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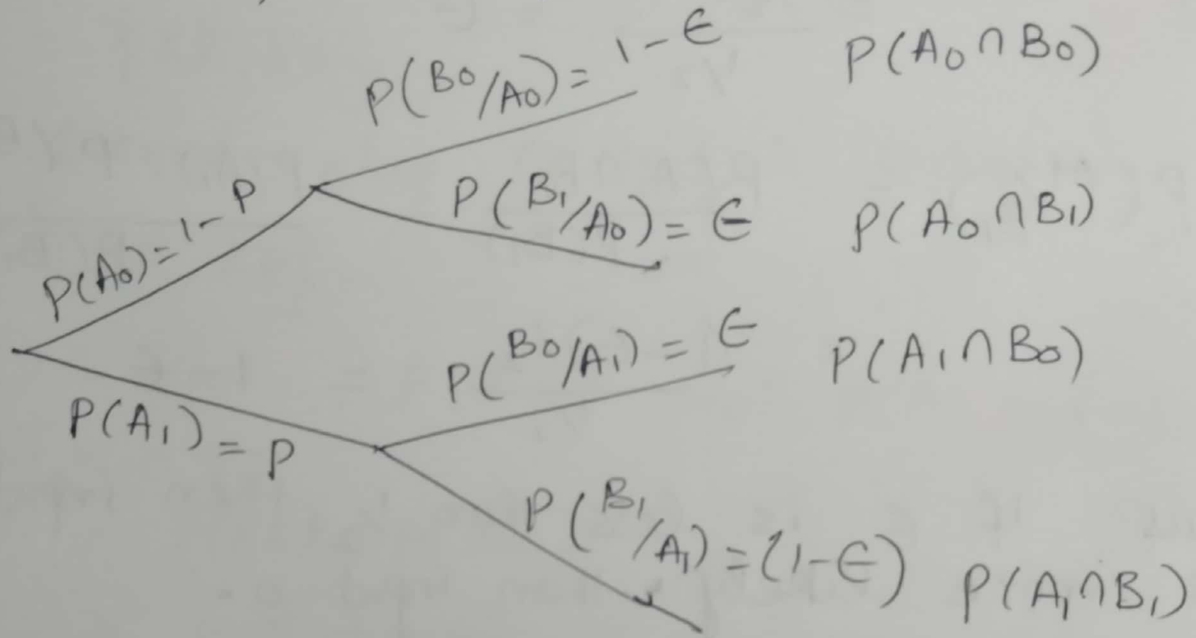
Example 2.26 & Example 2.29

Let

A_0: Input was 0 , A_1: Input was 1
B_0: Reciever decision 0 B_1: Reciever decision 1

$P(A_0) = 1-P$, $P(A_1) = P$



$P(A_0 \cap B_0) = (1-P)(1-\epsilon)$

$P(A_0 \cap B_1) = (1-P)\epsilon$

$P(A_1 \cap B_0) = P\epsilon$

$P(A_1 \cap B_1) = P(1-\epsilon)$

What is the probability that "reciever output was 1".

$$P(B_1) = P(A_0)\cdot P(B_1/A) + P(A_1)\cdot P(B_1/A)$$

$$= P(A_0 \cap B_1) + P(A_1 \cap B_1)$$

$$= (1-P)\epsilon + P(1-\epsilon)$$

$$= \frac{1}{2}\epsilon + \frac{1}{2}(1-\epsilon)$$

$$= \cancel{\frac{1}{2}\epsilon} + \frac{1}{2} - \cancel{\frac{1}{2}\epsilon}$$

$$\boxed{P(B_1) = \frac{1}{2}}$$

b) Find which ~~output~~ input is more probable given that the reciever has output a 1?

$$P(A_0/B_1) = \frac{P(A_0 \cap B_1)}{P(B_1)} = \frac{P(A_0) \cdot P(B_1/A_0)}{P(B_1)}$$

$$= \frac{\frac{1}{2}\varepsilon}{1/2} = \epsilon$$

$$P(A_1/B_1) = \frac{P(A_1 \cap B_1)}{P(B_1)} = \frac{P(A_1) \cdot P(B_1/A_1)}{P(B_1)}$$

$$= \frac{(1-\epsilon)/2}{1/2} = 1-\epsilon$$

Thus if ϵ is less than $1/2$, then input 1 is more likely than input 0.

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Example 1.15 & 1.16

Two circuits are tested.

A: Accepted
R: Rejected

~~S = {RR, RA, AR, AA}~~ $S = \{RR, RA, AR, AA\}$

B: First chip tested & rejected.

$B = \{RR, RA\}$

A: second chip tested & rejected.

$A = \{RR, AR\}$

$P(RR) = 0.01$, $P(RA) = 0.01$, $P(AR) = 0.01$

$P(AA) = 0.97$

$$P(A) = P(RR) + P(AR)$$
$$= 0.01 + 0.01$$
$$= 0.02$$

$$P(B) = P(RR) + P(RA)$$
$$= 0.01 + 0.01$$
$$= 0.02$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.01}{0.02}$$

$$\boxed{P(A/B) = 0.5}$$

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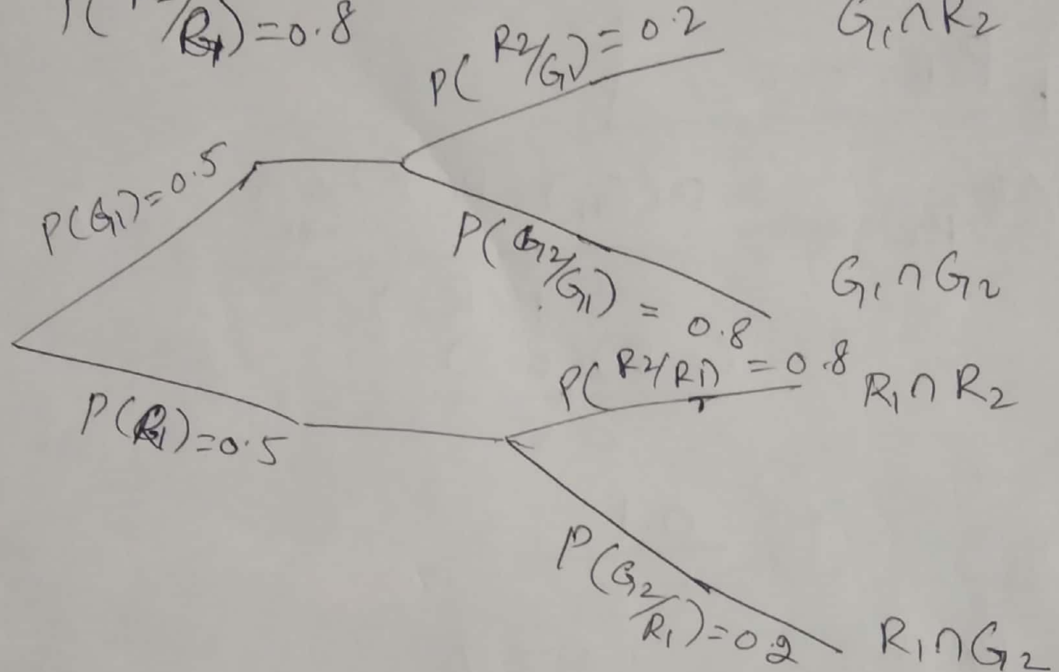
Example 1.25

$P(G_1) = 0.5$

$P(R_1) = 0.5$

$P(G_2/G_1) = 0.8$

$P(R_2/R_1) = 0.8$



$P(G_2) = P(R_1 \cap G_2) + P(G_1 \cap G_2)$

$= P(R_1) \cdot P(G_2/R_1) + P(G_1) \cdot P(G_2/G_1)$

$= (0.5)(0.2) + (0.5) \cdot (0.8)$

$= 0.1 + 0.4$

$= 0.5$

W = You wait for atleast one light.

$P(W) = P(R_1 \cap G_2) + P(G_1 \cap R_2) + P(R_1 \cap R_2)$

$= P(R_1) \cdot P(G_2/R_1) + P(G_1) \cdot P(R_2/G_1) + P(R_1) \cdot P(R_2/R_1)$

$= (0.5)(0.2) + (0.5)(0.2) + (0.5) \cdot (0.8)$

$= 0.1 + 0.1 + 0.4$

$= 0.6$

$$P(G_1/R_2) = \frac{P(G_1 \cap R_2)}{P(R_2)}$$

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$P(R_2) = P(G_1 \cap R_2) + P(R_1 \cap R_2)$

$= P(G_1) P(R_2/G_1) + P(R_1) \cdot P(R_2/R_1)$

$= (0.5)(0.2) + (0.5)(0.8)$

$= 0.1 + 0.4$

$\boxed{P(R_2) = 0.5}$

$$P(G_1/R_2) = \frac{P(G_1) \cdot P(R_2/G_1)}{P(R_2)}$$

$$= \frac{(0.5)(0.2)}{0.5}$$

$$= \frac{0.1}{0.5}$$

$\boxed{P(G_1/R_2) = 0.2}$

Ans