National University of Computer and Emerging Sciences

Lahore Campus

0246

# Linear Algebra (MT1004)

# Sessional-I Exam

**Date: September 21st 2024**

**Course Instructor(s)**

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**Total Time (Hrs):** 1 Hour

**Total Marks:** 40

**Total Questions:** 1

Roll No Section Student Signature

**Do not write below this line**

**Attempt all the questions.**

*CLO #1:* Use concept of elementary row operations to find the inverse of square matrices, determinant of a matrix and solving the system of linear equations.

**Q#1. (a):** Consider the following system in unknowns $x$, $y$ and $z$. For what value(s) of "a, b" does the following system have no solution? Unique solution? Infinitely many solutions? [10 marks]

no: $a = \pm 3$ and $b \neq \pm 3$ or $b \neq a$

uni: $a \neq \pm 3$

inf: $a = \pm 3$ and $b = \pm 3$ or $b = a$.

$$\begin{aligned} x - 2y &= 1 \\ x - y + az &= 2 \\ ay + 9z &= b \end{aligned}$$

$r_2 - 3r_1$, $r_2(-1/7)$, $r_1 - 4r_2$

**b):** Use **Inversion Algorithm** to find the inverse of $A = \begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$. Also write down $A^{-1}$ as a product of elementary matrices $A^{-1} = E_k E_{k-1} \ldots\ldots E_3 E_2 E_1$. [10 marks]

$\begin{bmatrix} -5/7 & 4/7 \\ 3/7 & -1/7 \end{bmatrix}$

**c):** Evaluate the given determinant by using row reduction method. [5 marks]

check.

$$\begin{vmatrix} 2 & -1 & 5 & 5 \\ 3 & 1 & 2 & 4 \\ -1 & -3 & 8 & 0 \\ 1 & 1 & 2 & -1 \end{vmatrix} = 14$$

**d):** Find the values of $\alpha$ for which $det\ A = 0$. [3 marks]

$\alpha = -2, \alpha = 4, \alpha = 3$

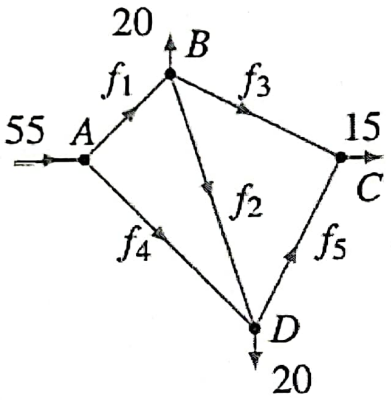
$$|A| = \begin{vmatrix} \alpha - 4 & 0 & 0 \\ 0 & \alpha & 2 \\ 0 & 3 & \alpha - 1 \end{vmatrix}$$

$r_3 - \frac{3}{\alpha} r_2$

**e):** Identify the row operation corresponding to $E$ and verify that the product $EM$ results from applying the row operation to M, where $E$ is an elementary matrix. [2 marks]

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M = \begin{bmatrix} 2 & -1 & 0 & -4 & -4 \\ 1 & -3 & -1 & 5 & 3 \\ 2 & 0 & 1 & 3 & -1 \end{bmatrix}$$ ✓

**f):** A proposed network of irrigation canals is described in the accompanying diagram. At peak demand, the flows at interchanges A, B, C, and D are as shown. [10 marks]




Set up a linear system whose solution provides the unknown flow rates and also solve the system for the unknown flow rates using Gauss Jorden elimination method.

$p \geq 55$ ×

$q \geq 15$ ×

$q - p \geq -20$ ✓

$f_1 = 55 - p$

$f_2 = 20 + q - p$

$f_3 = 15 - q$

$f_4 = p$

$f_5 = q$.