Digital Logic Design

Lecture 11

Don’t Care Conditions

- The minterms of a Boolean function specify all combinations of variable values for which the function is equal to 1. The function is assumed to be equal to 0 for the rest of the minterms.
- This assumption, however, is not always valid, since there are applications in which the function is not specified for certain variable value combinations.
- There are two cases in which this occurs.
  - In the first case, the input combinations never occur. As an example, the four-bit binary code for the decimal digits has six combinations that are not used and not expected to occur.
  - In the second case, the input combinations are expected to occur, but we do not care what the outputs are in response to these combinations.

Contd.

- In both cases, the outputs are said to be unspecified for the input combinations.

Incompletely specified functions

- Functions that have unspecified outputs for some input combinations are called incompletely specified functions.
- In most applications, we simply do not care what value is assumed by the function for the unspecified minterms. For this reason, it is customary to call the unspecified minterms of a function don’t-care conditions.
- These conditions can be used on a map to provide further simplification of the function.

Contd.

- It should be realized that a don't-care minterm cannot be marked with a 1 on the map, because that would require that the function always be a 1 for such a minterm.
- Likewise, putting a 0 in the square requires the function to be 0.
- To distinguish the don't-care condition from 1s and 0s, an **X** is used.

Simplification of an incompletely specified function

- When choosing adjacent squares to simplify the function in the map, the ×'s may be assumed to be either 0 or 1, whichever gives the simplest expression.
- An × need not be used at all if it does not contribute to covering a larger area.

Example 2.14

- Simplify the Boolean function F (A, B, C, D) = Σm(1,3,7,11,15)
- which has the don't-care conditions d (A, B, C, D) = Σm(0,2,5).

SOP: $F = C\,D + \overline{A}\,\overline{B}$, $F = C\,D + \overline{A}\,D$

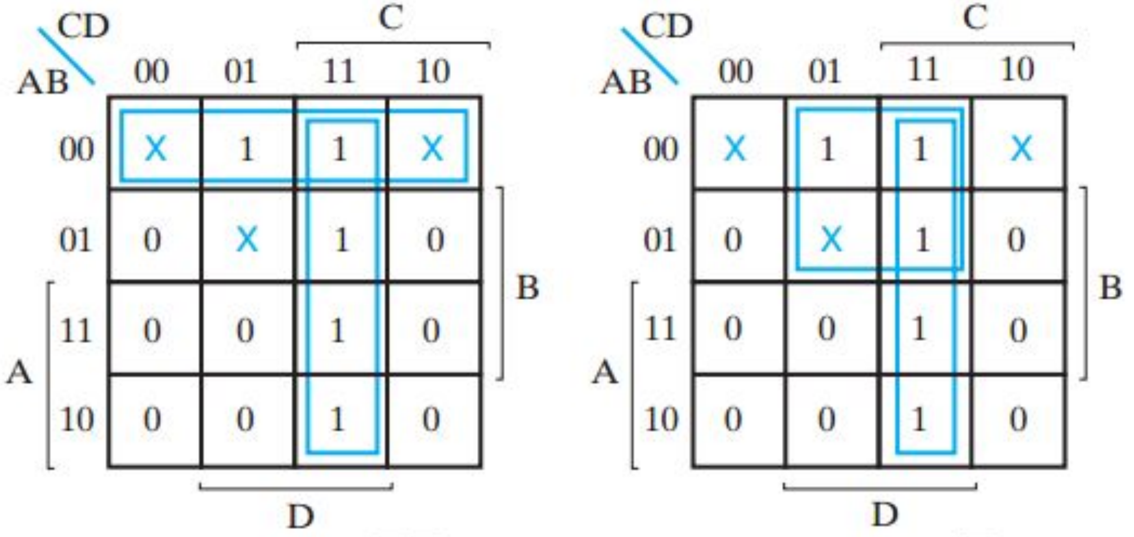
- It is also possible to obtain an optimized **product-of-sums** expression for the function.

POS: In this case, the way to combine the 0s is to include don't-care minterms 0 and 2 with the 0s, giving the optimized complemented function.

$\overline{F} = \overline{D} + A\,\overline{C}$

Taking the complement of F

$F = D(\overline{A} + C)$



(a) $F = CD + \overline{A}\,\overline{B}$

(b) $F = CD + \overline{A}D$

Exclusive OR/ Exclusive NOR

- **The *eXclusive OR* (*XOR*) function is an important Boolean function used extensively in logic circuits.**
- **The XOR function may be;**
  - **implemented directly as an electronic circuit (truly a gate) or**
  - **implemented by interconnecting other gate types (used as a convenient representation)**
- **The *eXclusive NOR* function is the complement of the XOR function**
- **By our definition, XOR and XNOR gates are complex gates.**

Exclusive OR/ Exclusive NOR

- **Uses for the XOR and XNORs gate include:**
  - **Adders/subtractors/multipliers**
  - **Counters/incrementers/decrementers**
  - **Parity generators/checkers**
- **Definitions**
  - **The XOR function is:** $$X \oplus Y = X\,\overline{Y} + \overline{X}\,Y$$
  - **The eXclusive NOR (XNOR) function, otherwise known as *equivalence* is:** $$\overline{X \oplus Y} = X\,Y + \overline{X}\,\overline{Y}$$
- **Strictly speaking, XOR and XNOR gates do no exist for more than two inputs. Instead, they are replaced by odd and even functions.**

Truth Tables for XOR/XNOR

- Operator Rules: XOR

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

XNOR

X	Y	$\overline{(X \oplus Y)}$ or $X \equiv Y$
0	0	1
0	1	0
1	0	0
1	1	1

- The XOR function means:

  X OR Y, but NOT BOTH

- Why is the XNOR function also known as the *equivalence* function, denoted by the operator ≡?

XOR/XNOR (Continued)

- **The XOR function can be extended to 3 or more variables. For more than 2 variables, it is called an *odd function* or *modulo 2 sum* (*Mod 2 sum*), not an XOR:**

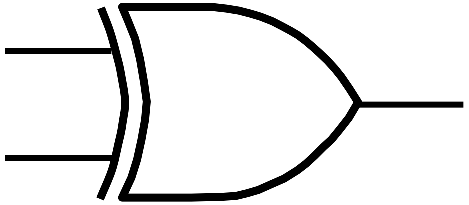
$$X \oplus Y \oplus Z = \overline{X}\,\overline{Y}\,Z + \overline{X}\,Y\,\overline{Z} + X\,\overline{Y}\,\overline{Z} + X\,Y\,Z$$

- **The complement of the odd function is the even function.**
- **The XOR identities:**

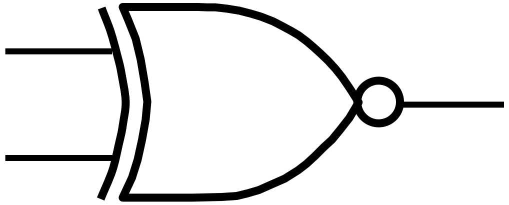
$$X \oplus 0 = X \qquad X \oplus 1 = \overline{X}$$
$$X \oplus X = 0 \qquad X \oplus \overline{X} = 1$$
$$X \oplus Y = Y \oplus X$$
$$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$$

Symbols For XOR and XNOR

- XOR symbol:



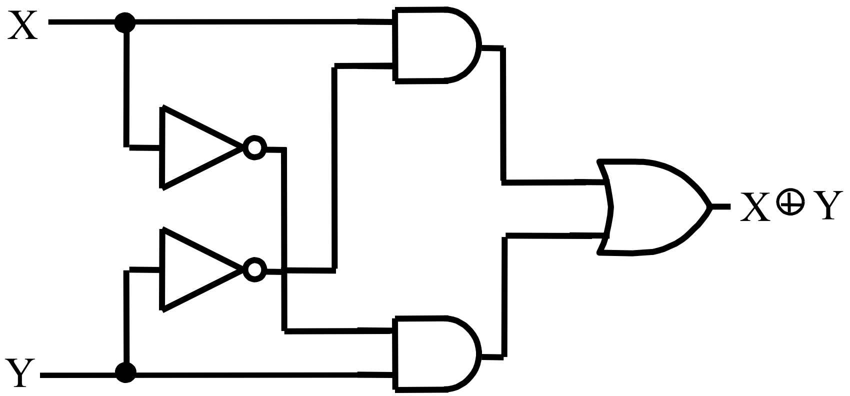
- XNOR symbol:



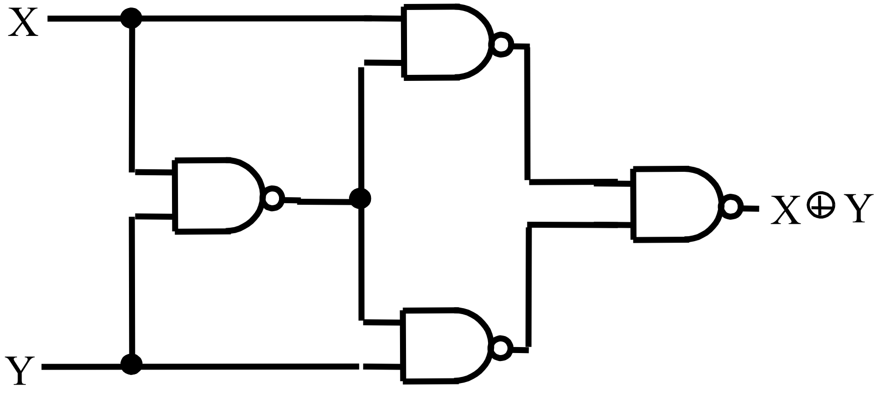
- Shaped symbols exist only for two inputs

XOR Implementations

- **The simple SOP implementation uses the following structure:**



- **A NAND only implementation is**:

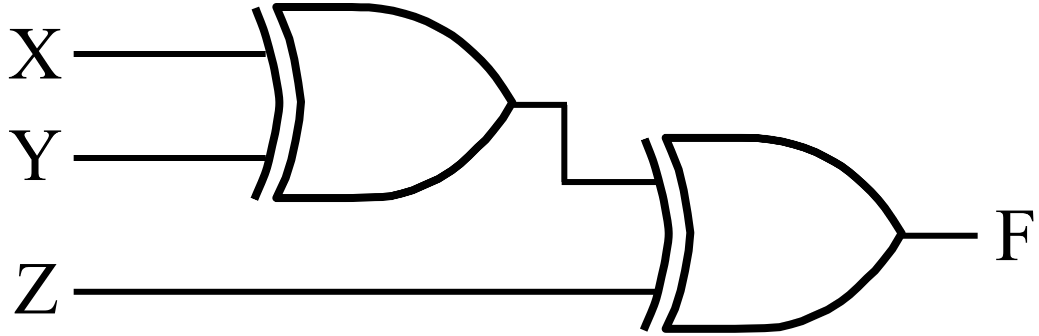


Odd and Even Functions

- **The odd and even functions on a K-map form "checkerboard" patterns.**
- **The 1s of an odd function correspond to minterms having an index with an odd number of 1s.**
- **The 1s of an even function correspond to minterms having an index with an even number of 1s.**
- **Implementation of odd and even functions for greater than four variables as a two-level circuit is difficult, so we use "trees" made up of :**
  - **2-input XOR or XNORs**
  - **3- or 4-input odd or even functions**

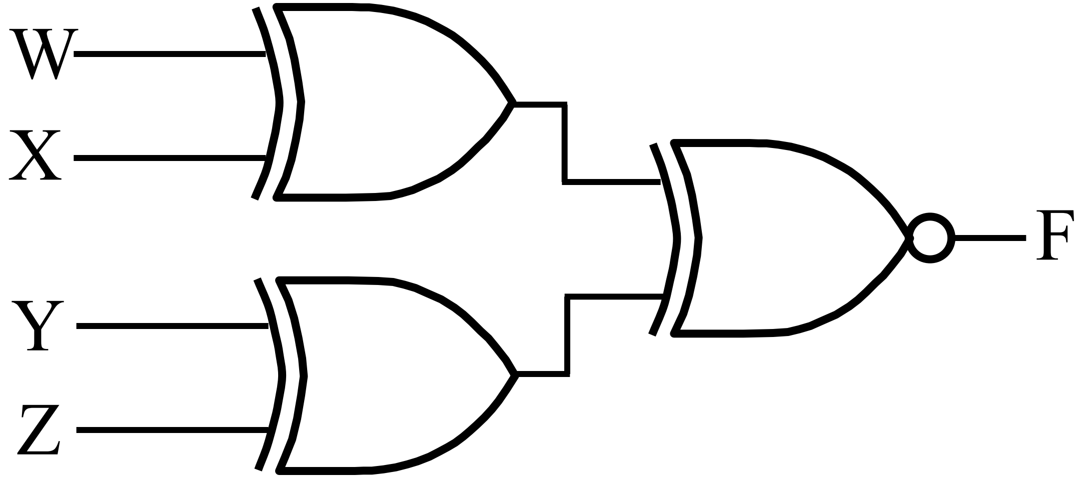
Example: Odd Function Implementation

- **Design a 3-input odd function $F = X \oplus Y \oplus Z$ with 2-input XOR gates**
- **Factoring, $F = (X \oplus Y) \oplus Z$**
- **The circuit:**



Example: Even Function Implementation

- **Design a 4-input odd function $F = W \oplus X \oplus Y \oplus Z$ with 2-input XOR and XNOR gates**
- **Factoring, $F = (W \oplus X) \oplus (Y \oplus Z)$**
- **The circuit:**



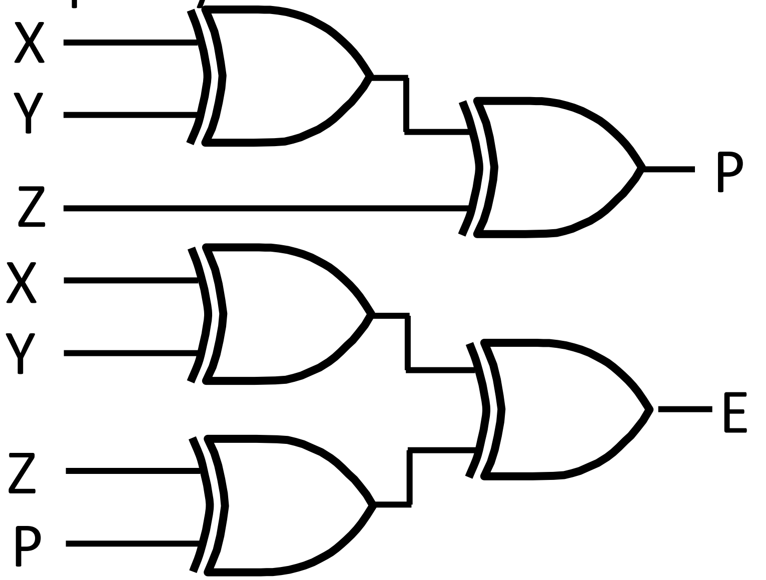
Parity Generators/ Checkers

- In Chapter 1, a parity bit added to n-bit code to produce an n + 1 bit code:
  - Add odd parity bit to generate code words with even parity
  - Add even parity bit to generate code words with odd parity
  - Use odd parity circuit to check code words with even parity
  - Use even parity circuit to check code words with odd parity
- Example: n = 3. Generate even parity code words of length four with odd parity generator:
- Check even parity code words of length four with

odd parity checker:

- Operation: (X,Y,Z) = (0,0,1) gives (X,Y,Z,P) = (0,0,1,1)

and E = 0. If Y changes from 0 to 1 between generator and checker, then E = 1 indicates an error.



Implementing functions with only OR and NOT gates

- $F = AB'C + A'C' + AB$
- $F = \overline{\overline{AB'C + A'C' + AB}}$
- $F = \overline{\overline{AB'C} + \overline{A'C'} + \overline{AB}}$
- By De Morgan's Laws
- $F = \overline{(A'+ B + C') \,.\, (A + C) \,.\, (A'+ B')}$
- $F = \overline{(A'+ B + C')} + \overline{(A + C)} + \overline{(A'+ B')}$

Implementing functions with only AND and NOT gates

- $F = (A+B'+C) \, . \, (A'+B+C) \, . \, (A'+C) \, . \, (B+C')$

- $F = \overline{\overline{(A+B'+C) \, . \, (A'+B+C) \, . \, (A'+C) \, . \, (B+C')}}$

- $F = \overline{\overline{(A+B'+C)} + \overline{(A'+B+C)} + \overline{(A'+C)} + \overline{(B+C')}}$

- $F = \overline{(A'.B.C') + (A.B'.C') + (A.C') + (B'.C)}$

- $F = \overline{(A'.B.C')} \, . \, \overline{(A.B'.C')} \, . \, \overline{(A.C')} \, . \, \overline{(B'.C)}$