National University of Computer and Emerging Sciences

Lahore Campus

Numerical Computing (CS2008) — **Final Exam**

Jan 3, 2025

Total Time (Hrs):	**3**
Total Marks:	**100**
Total Questions:	**7**

Course Instructor

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Roll No _____ Section _____ Student Signature _____

Do not write below this line

Attempt all questions on the answer book. Programmable calculators are not allowed.
Don't write anything on a question paper except your name and roll number.

Question#1 [5+5=10 marks, CLO-01]: (Rocket Propulsion)

The upward velocity of a rocket is given as a function of time in Table 1.

t (seconds)	0	10	15	20	35	50
v (ms^{-1})	0	227.04	362.78	517.35	602.97	901.67

Table 1

a) Estimate the velocity of the rocket after 1 minute by using most appropriate interpolation technique.

b) Determine the acceleration of the rocket at $t = 15$ seconds.

Note: Use all data points.

Hint: (i) To find velocity in part (a), convert minutes into seconds. (ii) Develop interpolating polynomial in part (a) and use the same polynomial in part (b).

Question#2 [10+5=15 marks, CLO-01]: (Surface Area)

The solid of revolution obtained by rotating the region under the curve $y = f(x)$ over the interval $a \leq x \leq b$ about the x-axis has surface area given by:

$$A = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}\,dx$$

a) Use $f(x) = x^3$ for $0 \leq x \leq 1$ to approximate the area using the Composite Simpson's rule for the step sizes $h = 0.25$ and $h = 0.5$.

b) Combine the two results from through Romberg Integration based on Simpson's rule to obtain $O(h^6)$ extrapolation. Compare the results of part (a) and (b) with the exact solution 0.56708845559 through absolute errors and draw conclusions.

Note: Truncate all results at 8 decimal places.

Question#3 [10 marks, CLO-02]: (Electrical Circuit Resonance)

A series RLC circuit has resistance R of 4 Ω, inductance L of 0.5 H, and a capacitance C of 0.01 F. Use Newton-Raphson Method to determine the resonant frequency ω in the circuit given by following equation:

$$f(\omega) = \tan\left(\frac{\omega L}{R}\right) - \omega CR - 1 = 0$$

Note: (i) Result should be 3 decimal places accurate. (ii) Consider ω to be in radians.

Question#4 [15 marks, CLO-02]: (Database Query Optimization)

A database query optimizer wants to find the optimal execution plan for a query that joins three tables, A, B, and C. The optimizer uses the following equations:

$$x + y + z = 100$$
$$x + 2y - z = 50$$
$$x - y + 2z = 70$$

Use Cholesky's method to find the costs of accessing tables A, B, and C i.e., $x, y, and\ z$ such that $x, y, z \geq 0$.

Question#5 [10 marks, CLO-03]: (Population Growth with Immigration)

The growth of population in a region with immigration is modeled through the following differential equation

$$\frac{dP}{dt} = rP + I$$

The term P represents the growth of the population, $r = 0.02$ is the intrinsic growth rate (a measure of how quickly the population grows in the absence of immigration) while the term $I = 0.1$ represents the constant influx of new individuals due to immigration. $P(0) = 10$ presents the initial

population in billions at zeroth day. Use Picard method to find the population at the fifth day of study $t = 5$.

Note: Use at least five Picard iterations to form the series.

Question#6 [15+5=20 marks, CLO-03]: (Spread of Information in a Social Network)
Information is spreading through a social network according to the differential equation:

$$\frac{dI}{dt} - I(\beta U - \gamma) = 0$$

$$\frac{dU}{dt} + \beta UI = 0$$

Where $I(t)$ is the number of people informed a time t, $U(t)$ is the number of unaware people at time t, $\beta = 0.4$ is the information transmission rate and $\gamma = 0.1$ is the information redundancy rate. $I(0) = 0$, $U(0) = 9$ are the initial number of people informed and unaware people at zeroth day, respectively. How many people are informed $I(t)$ and how many are unaware $U(t)$ at the first day $t = 1$?

a) Use RK1 and RK2 by taking step size $h = 0.5$ to solve the problem.
b) Compare the two results with exact solution, i.e. $I(1) = 0.133814209$and $U(1) = 0.954559449$. Draw conclusions.

Note: Truncate all results at 5 decimal places.

Question#7 [20 marks, CLO-03]: (Electrical Potential in a Sphere)
The electrical potential in a sphere is governed by the equation:

$$\frac{d^2u}{dr^2} + \frac{2}{r}\left(\frac{du}{dr}\right) = 0$$

Subject to boundary conditions

$$u(1) = 10$$

$$u(5) = 0$$

Use finite difference method with central difference approximation to solve the problem in (1,5) subdividing the range into four equal parts.