# Information Security

## CS3002

Lecture 5

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# Advanced Encryption Standard (AES)

# Origins

- Clear a replacement for DES was needed.
  - Have theoretical attacks that can break it.
  - Have demonstrated exhaustive key search attacks.
- US NIST issued call for ciphers in 1997.
- 15 candidates accepted in Jun 98.
- 5 were shortlisted in Aug-99.
- Rijndael was selected as the AES in Oct-2000.
- Issued as FIPS PUB 197 standard in Nov-2001 .

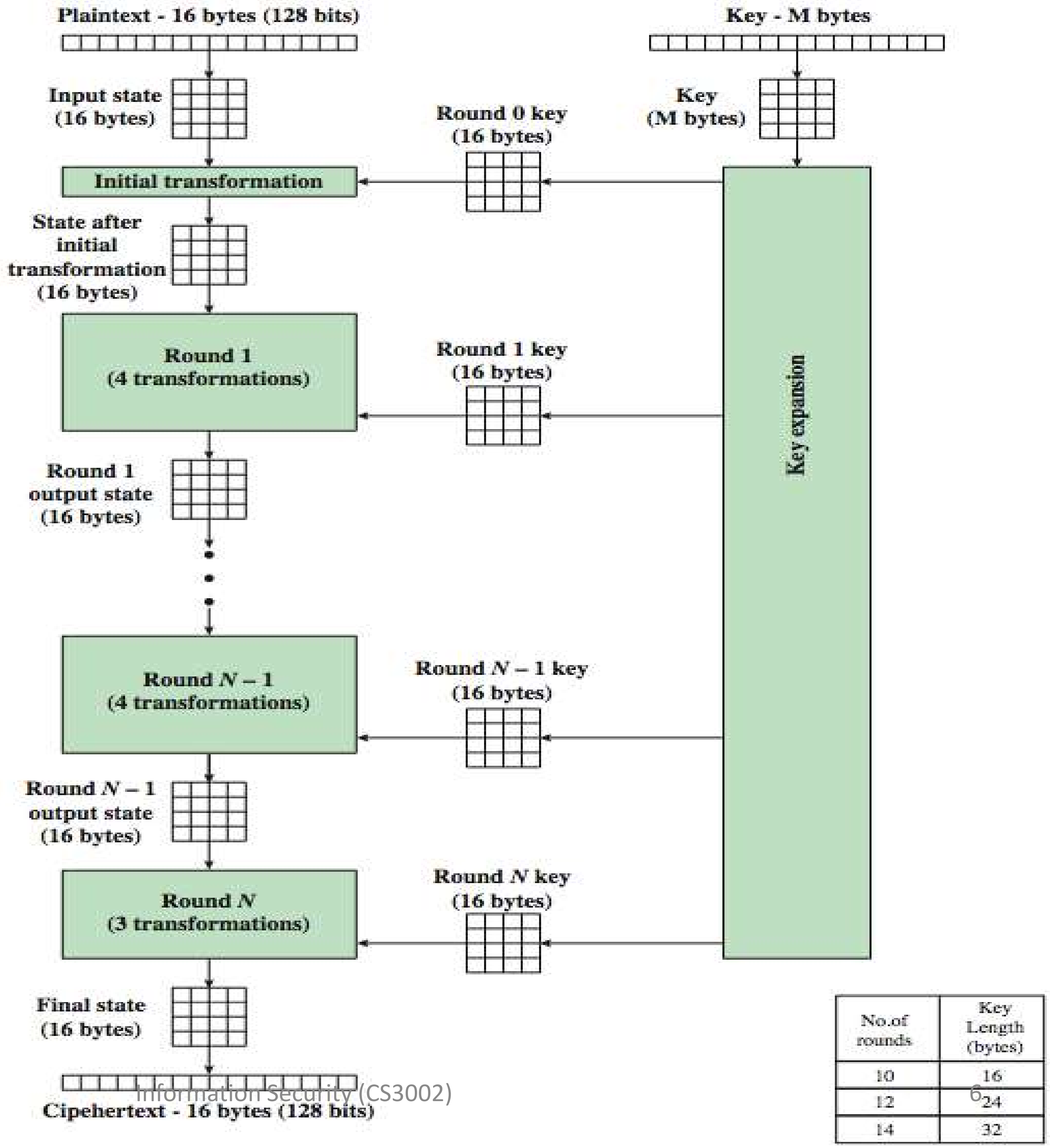
# The AES Cipher

- Allows *128, 192, and 256*-bit key sizes .
- Variable block length of *128, 192, or 256* bits. All nine combinations of key/block length possible.
  - A block is the smallest data size the algorithm will encrypt
- Vast speed improvement over DES in both hardware and software implementations
  - with fast XOR & table lookup implementation

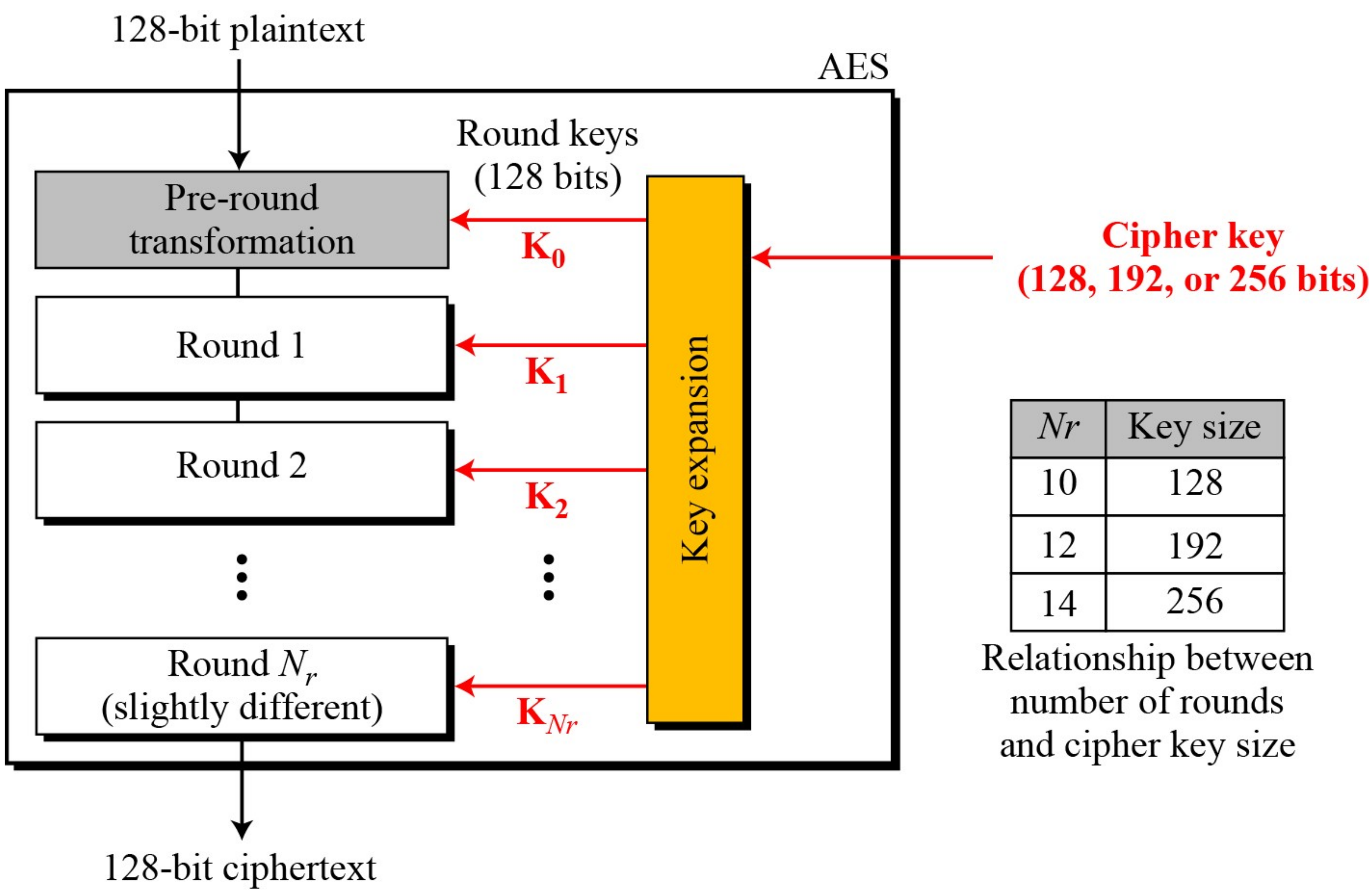
# The AES Cipher - Rijndael

- An **iterative** rather than **feistel** cipher
  - Processes data as block of 4 columns of 4 bytes.
  - Operates on entire data block in every round.
- Designed to be:
  - Resistant against known attacks.
  - Speed and code compactness on many CPUs.
  - Design simplicity.

# AES Encryption Process



# AES Encryption Process

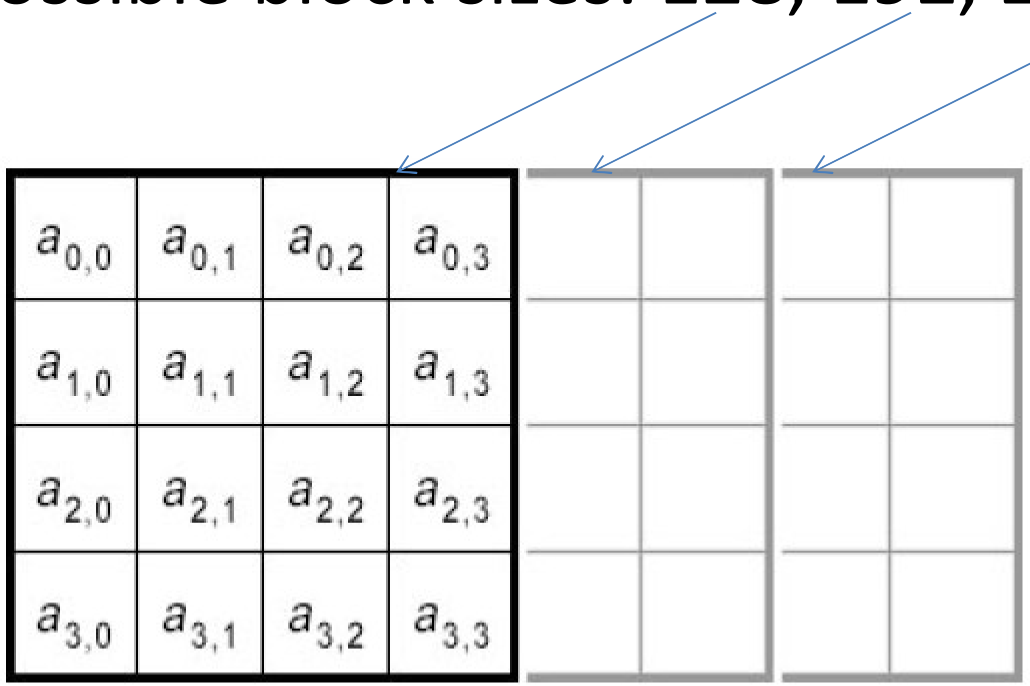



| *Nr* | Key size |
|---|---|
| 10 | 128 |
| 12 | 192 |
| 14 | 256 |

Relationship between number of rounds and cipher key size

# The AES Cipher- Input Text

- Possible block sizes: 128, 192, 256 bit




The state

# The AES Cipher

- Assume 128 bit block as input
- Input blocks represented as states at intermediates stages.

Input

| | | | |
|---|---|---|---|
| $in_0$ | $in_4$ | $in_8$ | $in_{12}$ |
| $in_1$ | $in_5$ | in9 | in13 |
| in2 | in6 | in10 | in14 |
| in3 | in7 | in11 | in15 |

State array

| | | | |
|---|---|---|---|
| $S_{0,0}$ | $S_{0,1}$ | $S_{0,2}$ | $S_{0,3}$ |
| $S_{1,0}$ | $S_{1,1}$ | $S_{1,2}$ | $S_{1,3}$ |
| $S_{2,0}$ | $S_{2,1}$ | $S_{2,2}$ | $S_{2,3}$ |
| $S_{3,0}$ | $S_{3,1}$ | $S_{3,2}$ | $S_{3,3}$ |

Output

| | | | |
|---|---|---|---|
| out0 | out 4 | out8 | out12 |
| out1 | out5 | out9 | out13 |
| out2 | out6 | out10 | out14 |
| out3 | out7 | out11 | out15 |

# The AES Cipher

- Key received as input array of 4 rows and $N_k$ columns.
- $N_k$ = 4,6, or 8, parameter which depends key size 128,192 or 256.
- Input key is expanded into an array of 44/52/60 words of 32 bits each depending upon key size.
- 4 different words serve as a key for each round.

| | | | |
|---|---|---|---|
| k0 | k4 | k8 | k12 |
| k1 | k5 | k9 | k13 |
| k2 | k6 | k10 | k14 |
| k3 | k7 | k11 | k15 |

Nk = 4

→

| | | | | | |
|---|---|---|---|---|---|
| w0 | w1 | w2 | …… | w42 | w43 |

# Steps in Rijndael (AES)

Confusion

Step1: Byte Substitution

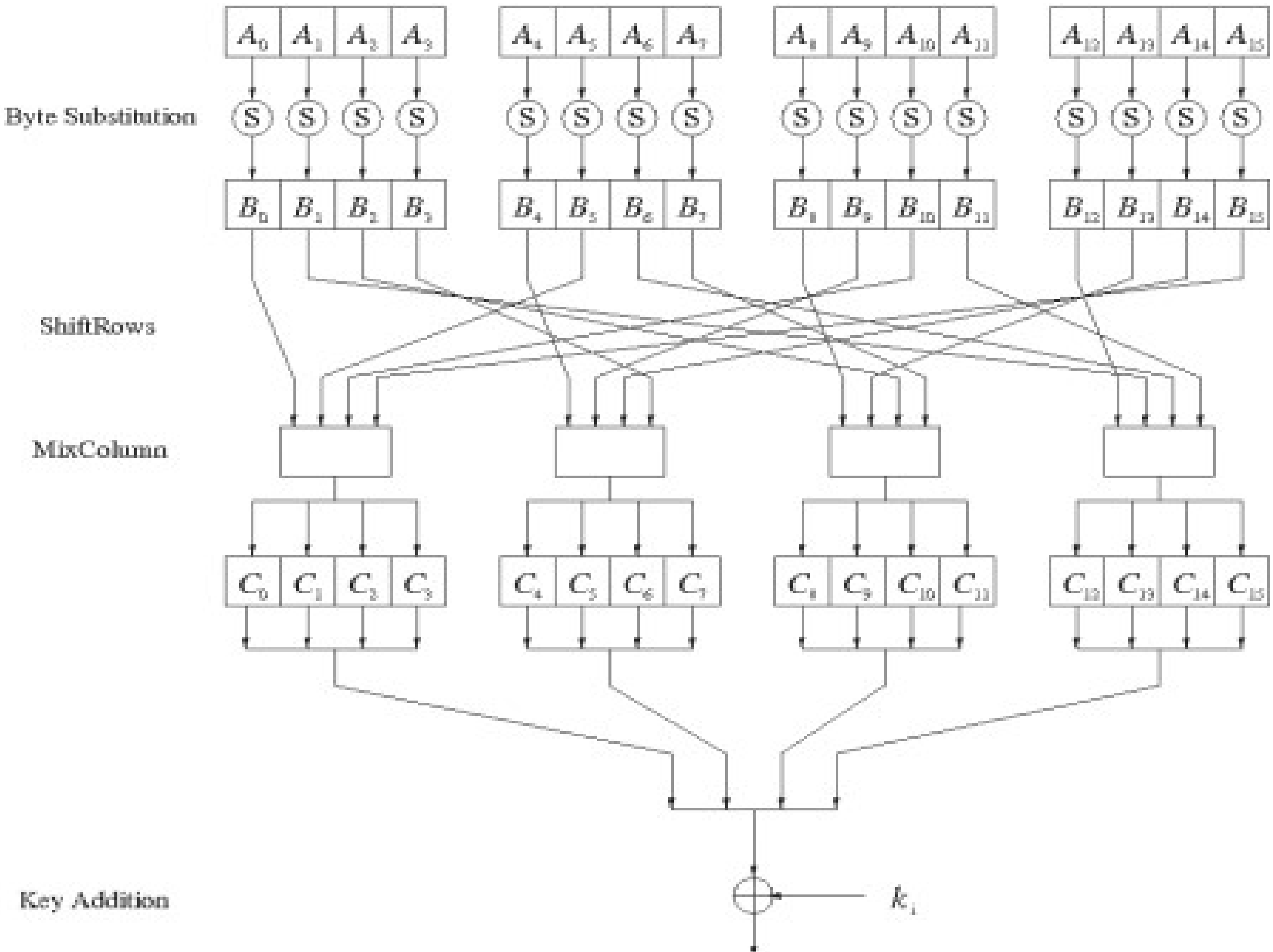
Diffusion

Step2: Shift Rows

Step3: Mix Cloumns

Step4: Round Key addition

Repeat these four steps 10,12,14 times




Courtesy: Book: Understanding cryptography

# Steps in Rijndael (AES)

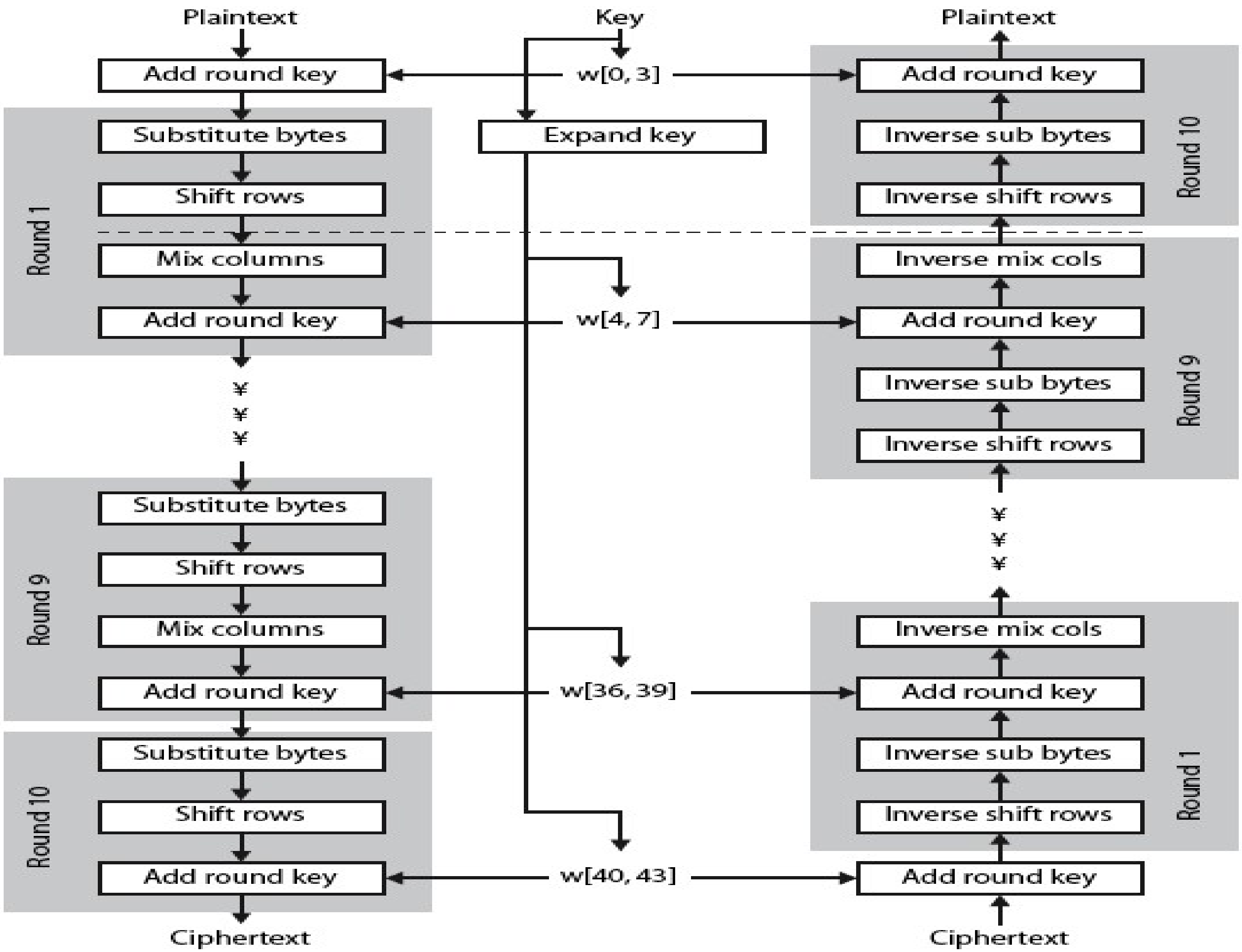
- SubBytes() – uses S-box to perform a byte-by-byte substitution of State, making use of arithmetic over GF(2^8).
- ShiftRows() – processes the State by cyclically shifting the last three rows of the State by different offsets.
- MixColumns() – takes all the columns of the State and mixes their data, independently of one another, making use of arithmetic over GF(2^8).
- AddRoundKey() – round key is added to the State using XOR operation.

# Rijndael (AES)

- Data block of 4 columns of 4 bytes is state and key is expanded to array of words.
- Has 10/12/14 rounds in which state undergoes:
  - Byte substitution (1 S-box used on every byte)
  - Shift rows (permute bytes between groups/columns)
  - Mix columns (subs using matrix multipy of groups)
  - Add round key (XOR state with key material)

    Diffusion
  - View as alternating XOR key & scramble data bytes
- Initial XOR key material (1op.) & incomplete last (3ops.) round (so 10 rounds each of 4 ops.)

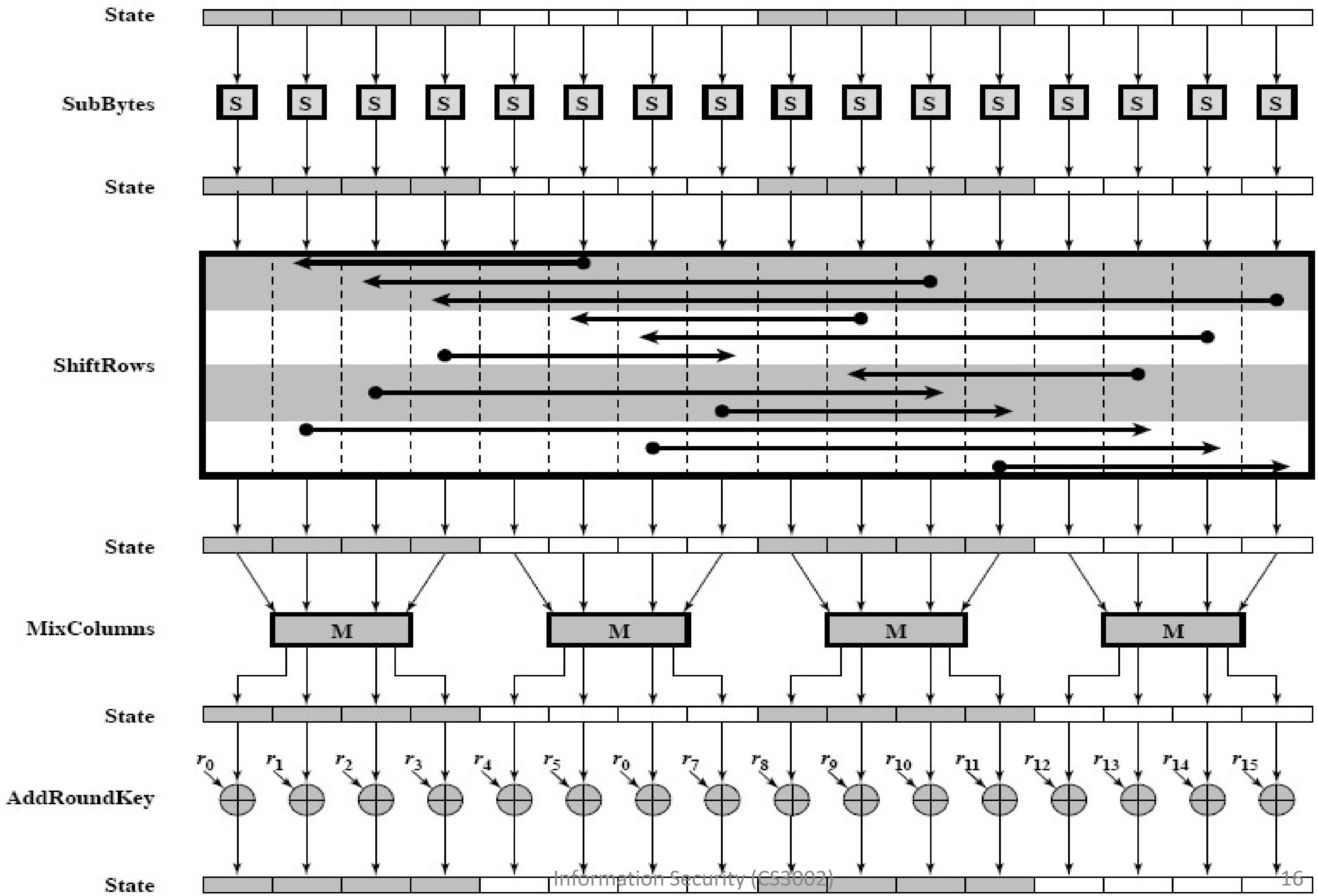
# Rijndael (AES)




(a) Encryption

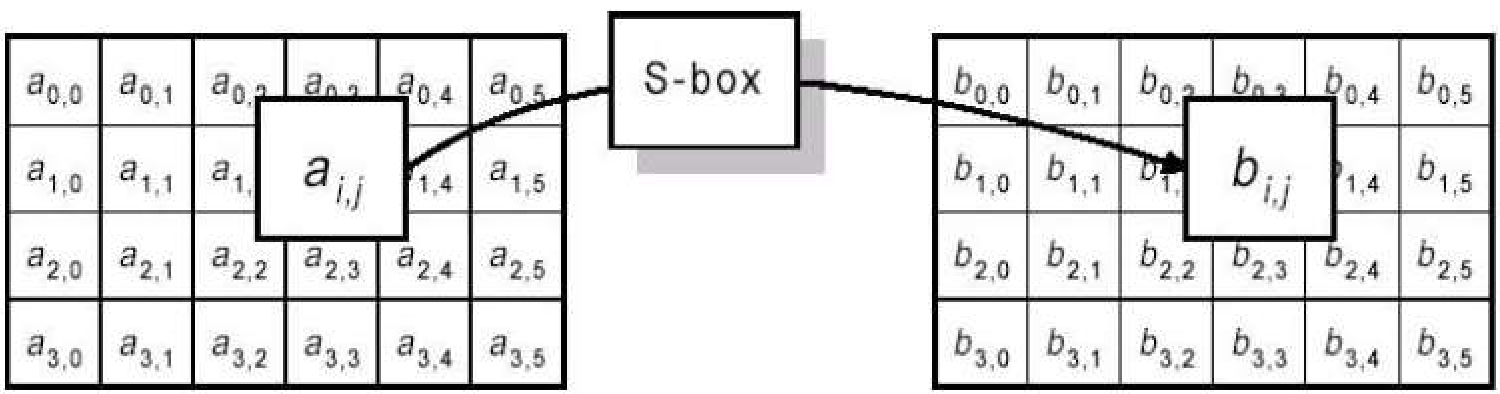
(b) Decryption

# AES Encryption Round



# 1. Byte Substitution

- Non linear byte substitution
  - Multiplicative inverse in $GF(2^8)$



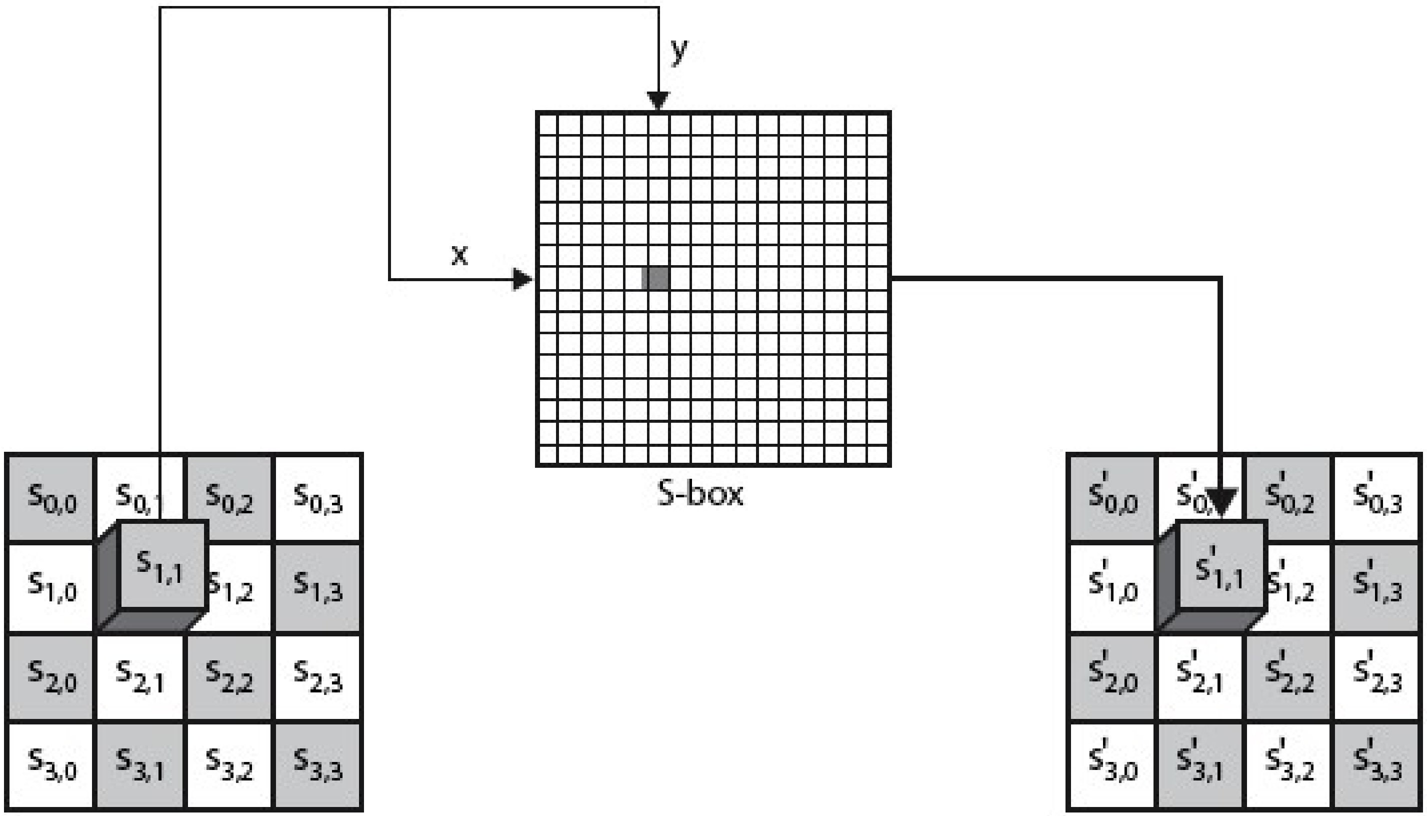

ByteSub acts on individual bytes of the state

Table 5.2 AES S-Boxes

| | | y | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
| x | 0 | 63 | 7C | 77 | 7B | F2 | 6B | 6F | C5 | 30 | 01 | 67 | 2B | FE | D7 | AB | 76 |
| | 1 | CA | 82 | C9 | 7D | FA | 59 | 47 | F0 | AD | D4 | A2 | AF | 9C | A4 | 72 | C0 |
| | 2 | B7 | FD | 93 | 26 | 36 | 3F | F7 | CC | 34 | A5 | E5 | F1 | 71 | D8 | 31 | 15 |
| | 3 | 04 | C7 | 23 | C3 | 18 | 96 | 05 | 9A | 07 | 12 | 80 | E2 | EB | 27 | B2 | 75 |
| | 4 | 09 | 83 | 2C | 1A | 1B | 6E | 5A | A0 | 52 | 3B | D6 | B3 | 29 | E3 | 2F | 84 |
| | 5 | 53 | D1 | 00 | ED | 20 | FC | B1 | 5B | 6A | CB | BE | 39 | 4A | 4C | 58 | CF |
| | 6 | D0 | EF | AA | FB | 43 | 4D | 33 | 85 | 45 | F9 | 02 | 7F | 50 | 3C | 9F | A8 |
| | 7 | 51 | A3 | 40 | 8F | 92 | 9D | 38 | F5 | BC | B6 | DA | 21 | 10 | FF | F3 | D2 |
| | 8 | CD | 0C | 13 | EC | 5F | 97 | 44 | 17 | C4 | A7 | 7E | 3D | 64 | 5D | 19 | 73 |
| | 9 | 60 | 81 | 4F | DC | 22 | 2A | 90 | 88 | 46 | EE | B8 | 14 | DE | 5E | 0B | DB |
| | A | E0 | 32 | 3A | 0A | 49 | 06 | 24 | 5C | C2 | D3 | AC | 62 | 91 | 95 | E4 | 79 |
| | B | E7 | C8 | 37 | 6D | 8D | D5 | 4E | A9 | 6C | 56 | F4 | EA | 65 | 7A | AE | 08 |
| | C | BA | 78 | 25 | 2E | 1C | A6 | B4 | C6 | E8 | DD | 74 | 1F | 4B | BD | 8B | 8A |
| | D | 70 | 3E | B5 | 66 | 48 | 03 | F6 | 0E | 61 | 35 | 57 | B9 | 86 | C1 | 1D | 9E |
| | E | E1 | F8 | 98 | 11 | 69 | D9 | 8E | 94 | 9B | 1E | 87 | E9 | CE | 55 | 28 | DF |
| | F | 8C | A1 | 89 | 0D | BF | E6 | 42 | 68 | 41 | 99 | 2D | 0F | B0 | 54 | BB | 16 |

(a) S-box

# Byte Substitution (cont.)



# Byte Substitution (cont.)

| | | | |
|---|---|---|---|
| EA | 04 | 65 | 85 |
| 83 | 45 | 5D | 96 |
| 5C | 33 | 98 | B0 |
| F0 | 2D | AD | C5 |

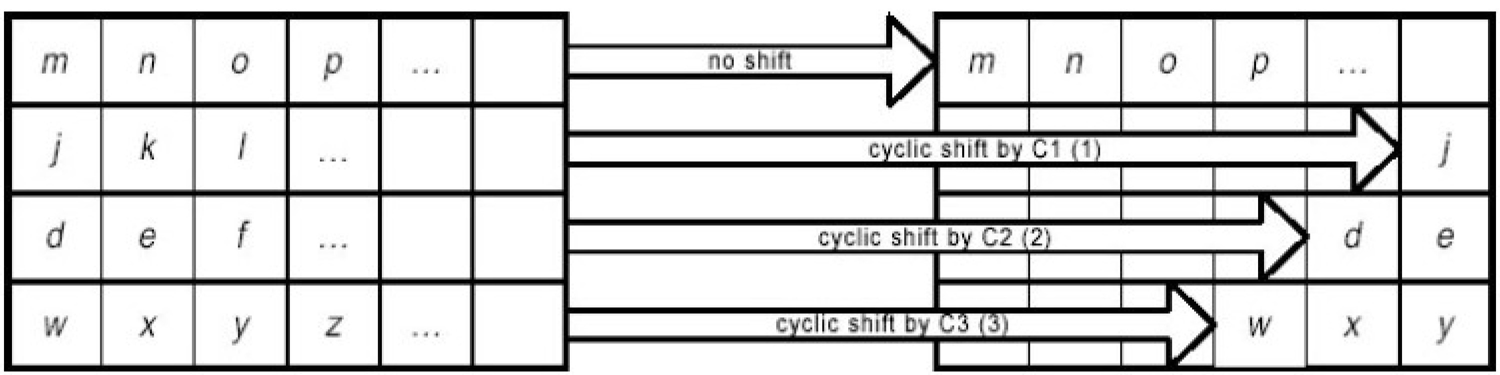
→

| | | | |
|---|---|---|---|
| 87 | F2 | 4D | 97 |
| EC | 6E | 4C | 90 |
| 4A | C3 | 46 | E7 |
| 8C | D8 | 95 | A6 |

# 2. Shift Rows

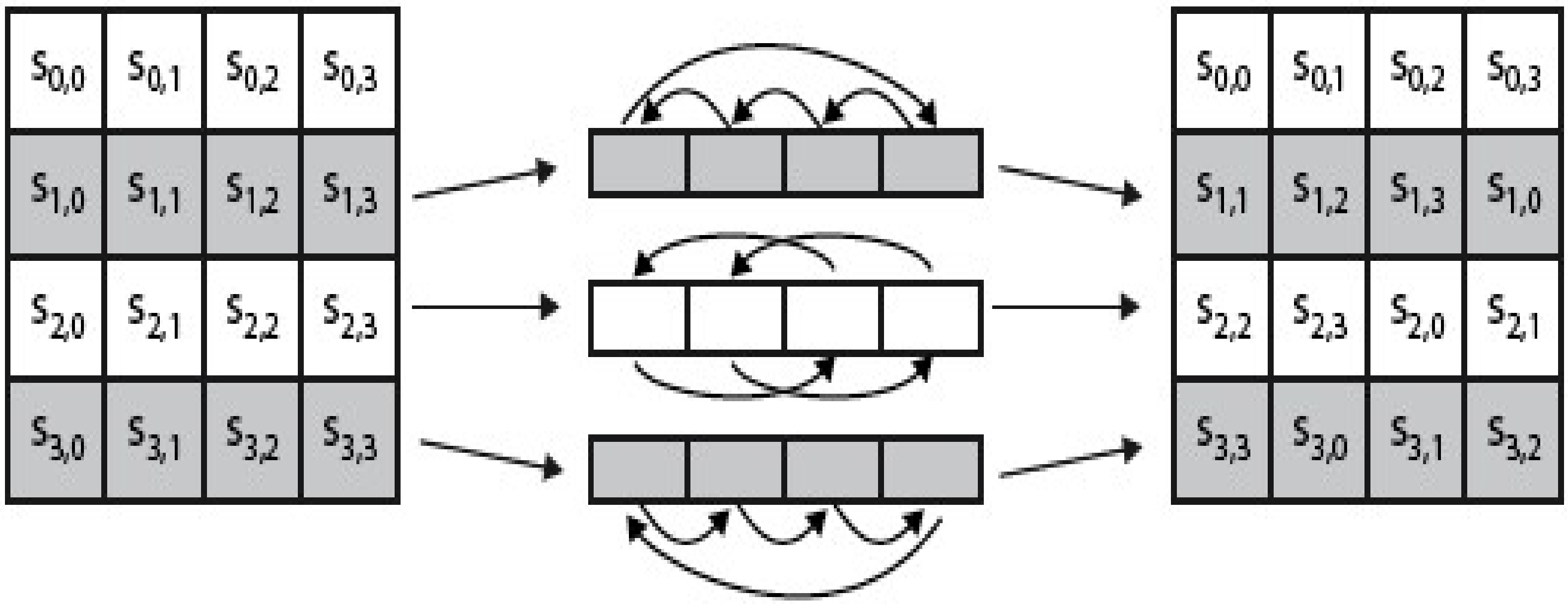
- rotating the rows with different offsets
  - Depending on block size
  - Cyclic Shift left

| BlockSize | C1 | C2 | C3 |
|---|---|---|---|
| 128 | 1 | 2 | 3 |
| 196 | 1 | 2 | 3 |
| 256 | 1 | 3 | 4 |




ShiftRow operates on the rows of the state

# Shift Rows (cont.)




- Decrypt inverts using *shifts to right*
- Since state is processed by columns, this step permutes bytes between the columns

# 3. Mix Columns

- Each column is processed separately.
- Each byte is replaced by a value dependent on all 4 bytes in the column.
- Can express each column as 4 equations
  - to derive each new byte in column.
- Decryption requires use of inverse matrix
  - with larger coefficients, hence a little harder

# Mix Columns (cont.)

$$
\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix}
\begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix}
=
\begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}
$$

$$
\begin{aligned}
s'_{0,j} &= (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j} \\
s'_{1,j} &= s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j} \oplus s_{3,j} \\
s'_{2,j} &= s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j}) \\
s'_{3,j} &= (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j})
\end{aligned}
$$

The following is an example of MixColumns:

| | | | |
|---|---|---|---|
| 87 | F2 | 4D | 97 |
| 6E | 4C | 90 | EC |
| 46 | E7 | 4A | C3 |
| A6 | 8C | D8 | 95 |

→

| | | | |
|---|---|---|---|
| 47 | 40 | A3 | 4C |
| 37 | D4 | 70 | 9F |
| 94 | E4 | 3A | 42 |
| ED | A5 | A6 | BC |

# Mix Columns (cont.)

$$(\{02\} \bullet \{87\}) \oplus (\{03\} \bullet \{6E\}) \oplus \{46\} \oplus \{A6\} = \{47\}$$

$$\{87\} \oplus (\{02\} \bullet \{6E\}) \oplus (\{03\} \bullet \{46\}) \oplus \{A6\} = \{37\}$$

$$\{87\} \oplus \{6E\} \oplus (\{02\} \bullet \{46\}) \oplus (\{03\} \bullet \{A6\}) = \{94\}$$

$$(\{03\} \bullet \{87\}) \oplus \{6E\} \oplus \{46\} \oplus (\{02\} \bullet \{A6\}) = \{ED\}$$

Multiplication and Addition is in GF(2^8).

# Mix Columns (cont.)

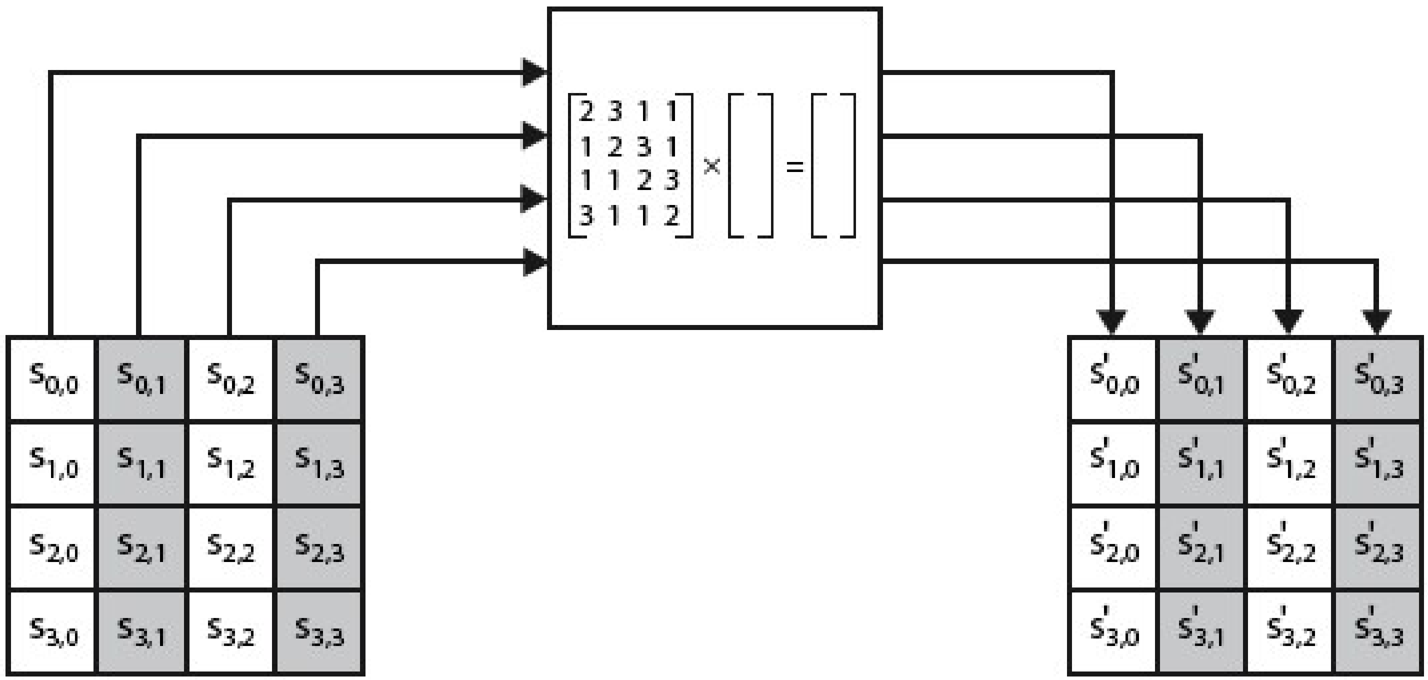



**Fig.5.5 (b)**

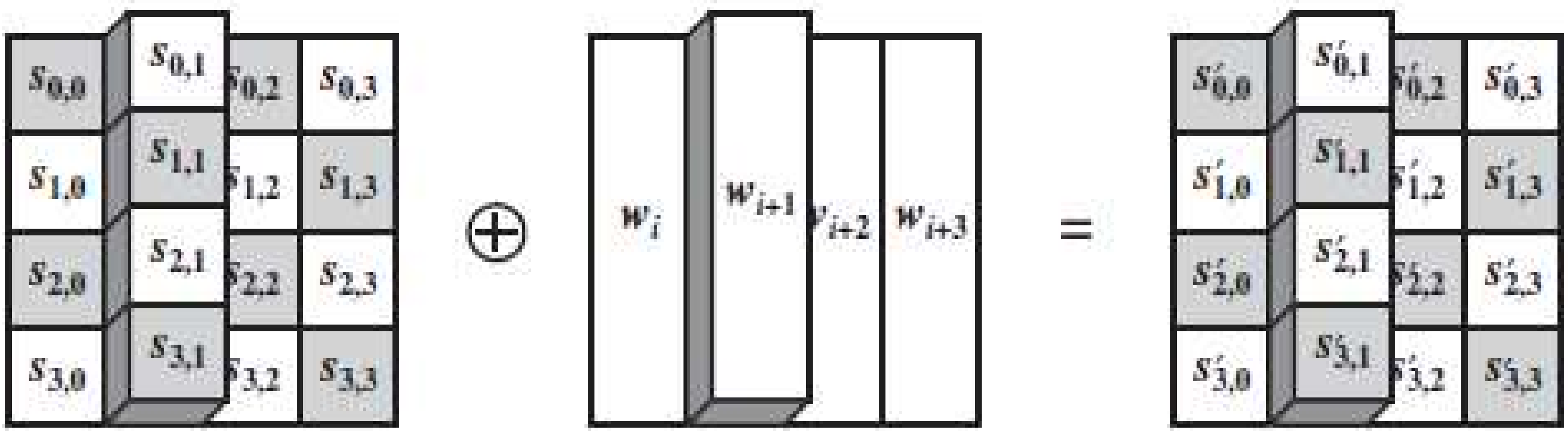
# 4. Add Round Key

- Apply the roundkey with a bitwise XOR.
- Size of roundkey equals block size.

$$\begin{bmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} & a_{0,4} & a_{0,5} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} \end{bmatrix} \oplus \begin{bmatrix} k_{0,0} & k_{0,1} & k_{0,2} & k_{0,3} & k_{0,4} & k_{0,5} \\ k_{1,0} & k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} \\ k_{2,0} & k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} \\ k_{3,0} & k_{3,1} & k_{3,2} & k_{3,3} & k_{3,4} & k_{3,5} \end{bmatrix} = \begin{bmatrix} b_{0,0} & b_{0,1} & b_{0,2} & b_{0,3} & b_{0,4} & b_{0,5} \\ b_{1,0} & b_{1,1} & b_{1,2} & b_{1,3} & b_{1,4} & b_{1,5} \\ b_{2,0} & b_{2,1} & b_{2,2} & b_{2,3} & b_{2,4} & b_{2,5} \\ b_{3,0} & b_{3,1} & b_{3,2} & b_{3,3} & b_{3,4} & b_{3,5} \end{bmatrix}$$

Bitwise round key addition

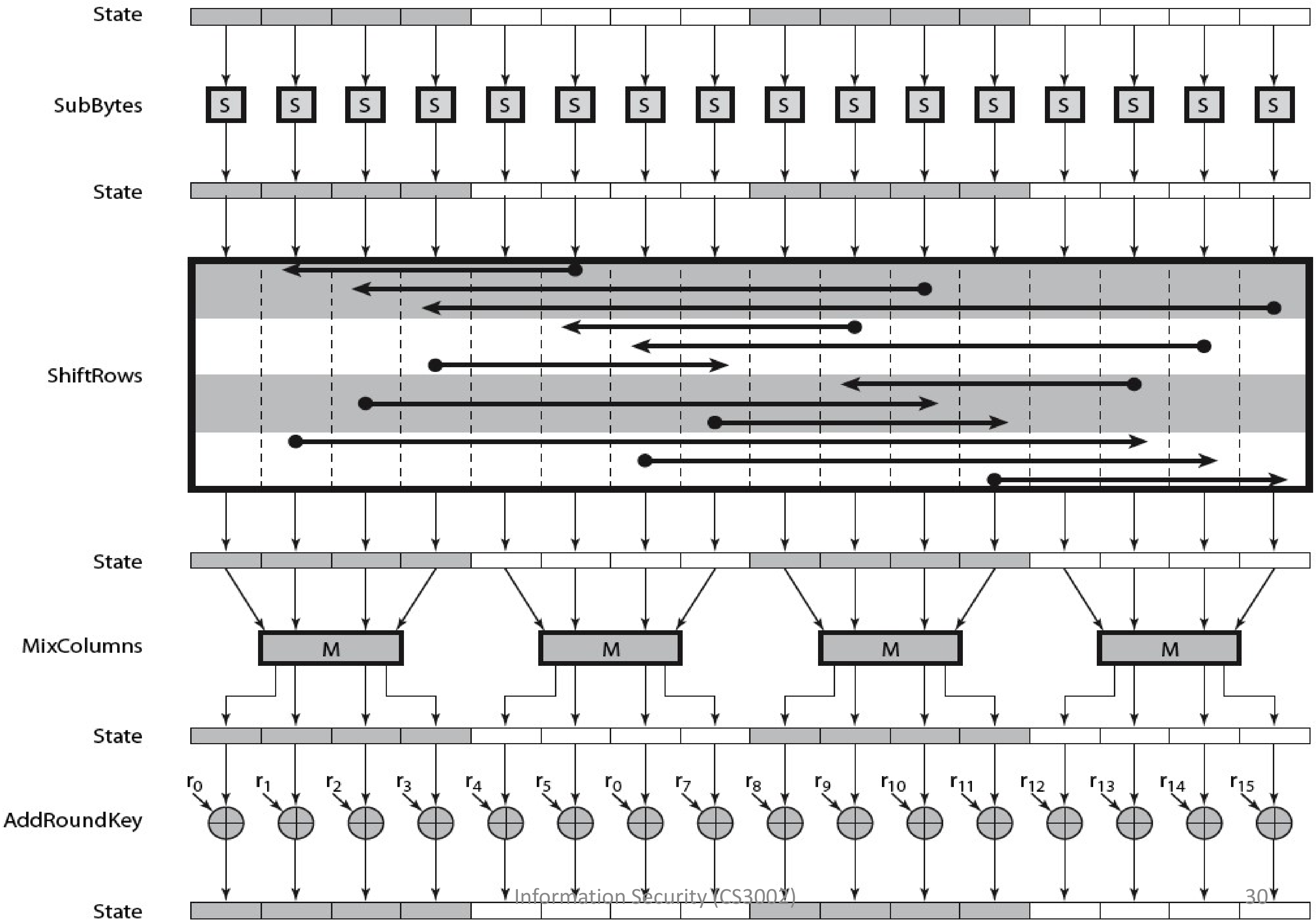
# Add Round Key (cont.)



# Add Round Key (cont.)

- Inverse for decryption identical.
  - Since XOR own inverse, with reversed keys.
- Designed to be as simple as possible.
  - A form of Vernam cipher on expanded key.
  - Requires other stages for complexity / security.

# AES Round


State
SubBytes
S
State
ShiftRows
State
MixColumns
M
State
AddRoundKey
r0 r1 r2 r3 r4 r5 r0 r7 r8 r9 r10 r11 r12 r13 r14 r15
State

# AES Key Expansion

Nk=4,6,8

Nr=10,12,14

# AES Key Expansion

- Takes as input a $N_k$ word key and produces a linear array of $N_k$ * ($N_r$+1) words.
- Expanded key provide a $N_k$ word round key for the initial AddRoundKey() stage and for each of the $N_r$ rounds of the cipher.
- The key is first copied into the first $N_k$ words, the remainder of the expanded key is filled $N_k$ words at a time.

# Key Expansion (cont.)
# Pseudo Code 16 byte key

```
KeyExpansion(byte key[16], word w[44])
{
   word temp;
   for (i = 0; i < 4; i++)  w[i] = (key[4*i], key[4*i+1],
                               key[4*i+2], key[4*i+3]);
   for (i = 4; i < 44; i++)
   {
       temp = w[i-1];
       if ( i mod 4 = 0 )    temp = SubWord(RotWord(temp))
                                     XOR Rcon[i/4];
       w[i] = w[i-4] XOR temp;
   }
}
```

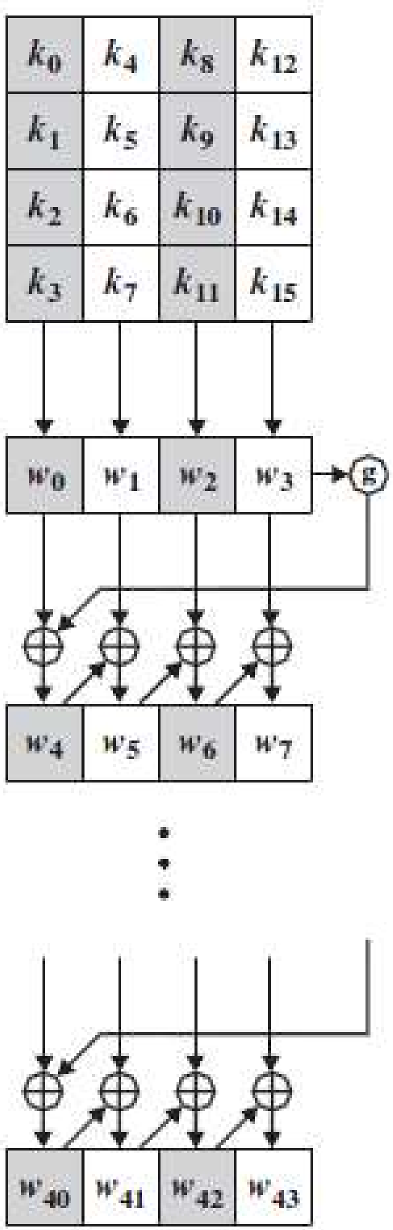
# Key Expansion (cont.)

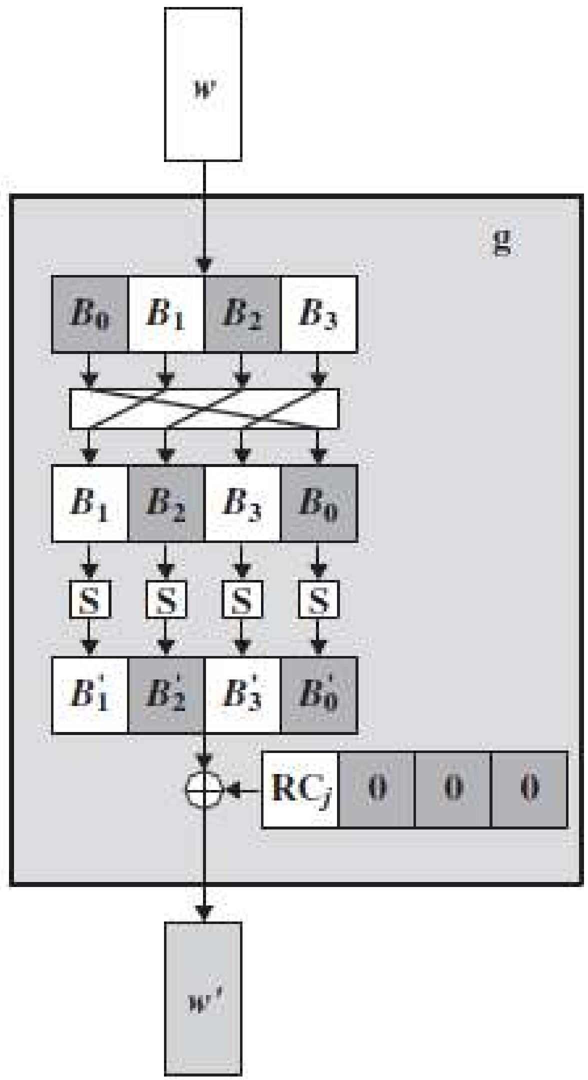
- **`RotWord`** performs a one byte circular left shift on a word. For example:

```
RotWord[b0,b1,b2,b3] = [b1,b2,b3,b0]
```

- **`SubWord`** performs a byte substitution on each byte of input word using the S-box (similar to SubBytes() )

- **`SubWord(RotWord(temp))`** is XORed with Rcon[j] – the round constant.



(b) Function g

| j | RC[j] |
|---|---|
| 1 | 01 |
| 2 | 02 |
| 3 | 04 |
| 4 | 08 |
| 5 | 10 |
| 6 | 20 |
| 7 | 40 |
| 8 | 80 |
| 9 | 1B |
| 10 | 36 |

Nk=4,6,8

Nr=10,12,14

# Pseudo Code for All Key Sizes

```
KeyExpansion(int* Key[4*Nk], int* EKey[Nb*(Nr+1)])
{
    for(i = 0; i < Nk; i++)
       EKey[i] = (Key[4*i],Key[4*i+1],Key[4*i+2],Key[4*i+3]);
    for(i = Nk; i < Nb * (Nr + 1); i++)
    {
       temp = EKey[i - 1];
       if (i % Nk == 0)
          temp = SubByte(RotByte(temp)) ^ Rcon[i / Nk];
       EKey[i] = EKey[i - Nk] ^ temp;
    }
}
```

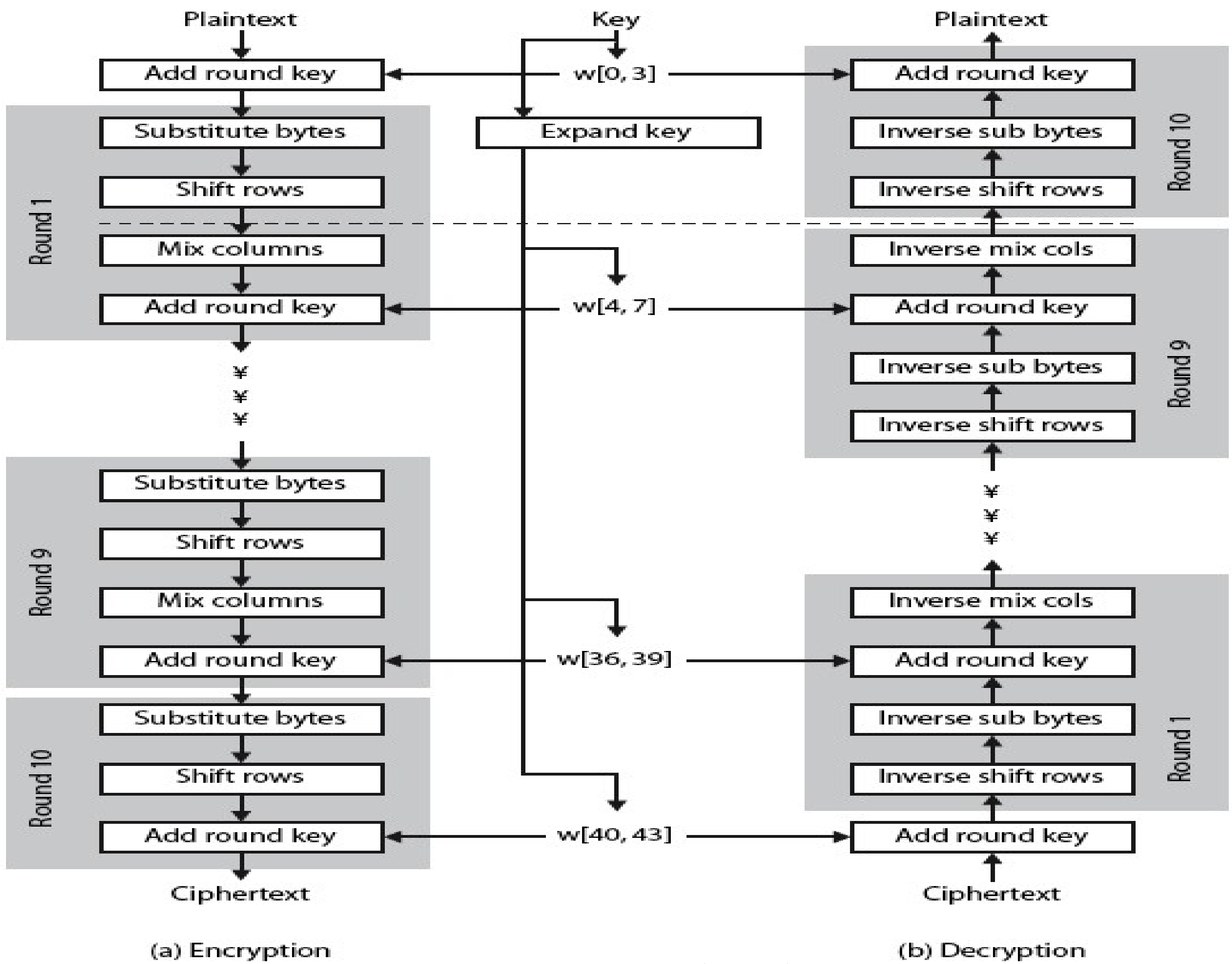
# Decryption of AES (Self-Study)

tweak -adjust finely

# AES Decryption

- AES decryption is not identical to encryption since steps done in reverse.
- But can define an equivalent inverse cipher with steps as for encryption.
  – but using inverses of each step
  – with a different key schedule
- Works since result is unchanged when
  – swap byte substitution & shift rows
  – swap mix columns & add (tweaked) round key

# Rijndael



Plaintext
Key
Plaintext
Add round key
w[0, 3]
Add round key
Substitute bytes
Expand key
Inverse sub bytes
Shift rows
Inverse shift rows
Round 1
Round 10
Mix columns
Inverse mix cols
Add round key
w[4, 7]
Add round key
Inverse sub bytes
Round 9
Inverse shift rows
¥
¥
¥
Substitute bytes
Shift rows
Round 9
Mix columns
Inverse mix cols
Add round key
w[36, 39]
Add round key
Substitute bytes
Inverse sub bytes
Round 10
Shift rows
Inverse shift rows
Round 1
Add round key
w[40, 43]
Add round key
Ciphertext
Ciphertext
(a) Encryption
(b) Decryption

# Equivalent Inverse Cipher

- The original sequence is :

Encryption:

**`SubBytes ShiftRows MixColumns AddRoundKey`**

Decryption:

**`InvShiftRows InvSubBytes AddRoundKey InvMixColumns`**

- Thus **`InvShiftRows`** needs to be interchanged with **`InvSubBytes`** and **`AddRoundKey`** with **`InvMixColumns`**.

# Equivalent Inverse Cipher

- **`InvShiftRows`** – Affects sequence of bytes but does not alter byte content and does not depend on the byte content to perform transformation.
- **`InvSubBytes`** – Affects content of bytes but does not alter byte sequence and does not depend on the byte sequence to perform transformation.
- Thus **`InvShiftRows and InvSubBytes`** can be interchanged. For given state **`S`**,

**`InvShiftRows(InvSubBytes(S))`**

**`=`**

**`InvSubBytes(InvShiftRows(S))`**

# Equivalent Inverse Cipher

- If key is viewed as sequence of words then both **`AddRoundKey`** and **`InvMixColumns`** operate on state one column at a time.

- These operations are linear with respect to the column input: State – S and key - w

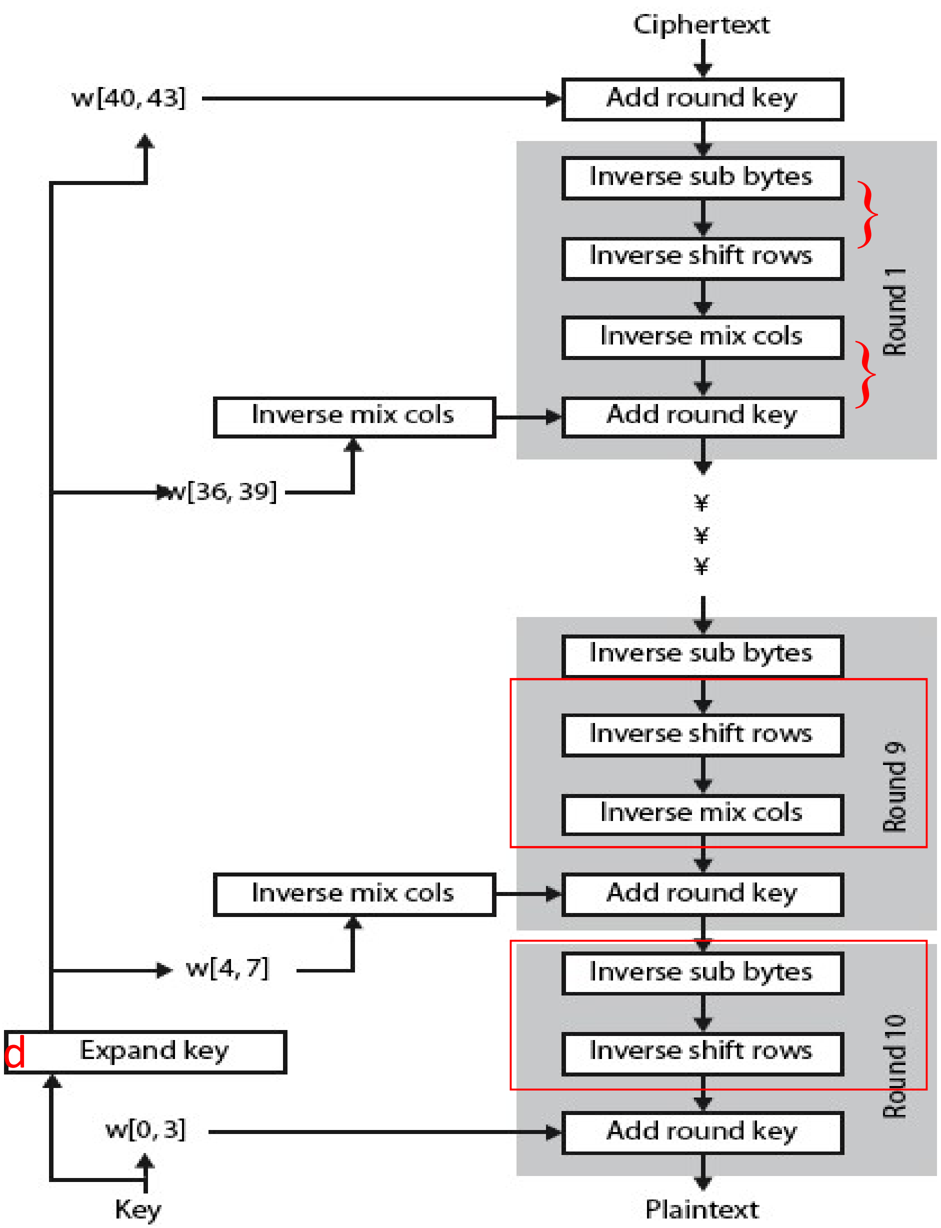
  **`InvMixColumns`**(S XOR w) =
  [**`InvMixColumns`**(S)] XOR **`InvMixColumns`**(w)]

# Equivalent Inverse Cipher

$$
\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix}
\begin{bmatrix} y0 \text{ XOR } k0 \\ y1 \text{ XOR } k1 \\ y2 \text{ XOR } k2 \\ y3 \text{ XOR } k3 \end{bmatrix}
=
\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix}
\begin{bmatrix} y0 \\ y1 \\ y2 \\ y3 \end{bmatrix}
+
\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix}
\begin{bmatrix} y0 \\ y1 \\ y2 \\ y3 \end{bmatrix}
$$

- Thus **`InvMixColumns`** and **`AddRoundKey`** can be interchanged.

# AES Decryption




- **InvShiftRows ↔ InvSubBytes** and
- **AddRoundKey ↔ InvMixColumns**.

# Two Decryptions compared

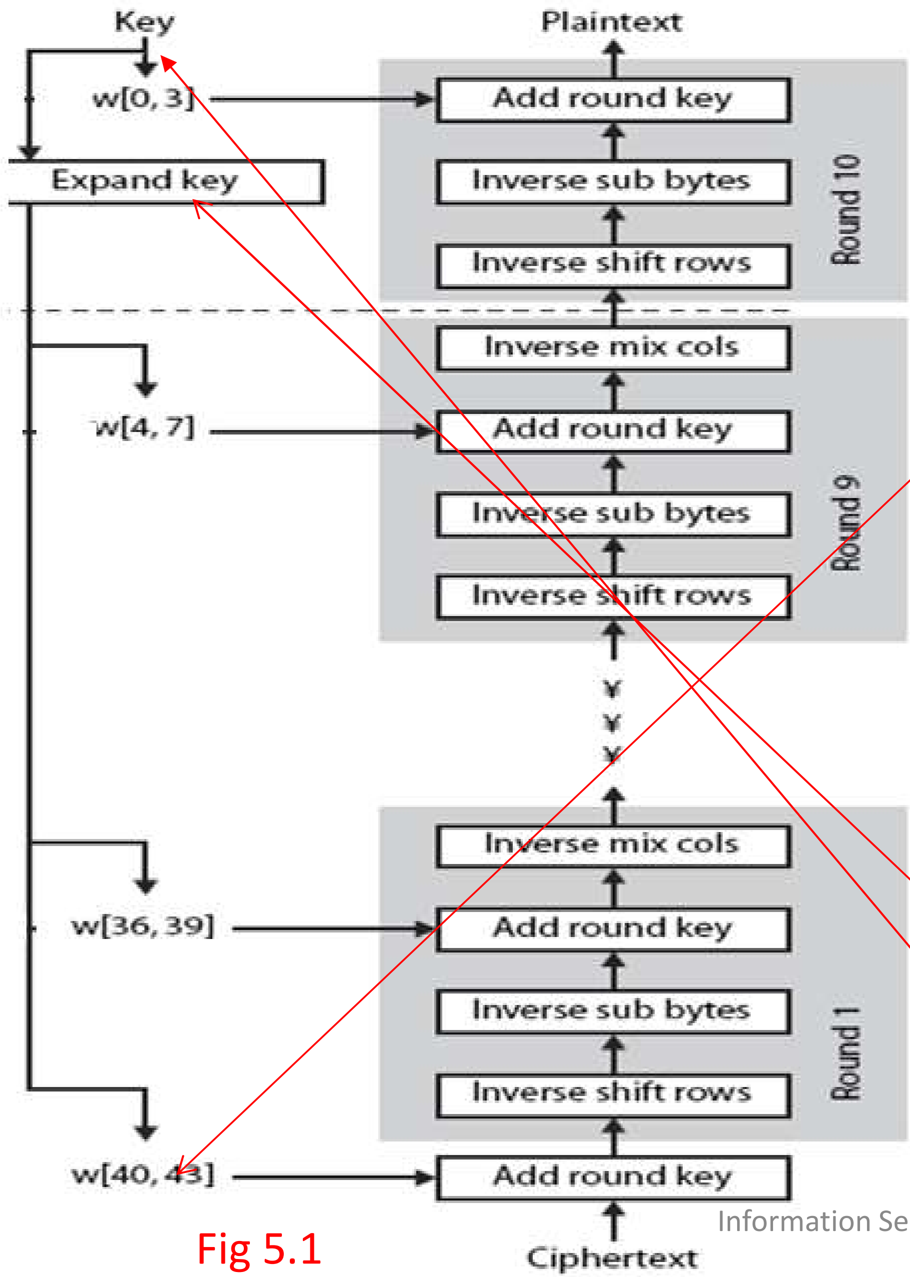



Fig 5.1

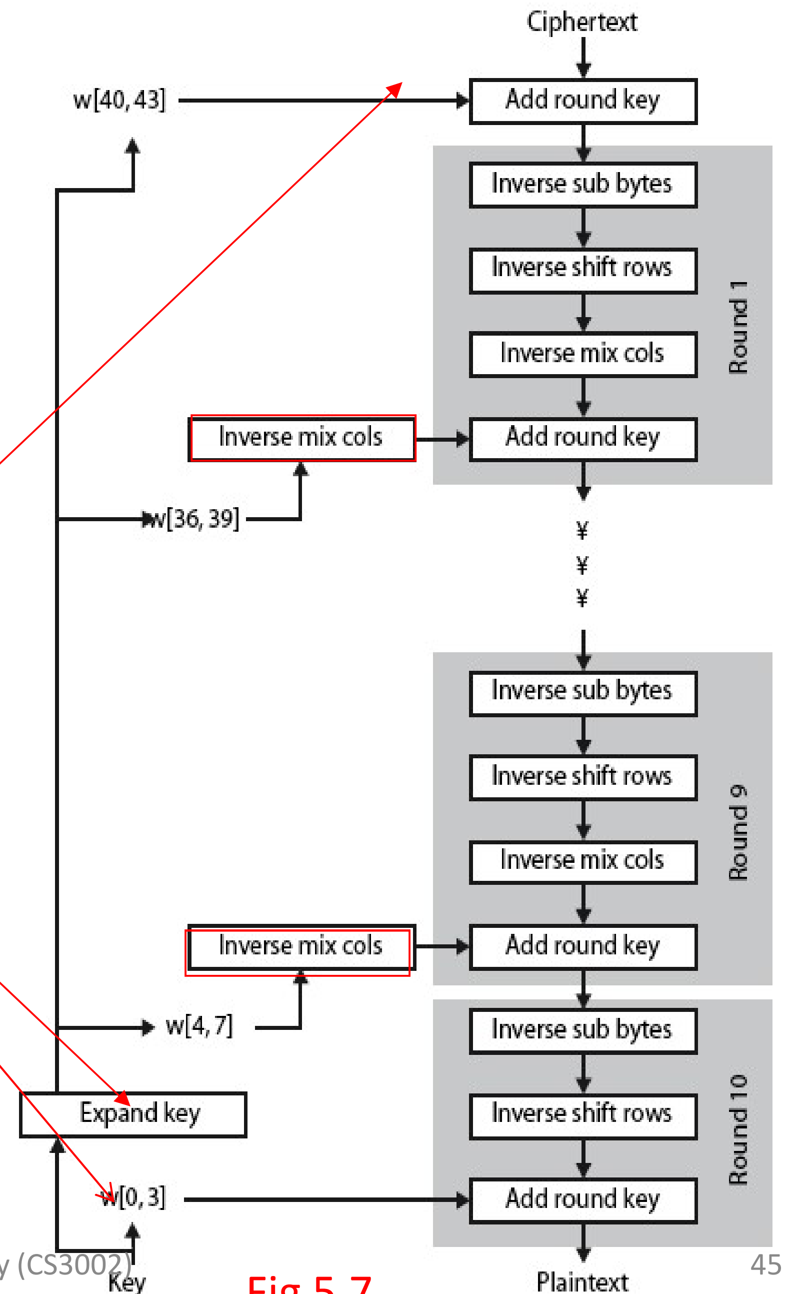



Fig 5.7

# Equivalent Inverse Cipher Advantages

- AS AES decryption cipher is not identical to the encryption cipher (Fig 5.1)
- Disadvantage – Two separate software or hardware modules are required if performing both encryption and decryption.
- So equivalent version of the decryption algorithm that has the same structure ( the same sequence of transformations) as the encryption algorithm, can use same software (Fig 5.7)