National University of Computer and Emerging Sciences, Lahore Campus

Course:	Linear Algebra	Course Code:	MT1004
Program:	BS (CS and Robotics)	Semester:	Fall 2023
Duration:	60 Minutes	Total Marks:	50
Paper Date:	11th November, 2023	Weight	12.5%
Section:	ALL	Page(s):	2
Exam:	Sessional-2	Roll No:	

Instruction/Notes:

1. Programmable calculators are not allowed.
2. Wrong calculation wok found (if any) at a step will not be further marked. Marks will be awarded till the correct calculations.
3. Do all the questions in the given order as mentioned in the paper.
4. Your kind cooperation will be appreciable for obeying the instructions.

Question # 1(a) (CLO-1) [5]: Determine whether the given planes are parallel, if so, then find the distance between them $x - 4y - 3z - 2 = 0$ and $3x - 12y - 9z - 7 = 0$.

Question # 1(b) (CLO-1)[5]: For the vectors $u = (-2, 0, 6)$, $v = (1, -3, 1)$ and $w = (-5, -1, 1)$, find the scalar triple product. Also, write formula for the volume of the parallelepiped using vectors.

Question # 1(c) (CLO-1)[5]: Define equation for the **Line** and **Plane** in $\mathbf{R}^n$ in vector and parametric form. Also, explain geometrically the solution space (sub-space) of the linear system $AX = 0$, given below:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Question # 2 (CLO-1) [10]: Let V be the set of all ordered pairs of real numbers, and consider u, v elements in V, if the following addition and scalar multiplication operations on $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are defined as

$$u + v = (u_1 + v_1 + 1,\ u_2 + v_2 + 1) \text{ and } ku = (0, ku_2), \text{ then}$$

check all the axioms for the non-empty set V to be a vector space. Identify the axioms which **holds** and **fails** for V to be a vector space/or not.

Question # 3 (CLO-2) [10]: Consider the vectors $u = (2, 1, 4)$, $v = (1, -1, 3)$ *and* $w = (3, 2, 5)$.

a. Show that whether the above given vectors **u, v** and **w** spans $V = R^3$.
b. Show that whether the given vectors **u, v** and **w** form the basis for $V = R^3$.
c. Show that the vector $\mathbf{x} = (-9, -7, -15)$ is a linear combination of the vectors u, v **and** w.

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Question # 4 (CLO-2) [15]: Let A be a matrix $A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$, then

a. Find the basis for the row space of A.
b. Find the basis for the column space of A.
c. Find the basis for the null space of A and explain the geometrically the solution space/sub-space/null space spanned by the basis for the null space of A.
d. Find rank and nullity for the given matrix A.

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