Multiple Random Variable

⇒ A random variable with only one variable is univariate.

⇒ A r-v involving two variables called Bivariate.

⇒ A r-v involving three or more variables called trivariate or multivariate.

Joint Distributions:-

The distribution of two or more r.v's which are observed simultaneously when an expeiment is performed is called joint distribution.

Bivariate Distribution function:-

Let X and Y be two r-v's defined on same sample space S then the function $F(x,y)$ defined as

$$F(x,y) = P(X \leq x \text{ and } Y \leq y)$$

where $F(x,y)$ gives the probability that X will take on a value less than or equal to x, at the same time, Y will take a value less than or equal to y, is called a bivariate or joint distribution function of X and Y.

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⇒ A bivariate distribution may be discrete when the possible values of (X, Y) are finite or countable infinite.

⇒ It is continuous if (X, Y) can assume can assume all values in some non-countable set of values.

⇒ A bivariate distribution is said to be mixed when one r.v is discrete and other is continuous.

Bivariate probability function :- (Probability mass function)

Let X & Y be two discrete r.v's defined on the same sample space, X takes values $x_1, x_2 \dots x_m$ and Y takes the values $y_1, y_2 \dots y_n$ then the probability that X takes on x_i and Y takes on y_j at the same time is called joint probability function or joint distribution of X and Y. It is denoted by $f(x_i, y_j)$.

$$f(x_i, y_j) = P(X = x_i \text{ and } Y = y_j)$$
$$i = 1, 2, \dots, m$$
$$j = 1, 2, \dots, n$$

⇒ Joint or bivariate probability distribution consisting of all pairs of values (x_i, y_i) and their associated probabilities $f(x_i, y_i)$ i.e the set of triples $[x_i, y_j, f(x_i, y_j)]$

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can either be shown in a two way table or be expressed by means of formula $f(x,y)$.

joint probability dist of X & Y

X \ Y	y_1	y_2	...	y_j	...	y_n	$P(X=x_i)$
x_1	$f(x_1,y_1)$	$f(x_1,y_2)$	...	$f(x_1,y_j)$	...	$f(x_1,y_n)$	$g(x_1)$
x_2	$f(x_2,y_1)$	$f(x_2,y_2)$	...	$f(x_2,y_j)$	...	$f(x_2,y_n)$	$g(x_2)$
$\vdots$	$\vdots$						$\vdots$
x_i	$f(x_i,y_1)$	$f(x_i,y_2)$	...	$f(x_i,y_j)$	...	$f(x_i,y_n)$	$g(x_i)$
$\vdots$	$\vdots$						$\vdots$
x_m	$f(x_m,y_1)$	$f(x_m,y_2)$	...	$f(x_m,y_j)$	...	$f(x_m,y_n)$	$g(x_m)$
$P(Y=y_j)$	$h(y_1)$	$h(y_2)$		$h(y_j)$		$h(y_n)$	1

A joint probability function has the following properties.

1) $f(x_i,y_j) \geq 0$ for all (x_i,y_j) i.e $i=1,2,\dots,m$
$j=1,2,\dots,n$

2) $\sum_i \sum_j f(x_i,y_j)=1$

<u>Marginal probability functions</u> :- We can obtain the individual probability functions of X and Y from the joint probability functions for X and Y.

⇒ Individual probability functions are called marginal probability functions.

⇒ let $f(x,y)$ be the joint probability function of two discrete r.v's X and Y. Then the marginal function of X is

$$g(x_i)=\sum_{j=1}^{n} f(x_i,y_j)$$

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$$g(x_i) = f(x_i, y_1) + f(x_i, y_2) + \cdots + f(x_i, y_n)$$
$$= f(X = x_i);$$ that is the individual probability function of X is found by adding over the rows of two-way table. Similarly the marginal function for Y is obtained by adding over the columns as

$$h(y_j) = \sum_{i=1}^{m} f(x_i, y_j)$$
$$= f(x_1, y_j) + f(x_2, y_j) + \cdots + f(x_m, y_j)$$
$$= f(Y = y_j)$$

$\Rightarrow$ The probabilities in each marginal probability function add to 1.

Conditional Probability Functions :-

Let X & Y be two discrete r.v's with joint probability function $f(x, y)$. Then the conditional probability function for X given $Y = y$, denoted as $f(x/y)$ is defined as

$$f(x_i/y_j) = P(X = x_i / Y = y_j)$$
$$= \frac{P(X = x_i \text{ and } Y = y_j)}{P(Y = y_j)}$$
$$= \frac{f(x_i, y_j)}{h(y_j)} \quad \text{for } i = 1, 2, 3, \ldots \quad j = 1, 2, 3, \ldots$$

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where $h(y)$ is marginal probability function and $h(y) > 0$.

Similarly, the conditional probability function for Y given $X = x$ is

$$f(y_j/x_i) = P(Y = y_j / X = x_i)$$

$$= \frac{P(Y = y_j \text{ and } X = x_i)}{P(X = x_i)}$$

$$= \frac{f(x_i, y_j)}{g(x_i)}$$

where $g(x) > 0$.

Independence:-

Two r.v's are said to be independent if and only if, all possible pairs of values (x_i, y_j) the joint probability function $f(x, y)$ can be expressed as the product of two marginal probability functions. That is, X and Y are independent if

$$f(x, y) = P(X = x_i \text{ and } Y = y_j)$$

$$= P(X = x_i) . P(Y = y_j)$$

$$= g(x_i)\, h(y) \quad \text{for all } i \text{ and } j$$

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Example 7·6 :-

B	R	G	
3	2	3	= 8

X: No. of black balls

Y: No. of Red balls

$S \cdot S = \binom{8}{2}$ ∵ two balls are selected

i) The joint probability function $f(x, y)$

$X = 0, 1, 2$

$Y = 0, 1, 2$

$(0,0)\ (0,1)\ (0,2)\ (1,0)\ (1,1)\ (2,0)$

$$f(x, y) = \binom{3}{x}\binom{2}{y}\binom{3}{2-x-y} \Big/ \binom{8}{2}$$

Joint probability Distribution.

X \ Y	0	1	2	g(x)
0	3/28	6/28	1/28	10/28
1	9/28	6/28	0	15/28
2	3/28	0	0	3/28
h(y)	15/28	12/28	1/28	1

ii) $P(X + Y \leq 1) = f(0,0) + f(0,1) + f(1,0)$

$$= \frac{3}{28} + \frac{6}{28} + \frac{9}{28}$$

$$= 18/28 = 9/14$$

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(iii) The marginal functions of x alone and y alone are

$g(x) = \sum_y f(x,y)$ and $h(y) = \sum_x f(x,y)$

x	$g(x)$
0	$10/28$
1	$15/28$
2	$3/28$
	1

$g(0) = f(0,0) + f(0,1) + f(0,2) = \frac{10}{28}$

$g(1) = f(1,0) + f(1,1) + f(1,2) = \frac{15}{28}$

$g(2) = f(2,0) + f(2,1) + f(2,2) = \frac{3}{28}$

similarly

y	$h(y)$
0	$15/28$
1	$12/28$
2	$1/28$
	1

iv) The conditional prob. dist $f(x/1)$

$$f(x/1) = P(X=x/Y=1)$$

$$= \frac{P(X=x \text{ and } Y=1)}{P(Y=1)}$$

$$= \frac{f(x,1)}{h(1)} \qquad x = 0, 1, 2$$

$h(1) = f(0,1) + f(1,1) + f(2,1)$

$$h(1) = \frac{6}{28} + \frac{6}{28} + 0$$

$$= \frac{12}{28} = \frac{3}{7}$$

$$f(0/1) = \frac{6/28}{3/7} = 6/28 \times 7/3 = 1/2$$

$$f(1/1) = \frac{7}{3} f(1,1) = \frac{7}{3} \times 6/28 = 1/2$$

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$$f(2/1) = \frac{7}{3} f(2,1) = \frac{7}{3} \times 0 = 0$$

Hence

$$f(0/1) = P(X=0/Y=1)$$

$$= \frac{f(0,1)}{h(1)}$$

$$= \frac{1}{2}$$

conditional probability distribution is

X	$f(x/1)$
0	1/2
1	1/2
2	0

vi) Are X and Y independent?

$$f(0,1) = \frac{6}{28}$$

$$g(0) = \frac{3}{28} + \frac{6}{28} + \frac{1}{28} = \frac{10}{28}$$

$$h(1) = \frac{6}{28} + \frac{6}{28} + 0 = \frac{12}{28}$$

$$f(0,1) \neq g(0)h(1)$$

$$\frac{6}{28} \neq \frac{10}{28} \times \frac{12}{28}$$

and therefore X and Y are not statistically independent.

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