Roll Number: _______________

National University of Computer and Emerging Sciences, Lahore Campus

	Course Name:	Theory of Automata	**Course Code:**	CS 301
	Program:	BS Computer Science	**Semester:**	Fall 2019
	Duration:	180 Minutes	**Total Marks:**	75
	Paper Date:	December 16, 2019.	**Weight**	45
	Section:	N/A	**Page(s):**	6
	Exam Type:	Final Exam		

Student : Name:_______________________________ **Roll No.**_________________

Instruction/ Notes:

1. Solve in the space provided. Extra sheets will NOT be collected or marked.
2. One A4 handwritten help sheet is allowed.
3. In case of any ambiguity make a reasonable assumption.

Good luck!

Problem 1 **(Marks: 1+2+2)**

Given the regular expressions: $R_1 = (a^*+b)^*ab$ and $R_2 = (a+b^*)^*a^*b$

a. Write the three shortest strings generated by R_1.

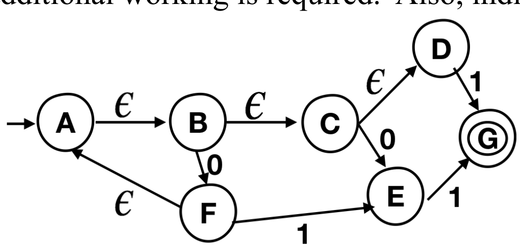
b. Give a regular expression for $L(R_1) \cap L(R_2)$

c. Which of the expressions are correct?

i. $L(R_1) \cup L(R_2) = \Sigma^*$ ii. $L(R_1) \subseteq L(R_2)$ iii. $L(R_1)L(R_2) = (a+b)^*ab(a+b)^*(a^*b^*)b$

Problem 2 **(Marks: 5)**

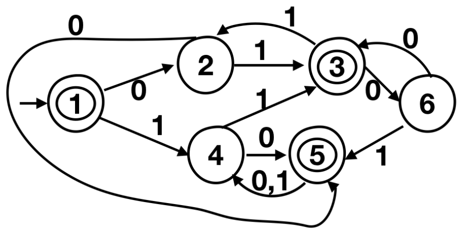
Remove all the ϵ-transitions from this NFA-ϵ and make the state transition diagram of the resulting NFA. No additional working is required. Also, indicate the **start and final states.**



State	0	1

Problem 3 **(Marks: 5)**

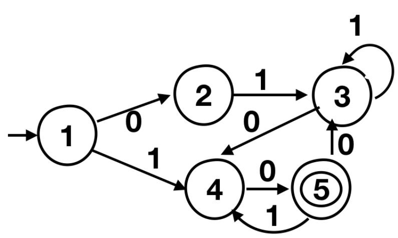
Minimize the following DFA. Fill out the table. Draw the resulting DFA and **clearly indicate which states were merged.**



6						x
5					x	x
4				x	x	x
3			x	x	x	x
2		x	x	x	x	x
1	x	x	x	x	x	x
	1	2	3	4	5	6

Problem 4 **(Marks: 5)**

Find the regular expression corresponding to the given automaton using the method of state elimination. Show all steps.



Problem 5 **(Marks: 5)**

L = $\{(0^n 1^{2n})* \mid n \geq 0\}$

Write the three shortest strings of L and express this language as a CFG. Clearly mark the start symbol.

Problem 6 **(Marks: 5)**

Use the CYK algorithm to determine if the string 0110 is generated by the given CFG (S is start symbol). Show working.

S → AZ | ZA

Z → 0 | AZ

A → 1 | ZA

Problem 7 **(Marks: 5)**

Prove that $n^4 2^{n^3} = 2^{O(n^3)}$. All steps are required.

Problem 8 **(Marks: 5)**

Prove that if we find a language A ∈ P and we also find that A is NP-complete then P=NP

Problem 9 **(Marks: 5)**

Reduce the following instance of 3SAT to an instance of CLIQUE. Clearly specify the graph G and k.

CLIQUE = {<G,k> | G is an undirected graph with a k-clique}

$(x_1 \land x_1 \land x_2) \lor (\bar{x}_1 \land \bar{x}_2 \land \bar{x}_2)$

Problem 10 **(Marks: 5)**

Prove that the following language belongs to NP complexity class

L = {<G,s,t,k> | G is an undirected graph and there is a path from s to t of length at least k and no node repeats itself in the path}

Problem 11 **(Marks: 2x8=16)**

For each part, give a **one/two line explanation** or counter example. **Only yes/no answer scores zero.**

i. Can an NFA can be reduced to a DFA in polynomial time or not?

ii. If L_1 and L_2 are context free languages then is $L_1 \cap L_2$ also a context free language?

iii. Are all context free languages member of NP? Are they NP-complete?

iv. If $L_1 \subseteq L_2$ and L_1 is not a regular language then is L_2 also non-regular?

v. Are there any recursively enumerable languages member of NP?

vi. L = {<M,w> | M is a Turing machine and M halts on input w}. Is L Turing recognizable?

vii. Is $P \subseteq NP$?

viii. Can we simulate a PDA using a queue, which is a first in first out structure, instead of a stack?

Problem 12 **(Marks: 2+2)**

i. Write the complement of L defined over the alphabet $\{0,1\}$. L = $\{0^n 1^n \mid n \geq 0\}$

ii. Suppose $L_1 = \{0^n 1^n \mid n \geq 0\}$ and $L_2 = \{1^n 2^n \mid n \geq 0\}$

Write down $L_1 L_2$

Problem 13 **(Marks: 5)**

Make a deterministic single tape Turing machine to decide the following language:

L = {x | x $\in$ {0,1}* and length of x is odd and there is a zero in the center of the string}

Examples of string in L are 11**0**10, 0111**0**0011, 0**0**1, etc.

Label each edge using Sipser's notation: Symbol read -> symbol written, {L,R} (you can omit symbol written if input symbol is not changed)