Addition law of Probability:-

If A and B are two mutually exclusive events then

$P(A \text{ or } B) = P(A) + P(B)$

$P(A \cup B) = P(A) + P(B)$

⇒ If A and B are two not mutually exclusive events then

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

Question:-

Two fair dice are thrown. A prize is won if the total score on the two rolls is 4 or if each individual score is over 4.

$$S.S = \begin{Bmatrix} (1,1)(1,2)(1,3)(1,4)(1,5)(1,6) \\ (2,1)(2,2)(2,3)(2,4)(2,5)(2,6) \\ (3,1)(3,2)(3,3)(3,4)(3,5)(3,6) \\ (4,1)(4,2)(4,3)(4,4)(4,5)(4,6) \\ (5,1)(5,2)(5,3)(5,4)(5,5)(5,6) \\ (6,1)(6,2)(6,3)(6,4)(6,5)(6,6) \end{Bmatrix}$$

each having probability $1/36$

A: Total Score is 4

B: Each roll of the dice gives a score over 4.

$A = \{(1,3)(2,2)(3,1)\}$ $B = \{(5,5)(5,6)(6,5)(6,6)\}$

$P(A) = \frac{3}{36}$, $P(B) = 4/36$

$P(A \cup B) = P(A) + P(B) = \frac{3}{36} + 4/36 = 7/36$

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b) A prize is won if the total is 10 or if each individual score is over 4. Find the probability that a prize is won. ②

C: Total score on two rolls is 10

$C = \{(5,5)(4,6)(6,4)\}$

$B = \{(5,5)(5,6)(6,5)(6,6)\}$

$P(B) = 4/36 \qquad P(C) = \frac{3}{36}$

$P(B \cap C) = \{(5,5)\} = \frac{1}{36}$

$P(B \text{ or } C) = P(\text{Prize won})$

$= P(B) + P(C) - P(B \cap C)$

$= \frac{4}{36} + \frac{3}{36} - \frac{1}{36}$

$= 6/36$

Question 6.27(c) (Sher Muhammad ch)

G: choosen person wear Glasses

G': " " doesn't wear glasses

W: " " is woman

M: " " Man

$P(W) = 7/20 \quad P(G) = 6/20 \quad P(W \cap G) = 4/20$

P(women or someone who wear glasses) = ?

$P(W \cup G) = P(W) + P(G) - P(W \cap G)$

$P(W \cup G) = 7/20 + 6/20 - 4/20$

$\boxed{P(W \cup G) = 9/20}$

Independent Events:-

Two events A and B are said to be independent if the probability that one event occurs, is not affected by whether the other event has or has not occured. (which has no effect on one another)

Multiplication law

If A and B are two events then

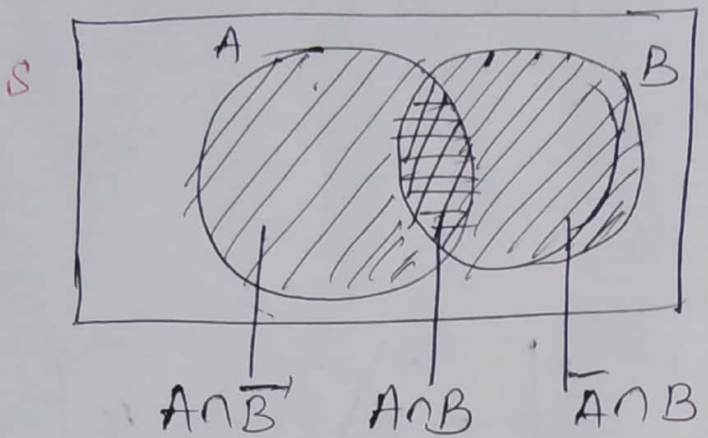
$P(A \text{ and } B) = P(A) \cdot P(B)$

$P(A \cap B) = P(A) \cdot P(B)$ [Independent events]

$P(A \cap B) = P(A) \cdot P(B/A)$ [Dependent events]

Addition Law:- If A and B are two events defined in a sample space S, then.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



The event $A \cup B$ may be written As

$$A \cup B = A \cup (\bar{A} \cap B)$$

$$P(A \cup B) = P(A) + P(B \cap \bar{A}) \quad \text{——— (1)}$$

B can be written As

$$B = (A \cap B) \cup (\bar{A} \cap B)$$

$$P(B) = P(A \cap B) + P(\bar{A} \cap B) \quad \text{——— (2)}$$

subtracting (2) from (1)

$$P(A \cup B) - P(B) = P(A) + \cancel{P(\bar{A} \cap B)} - P(A \cap B) - \cancel{P(\bar{A} \cap B)}$$

$$P(A \cup B) - P(B) = P(A) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Now if A & B are mutually exclusive then $P(A \cap B) = 0 = P(\emptyset)$

So

$$P(A \cup B) = P(A) + P(B)$$

Hence proved.

Conditional probability :-

The probability has been calculated on the basis of an extra 'condition' which you have been given. consider a die throwing example with Sample space $S=\{1,2,3,4,5,6\}$ - Suppose we wish to know the probability of the outcome that the die shows a 6, say event A. If before seeing the outcome we are told that the die shows an even no of dots, say event B, then the information that the die shows an even number excludes the outcomes 1, 3, 5 thereby reduces the original S.S to a sample space that consists of 3 outcomes 2, 4, 6. then the desired prob in the reduced S.S B is 1/3, since each outcome in the reduced S.S is equally likely. we call 1/3 as the conditional prob of event A because it is computed under the condition that the die has shown even no of dots. In other words.

P(die shows 6 / die shows an even no) = 1/3

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

"The sample space for an experiment must often be changed when some additional information pertaining to the outcome of the experiment is recieved. The effect of

(6)

Such information is to reduce the sample space by excluding some outcomes as being impossible which before recieving the information were believed possible. The probabilities associated with such a reduced S.S are called conditional probabilities.

Example 1 (Pg 72)

	G	B	Total
Left H	5	6	11
Right H	12	7	19
	17	13	30

L: left handed person is choosen

G: Girl is choosen

$P(L) = 11/30$

$P(G) = \frac{17}{30}$

$P(L/G)$ stands for probability that person choose is left-handed given that person choosen is girl.

$$P(L/G) = \frac{P(L \cap G)}{P(G)} = \frac{5/30}{17/30} = \frac{5}{17}$$

If A and B are two events $P(A) > 0$ then the conditional prob of B given A is

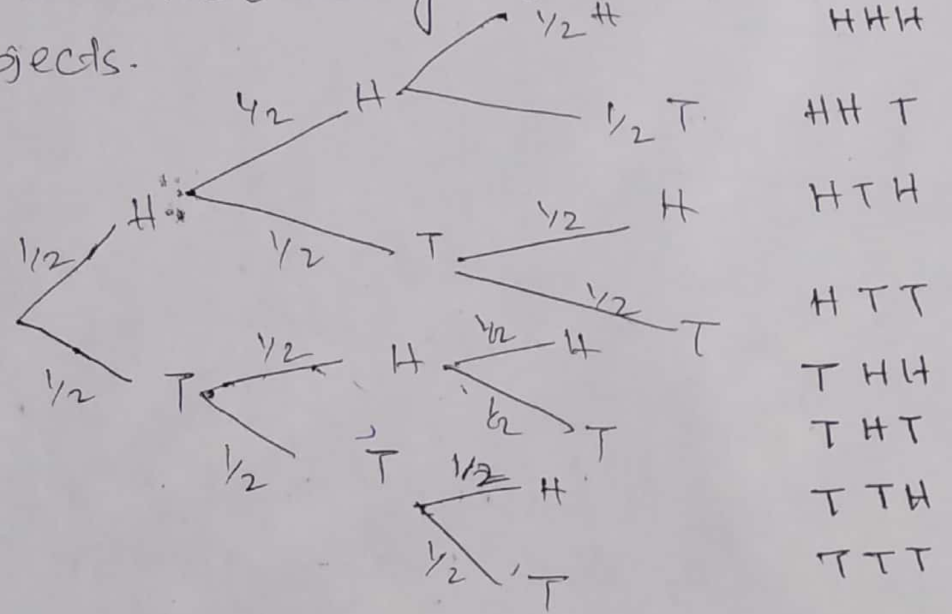
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$P(A \text{ and } B) = P(A) \cdot P(B/A)$

Which is known as multiplication law of probability.

Tree Diagram :- A way of representing a sequence of events used in probability since they record all possible outcomes in a clear and uncomplicated manner.

⇒ You can use tree diagrams in any problem in which there is a clear sequence to the outcomes including problems which are not neccessarily to do with the selection of objects.



A tree diagram which represents a coin being tossed three times

Example 2 :- (W.O.R)

R	W	
7	4	= 11

2 discs are selected W.O.R

R_1: First disc is Red

R_2: Second disc is Red

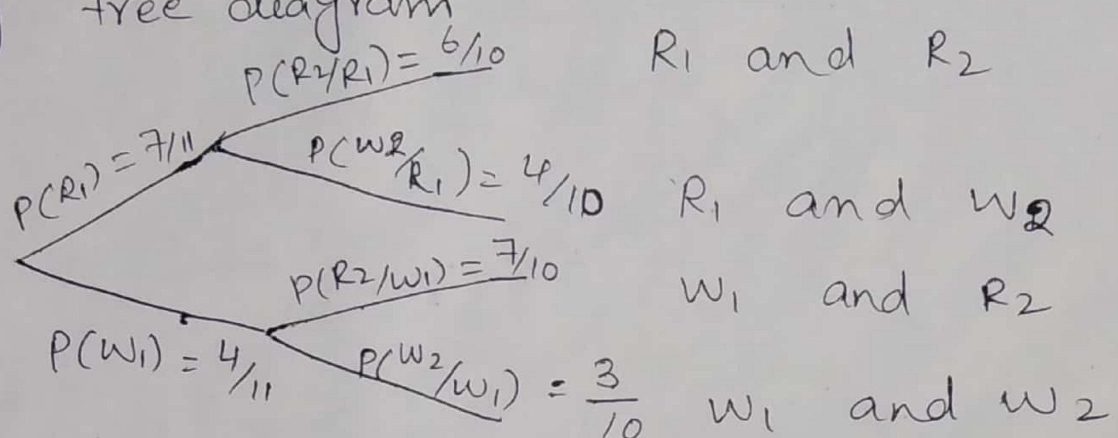
W_1: First disc is white

W_2: Second disc is white

$P(R_1' \text{ and } R_2) = ?$

$$P(R_1 \cap R_2) = P(R_1) \cdot P(R_2/R_1)$$
$$= \frac{7}{11} \cdot \frac{6}{10}$$
$$= \frac{42}{110}$$

using tree diagram



$$P(W_1 \cap R_2) = P(W_1) \cdot P(R_2/W_1)$$
$$= \frac{4}{11} \cdot \frac{7}{10} = \frac{14}{55}$$

P(Both discs are of same colour)

$$= P(R_1 \cap R_2) \text{ or } P(W_1 \cap W_2)$$
$$= P(R_1) \cdot P(R_2/R_1) + P(W_1) \cdot P(W_2/W_1)$$
$$= \frac{7}{11} \cdot \frac{6}{10} + \frac{4}{11} \times \frac{3}{10}$$
$$= \frac{27}{55}$$

Example#4 (pg 75)

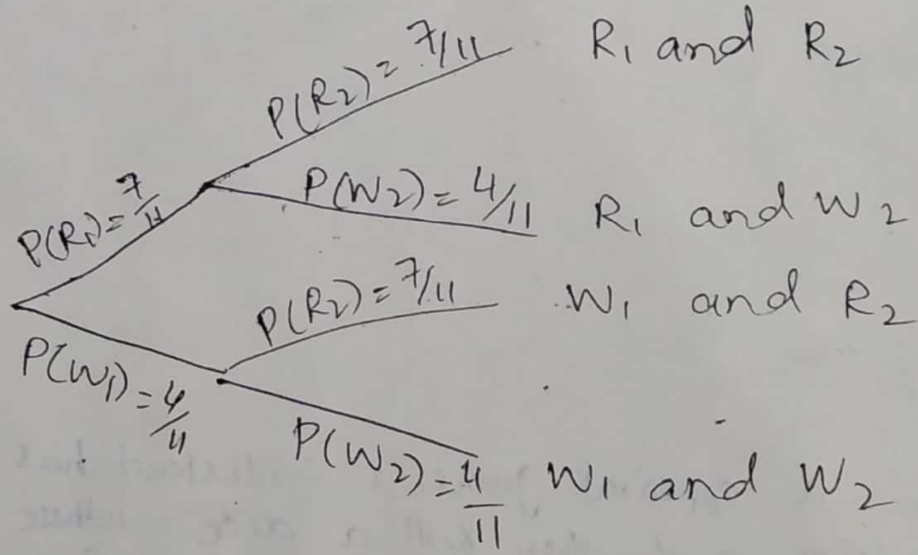
R W

7 4 = 11

2 discs are selected w.R

$$P(R_2) = P(R_1 \cap R_2) \text{ or } P(W_1 \cap R_2)$$

$$= P(R_1 \cap R_2) + P(W_1 \cap R_2)$$



$$P(R_2) = P(R_1) \cdot P(R_2) + P(W_1) \cdot P(R_2)$$

$$P(R_2) = \frac{7}{11} \cdot \frac{7}{11} + \frac{4}{11} \cdot \frac{7}{11}$$

$$\boxed{P(R_2) = \frac{7}{11}}$$

In this case $P(R_2) = P(R_2/R_1)$; which means

that the first disc's being red has no effect on the chance of second disc being red. (11)
(Since the first disc was replaced before the second removed).

$\Rightarrow$ In case of independent $P(B/A) = P(B)$

so as $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

$$P(B) = \frac{P(A \cap B)}{P(A)} \qquad \because P(B/A) = P(B)$$

$$P(B) \cdot P(A) = P(A \cap B)$$

Example 4.5.1 (Pg 76)

P(Prize won) = P (coin shows heads) and (dice score is less than 3).

(As coin shows heads and score on the dice have no effect on other so independent case)

P(Won) = P (coin shows heads) x P (dice score is less than 3)

$$= \frac{1}{2} \times \frac{2}{6} = 1/6$$

Question:- In a carnival game, a contestant has to first spin a fair coin and then roll a dice whose faces are numbered one to six. The contestant wins a prize if the coin shows heads and the dice shows score below 3. Find the prob that a contestant wins a prize.

Example 1:-

Consider a class of 30 students, of whom 17 are girls and 13 are boys. Suppose further that five of the girls and six of the boys are left handed, and all of the remaining Students are right handed. Suppose if a student selected at random is a girl, what is the probability that she is left handed.

Example 2:-

Suppose a jar contains seven red discs and four white discs. Two discs are selected w.o.r. What is the probability that

a) both are red.

b) First white and second red.

c) Both discs are of same colour.

Example 3:-

Weather records indicate that the probability that a particular day is dry is $3/10$. Arid F.C. is a football team whose record of success is better on dry days than on wet days. The probability that Arid win on a dry day is $3/8$, whereas the probability that they win on a wet day is $3/11$. Arid are due to play their next match on saturday.

a) What is the probability that Arid will win?

b) Three Saturdays ago Arid won their match, what is the probability that it was a dry day?

Example # 4:-

Suppose a jar contains seven red discs and four white discs. Two discs are selected w.r. What is the probability that second disc is red.

	W	M	Total
G	4	2	6
G'	3	11	14
	7	13	20

$P(W) = 7/20$ $\quad P(G) = \frac{6}{20}$ $\quad P(W \cap G) = \frac{4}{20}$

P(woman or some one who wear Glasses)

$= P(W) + P(G) - P(W \cap G)$

$= \frac{7}{20} + \frac{6}{20} - \frac{4}{20} = \frac{9}{20}$