

CS 201: DISCRETE STRUCTURES

SECTION G

November 27, 2018

Quiz 5

PROBLEM

Solve the following recurrence:

$$a_n = 5a_{n-1} - 4a_{n-2} + 1 \quad (1)$$

With the following initial conditions:

$$a_0 = 0$$

$$a_1 = 1$$

SOLUTION

The characteristic equation is given by:

$$r^2 - 5r + 4 = 0$$

$$r = 1, 4$$

The solution of the corresponding linear homogeneous recurrence relation is:

$$a_n^{(h)} = \alpha_1 4^n + \alpha_2$$

The particular solution is given by:

$$a_n^{(p)} = nP_0$$

Substitute $a_n^{(p)}$ above in (1)

$$nP_0 = 5(n-1)P_0 - 4(n-2)P_0 + 1$$

$$0 = 3P_0 + 1$$

$$P_0 = -1/3$$

The solution to the recurrence is given by:

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$a_n = \alpha_1 4^n + \alpha_2 - (1/3)n$$

Substitute the initial conditions

With the following initial conditions:

$$a_0 = 0$$

$$0 = \alpha_1 + \alpha_2 - 1/3(0) \quad (2)$$

$$a_1 = 1$$

$$1 = 4\alpha_1 + \alpha_2 - 1/3 \quad (3)$$

Solve (2) and (3) yourself to get the following solution:

$$a_n = (4/9)4^n - (4/9) - (1/3)n$$