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Question # [illegible]

Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a transformation. Find the standard matrix A that:

a. Rotate each vector that belongs to the domain in counter clockwise direction through an angle [illegible] about origin.

$e_1 = \begin{bmatrix}1\\0\end{bmatrix}, e_2 = \begin{bmatrix}0\\1\end{bmatrix}$

After Reflection

b. Gives an horizontal shear transformation that maps e_2 into $e_2 + 2e_1$ but leaves the vector e_1 unchanged.

[illegible] $e_1 = \begin{bmatrix}1\\0\end{bmatrix}, e_2 = \begin{bmatrix}0\\1\end{bmatrix}$ [illegible]

S.T

After transformation

c. Gives an Projection onto the x_2 = [illegible]

$e_1 = \begin{bmatrix}1\\0\end{bmatrix}, e_2 = \begin{bmatrix}0\\1\end{bmatrix}$

S.P

After Projection

S.M = [illegible]

d. First reflects points through the vertical x_2-axis and then rotates points through an angle of $\pi/2$ radians about origin in counterclockwise direction.

$e_1 = \begin{bmatrix}1\\0\end{bmatrix}, e_2 = \begin{bmatrix}0\\1\end{bmatrix}$

After Reflection

Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} x_1 - 5x_2 + 4x_3 \\ x_2 - 6x_3 \end{bmatrix}$

a. Determine whether the given transformation is linear or not.

$T: \mathbb{R}^3 \to \mathbb{R}^2$

$T(cx + dy + ez)$

$= \begin{bmatrix} (cx_1 + dy_1 + ez_1) - 5(cx_2 + dy_2 + ez_2) + 4(cx_3 + dy_3 + ez_3) \\ (cx_1 + dy_1 + ez_1) \cdot 0 + (cx_2 + dy_2 + ez_2) - 6(cx_3 + dy_3 + ez_3) \end{bmatrix}$

$= \begin{bmatrix} cx_1 + dy_1 + ez_1 & -5cx_2 - 5dy_2 - 5ez_2 & 4cx_3 + 4dy_3 + 4ez_3 \\ 0 & cx_2 + dy_2 + ez_2 & -6cx_3 - 6dy_3 - 6ez_3 \end{bmatrix}$

$= \begin{bmatrix} cx_1 - 5cx_2 + 4cx_3 \\ 0 + cx_2 - 6cx_3 \end{bmatrix} + \begin{bmatrix} dy_1 - 5dy_2 + 4dy_3 \\ 0 + dy_2 - 6dy_3 \end{bmatrix} + \begin{bmatrix} ez_1 - 5ez_2 + 4ez_3 \\ 0 + ez_2 - 6ez_3 \end{bmatrix}$

$= cT(x) + dT(y) + eT(z)$

So: Transformation is linear.

b. Find matrix of transformation.

$\begin{bmatrix} x_1 & -5x_2 & 4x_3 \\ 0 & x_2 & -6x_3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ [illegible] & [illegible] \\ [illegible] & 0 \end{bmatrix}$

$\begin{bmatrix} \boxed{1} & -5 & 4 & 1 & 0 \\ 0 & \boxed{1} & -6 & 1 & 0 \end{bmatrix}$

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[illegible]

Yes all the Rows (r_1, r_2) are pivot and there exist one pivot in each column in Row. T is not one to one in R_2.

[illegible]

Yes as the rows are pivot so T maps onto R_2.

Find a vector x whose image is [illegible]

$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ [illegible]

PTO

7. Find the image of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} x_1 & & -5x_2 & +4x_3 \\ 0 & & x_2 & -6x_3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & -10x_2 & +12x_3 \\ 0 & +2x_2 & -18x_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -10 & +12 \\ 0 & +2 & -18 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -16 \end{bmatrix}$$

8. Determine whether the columns of A are linearly dependent or independent.

As there are 3 columns and 2 rows and last column is 0 so A is linearly dependent !!

9. Is [illegible] diagonalizable? Explain in brief.

A: [illegible] [illegible] [illegible] not equal [illegible]

[illegible]

Question # 5

a. If $A = \begin{bmatrix} [illegible] \end{bmatrix}$. Give geometric description of the transformation. Discuss the properties of A.

Properties of A.

i) System A is consistent.

ii) Solution of A is trivial

iii) System is linearly dependent

iv) System is one to one in A.

v) System is onto A.

FAST

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[illegible]

As given

$A = \begin{bmatrix} [illegible] \end{bmatrix}$

As we are given that A maps R^3 onto R^3 and there are all pivot position so transformation must be one to one.

Find the standard matrix A that gives a reflection through the plane [illegible]