3A Quiz (3)

National University of Computer and Emerging Sciences, Lahore Campus

Course:	Linear algebra	Course Code:	
Program:		Semester:	
Duration:		Total Marks:	(03)
Paper Date:	16/12/2021	Weight	
Section:	BS SE 3A	Page(s):	
Exam:		Roll No:	
Instruction/Notes:	Attempt All Questions		

Q① Find the characteristic equation & eigenvalues of the matrix $A = \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$

Characteristic Eq.:

$\det(\lambda I - A) = 0$

$\left| \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix} \right| = 0$

$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix} \right| = 0$

$\begin{vmatrix} (\lambda+2) & 7 \\ -1 & (\lambda-2) \end{vmatrix} = 0$

2.6 / 3.0

$(\lambda+2)(\lambda-2) - (-1)(7) = 0$

$\lambda^2 - 2\lambda + 2\lambda - 4 + 7 = 0$

$\lambda^2 + 3 = 0, \quad \lambda^2 = -3$

← characteristic eq.

No real eigen values

1.0

Q② Find matrix P that diagonalizes A & check your work by computing $P^{-1}AP$. ; $A = \begin{bmatrix} 4 & 0 \\ 2 & 4 \end{bmatrix}$

Characteristic Eq.

$\det(\lambda I - A) = 0$

$\left| \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 2 & 4 \end{bmatrix} \right| = 0$

$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 2 & 4 \end{bmatrix} \right| = 0$

$\begin{vmatrix} (\lambda-4) & 0 \\ -2 & (\lambda-4) \end{vmatrix} = 0$

$(\lambda-4)(\lambda-4) - 0 = 0$

$(\lambda-4)^2 = 0$ (characteristic equation)

$\lambda = 4, 4$ (eigen values)

(P.T.O)

For $\lambda = 4$

$$Ax = \lambda x$$

$$\begin{bmatrix} 4 & 0 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 4 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$4x_1 = 4x_1$$

$$2x_1 + 4x_2 = 4x_2$$

$$2x_1 = 4x_2 - 4x_2$$

$$2x_1 = 0$$

$$\boxed{x_1 = 0}$$ Let $x_2 = t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} 0 \\ t \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boxed{\text{basis } P_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

~~basis $P_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$~~

1.6

$$P = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

To Find

$P^{-1} A P$

$P^{-1} = \frac{1}{\det}$

$\det = 0$

inverse is not possible