

# Outline

---

- Informed Search
- Uninformed Versus Informed Search
- Best First Search
- Greedy Search
- A\* Search

# Informed Search

---

- Informed searches use domain knowledge to guide the selection of the best path to continue searching
- **Heuristics** are used, which are informed guesses
- Heuristic means "serving to aid discovery"

# Informed Search

---

- Define a **heuristic** function,  $h(n)$ 
  - uses domain-specific information in some way
  - is computable from the current state description
  - it estimates
    - the "goodness" of node  $n$
    - how close node  $n$  is to a goal
    - the cost of *minimal* cost path from node to a goal state

# Informed Search

---

- $f(n) \geq 0$  for all nodes
- $h(n)$  close to 0 means we think  $n$  is close to a goal state
- $h(n)$  very big means we think  $n$  is far from a goal state
- All domain knowledge used in the search is encoded in the heuristic function,
- An example of a “*weak method*” for AI because of the limited way that domain-specific information is used to solve a problem

# Uninformed versus Informed

Uninformed search	Informed search
<ul style="list-style-type: none"><li>• does not have any additional information on the <u>quality</u> of states.</li><li>• So, it is impossible to determine <u>which state is the better than others</u>. As a result, search efficiency depends only on the structure of a state space</li></ul>	<ul style="list-style-type: none"><li>• heuristically informed search uses a certain kind of information about states in order to guide search along <u>promising</u> branches within a state space.</li><li>• Using problem-specific knowledge as hints to guide the search.</li></ul>

# Uninformed versus Informed(cont)

Uninformed search	Informed search
<ul style="list-style-type: none"><li>• look for solutions by <u>systematically</u> generating new states and checking each of them against the goal.</li></ul> <ol style="list-style-type: none"><li>1. It is very <u>inefficient</u> in most cases.</li><li>2. Most successor states are “obviously” a bad choice.</li><li>3. Such strategies do not use problem-specific knowledge</li></ol>	<ol style="list-style-type: none"><li>1. They are almost always <u>more efficient</u> than uninformed strategies.</li><li>2. May reduce time and space complexities.</li><li>3. Evaluation function <math>f(n)</math> measures distance to the goal.</li><li>4. Order nodes in Frontier according to <math>f(n)</math> and decide which node to expand next.</li></ol>

# Best-First Search

---

An **informed search strategy** uses problem-specific knowledge beyond the definition of the problem itself, so it can find solutions more efficiently than an uninformed strategy.

**Best-first search** is an instance of the general TREE-SEARCH or GRAPH-SEARCH algorithm in which a node is selected for expansion based on an **evaluation function,  $f(n)$** .

- A key component of these algorithms is a **heuristic function** denoted  $h(n)$  :

*$h(n)$  = estimated cost of the cheapest path from node  $n$  to a goal node, if  $n$  is a goal node, then  $h(n) = 0$ .*

# Best-First Search

---

Special cases:

- Greedy Best first search
  - “Always chooses the successor node with the best  $f$  value” where  $f(n) = h(n)$
  - We choose the one that is nearest to the final state among all possible choices
- A\* search
  - Best first search using an “admissible” heuristic function  $f$  that takes into account the current cost  $g$
  - Always returns the optimal solution path



---

# Greedy Best First Search

*eval-fn*:  $f(n) = h(n)$

# Greedy Best-First Search

---

- Greedy best-first search tries to expand the node that is closest to the goal because it is likely to lead to a solution quickly.
- Thus, it evaluates nodes by using just the **heuristic function**; that is,  $f(n)=h(n)$ .
- Evaluation function  
 $f(n) = h(n)$  (heuristic) = estimate of cost **from  $n$  to goal**
- Assume **hold(n)** = **straight line distance** from **n** to **Bucharest**, so Greedybest-first search expands the node that appears to be closest to the goal

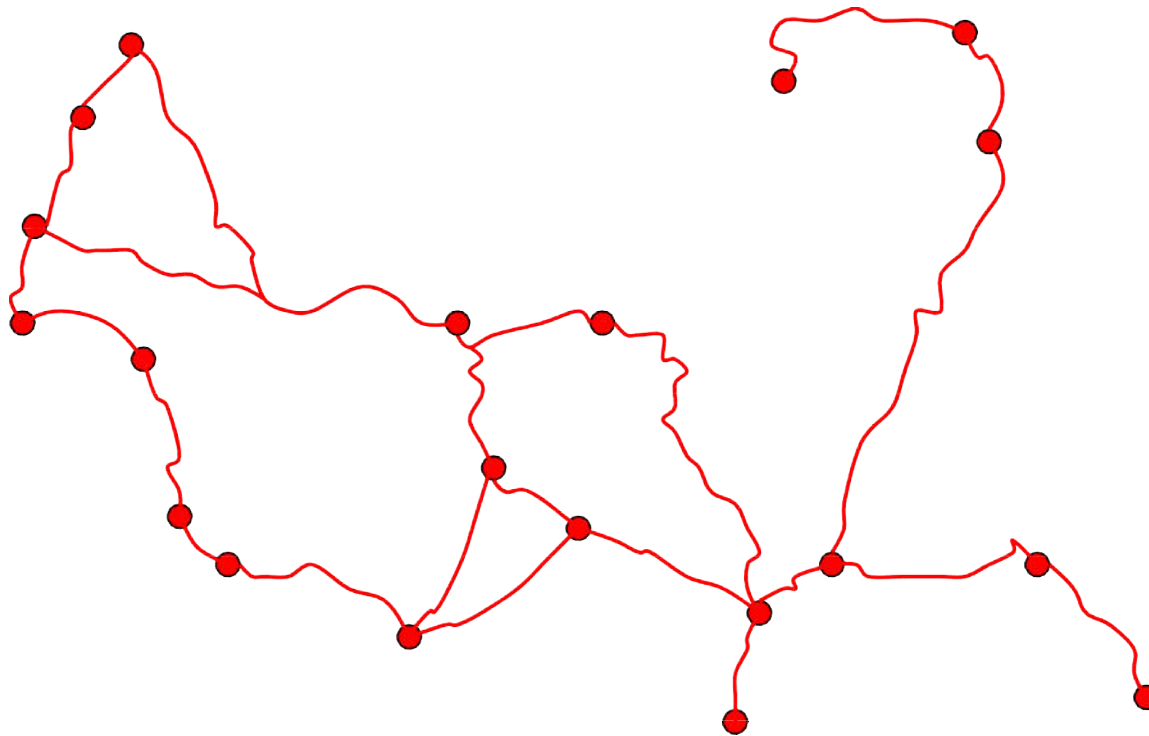
# Romania



# Romania

A map of Romania with major cities labeled. A red line with circular markers indicates a travel route. The route starts in the northwest near Satu Mare, goes south through Oradea, Timisoara, and Arad, then east through Sibiu, Brasov, and Cluj Napoca. It continues south through Rimnicu Vilcea, Pitesti, and Targoviste, then east through Ploiesti, Bucharest, and Giurgiu. From Giurgiu, it goes north through Iasi, Vaslui, Bacau, and Neamt, then south through Focșani, Galati, Braila, Tulcea, Constanta, Calarasi, Slobozia, and ends near Drobeta Turnu Severin. The Black Sea is visible to the east.

# Romania



# Map of Romania

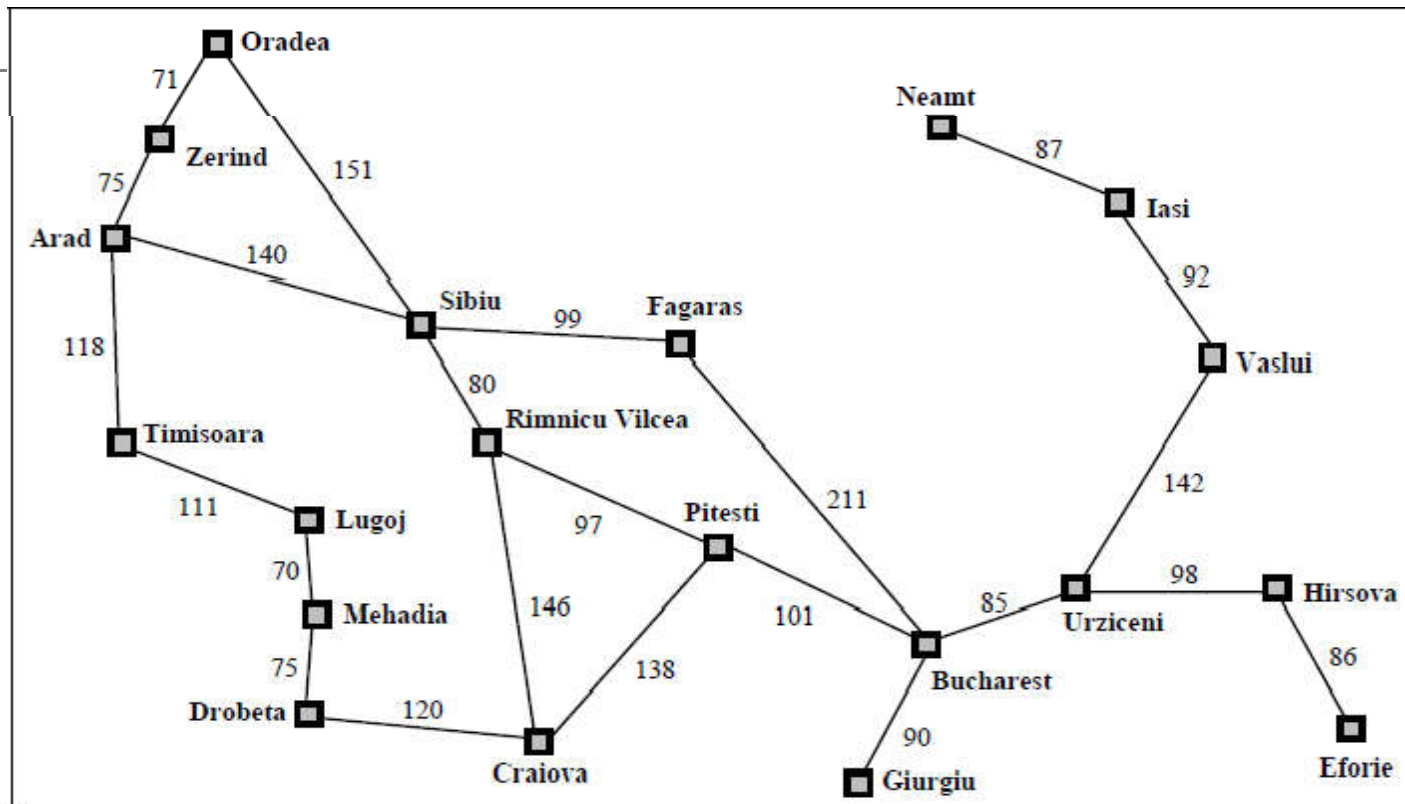
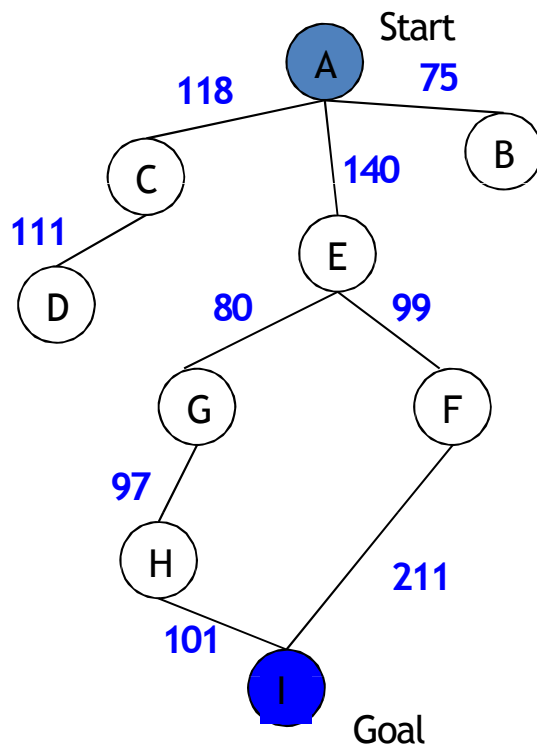


Figure 3.2 A simplified road map of part of Romania.

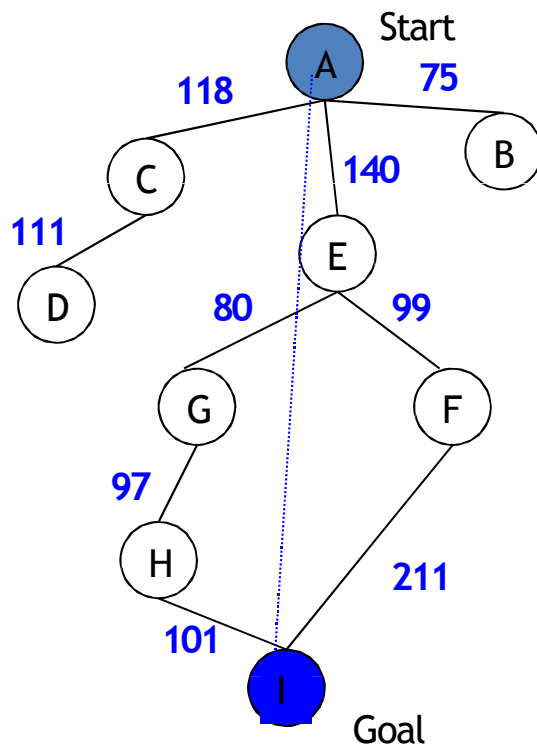
# Greedy Search



State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

$f(n) = h(n) = \text{straight-line distance heuristic}$

# Greedy Search

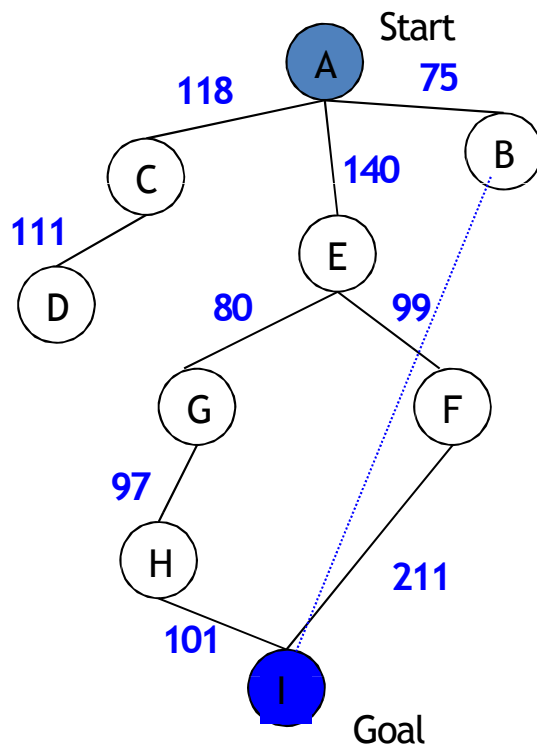


State	Heuristic: h(n)
<b>A</b>	<b>366</b>
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

$f(n) = h(n) = \text{straight-line distance heuristic}$



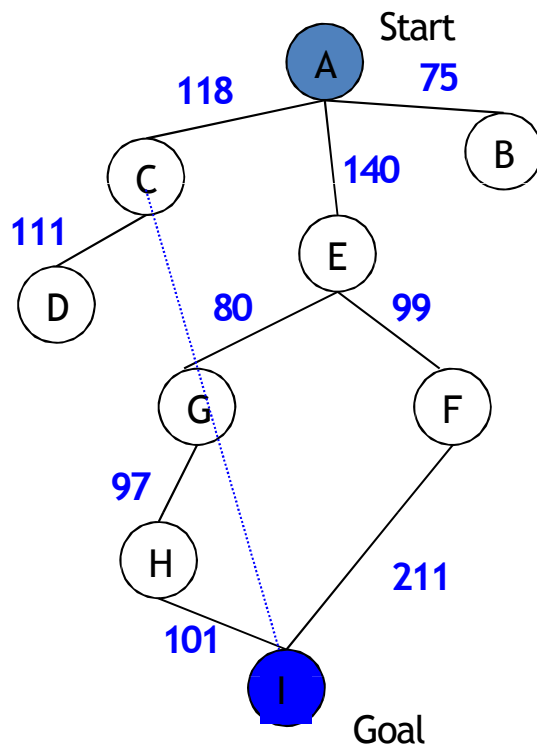
# Greedy Search



State	Heuristic: h(n)
A	366
<b>B</b>	<b>374</b>
C	329
D	244
E	253
F	178
G	193
H	98
I	0

$f(n) = h(n) = \text{straight-line distance}$   
heuristic

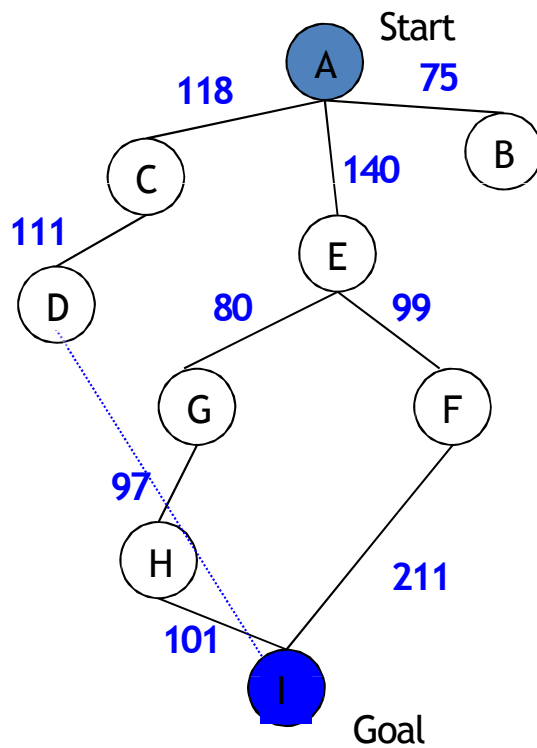
# Greedy Search



State	Heuristic: h(n)
A	366
B	374
<b>C</b>	<b>329</b>
D	244
E	253
F	178
G	193
H	98
I	0

$f(n) = h(n) = \text{straight-line distance heuristic}$

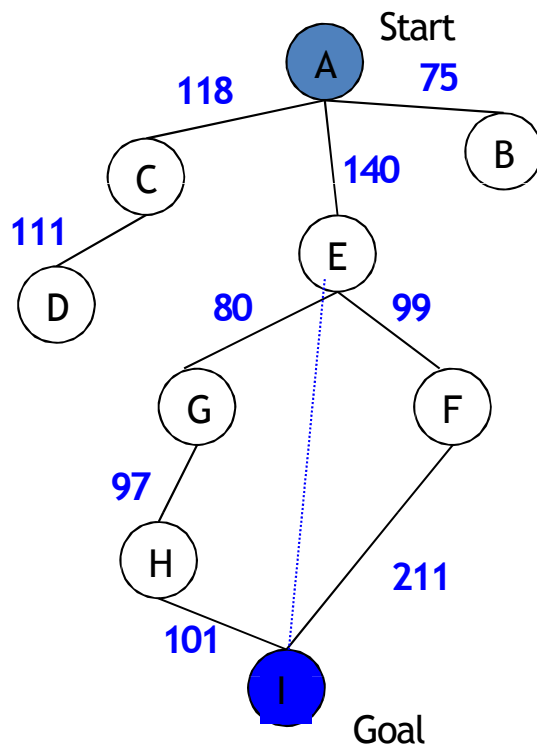
# Greedy Search



State	Heuristic: h(n)
A	366
B	374
C	329
<b>D</b>	<b>244</b>
E	253
F	178
G	193
H	98
I	0

$f(n) = h(n) = \text{straight-line distance}$   
heuristic

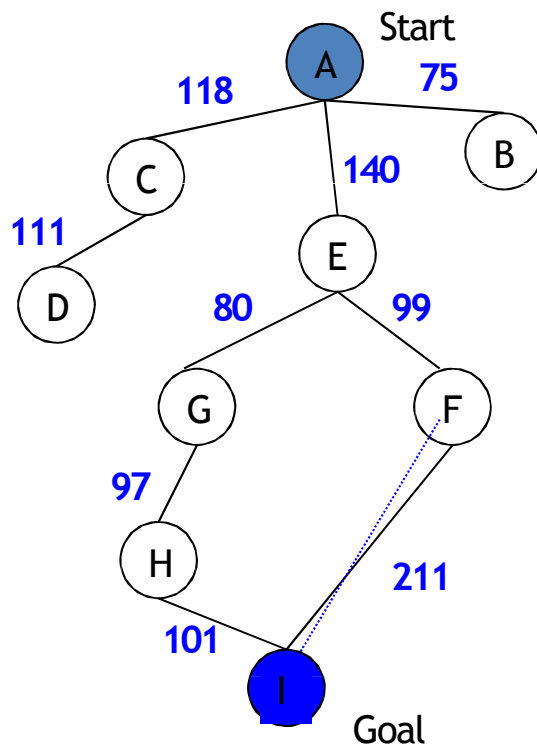
# Greedy Search



State	Heuristic: h(n)
A	366
B	374
C	329
D	244
<b>E</b>	<b>253</b>
F	178
G	193
H	98
I	0

$f(n) = h(n) = \text{straight-line distance}$   
heuristic

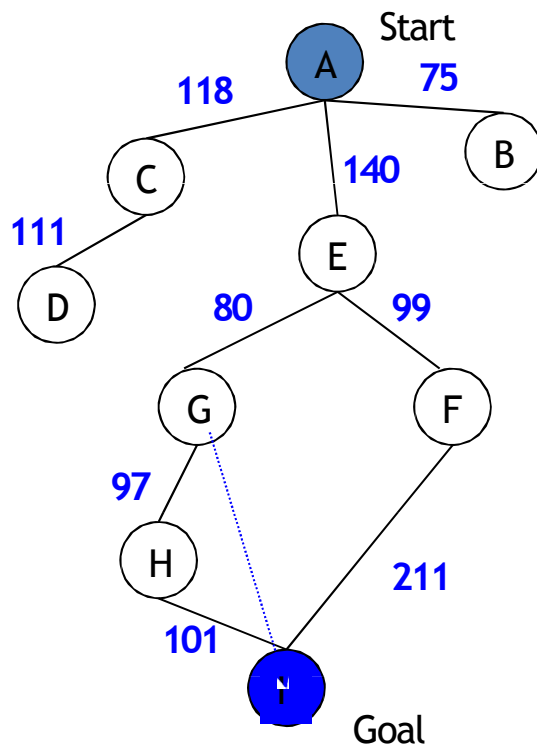
# Greedy Search



State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
<b>F</b>	<b>178</b>
G	193
H	98
I	0

$f(n) = h(n) = \text{straight-line distance}$   
heuristic

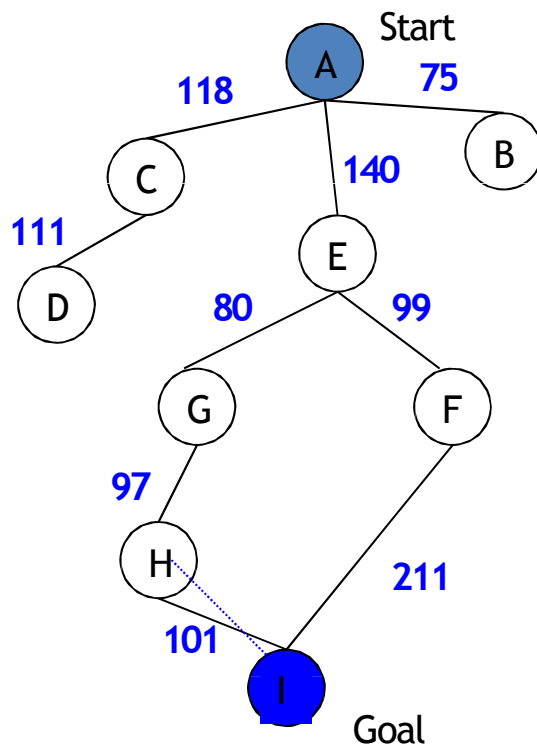
# Greedy Search



State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
F	178
<b>G</b>	<b>193</b>
H	98
I	0

$f(n) = h(n) = \text{straight-line distance}$   
heuristic

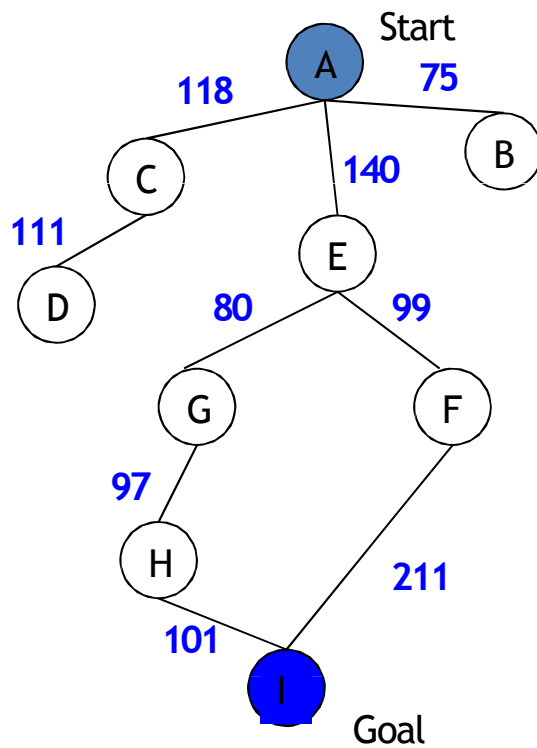
# Greedy Search



State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
<b>H</b>	<b>98</b>
I	0

$f(n) = h(n) = \text{straight-line distance}$   
heuristic

# Greedy Search



State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
<b>I</b>	<b>0</b>

$f(n) = h(n) = \text{straight-line distance}$   
heuristic

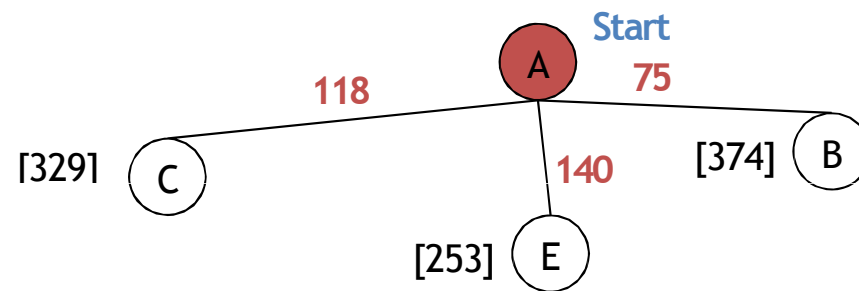


# Greedy Search – Tree Search

(A) Start

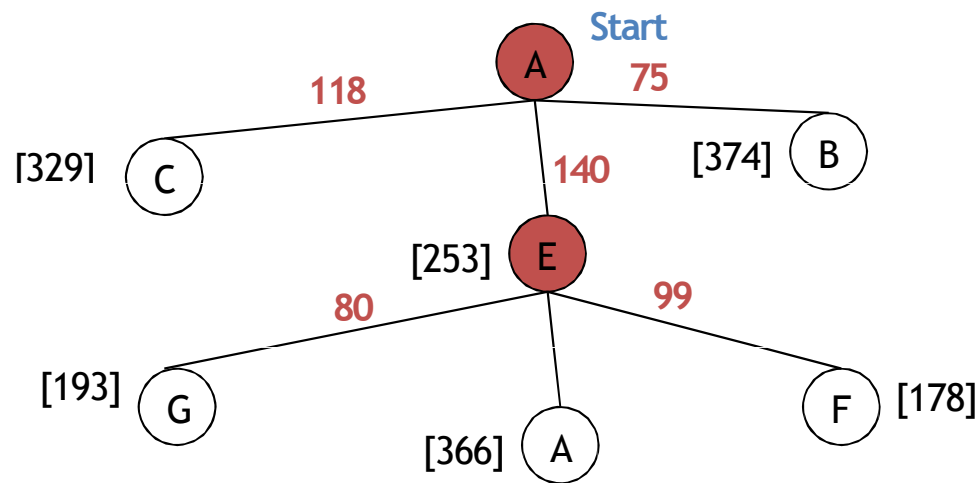
State	Heuristic : $h(n)$
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
<b>I</b>	<b>0</b>

# Greedy Search – Tree Search



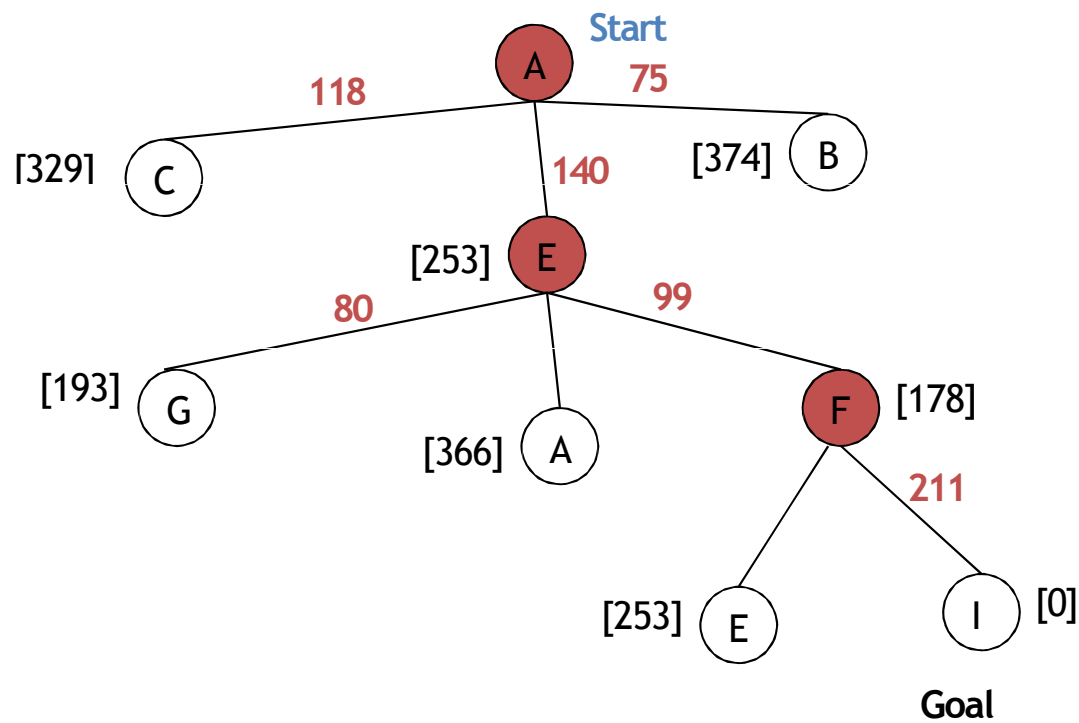
State	Heuristic : h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

# Greedy Search – Tree Search



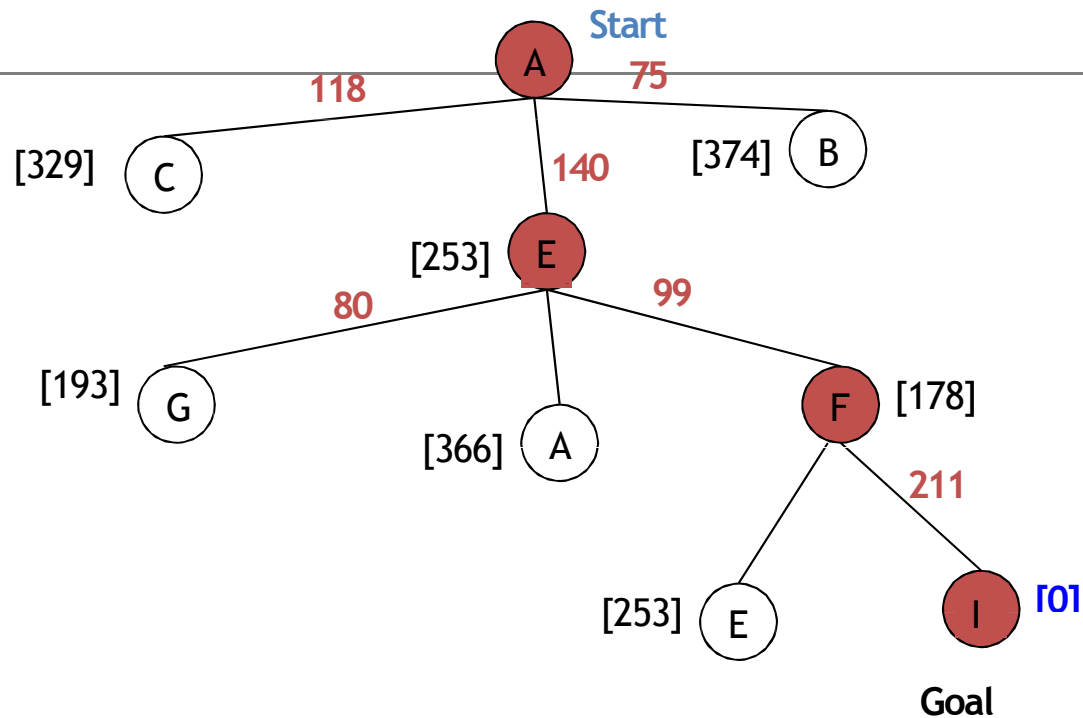
State	Heuristic : h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
<b>I</b>	<b>0</b>

# Greedy Search – Tree Search



State	Heuristic : h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
<b>I</b>	<b>0</b>

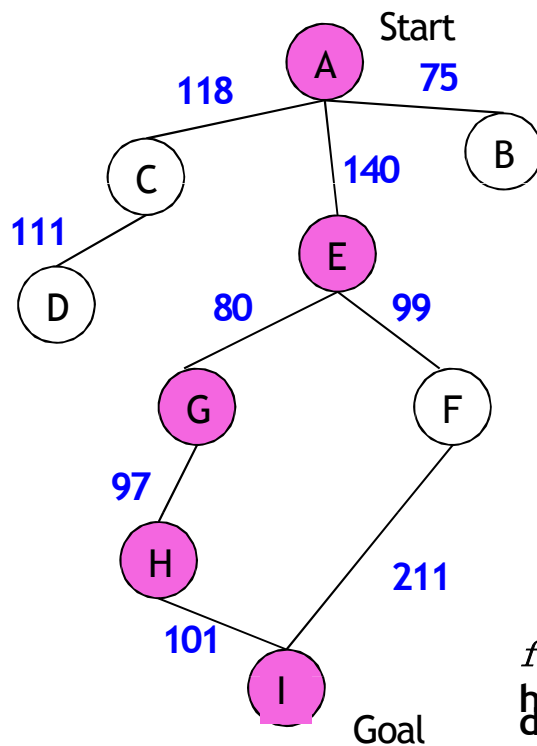
# Greedy Search – Tree Search



$$\text{dist}(A-E-F-I) = 140 + 99 + 211 = 450$$

State	Heuristic : h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
<b>I</b>	<b>0</b>

# Greedy Search: Optimal?



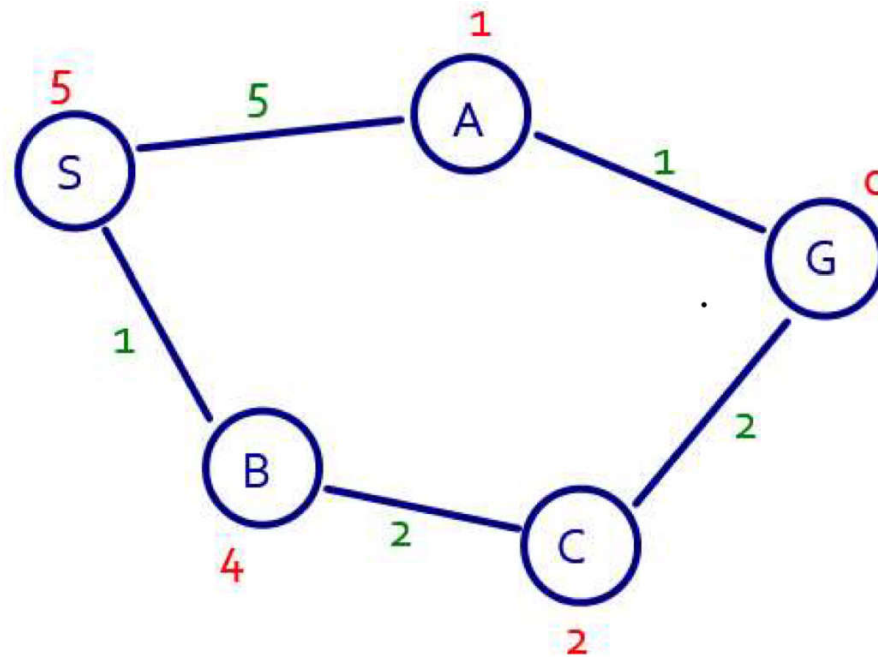
State	Heuristic: h(n)
A	366
B	374
C	329
D	244
<b>E</b>	<b>253</b>
F	178
<b>G</b>	<b>193</b>
<b>H</b>	<b>98</b>
I	0

$f(n) = h(n) = \text{straight-line distance}$

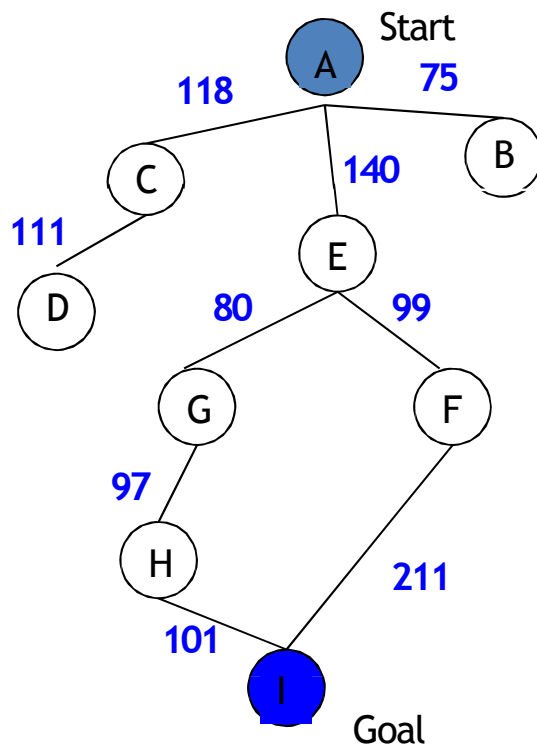
heuristic  
 $\text{dist}(A-E-G-H-I) = 140 + 80 + 97 + 101 = 418$

# Greedy Search: Optimal?

---



# Greedy Search: Complete?



State	Heuristic: h(n)
A	366
B	374
** C	<b>250</b>
D	244
E	253
F	178
G	193
H	98
I	0

$f(n) = h(n) = \text{straight-line distance heuristic}$

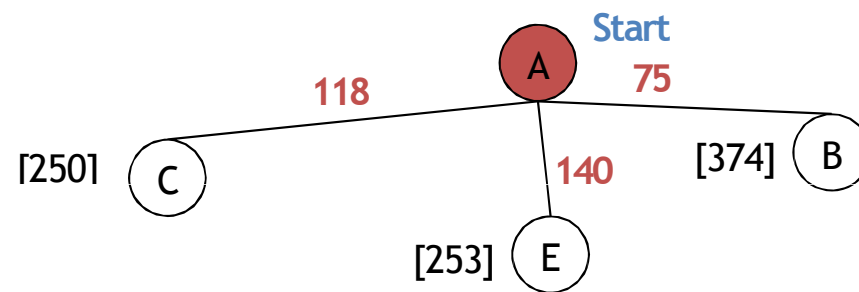


# Greedy Search: Tree Search

(A) Start

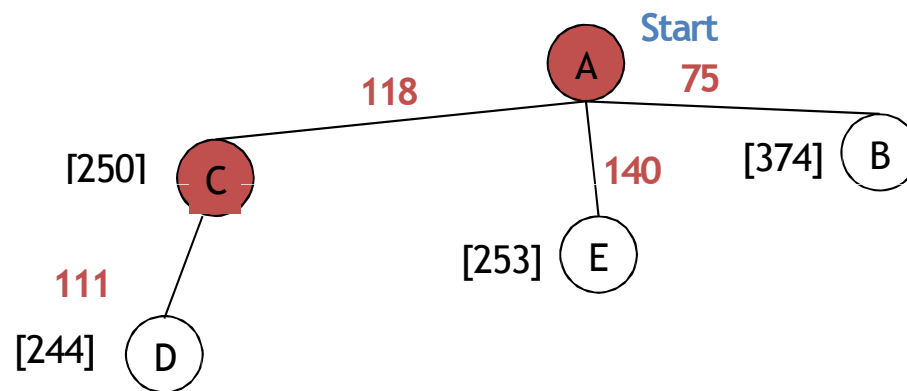
State	Heuristic: $h(n)$
A	366
B	374
** C	<b>250</b>
D	244
E	253
F	178
G	193
H	98
I	0

# Greedy Search: Tree Search



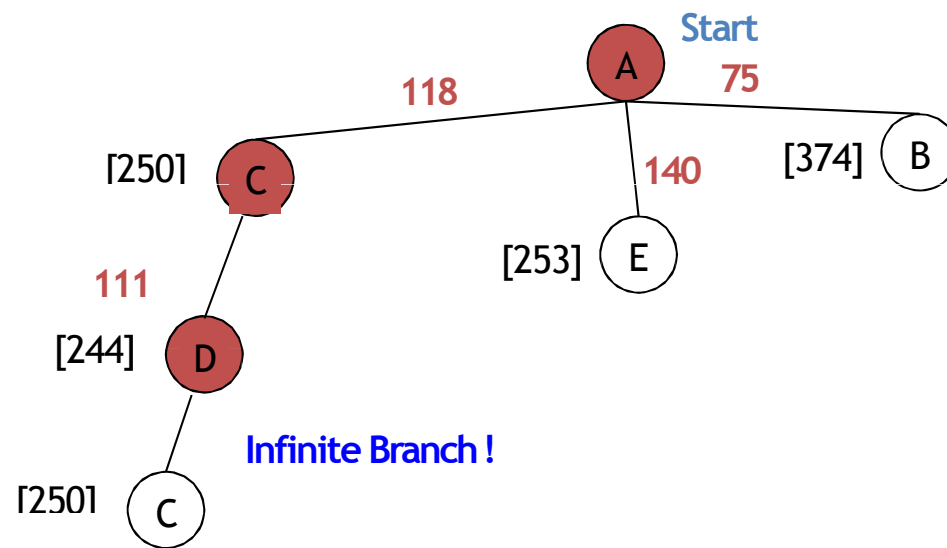
State	Heuristic: h(n)
A	366
B	374
<b>** C</b>	<b>250</b>
D	244
E	253
F	178
G	193
H	98
I	0

# Greedy Search: Tree Search



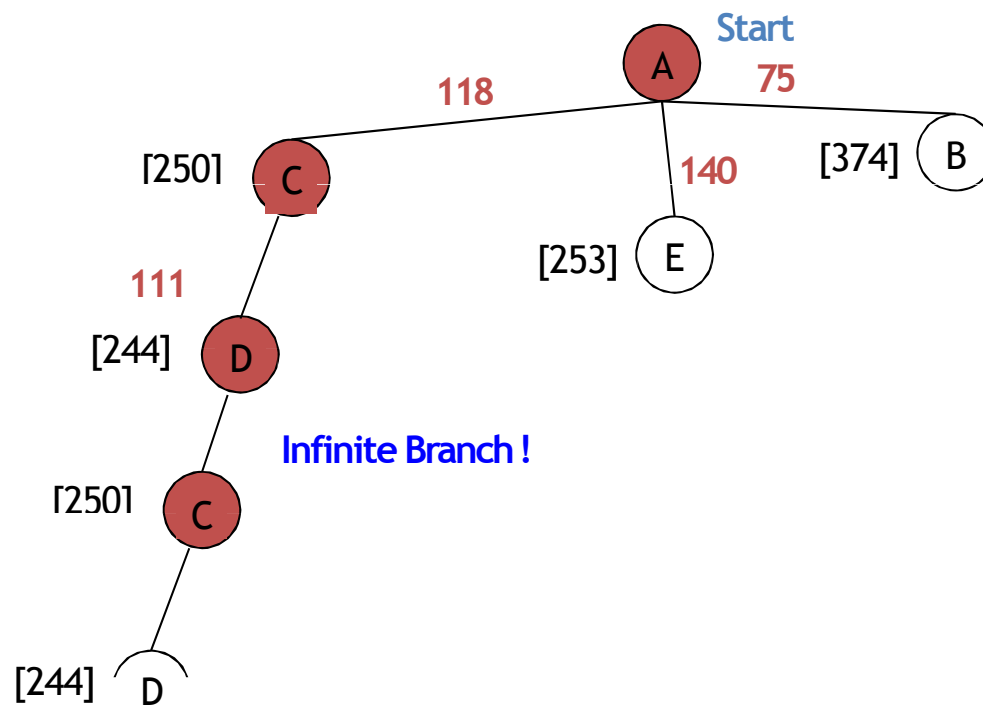
State	Heuristic: h(n)
A	366
B	374
<b>** C</b>	<b>250</b>
D	244
E	253
F	178
G	193
H	98
I	0

# Greedy Search: Tree Search



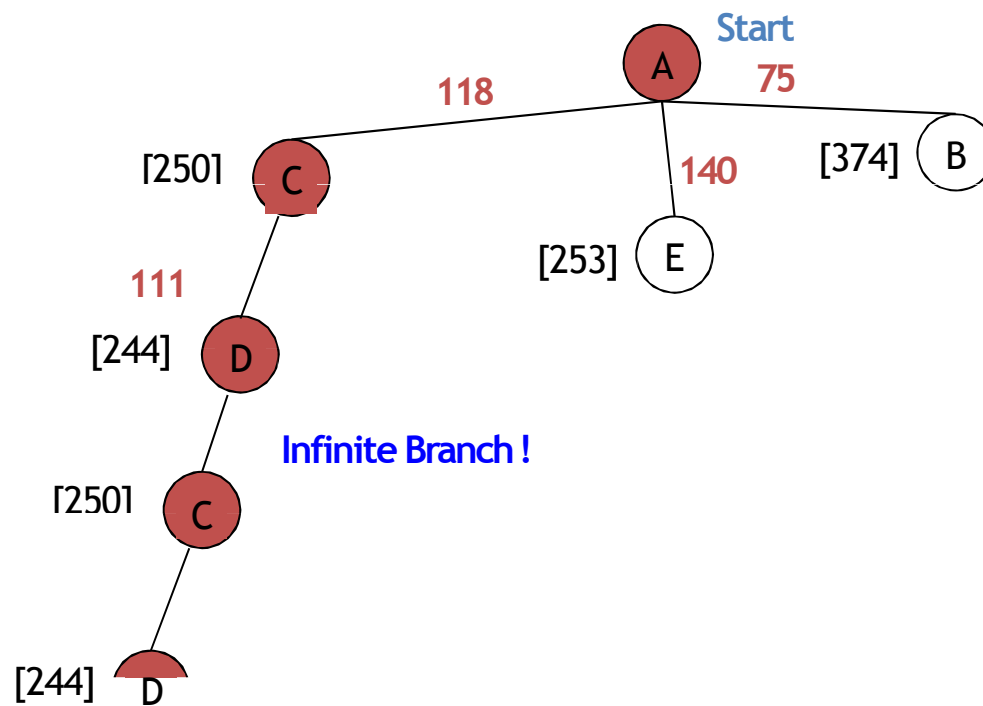
State	Heuristic: $h(n)$
A	366
B	374
** C	<b>250</b>
D	244
E	253
F	178
G	193
H	98
I	0

# Greedy Search: Tree Search



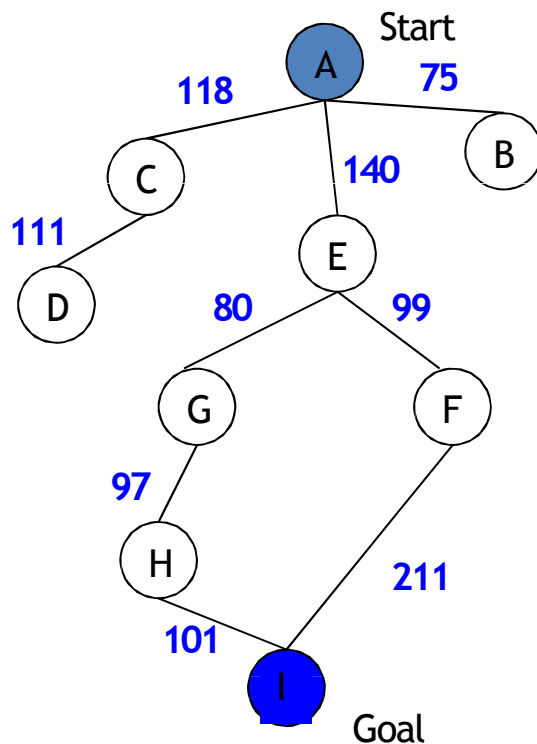
State	Heuristic: $h(n)$
A	366
B	374
** C	<b>250</b>
D	244
E	253
F	178
G	193
H	98
I	0

# Greedy Search: Tree Search



State	Heuristic: $h(n)$
A	366
B	374
** C	<b>250</b>
D	244
E	253
F	178
G	193
H	98
I	0

## Greedy Search: Time and Space Complexity ?



- Greedy search is not optimal.

- Greedy search is incomplete without systematic checking of repeated states.

- In the worst case, the Time and Space Complexity of Greedy Search are both  $O(b^m)$

Where  $b$  is the branching factor and  $m$  the maximum path length

---

## A\* Search

*eval-fn*:  $f(n) = g(n) + h(n)$



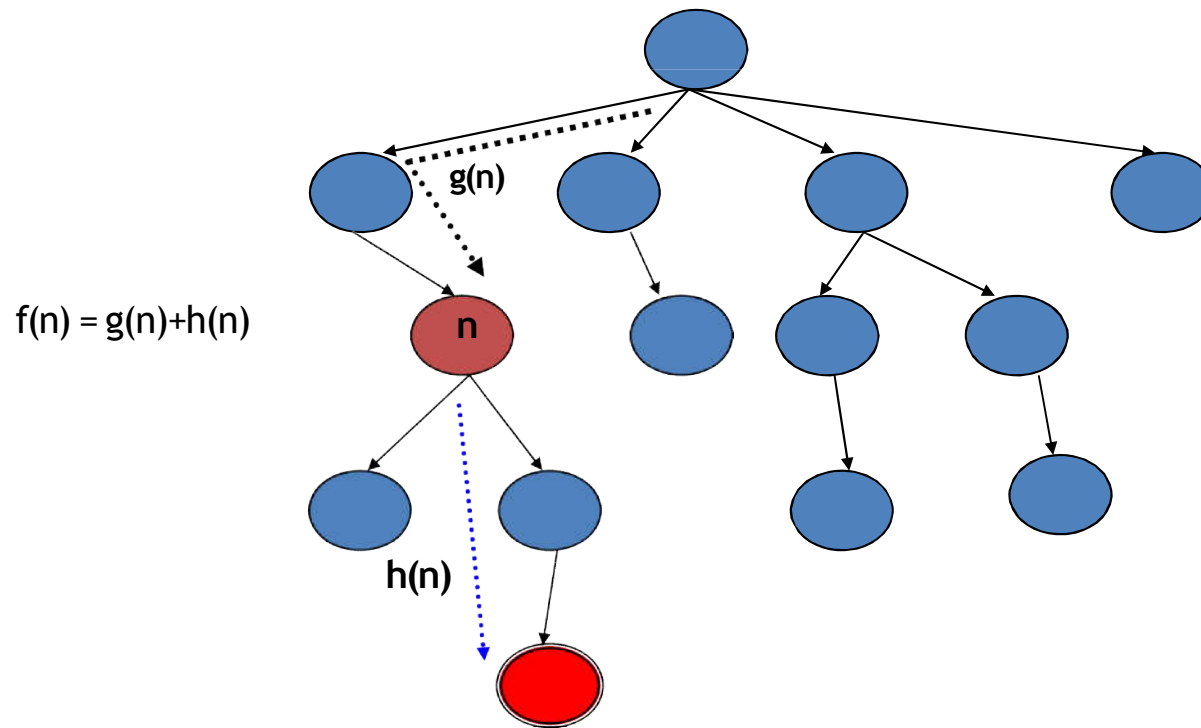
# A\* (A Star)

- Greedy Search minimizes a heuristic  $h(n)$  which is an estimated cost from a node  $n$  to the goal state. Greedy Search is efficient but it is not optimal nor complete.
- Uniform Cost Search minimizes the cost  $g(n)$  from the initial state to  $n$ . UCS is optimal and complete but not efficient.
- **New Strategy:** Combine Greedy Search and UCS to get an **efficient** algorithm which is **complete and optimal**.

## A\* (A Star)

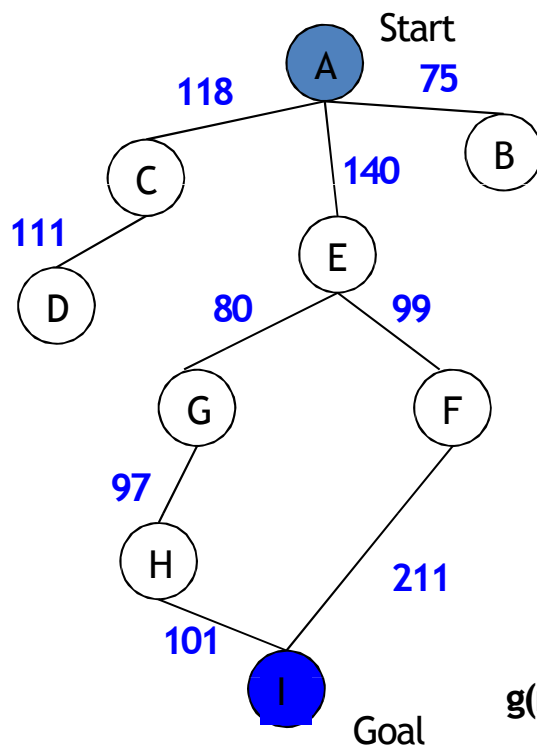
- A\* uses a heuristic function which combines  $g(n)$  and  $h(n)$ :  $f(n) = g(n) + h(n)$
- **$g(n)$**  is the exact cost to reach node  $n$  from the initial state.
- **$h(n)$**  is an estimation of the remaining cost to reach the goal.

# A\* (A Star)



# A\*

## Search



State	Heuristic: $h(n)$
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

$$f(n) = g(n) + h(n)$$

$g(n)$ : is the exact cost to reach node  $n$  from the initial state.

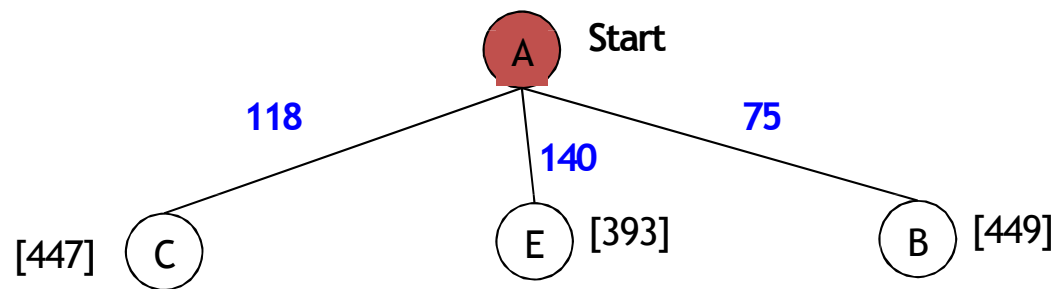
# A\* Search: TreeSearch

---

(A) Start

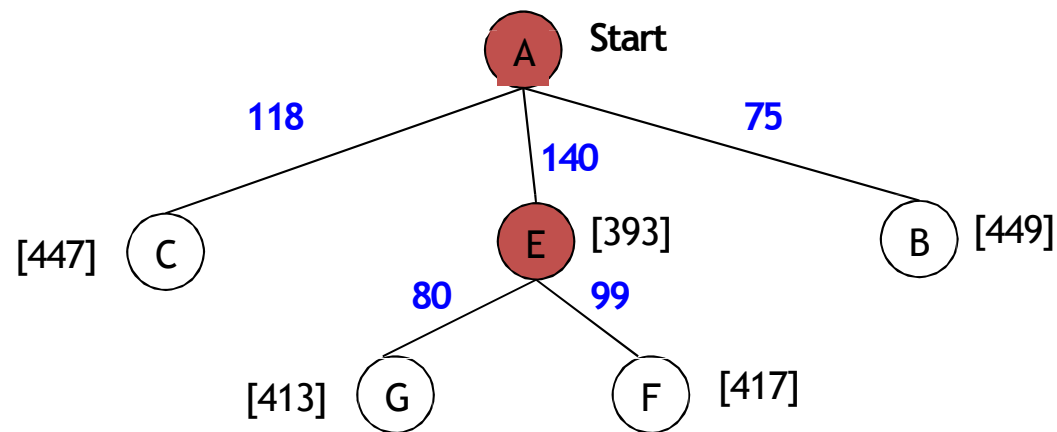
State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

# A\* Search: TreeSearch



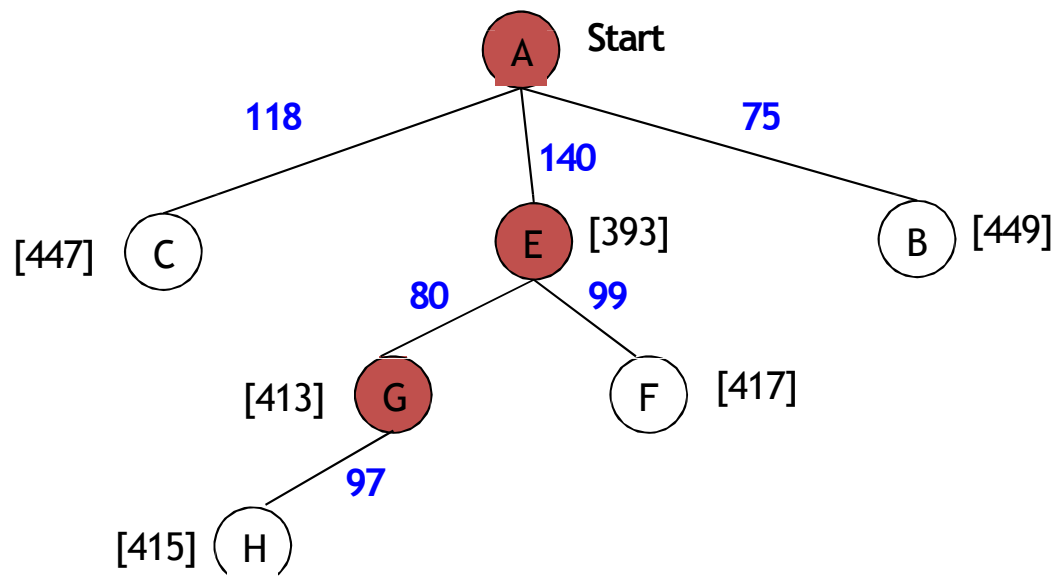
State	Heuristic: $h(n)$
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

# A\* Search: TreeSearch



State	Heuristic: $h(n)$
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

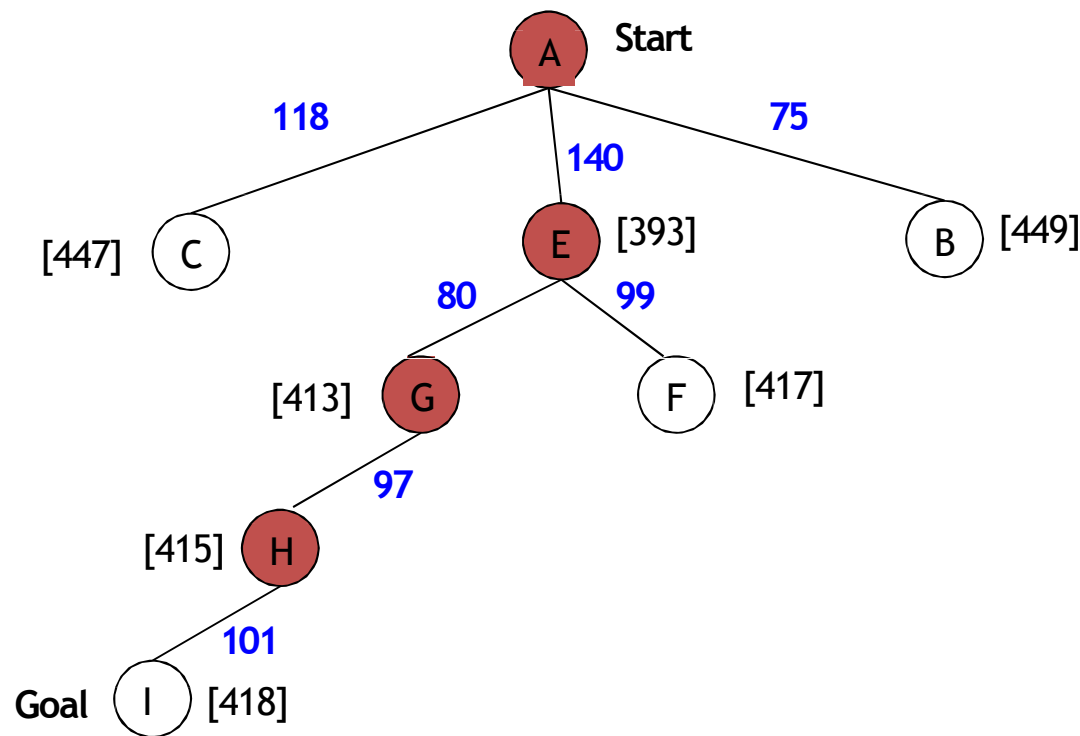
# A\* Search: TreeSearch



State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

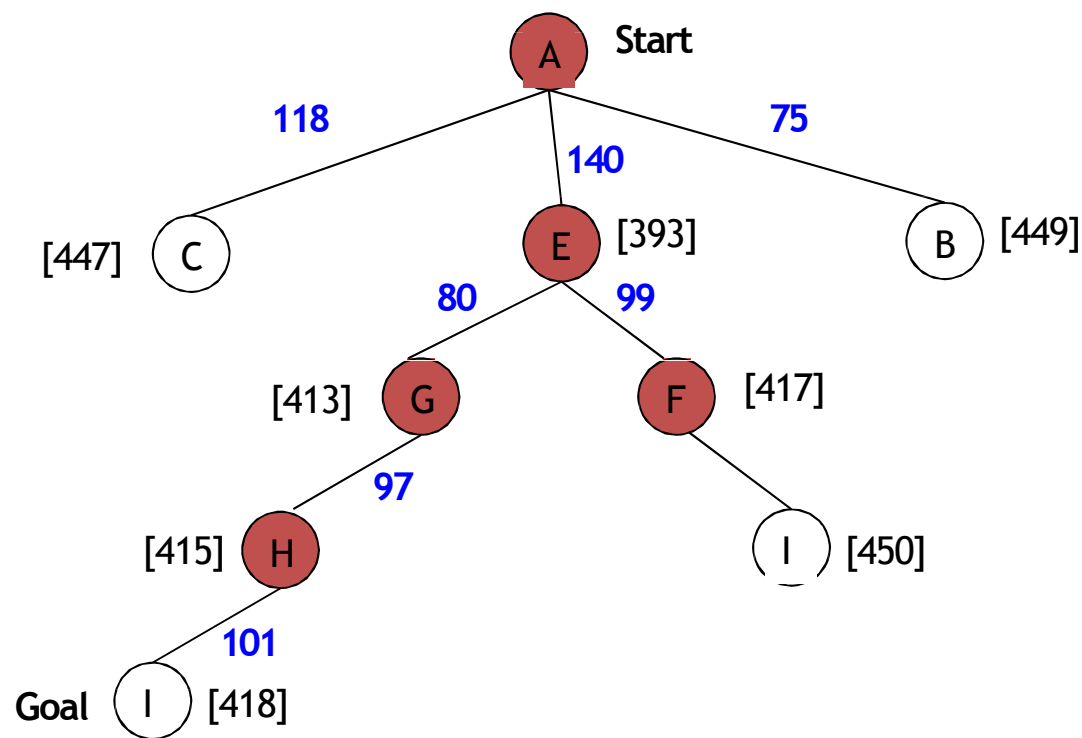


# A\* Search: TreeSearch



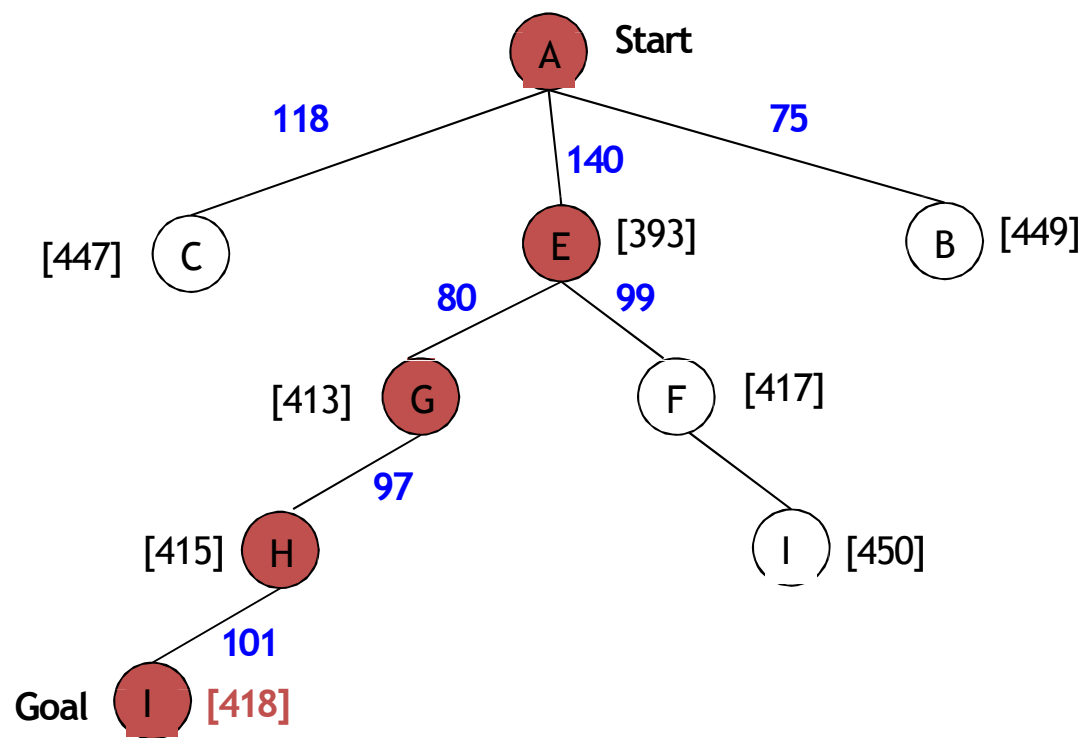
State	Heuristic: $h(n)$
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

# A\* Search: TreeSearch



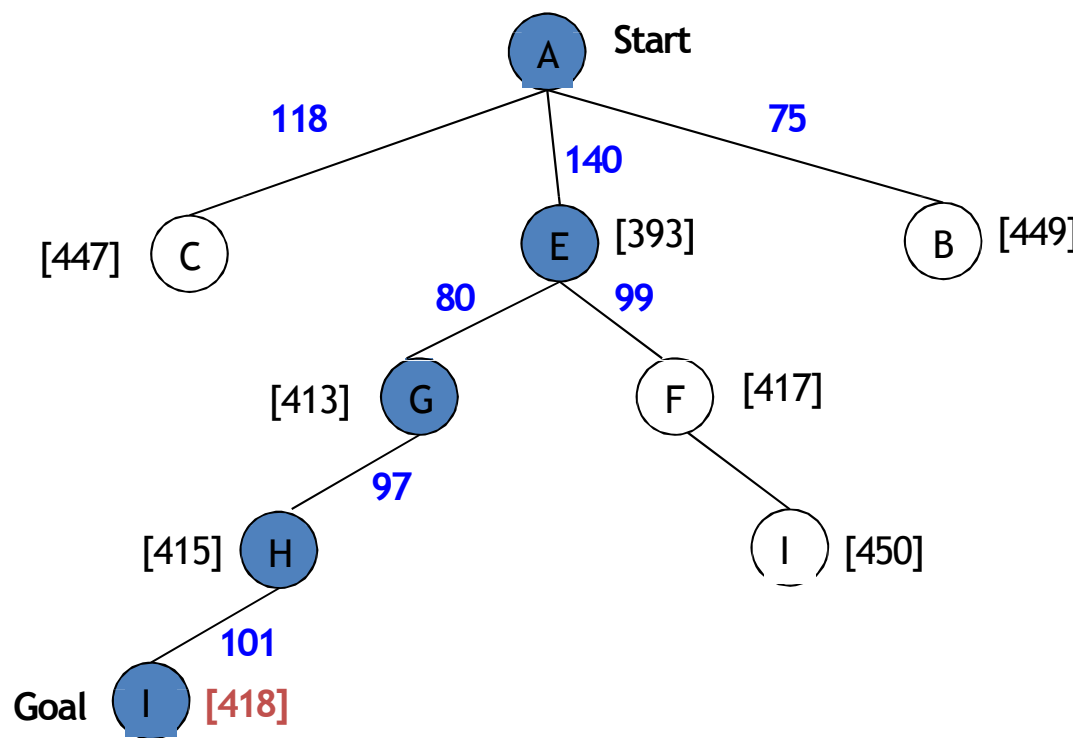
State	Heuristic: $h(n)$
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

# A\* Search: TreeSearch



State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

# A\* Search: TreeSearch



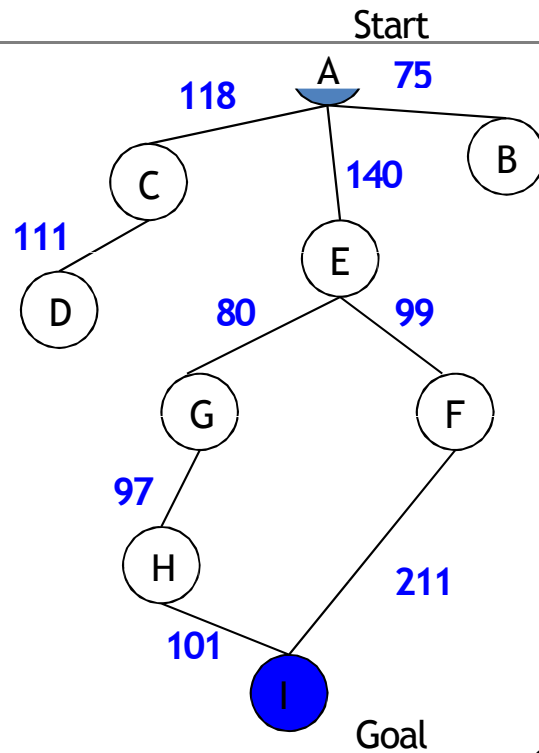
State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	0

## A\* with $f()$ not Admissible

---

- $h()$  overestimates the cost to reach the goal state
- The heuristic function  $h(n)$  is called admissible if  $h(n)$  is never larger than  $h^*(n)$ , namely  $h(n)$  is always less or equal to the true cheapest cost from  $n$  to the goal.
- A\* is admissible if it uses an admissible heuristic, and  $h(\text{goal}) = 0$ .
- If the heuristic function,  $h$  always underestimates the true cost ( $h(n)$  is smaller than  $h^*(n)$ ), then A\* is guaranteed to find an optimal solution.

# A\* Search: $h$ not admissible !



State	Heuristic: $h(n)$
A	366
B	374
C	329
D	244
E	253
F	178
G	193
<b>H</b>	<b>138</b>
I	0

$$f(n) = g(n) + h(n) - \text{(H-I) Overestimated}$$

$g(n)$ : is the exact cost to reach node  $n$  from the initial state.

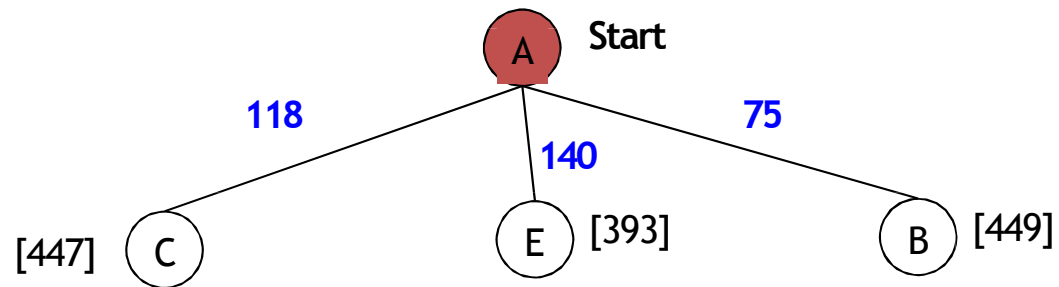
# A\* Search: TreeSearch

---

A Start

# A\* Search: TreeSearch

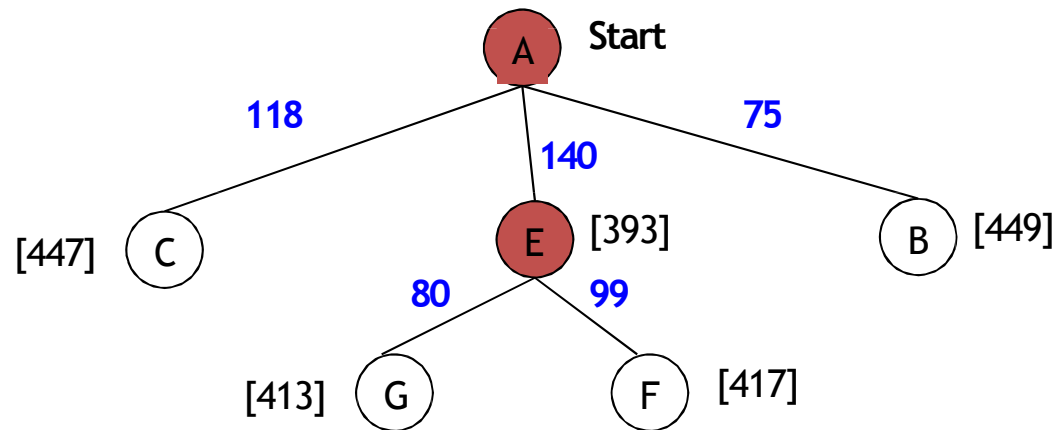
---



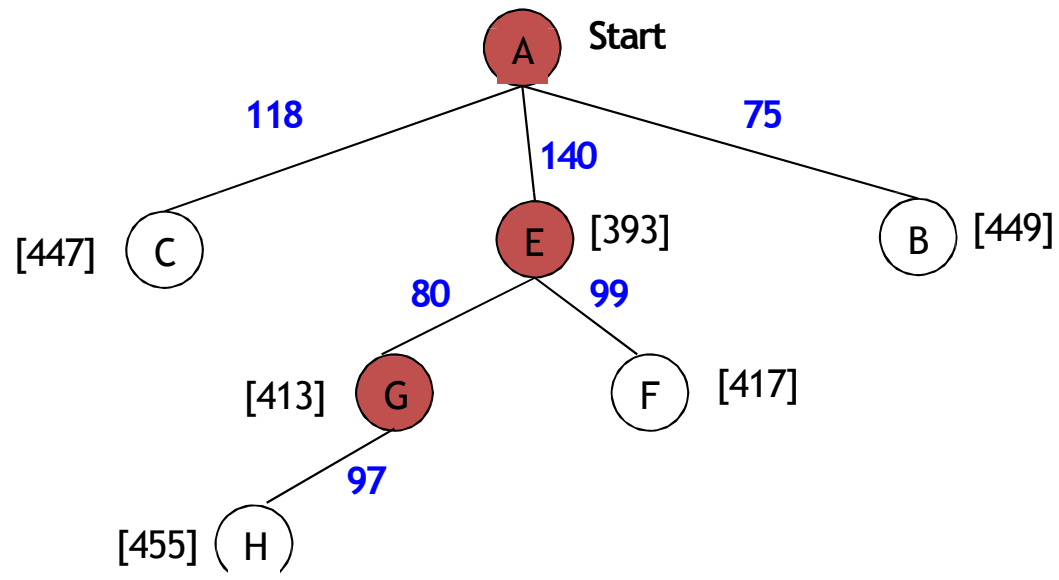


# A\* Search: TreeSearch

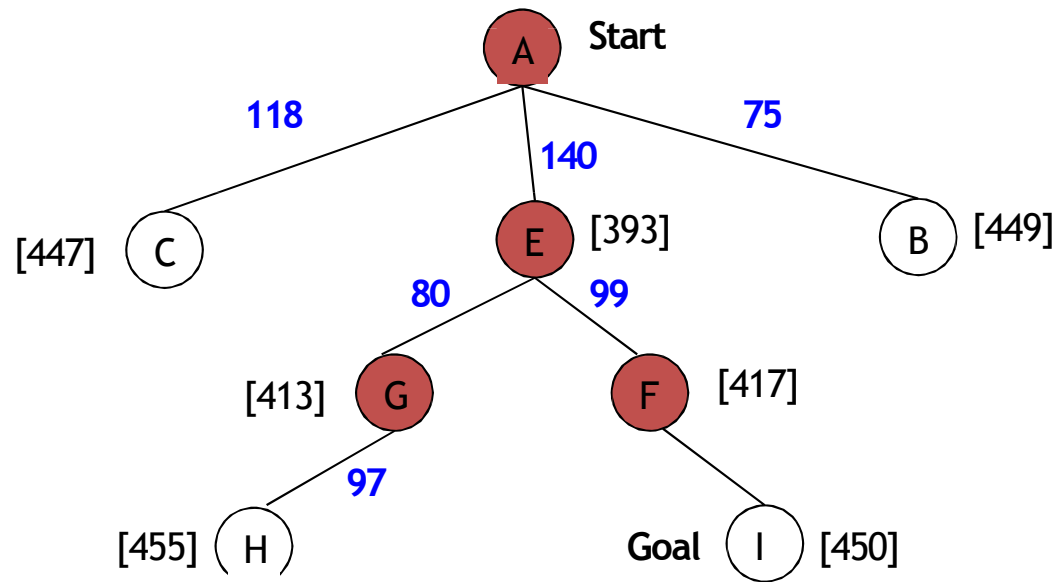
---



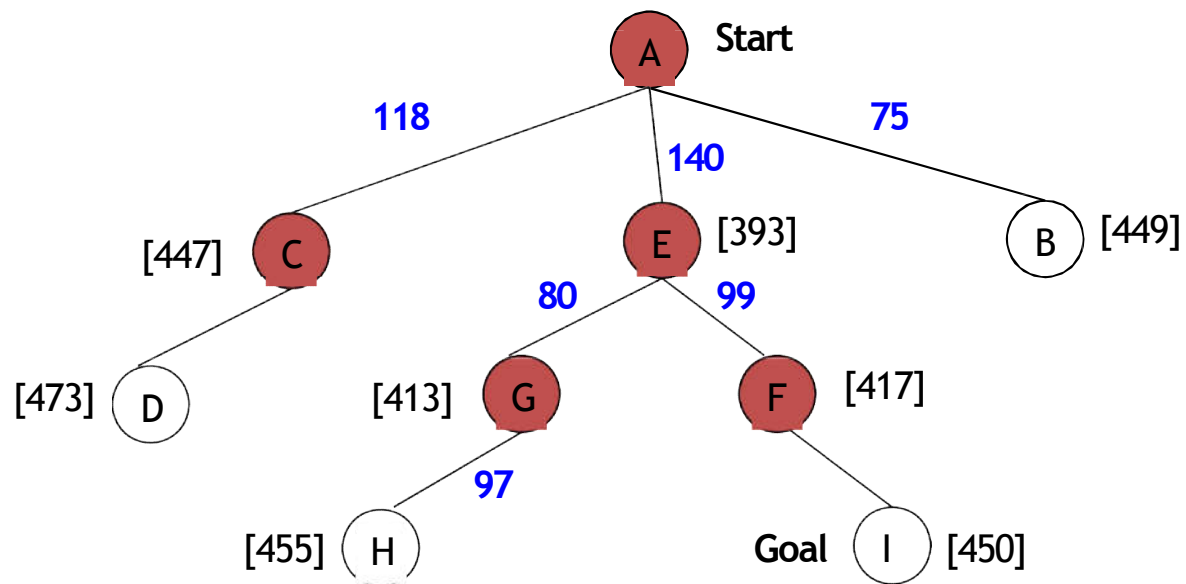
# A\* Search: TreeSearch



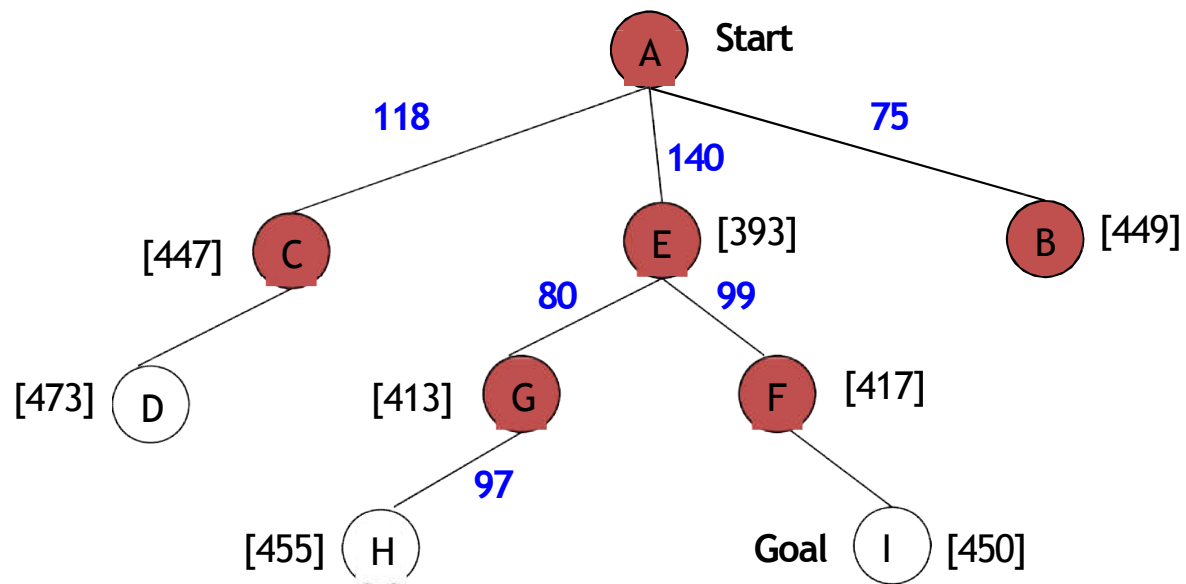
# A\* Search: TreeSearch



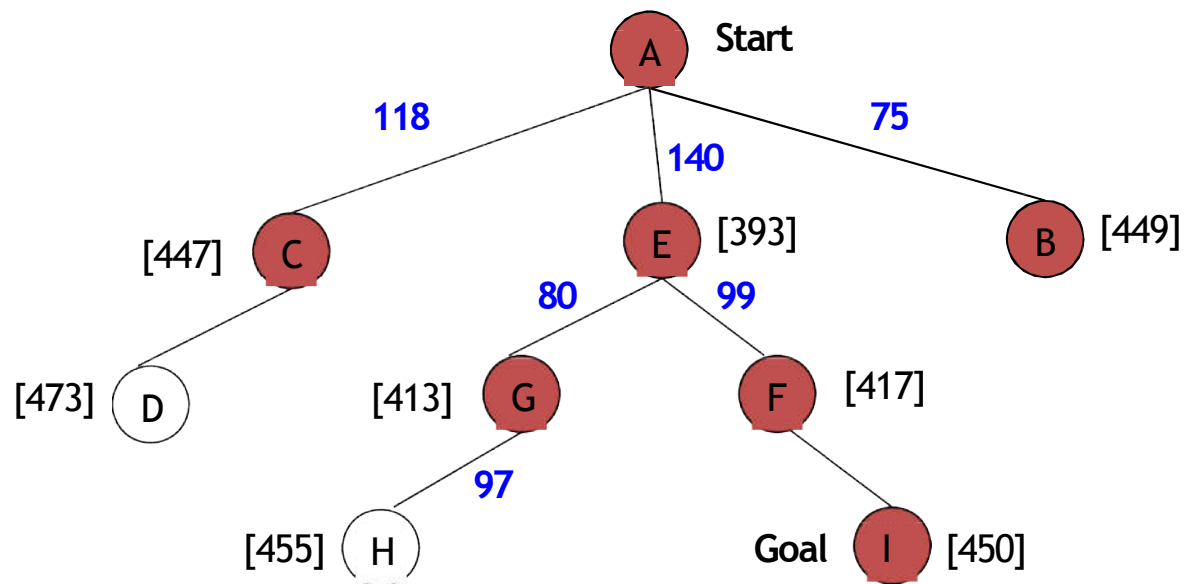
# A\* Search: TreeSearch



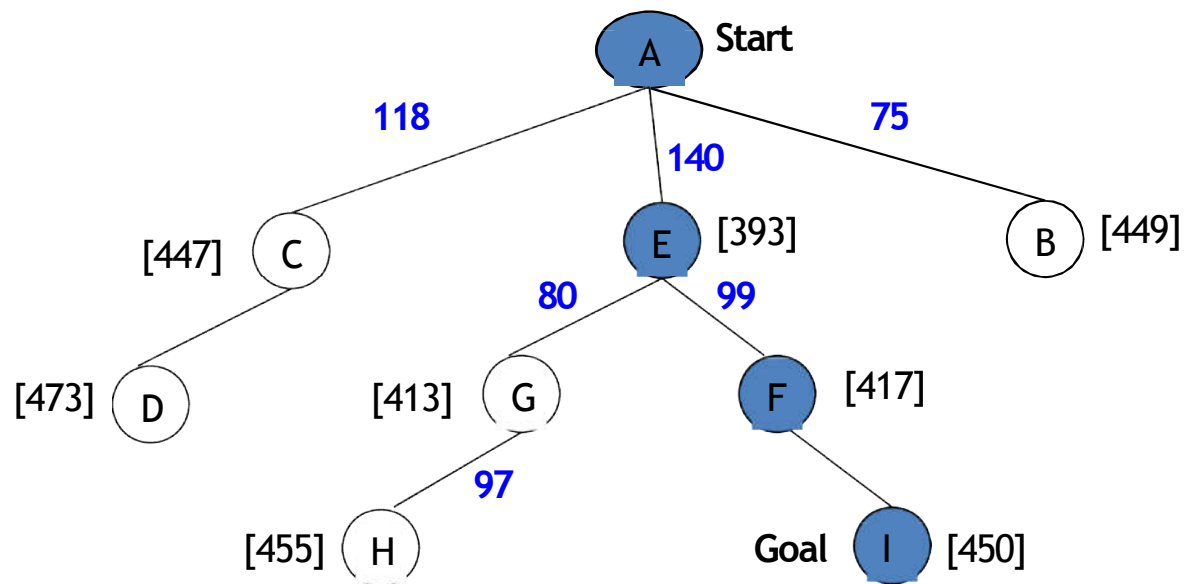
# A\* Search: TreeSearch



# A\* Search: TreeSearch



# A\* Search: TreeSearch



A\* not optimal !!!

# A\* Search

---

A\* with systematic checking for repeated states ...



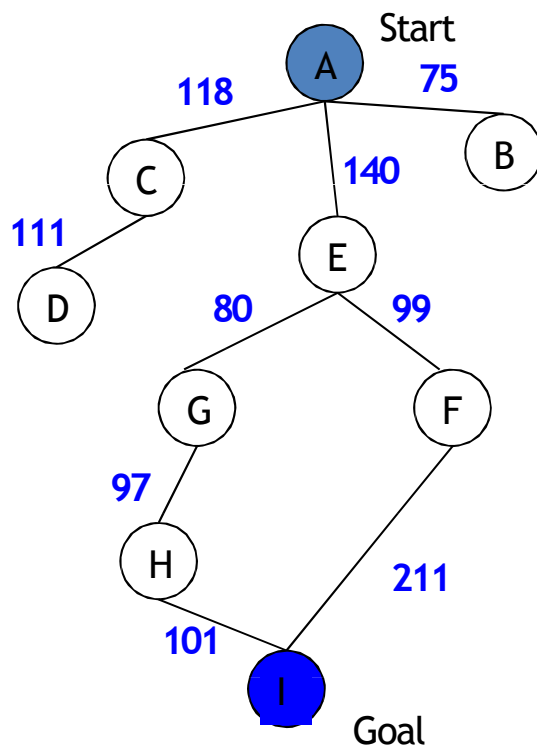


# A\* Algorithm

---

1. Search queue Q is empty.
2. Place the start state s in Q with f value h(s).
3. If Q is empty, return failure.
4. Take node n from Q with the lowest f value.  
(Keep Q sorted by f values and pick the first element).
5. If n is a goal node, stop and return solution.
6. Generate successors of node n.
7. For each successor n' of n do:
  - a) Compute  $f(n') = g(n) + \text{cost}(n, n') + h(n')$ .
  - b) If n' is new (never generated before), add n' to Q.
  - c) If node n is already in Q with a higher f value, replace it with current f(n') and place it in sorted order in Q.End for
8. Go back to step 3.

# A\* Search: Analysis



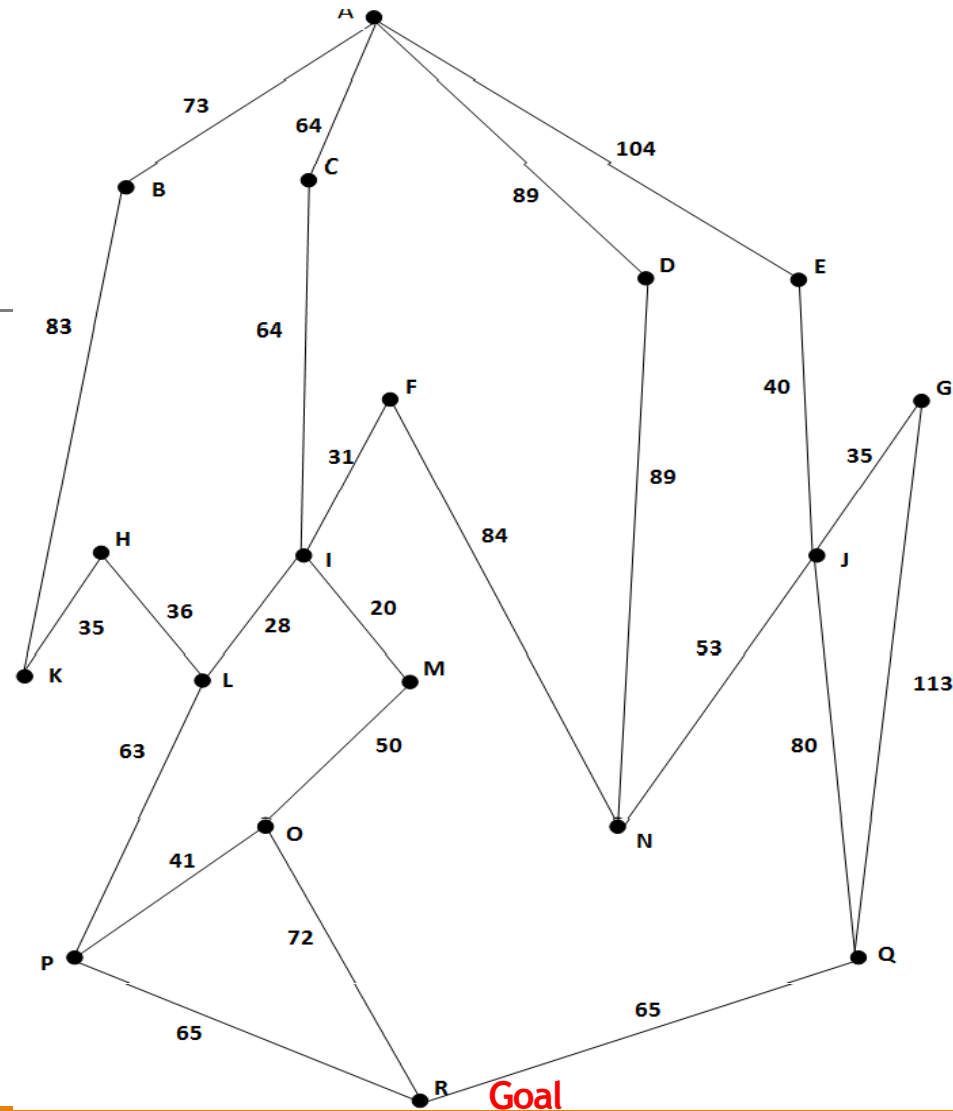
- A\* is complete except if there is an infinity of nodes with  $f < f(G)$ .
- A\* is optimal if heuristic  $h$  is admissible.
- Time complexity depends on the quality of heuristic but is still exponential.
- For space complexity, A\* keeps all nodes in memory. A\* has worst case  $O(b^d)$  space complexity, but an iterative deepening version is possible (IDA\*).

# Conclusions

---

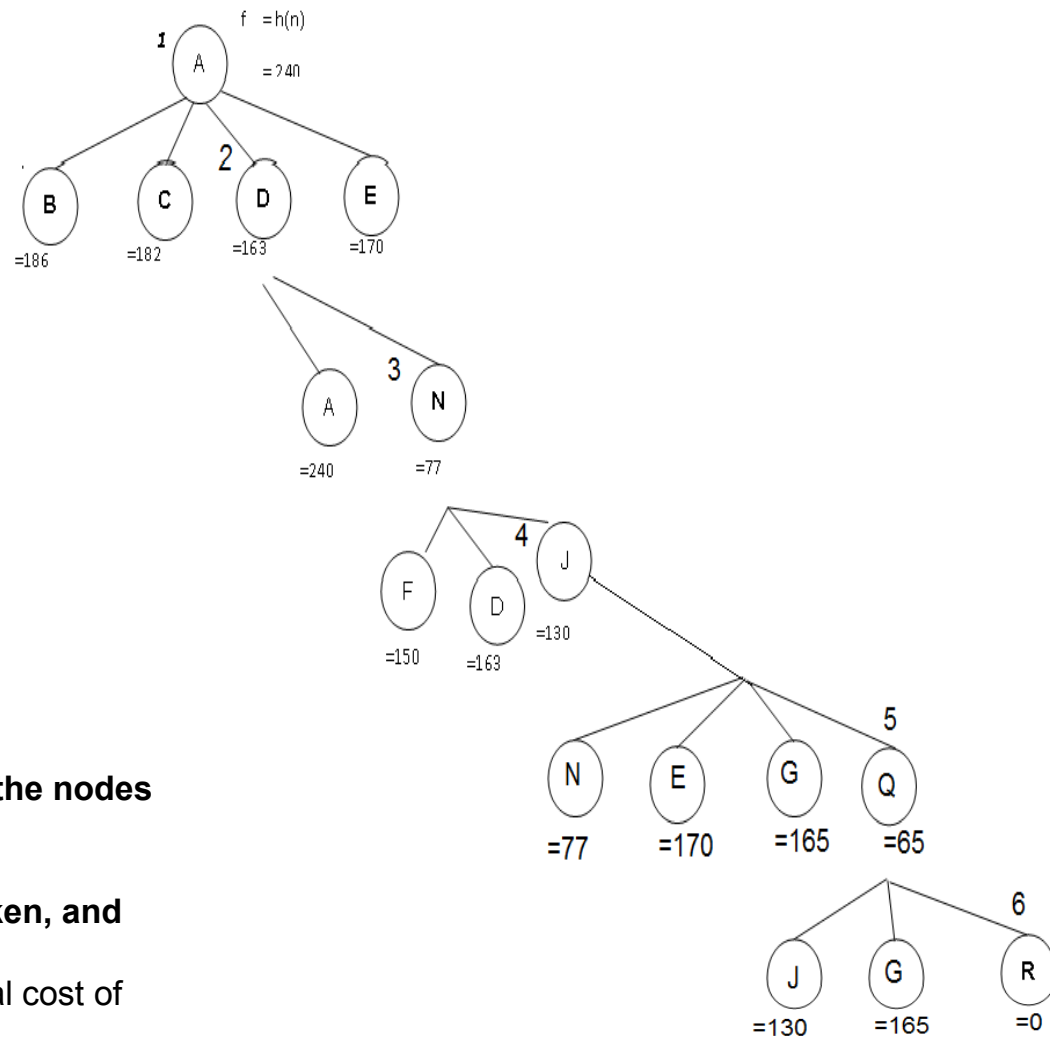
- Frustration with *uninformed* search led to the idea of using domain-specific knowledge in a search so that one can intelligently explore only the relevant part of the search space that has a good chance of containing the goal state. These new techniques are called informed (heuristic) search strategies.
- Even though heuristics improve the performance of informed search algorithms, they are still time-consuming, especially for large-size instances.

# Exercise



A	240
B	186
C	182
D	163
E	170
F	150
G	165
H	139
I	120
J	130
K	122
L	104
M	100
N	77
O	72
P	65
Q	65
R	0

**The Search Tree Using  
Greedy best first  
Algorithm**



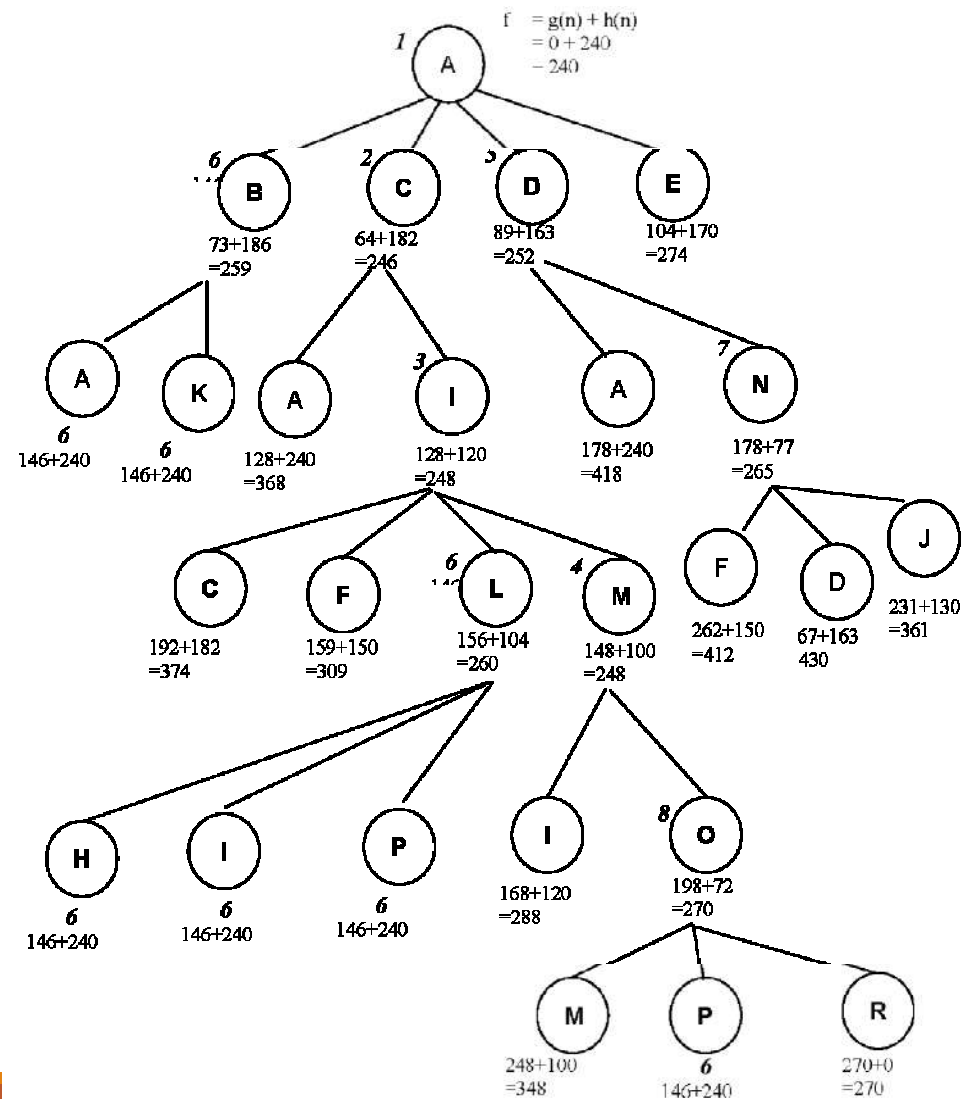
**State the order in which the nodes  
were expanded**

A, D, N, J, Q, R

**State the route that is taken, and  
give the total cost**

A, D, N, J, Q, R, with a total cost of  
376

## The Search Tree Using A\*



**State the order in which the nodes were expanded**

A, C, I, M, D, B, N, O, L

**State the route that is taken, and give the total cost**

A, C, I, M, O, R, with a total cost of 270