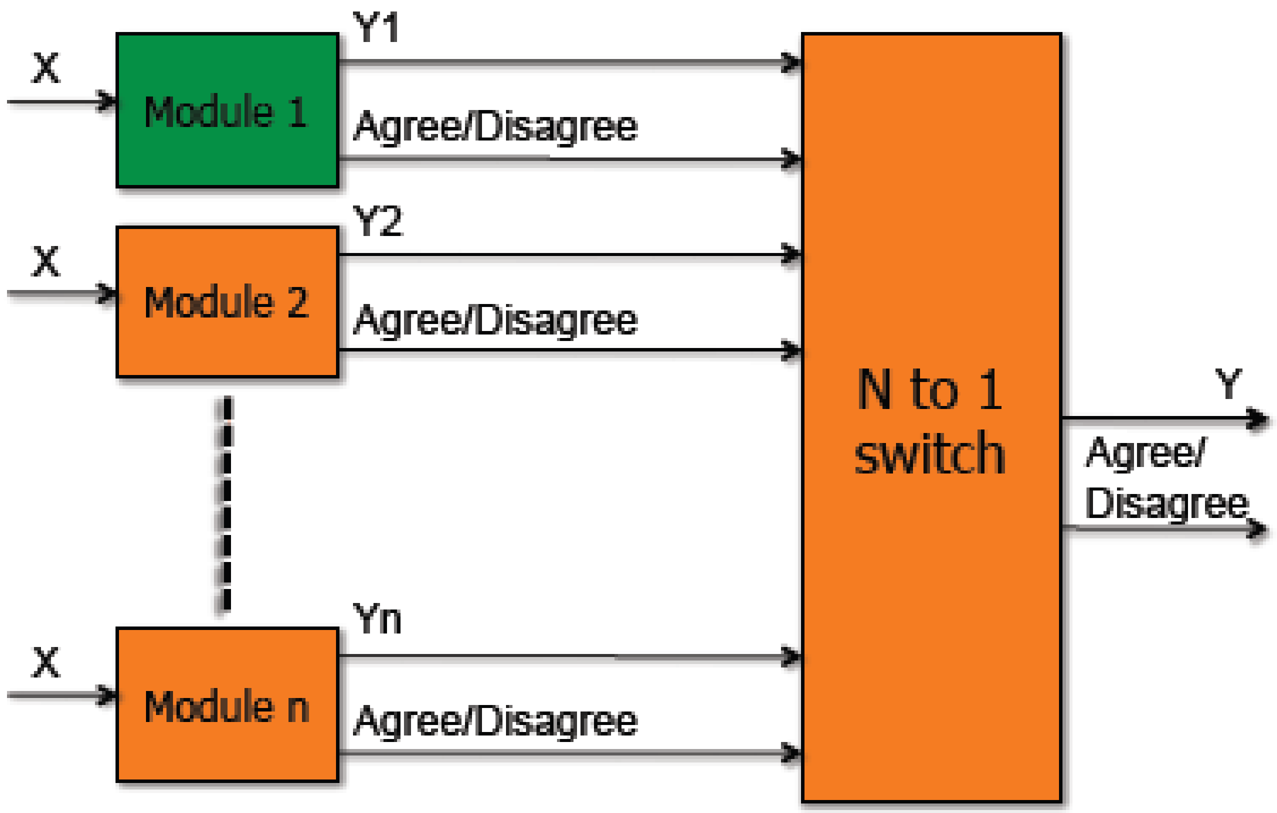


- Duplication with comparison

Active Redundancy

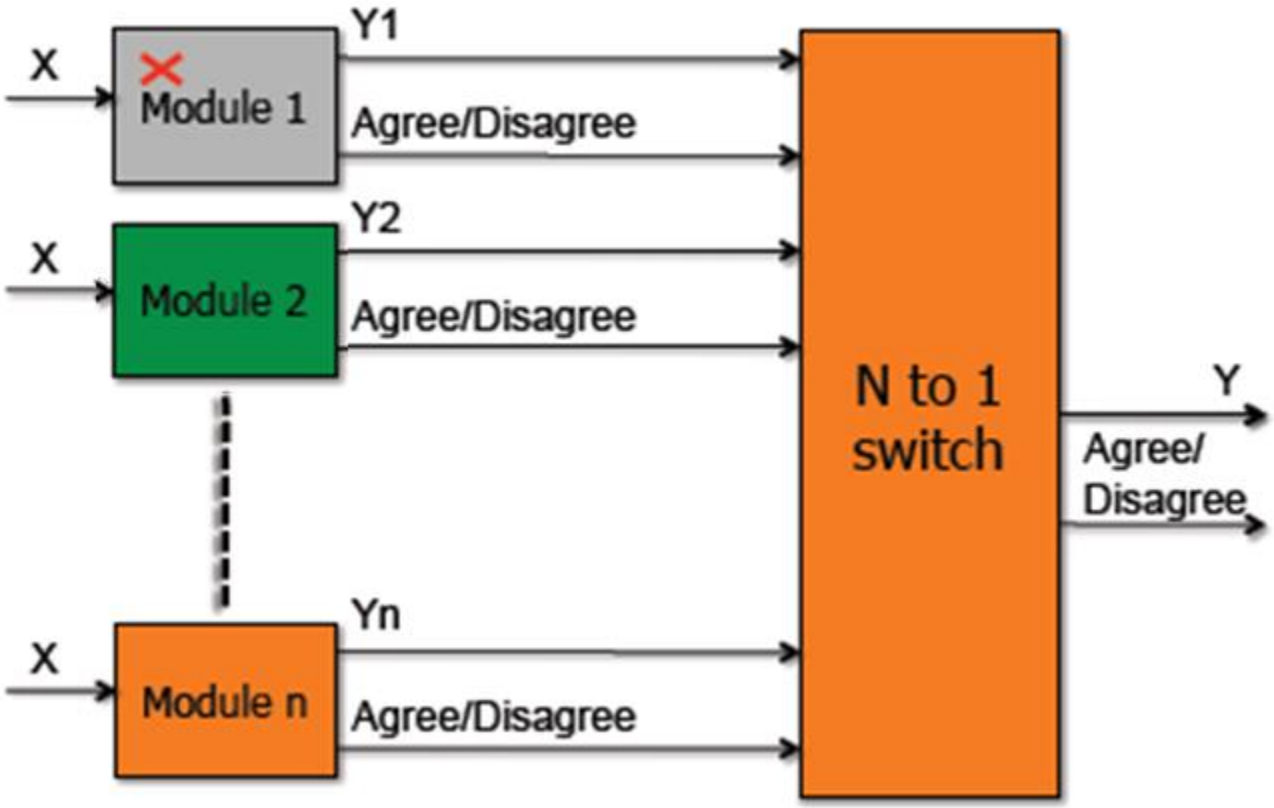
- The purpose of *error localization, containment and recovery* is to reconfigure the system *upon error detection* so that the correct system functionalities are recovered after a certain recovery time
- During recovery the *system is unavailable*:
- The system is not able to respond to inputs
- Maximum allowed recovery time is a function of the application
- Can be implemented using:
  - Hot standby sparing
  - Cold standby sparing

Hot standby sparing



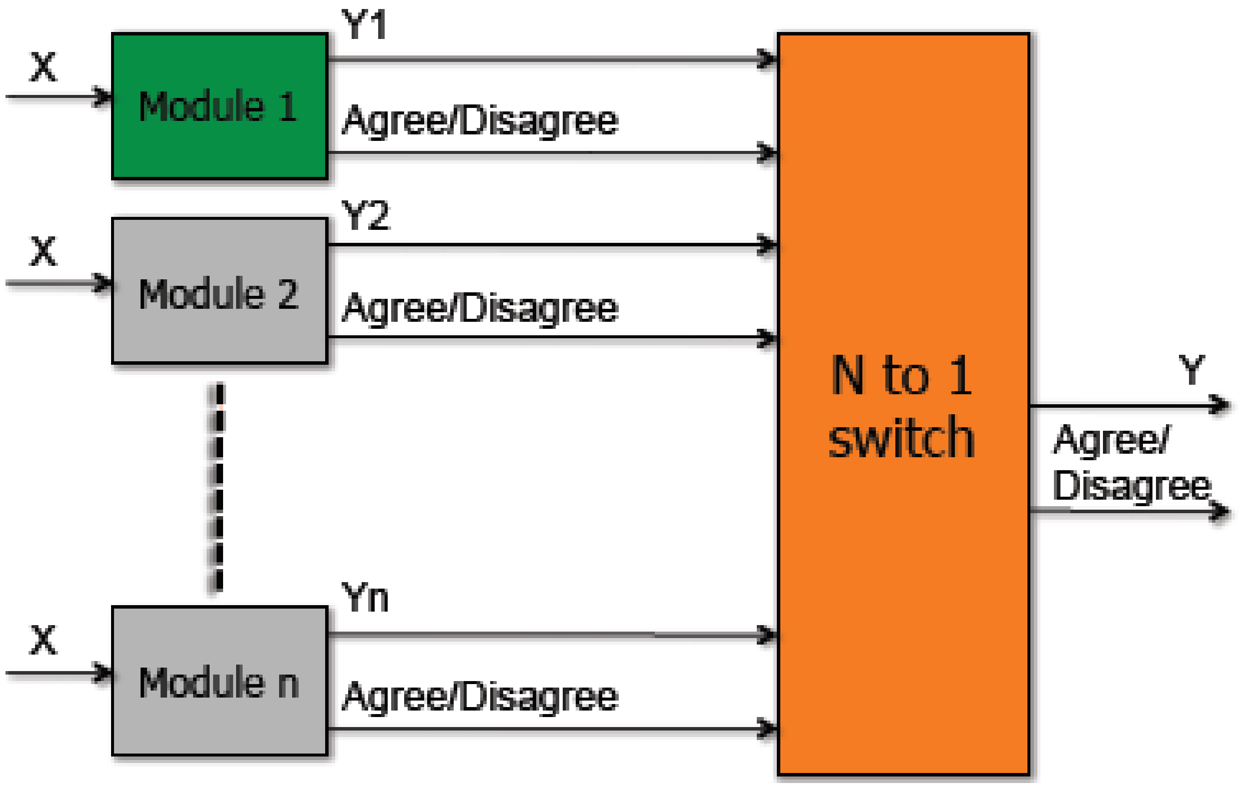
- N modules are used all active at the same time
- One module is the primary module, others are spare modules

Hot standby sparing



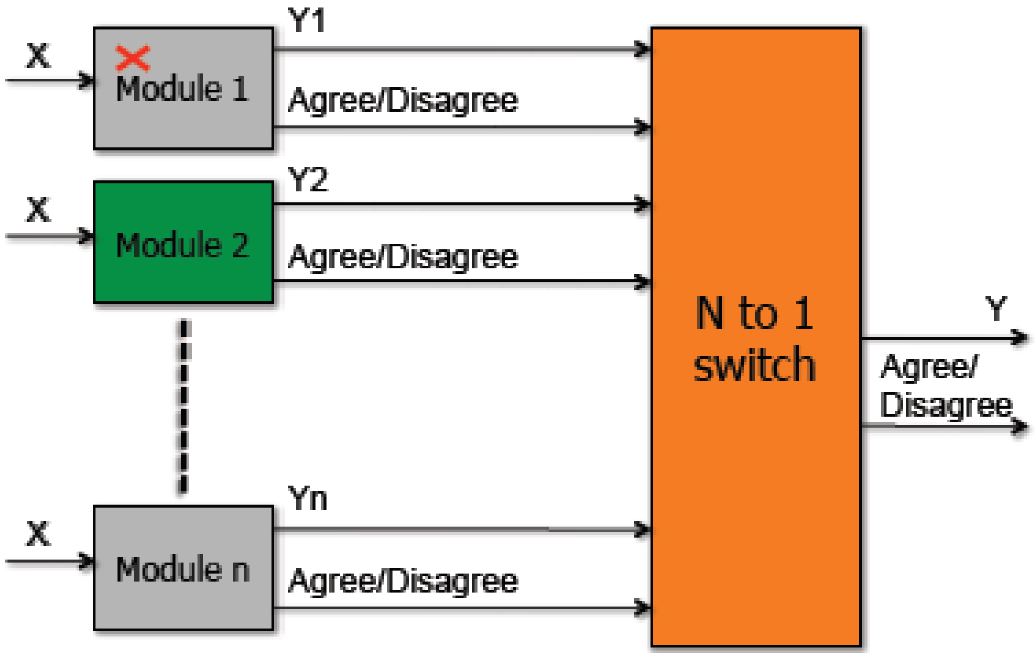
- When an error is detected, the switch sets the faulty primary module to *not-used state*, and it selects a new primary module among the *spares*

Cold standby sparing



- N modules are used: one active, the *primary module*, the others powered-off, the *spare modules*.

Cold standby sparing



- When an error is detected, it *powers-off* the *primary module*, selects a new primary module among the *spares*, and then turns it on.

Dealing with faulty modules

- In hot standby two methods can be used for dealing with the faulty module
- Permanently evicted:
  - When detected as faulty, it is powered-off and no longer used
  - The number of spares decreases over time
- Temporarily evicted:
  - When detected as faulty, it is set to idle but not powered off
  - The switch monitors the status of the idle module and, in case it is no longer affected by faults, it is promoted to the role of spare module
  - In case of transient faults the number of spares remain constant over time

Tradeoffs

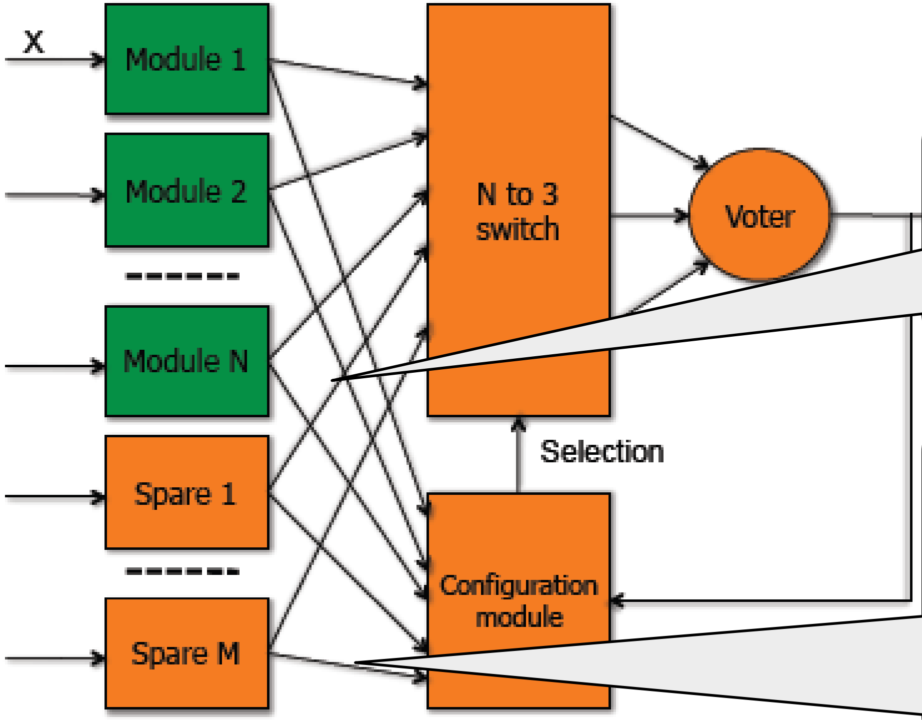
- Active redundancy for error detection only is less expensive than TMR, but does not provide masking
- Standby sparing is more expensive than TMR but it provides error correction:
  - It can survive up to N-1 permanent errors, and provide error detection in case of N permanent faults
  - TMR is able to survive 1 permanent fault, only

Cold sparing vs. hot sparing

- Area: same
- Energy: cold sparing uses 1/n of the energy of hot sparing
- Availability: in hot sparing, availability is higher than in cold one

Hybrid Redundancy

N primary modules and M spare modules



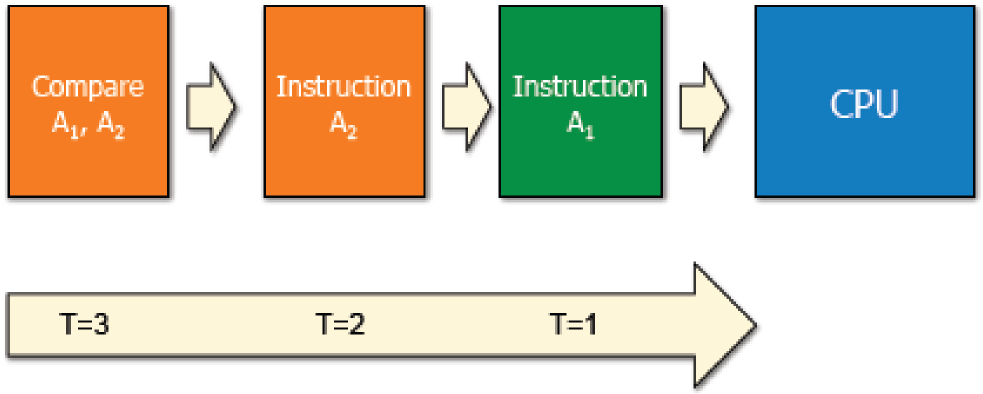
N to 3 switch selects 3 out of N primary outputs to be voted by the voter.

The configuration module detects the primary modules whose outputs differ from the actual one (Y).
Each faulty primary module is replaced by a spare one.

- Combines passive and active redundancies to achieve:
  - Error masking
  - Error correction
- Combining TMR with sparing we can have N modular redundancy with spares

Time Redundancy

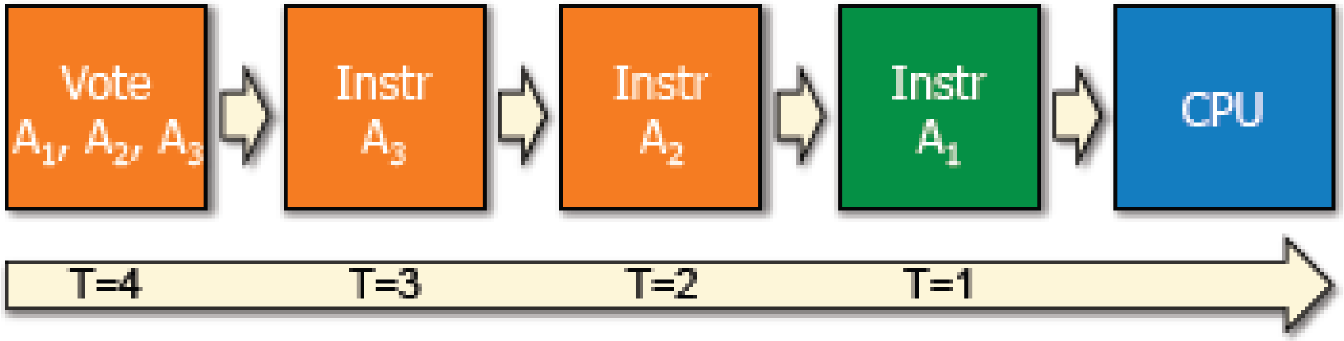
- More time than that needed for processing an input is used
- The additional time is devoted to detect and possibly correct errors occurred during the processing



- The same instruction is executed twice, and a comparison detects the occurrence of errors during the computations

Time Redundancy

- Error Correction:



- Cost:
  - Area: almost negligible
  - Time: ~3x for detection, ~4x for correction

Information Redundancy

- More data than those needed by the application are stored

- The *redundant data* added to the original data is used to detect and possibly *correct errors affecting the original data*
- The redundant data is a function of the original one

Purpose of information redundancy

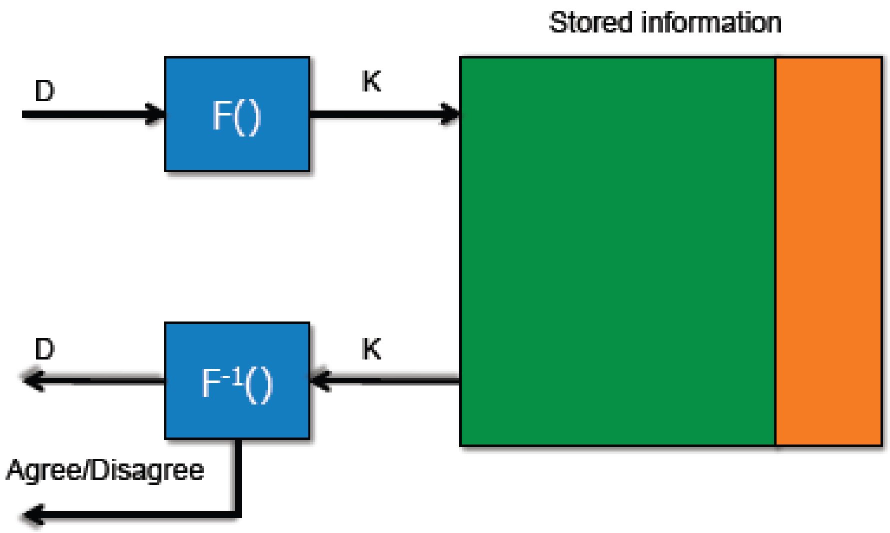
- Vulnerable components
  - Channels
  - Processes (clients, servers)

- Security properties:
  - Authentication
  - Authorization
  - Confidentiality
  - Integrity
  - Availability

Cryptography

- Encoding operation: $K=F(D)$
  - D is the original data over N bits
  - F() is the encoding function
  - K is the encoded information, codeword, over M>N bits
- Decoding operation: $D=F^{-1}(K)$
  - $F^{-1}()$ is the decoding function
- In case there is some error during transmission then codeword K will be transformed to K* and therefore for each D, $K^* \neq F(D)$

Information redundancy



- Several redundancy schemes are possible that provides:
  - Error detection
  - Error correction
- Schemes differ in the type of error addressed:
  - Single error
  - Multiple error

Information redundancy

- Examples of information redundancy are:
  - Parity codes
  - Hamming codes
  - Reed Solomon codes

Parity Codes

- N data bits are coded in a N+1 codeword
  - The codeword is such that:
  - The number of ‘1’ in the codeword is even: even parity
  - The number of ‘1’ in the codeword is odd: odd parity
  - Any single error in the codeword can be detected but not corrected

References

1. Slides of Dr. Rana Asif Rehman & Dr. Haroon Mahmood
2. Kumar, V., Grama, A., Gupta, A., & Karypis, G. (1994). *Introduction to parallel computing* (Vol. 110). Redwood City, CA: Benjamin/Cummings.
3. Quinn, M. J. Parallel Programming in C with MPI and OpenMP, (2003).

Helpful Links:

1. https://mpitutorial.com/tutorials/mpi-send-and-receive/
2. http://boron.physics.metu.edu.tr/ozdogan/GraduateParallelComputing.old/week11/node2.html (Odd-Even Sort)
3. https://mpitutorial.com/tutorials/mpi-scatter-gather-and-allgather/