National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	**Discrete Structures**	**Course Code:**	**CS-1005**
Degree Program:	**Bachelor of CS & DS**	**Semester:**	**Spring 2022**
Exam Duration:	**1 Hour**	**Total Marks:**	**45**
Paper Date:	**November 11, 2022**	**Weight**	**20%**
Section:	**All**	**Page(s):**	**10**
Exam Type:	**MID-II**		

Stu

1. Verify at the start of the exam that you have a total of four (4) questions printed on ten (10) pages including this title page.
2. Attempt all questions on the question-book and in the given order.
3. The exam is closed books, closed notes. Please see that the area in your threshold is free of any material classified as 'useful in the paper' or else there may a charge of cheating.
4. Read the questions carefully for clarity of context and understanding of meaning and make assumptions wherever required, for neither the invigilator will address your queries, nor the teacher/examiner will come to the examination hall for any assistance.
5. Fit in all your answers in the provided space. You may use extra space on the last page if required. If you do so, clearly mark question/part number on that page to avoid confusion.
6. Use only your own stationery and calculator. If you do not have your own calculator, use manual calculations.
7. Use only permanent ink-pens. Only the questions attempted with permanent ink-pens will be considered. Any part of paper done in lead pencil cannot be claimed for rechecking.

	Q-1	Q-2	Q-3	Q-4	Total
Total Marks	6	7	12	20	**45**
Marks Obtained	04	07	12	09	32

$p \to q$
p
$\therefore q$

$p \to q$
$\sim q$
$\sim p$

1. State which of the following arguments are valid by universal modus Ponens or universal modus Tollens, and which are invalid and exhibit the inverse or converse error. Justify your answers.

a) If a product of two numbers is 0, then at least one of the numbers is 0.

$\sim q$ For a particular number x, neither $(2x + 1)$ nor $(x - 7)$ equals 0.

$\sim p$. $\therefore$ The product $(2x + 1)(x - 7)$ is not 0.

b) If an infinite series converges, then the terms go to 0.

q The terms of the infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ go to 0.

p. $\therefore$ The infinite series $\sum_{n=1}^{\infty} \frac{1}{n}$ converges.

c) If n is a real number with $n > 2$, then $n^2 > 4$.

$\sim p$. n is real number and $n \leq 2$.

$\sim q$ $\therefore n^2 \leq 4$.

[2+2+2]

a) Valid
q: at least one of no. is 0
$\sim q$: neither of no. is Zero

$p \to q$ Universal Modus Ponens
$\sim q$
$\therefore \sim p$

(b). Converse error

because
$p \to q$.
q
$\therefore p$
} This is wrong-error.

p: infinite series converge
q: term goes to Zero.

(c). Inverse Error.

$p \to q$
$\sim p$
$\therefore \sim q$
} this is wrong-error

p: n is a real no. $n > 2$
q: $n^2 > 4$

2. Check whether the following statement is true or not. If true then prove it and if false then find a counterexample

a) If $3|(x+y)$ then $3|(x-y)$.

b) For every real number x, $x - \lfloor x \rfloor < \frac{1}{2}$ then $\lfloor 2x \rfloor = 2\lfloor x \rfloor$.

[2+5]

floor definition.
$n \leq x < n+1$

① $\lfloor x \rfloor \leq x$
multiply 2 → $2\lfloor x \rfloor \leq 2x$

~~② $x < \lfloor x \rfloor + 1$~~
~~multiply by 2~~
~~$2x < 2\lfloor x \rfloor + 2$.~~

B : True

$x - \lfloor x \rfloor < \frac{1}{2}$

multiply by 2

$2x - 2\lfloor x \rfloor < 1$

$2x < 2\lfloor x \rfloor + 1 \longrightarrow$ ①

$\lfloor x \rfloor \leq x$

multiply by 2

$2\lfloor x \rfloor \leq 2x \longrightarrow$ ②

$2\lfloor x \rfloor \leq 2x < 2\lfloor x \rfloor + 1$

definition of floor $n \leq x < n+1$

hence proved
~~So~~ $\lfloor 2x \rfloor = 2\lfloor x \rfloor$

3.

a) Use method of contraposition to prove that, for all integers a, b, and c, if $a \nmid bc$ then $a \nmid b$.

b) Use method of contradiction to show that, for all odd integers a and b, $a^2 - b^2 \neq 4$.

[5+7]

$p \to q$
$\sim q \to \sim p$

(a) For all integers, a, b, and c if $a \nmid bc$ then $a \nmid b$

Contrapositive: For all integers a, b and c if $a \mid b$ then $a \mid bc$

if $a \mid b$

$b = ar$ (r is some integer).

multiply both sides by c

$bc = arc$

$bc = a(r.c)$

r.c = some integer because both r and c are integers and product of integers is integer

$bc = a$ (some integer)

hence proved $a \mid bc$

So the statement is true as contrapositive is true.

Negation. Suppose not.

b) There is an ~~integer~~ odd integer a & b so $a^2-b^2=4$

$$a^2-b^2=(a+b)(a-b)=4$$

unique factorization theorem $2.2=4$ or $4.1=4$.

Case I: $a=b=2$

$\cancel{a+b=a-b}$

$a+b=2$ $\quad$ $a-b=2$

$a=2-b$ $\quad$ $a=2+b$

$\cancel{2}-b=\cancel{2}+b$

$-b=b$ so $b=0$ which is not odd integer.

Case II $a+b=4$ & $a-b=1$

$a=4-b$ $\quad$ $a=1+b$

$4-b=1+b$

$4-1=b+b$

$3/2=b$

$3/2$ is not an odd integer.

$b=4-a$ $\quad$ $a-1=b$

$4-a=a-1$

$4+1=2a$

Case III $a+b=1$ & $a-b=4$

$a=1-b$ $\quad$ $a=4+b$

$1-b=4+b$

$1-\cancel{3}4=b+b$

$-3=2b$

$b=-3/2$

b is not an odd integer.

4.

a) Prove by mathematical induction that for every integer $n \geq 2$,

$$\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots \ldots\ldots\ldots \frac{1}{\sqrt{n}}$$

b) Use strong mathematical induction to prove that every integer greater than 1 is either a prime number or a product of prime numbers $n \geq 2$.

This Question is continued on previous page .... pg#8

[10+10]

Base Case: $n = 2$.

$$\sqrt{2} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}}.$$

$1.41 < 1.707$ True base case true

Inductive Step: for $k \geq 2$ if $P(k)$ is true then $P(k+1)$ is true.

$P(k)$: $\sqrt{k} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \cdots + \frac{1}{\sqrt{k}}$ True.

$P(k+1)$: $\sqrt{k+1} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \cdots\cdots \frac{1}{\sqrt{k+1}}$

$$\sqrt{k+1} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \cdots \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}}$$

Let $\boxed{\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} \cdots + \frac{1}{\sqrt{k}} = r}$

$$\sqrt{k} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{k}}$$

Square both sides.

$k < r^2$

$$k+1 < \left(r + \frac{1}{\sqrt{k+1}}\right)^2$$

$$k+1 < r^2 + 2r\left(\frac{1}{\sqrt{k+1}}\right) + \left(\frac{1}{\sqrt{k+1}}\right)^2$$

$\cancel{r^2 + k < r^2 + 2r(1 \ldots}$