Properties of Expectation :-

1) If a is a constant then

$E(a) = a$

2) If x is discrete r.v and a & b are constants then

$E(ax+b) = a\,E(x) + b$

3) $E(x+y) = E(x) + E(y)$

4) $E(xy) = E(x) \cdot E(y)$

Example 4.4 $\quad \mu_{g(x)} = E[g(x)] = \sum_x g(x) f(x)$

x	$f(x)$	$(2x-1)$	$(2x-1)f(x)$	$xf(x)$
4	$1/12$	7	$7/12$	$4/12$
5	$1/12$	9	$9/12$	$5/12$
6	$3/12$	11	$33/12$	$18/12$
7	$3/12$	13	$39/12$	$21/12$
8	$2/12$	15	$30/12$	$16/12$
9	$2/12$	17	$34/12$	$18/12$
			$152/12$	$82/12$

$$E[g(x)] = E(2x-1) = \frac{152}{12} = 12.67$$

4.17 $\quad E(ax+b) = aE(x) + b$

$$E(x) = \sum x f(x) = \frac{82}{12} = 6.84$$

$$E(2x-1) = 2E(x) - 1$$
$$= 2(6.84) - 1$$
$$12.68 = 12.68$$

Hence proved.

(3) Example 4.5 $E(4x+3) = ?$ $\mu_{g(x)} = E[g(x)] =$

$$E(4x+3) = \int_{-1}^{2} (4x+3)\frac{x^2}{3}\,dx$$

$$= \frac{1}{3}\int_{-1}^{2} 4x^3 + 3x^2\,dx$$

$$= \frac{1}{3}\left| \cancel{4}\frac{x^4}{\cancel{4}} + \cancel{3}\frac{x^3}{\cancel{3}} \right|_{-1}^{2}$$

$$= \frac{1}{3}\left| x^4 + x^3 \right|_{-1}^{2}$$

$$= \frac{1}{3}\left[(16-1) + (8+1)\right] = \frac{1}{3}(24)$$

$$E(4x+3) = 8$$

Example 4.18

$$E(4x+3) = 4E(x)+3$$

$$E(x) = \int_{-1}^{2} x\cdot\left(\frac{x^2}{3}\right)dx = \int_{-1}^{2} \frac{x^3}{3}\,dx$$

$$= \frac{1}{3}\left|\frac{x^4}{4}\right|_{-1}^{2} = \frac{1}{12}(16-1)$$

$$= \frac{15}{12} = \frac{5}{4}$$

As

$$E(4x+3) = 4E(x)+3$$

$$= \cancel{4}\left(\frac{5}{\cancel{4}}\right)+3$$

$$8 = 8$$

Hence proved.

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Mean and variance of binomial distribution

As

$$f(x) = \binom{n}{x} p^x q^{n-x} \qquad x = 0, 1, 2, \ldots, n$$

$$\mu = E(x) = \Sigma x f(x)$$

so

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n}{x} \, {}^{n-1}C_{x-1} \, p^x q^{n-x}$$

taking np as common

$$= np \sum_{x=0}^{n} \frac{x}{x} \, {}^{n-1}C_{x-1} \, p^{x-1} q^{n-x}$$

$$= np \, (q+p)^{n-1} \qquad \because \sum_{x=0}^{n} {}^{n}C_x p^x q^{n-x} = (q+p)^n$$

$$= np \, (1)^{n-1}$$

$$\boxed{E(x) = np}$$

Rough Explanation

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$5! = 5 \times 4!$ or $5! = 5(5-1)!$

$n! = n(n-1)!$ or $n(n-1)(n-2)!$

$n! = n(n-1)(n-2)(n-3)!$

$${}^{n}C_x = \frac{n!}{x!\,(n-x)!}$$

$${}^{n}C_x = \frac{n(n-1)!}{x(x-1)!\,(n-x)!} \quad \text{or} \quad \frac{n}{x} \, {}^{n-1}C_{x-1}$$

$${}^{n}C_x = \frac{n}{x} \, \frac{n-1}{x-1} \, {}^{n-2}C_{x-2}$$

$$(q+p)^n = {}^nc_0 p^0 q^{n-0} + {}^nc_1 p^1 q^{n-1} + \cdots + {}^nc_n p^n q^{n-n}$$

$$= \sum_{x=0}^{n} {}^nc_x p^x q^{n-x}$$ (Binomial expansion)

$$var(x) = E(x^2) - [E(x)]^2$$

$$E(x^2) = \Sigma x^2 f(x)$$

we may write

$$x^2 = x(x-1) + x \Rightarrow E(x^2) = E[x(x-1)] + E(x)$$

so

$$E(x^2) = \sum_{x=0}^{n} [x(x-1)+x]\, {}^nc_x p^x q^{n-x}$$

$$E(x^2) = \sum_{x=0}^{n} x(x-1)\, {}^nc_x p^x q^{n-x} + \sum_{x=0}^{n} x\, {}^nc_x p^x q^{n-x}$$

$$E[x(x-1)] = \sum_{x=0}^{n} x(x-1) \frac{n(n-1)}{x(x-1)}\, {}^{n-2}c_{x-2}\, p^x \cdot q^{n-x}$$

taking $n(n-1)p^2$ as common

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{\cancel{x(x-1)}}{\cancel{x(x-1)}}\, {}^{n-2}c_{x-2}\, p^{x-2} q^{n-x}$$

$$E(x^2) = E[x(x-1)] + E(x)$$

$$E(x^2) = n(n-1)p^2 (q+p)^{n-2} + np \quad \because E(x) = np$$

$$= n(n-1)p^2 (1)^{n-2} + np$$

$$\boxed{E(x^2) = n(n-1)p^2 + np}$$

$$var(x) = E(x^2) - [E(x)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= np[(n-1)p + 1 - np]$$

$$= np[\cancel{np} - p + 1 - \cancel{np}]$$

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$$var(x) = np[1-p]$$

$$\boxed{var(x) = npq}$$

Hence proved.

<u>Mean and variance of poisson dist:-</u>

$$f(x) = \frac{e^{-\mu}\mu^x}{x!} \qquad x = 0, 1, 2 \ldots$$

As $E(x) = \sum x f(x)$

$$= \sum x \frac{e^{-\mu}\mu^x}{x!}$$

taking μ as common

$$E(x) = e^{-\mu}\mu \sum_{x=1}^{\infty} \frac{\cancel{x}}{\cancel{x}} \frac{\mu^{x-1}}{(x-1)!}$$

$$= e^{-\mu}\mu\left[1 + \mu + \frac{\mu^2}{2!} + \frac{\mu^3}{3!} + \cdots\right]$$

$$= \cancel{e^{-\mu}}\,\mu\,\cancel{e^{+\mu}}$$

$$\boxed{E(x) = \mu}$$

Now

$$E(x^2) = \sum_{x=0}^{\infty} x^2 f(x)$$

$$= \sum_{x=0}^{\infty}\left[x(x-1) + x\right] \frac{e^{-\mu}\mu^x}{x!}$$

$$= \sum_{x=0}^{\infty} x(x-1)\frac{e^{-\mu}\mu^x}{x!} + \sum_{x=0}^{\infty} \frac{e^{-\mu}\mu^x}{x!}$$

$$E[x(x-1)] = \sum_{x=0}^{\infty} x(x-1)\frac{e^{-\mu}\mu^x}{x!}$$

taking μ^2 as common

$$= e^{-\mu}\mu^2 \sum_{x=2}^{\infty} \frac{\cancel{x(x-1)}}{\cancel{x(x-1)}} \frac{\mu^{x-2}}{(x-2)!}$$

So

$$E[x(x-1)] = e^{-\mu}\mu^2\left[1+\mu+\frac{\mu^2}{2!}+\frac{\mu^3}{3!}+\cdots\right]$$

$$= \cancel{e^{-\mu}}\,\mu^2\,\cancel{e^{+\mu}}$$

$$= \mu^2$$

As $E(x^2) = E[x(x-1)] + E(x)$

$$= \mu^2 + \mu \qquad \because E(x) = \mu$$

So $\mathrm{var}(x) = E(x^2) - [E(x)]^2$

$$= \mu^2 + \mu - (\mu)^2$$

$$= \cancel{\mu^2} + \mu - \cancel{\mu^2}$$

$$\boxed{\mathrm{var}(x) = \mu}$$