SOP optimization with simulate annealing

- ## Problem Description:

The Sequential Ordering Problem (SOP) with precedence constraints consists of finding a minimum weight Hamiltonian path on a directed graph with weights on the arcs and on the nodes, subject to precedence constraints among nodes.

- ## Instances Description:

Instances are provided by TSPLIB that it is a library of sample instances for the TSP (and related problems like SOP, ATSP, HCP) from various sources and of various types.

Each instance file consists of two part as **specification part** that contains information about the instance data and **data part**.

- ## Algorithm Description:

The algorithm designed base on the related paper as *(An Ant Colony System Hybridized with a New Local Search for the Sequential Ordering Problem)*.

Constructive heuristic used for generating initial solution in the way that from the bingeing each time minimum possible length edge based on precedence condition selected.

For neighboring method to move from current solution to another, **Lexicographic Search** using **forwarding and back warding path-preserving-3-exchange** applied and best solution selected among them.

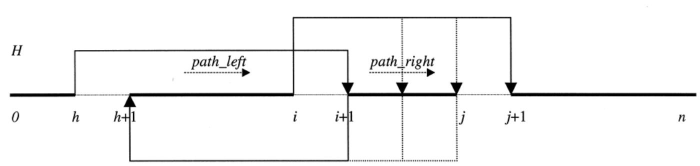


Figure 6. Lexicographic forward path-preserving-3-exchange.

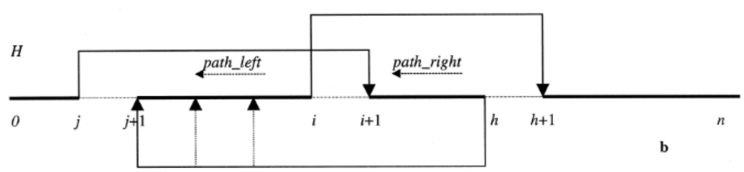


Figure 7. Lexicographic backward path-preserving-3-exchange.

The only difference is that in this algorithm lexicographic search doesn't applied on whole search space by iteratively change the parameters "h, i, j", instead random "h" generated and according to that random "i,j" created to do the search.

With the use of loop with size half of dimension forward and with same size loop backward exchanging applied.

It means that in each simulated annealing iteration best solution selected from a list of solutions with size of problem dimension.

- Initial Constructive heuristic
- O(dimension/2) forward searching with random "h, i, j" parameters.
- O(dimension/2) backward searching with random "h, i, j" parameters.
- Selecting the best from search as next solution

Algorithm time complexity:

```
for it in range(int(dimension/2)):
        h = randrange(0, dimension-3)
        i = h + 1
        ...
        for j in range(i, len(solution)):
                for dep in deps[solution[j]]:
                        ...
```

As code shows the forward and backward search consist of 3 loops so the time complexity is $O(n^3)$.

```
def get_neighbor(problem, dependencies, state, cost):
  ...
  new_state1 = fpp3exchange(problem, dependencies, state)
  new_state2 = bpp3exchange(problem, dependencies, state)
  ...
```

and the neighboring function calling both of them for selecting new solution so the searching algorithm complexity is O(n3).

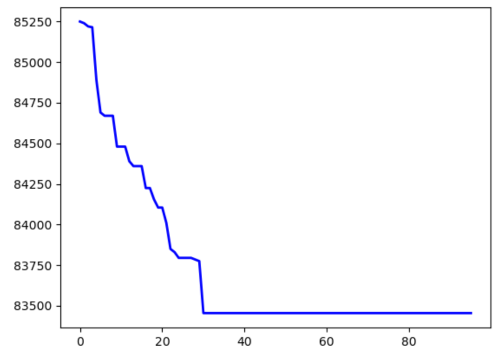
For updating the temperature 3 methods (**linear** and **logarithmic** and **exponential**) applied to find the best to work with.

Algorithm Progress:

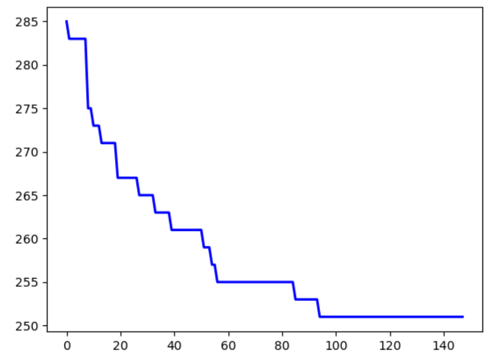
```
T = 1
ALPHA = 0.8                (for using in temperature updating)
TEMP_MODE = EXP (temperature updating method)
```

INIT_HEURISTIC = True (using initial heuristic)
NUM_ITERATIONS = 500

- Algorithm progress plot for sample instances:



p43.4.sop



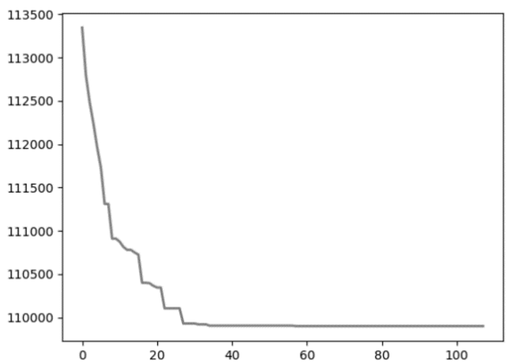
jpeg.4753.54.sop

The whole results (main, max, avg) came at the end.

- ## Initial methods comparison:

T = 1
ALPHA = 0.8
TEMP_MODE = EXP
NUM_ITERATIONS = 500

- Random



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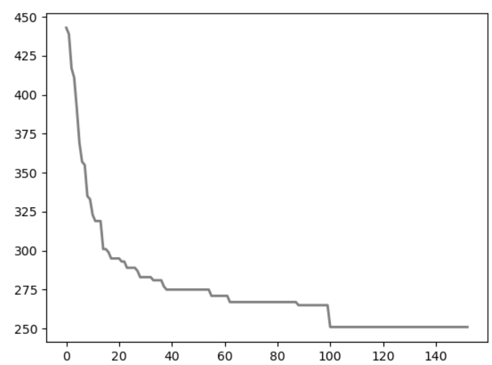

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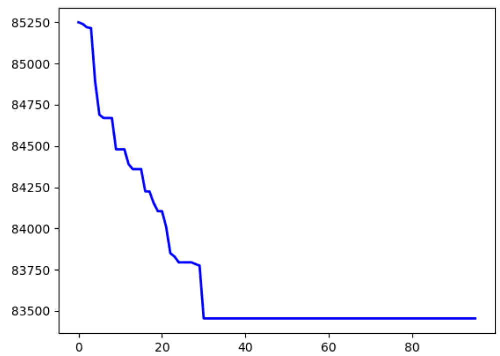
- Heuristic:



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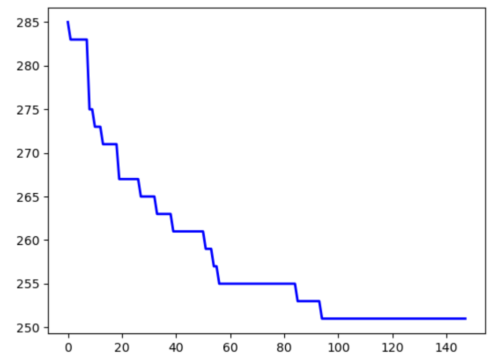

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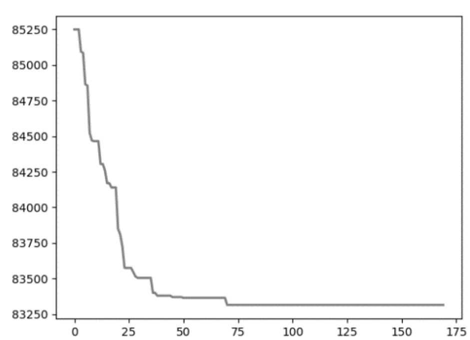
as result shows with heuristic method algorithm start from much better initial solution and in some cases leads to better final solution.
For 10 instances as test heuristic method gave better solution.

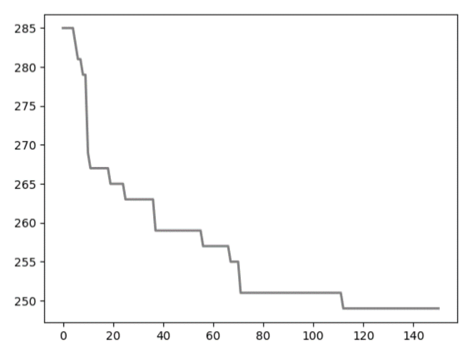
- ## Temperature update methods comparison:

  T = 1
  ALPHA = 0.9
  INIT_HEURISTIC = True
  NUM_ITERATIONS = 500

  - Linear (ALPHA * T):



p43.4.sop



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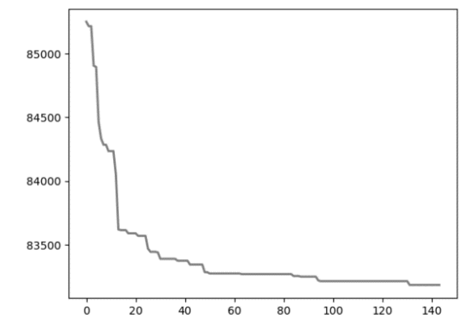
- Logarithmic (T0 / math.log(step)):



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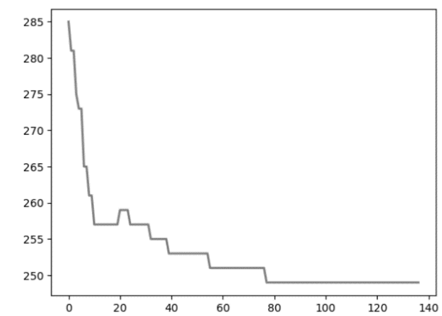

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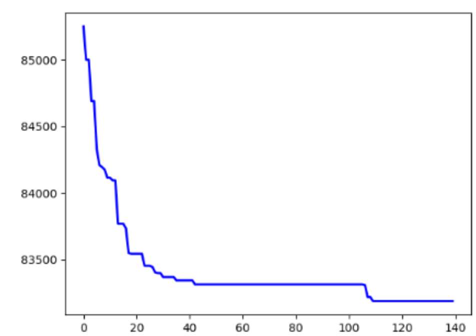
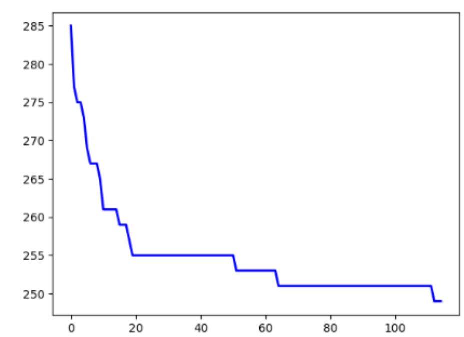
- Exponential exp(-ALPHA * step)*T0:



p43.4.sop



jpeg.4753.54.sop

as plots show exponential method perform a little bit better search in compare with other.

- ## Comparison with BKSs:

Instances run with bellow config:

```
T = 1
ALPHA = 0.9
TEMP_MODE = EXP
INIT_HEURISTIC = True
NUM_ITERATIONS = 500
```

Instances with run time under 30 seconds ran 20 times and other with ran 10 times.

On the “E” instances folder, results were near to the BK answers except bellow instance types: kro124p.*, prob.100, prob.7. *, rbg109a.sop.
it seems that from view of this algorithm, these problems were harder than other.

On the “H” instances folder, results were almost similar to the BK answers (with maximum difference equal to 7).

the “M” instances were much more time consuming and the results weren’t as good as “H” folder.

The whole results (main, max, avg) came at the end.

- ## Algorithm analysis:

Strength:

this algorithm is much faster that algorithm explained in the related origin paper cause instead of searching whole space with time complexity O(n^3), perform the search just for ***“problem.dimension”*** times.

The results are really close to the paper method in most of the cases.

Weakness:

because of searching the less problem area that related paper method, in some instances it reaches a little bit worst result.

In overall the algorithm is a less time-consuming version of paper method with good acceptable results.

- Results:

Instance	BKS	BEST	AVG	MAX	min_time	avg_time	max_time	Diff of Best
br17.1.sop	41	41	42.3	47	0.1416	0.1446	0.1496	0
br17.10.sop	55	55	57.8	65	0.1107	0.11877	0.1496	0
br17.12.sop	55	55	59.2	63	0.1077	0.116270	0.1625	0
ESC78.sop	18230	18250	18400.5	18535	1.7354	1.7924	1.9119	20
ESC98.sop	2125	2125	2125.0	2125	4.1928	4.4230	4.6745	0
ft53.2.sop	8026	9473	10312.8	11041	1.1050	1.2013	1.3194	1447
ft70.2.sop	40419	44076	45597.4	46640	2.0604	2.3552	2.7556	3657
kro124p.1.sop	39420	46934	48537.15	49917	7.9588	8.9360	9.8073	7514
kro124p.3.sop	49499	59438	62762.4	66501	3.4378	4.8182	5.6962	9939
p43.1.sop	28140	28290	28598.75	28810	0.8178	0.9116	1.0894	150
p43.4.sop	83005	83140	83270.5	83445	0.3390	0.3805	0.4697	135
prob.100.sop	1123	3158	3870.05	4849	4.0428	4.2812	4.5359	2035
prob.5.sop	243	421	548.15	682	0.7549	0.8216	0.9136	178
prob.7.40.sop	1071	1788	2301.3	2981	0.6532	0.7296	0.8986	717
prob.7.60.sop	912	1952	2545.5	2968	1.4327	1.5482	1.6655	1040
prob.7.70.sop	879	2310	2881.25	3525	1.9058	2.1013	2.2778	1431
rbg050a.sop	400	407	439.4	478	0.8008	0.9093	1.0082	7
rbg050b.sop	397	403	432.75	463	0.8028	0.9041	1.1270	6
rbg050c.sop	467	468	480.9	494	0.7749	0.8580	0.9614	1
rbg105a.sop	1023	1064	1104.55	1143	1.9378	2.1143	2.4140	41
rbg118a.sop	1423	1424	1450.35	1507	1.8151	1.9227	2.1562	1
rbg124a.sop	1361	1366	1397.25	1436	1.8051	1.9065	2.0226	5
rbg126a.sop	1381	1398	1421.6	1481	1.9942	2.2430	2.6624	17
rbg143a.sop	1765	1774	1801.35	1832	2.1233	2.2379	2.4345	9
rbg219a.sop	2544	2578	2605.35	2632	6.6901	7.2614	7.8829	34
rbg247a.sop	3062	3101	3140.35	3187	8.1642	8.7001	9.8681	39
rbg341a.sop	2568	3117	3217.9	3342	26.782	29.577	33.403	549
ry48p.2.sop	16666	18290	20884.7	23105	0.9273	1.0249	1.1865	1624
ry48p.3.sop	19894	22029	23826.5	25251	0.7129	0.8665	1.0328	2135
prob.7.65.sop	915	1649	1930.65	2188	1.6960	2.2015	2.6070	734
rbg109a.sop	198	1046	1081.1	1110	2.0425	2.5092	2.7716	848
rbg117a.sop	1494	1497	1516.55	1548	1.4388	1.6960	1.8951	3
rbg150a.sop	1750	1783	1829.5	1866	3.7965	4.4883	5.2027	33
rbg174a.sop	2033	2059	2114.5	2146	5.1369	6.4195	7.2307	26
rbg190a.sop	2241	2269	2290.0	2311	5.2462	6.6285	7.6026	28
rbg285a.sop	3482	3557	3604.55	3668	14.519	15.807	18.102	75
rbg358a.sop	2545	2884	3001.15	3141	37.397	41.883	47.849	339
gsm.153.124.sop	1109	1110	1121.05	1129	0.6336	0.7397	0.9579	1
gsm.462.77.sop	577	578	581.45	587	0.5404	0.5812	0.6931	1
jpeg.3184.107.sop	791	798	808.0	817	0.8498	0.9889	1.1526	7

jpeg.4753.54.sop	245	247	256.5	269	0.3554	0.4670	0.7355	2
susan.260.158.sop	1016	1022	1035.65	1055	1.7578	2.1034	2.4285	6
typeset.15577.36.sop	155	155	160.65	171	0.2309	0.2598	0.3679	0
typeset.1723.25.sop	64	64	69.85	78	0.1578	0.1951	0.3143	0
typeset.19972.246.sop	2018	2018	2021.6	2034	1.3684	1.4857	1.7511	0
typeset.4724.433.sop	3466	3468	3478.2	3496	6.0954	6.8138	8.1372	2
typeset.16000.68.sop	84	84	85.2	90	0.6667	0.8752	1.1691	0
typeset.10835.26.sop	127	127	130.9	137	0.1950	0.2186	0.2806	0
R.200.100.1.sop	61	340	402.3	453	27.408	28.773	30.956	279
R.200.100.60.sop	71749	72804	74300.15	75808	1.8221	1.9594	2.2568	1055
R.200.1000.30.sop	41196	46190	49303.0	52981	2.2330	2.5983	3.2566	4994
R.200.1000.60.sop	71556	72846	74722.2	76561	1.9925	2.5362	2.8859	1290
R.300.1000.60.sop	109471	110993	112747.95	114203	5.2474	6.6597	9.1146	1522
R.400.1000.15.sop	38963	64354	66147.15	68407	21.304	22.863	25.351	25391
R.500.1000.1.sop	1316	3532	3733.14	3926	631.41	738.70	858.61	2216
R.600.100.60.sop	23293	24300	24479.8	24711	42.649	49.824	66.338	1007
R.600.1000.1.sop	1337	3676	3681.5	3687	1073.2	10774.5	1081.7	2339
R.600.1000.60.sop	214608	224197	226373.6	228394	29.580	33.008	39.903	9589
R.700.1000.15.sop	65678	121526	123669.0	126399	77.331	81.251	92.250	55848
R.700.1000.60.sop	245589	257974	259705.3	261393	80.584	92.586	100.08	12385