National University of Computer and Emerging Sciences, Lahore Campus

	Course Name:	**Discrete Structures**	**Course Code:**	**CS-211**
	Program:	**Computer Science**	**Semester:**	**Fall 2018**
	Duration:	**60 Minutes**	**Total Marks:**	**4+4+4+4+4**
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	Section:	**ALL**	**Page(s):**	**2**
	Exam Type:	**Sessional - I Solution**		

QUESTION 1: Translate the following English sentences into propositional logic using the relevant propositions and predicates:

W: Planet has water	O: Planet has oxygen	N: Planet has nitrogen
F: Planet has food	L: Planet has life	

Operators you are allowed to use: {^, v, ¬, →, ↔}. **No other operators or propositional/predicate symbols are allowed.**

SOLUTION

a. Whenever there is oxygen and water on a planet, it has food: O ^ W → F

b. A planet has neither water nor oxygen but it has nitrogen: ¬W ^¬O ^ N

c. A planet that has water can have either nitrogen or food but not both: W → (¬(N↔F))

d. It is necessary to have oxygen and water to have life on a planet : L → O ^ W

QUESTION 2: Translate the following to predicate calculus. You are allowed to use the quantifiers: {∃, ∀} and the connectives: {^, v, ¬, →, ↔}. **No other operators or propositional/predicate symbols are allowed.**

A(x): x is an astronaut	V(x,y): x visits y	S(x): x is a star
P(x): x is a planet	M(x): x is a mathematician	

SOLUTION

a. Every astronaut is also a mathematician: ∀x (A(x) → M(x))

b. There are some astronauts who are mathematicians and have visited some planet: ∃x (A(x) ^ M(x) ^ V(x,y) ^ P(y))

c. There are some mathematicians who have not visited any star: ∃x (M(x) ^ ∀y(S(y) → ¬V(x,y)))

d. All astronauts who have visited a star have also visited all planets: ∀x (A(x) ^ ∃y((V(x,y) ^ S(y)) ^ (∀z(P(z) → V(x,z)))

QUESTION 3: Translate each expression into proper English statements.
M(x) = x is Maroon, G(x) = x is green, R(x) = x is Russian. ¬ is the not operator

SOLUTION

a. ∀x (M(x) → ¬G(x)): All Maroon are not green
b. ∃x¬ ((M(x) ∧ G(x)) → R(x)) : There are some Maroon and Green which are not Russian
c. ∀x (M(x) ∧ R(x)): Everything is Maroon and Green
d. ∀x(M(x) → (G(x) ∧ R(x))): All Maroon are Green and Russian

QUESTION 4: There are 4 students: S,M,N,P. If M enrolls in discrete math then N also enrolls in discrete math. P and S always stay together, so if one enrolls then the other one also enrolls and if one of them does not enroll then the other one also does not enroll. Either S or M but not both enroll in discrete math. N is not enrolled in discrete math.

Translate all the above facts to propositional logic using the set of connectives: {^, v, ¬, →, ↔} and next use the concept of satisfiability and rules of inference to determine who is enrolled in discrete math. Truth table, and informal reasoning will not be accepted and marks will be given only on the quality of your answer.

SOLUTION

The given facts are:

$M \rightarrow N$ (1)
$P \leftrightarrow S$ (2)
$\neg (S \leftrightarrow M)$ (3)
$\neg N$ (4)

using (1) and (4) and apply Modus Tolens

$\neg M$ (5)

using (3) using logical identity for biconditional:

$(S \rightarrow \neg M) \wedge (\neg M \rightarrow S)$ (6)

using simplification on (6)

$(\neg M \rightarrow S)$ (7)

Using Modus Ponens on (5) and (7), we conclude

S (8)

Using (2) and (8)

P (9)

From the above we conclude that M and N are not enrolled (4) and (5)
and P and S are enrolled in discrete math (8) and (9)

QUESTION 5: Show that $[\, p \wedge (p \rightarrow q) \,] \rightarrow q$ is a tautology using logical equivalences (and not truth table) .

SOLUTION

$[\, p \wedge (p \rightarrow q) \,] \rightarrow q$
$\equiv \neg [\, p \wedge (p \rightarrow q) \,] \vee q$ (from logical identity of implication)
$\equiv \neg [\, p \wedge [\neg p \vee q \,] \,] \vee q$ (from logical identity of implication)
$\equiv \neg [\, q \,] \vee q$ (apply resolution on (p) and $(\neg p \vee q)$)
$\equiv T$ (negation law)

Hence proved that the given expression is a tuatology