Analyzing Matrix inversion algorithm

* Algorithm:

For calculate matrix inversion we use LUP decomposition method which describe as bellow:

For given matrix A:

LU factorization with partial pivoting as:

***PA = LU***

***L*** and ***U*** are lower and upper triangular matrices. unique factorization for matrix ***A***

require the lower triangular matrix *L* to be a unit triangular matrix.

***P*** is a permutation matrix which reorders the rows of ***A***.

Then for calculating matrix invers we solve bellow expression in defined manner as bellow:

***PA = LU => AA-1 = LU A-1 = PI:***

We Iteratively move over columns of ***I*** as ***b*** and solve equations:

1. First, we solve the equation ***Ly = Pb*** for y.
2. Second, we solve the equation ***Ux = y*** for x.

* Implementation:

Our code has two main methods:

*static int LUPdecompose(int size, Type A[MAX][MAX], int P[MAX]);*

which return LU matrix in A and permutation matrix in P.

*static int LUPinverse(int size, int P[MAX], Type LU[MAX][MAX],*

*Type B[MAX][MAX], Type X[MAX], Type Y[MAX]);*

which return invers of matrix in A in LU.

* Compiling:

Space complexity of algorithm is ***O(n2)*** which for large size of ***n*** may cause problem due to default stack size per application as ***2MB*** in my OS and compiler base config.

Because of that I preserve more space for stack size to prevent segment fault of code that cause sudden execution termination at the start of running.

|  |
| --- |
| gcc -Wl,--stack,4000000 -Wall -pg lup\_matrix\_inverse.c -o0 -o int\_500\_out.exe |

-Wl,option: pass option as an option to the linker.

--stack, <size>: where <size> is in bytes to set the stack size.

-o: specify output exe file name

-o0: without optimizing

-pg: Generate extra code to write profile information suitable use gprof.

* System Information’s:

CPU: core i5 8th generation

RAM: 8GB

OS: windows 10

Cache: 1L = 256KB, 2L = 1MB, 3L = 6MB

* Performance profiling with Gprah:

|  |
| --- |
| gprof int\_500\_out.exe > int\_500\_profile-data.txt |

(500 \* 500) Matrix

int data type -> ***4MB*** stack size

float data type -> ***4MB*** stack size

double data type -> ***7MB*** stack size

(1000 \* 1000) Matrix

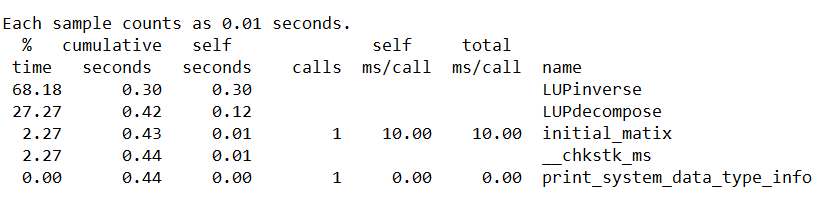
int data type -> ***12MB*** stack size

float data type -> ***12MB*** stack size

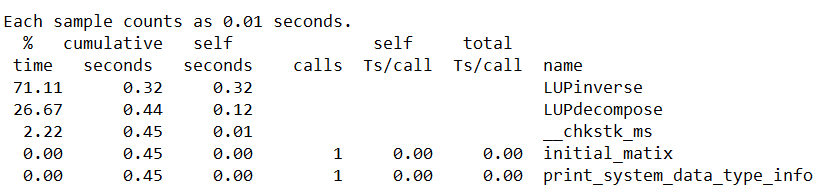
double data type -> ***28MB*** stack size

(500 \* 500) Matrix

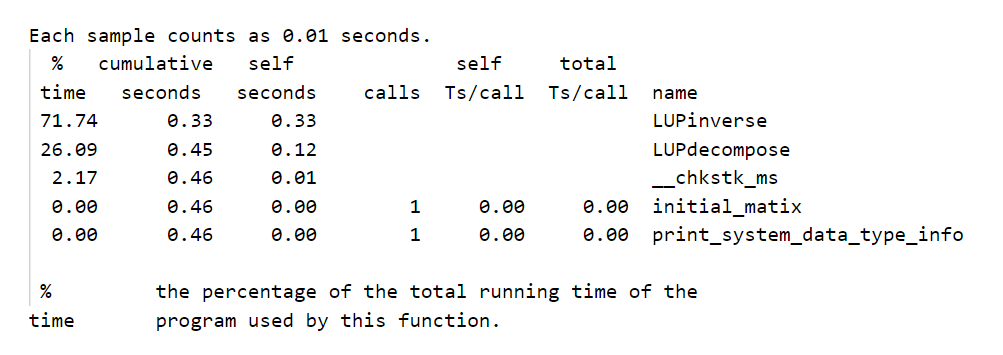
**int data type:**



**float data type:**

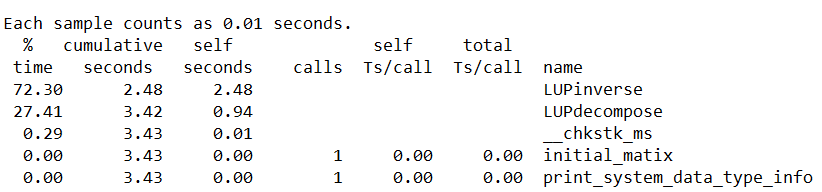


**double data type:**

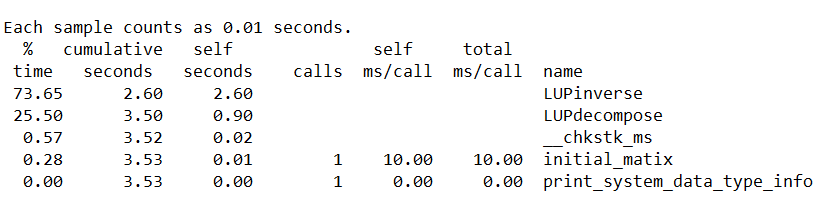


(1000 \* 1000) Matrix

**float data type:**



**double data type:**



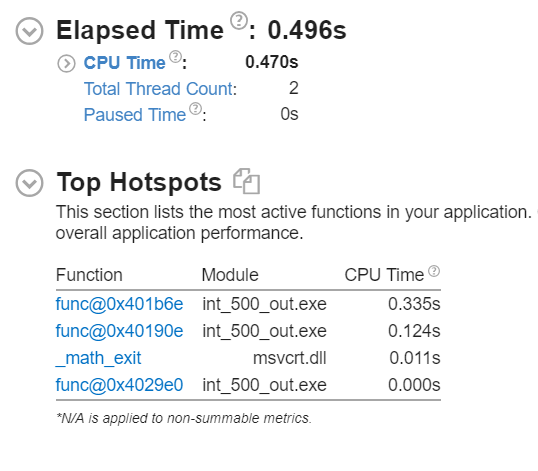
* Performance profiling with Vtune:

The first function address always is = ***LUPinverse***

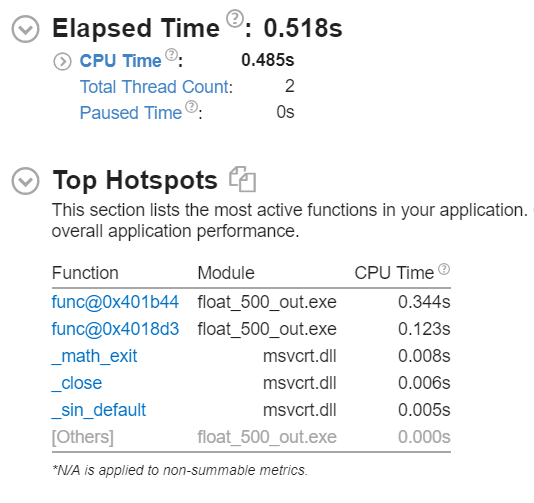
The second function address always is = ***LUPdecompose***

(500 \* 500) Matrix

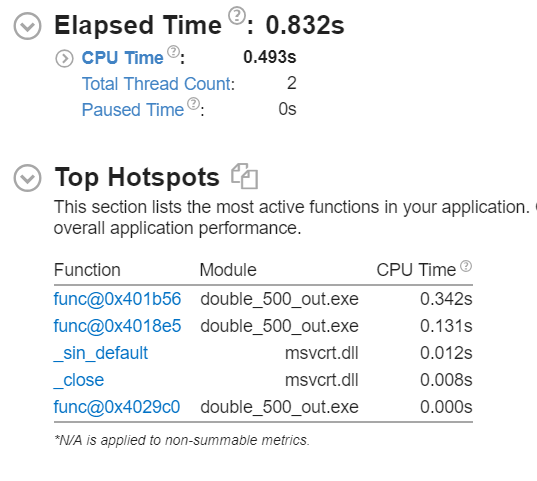
**int data type:**



**float data type:**

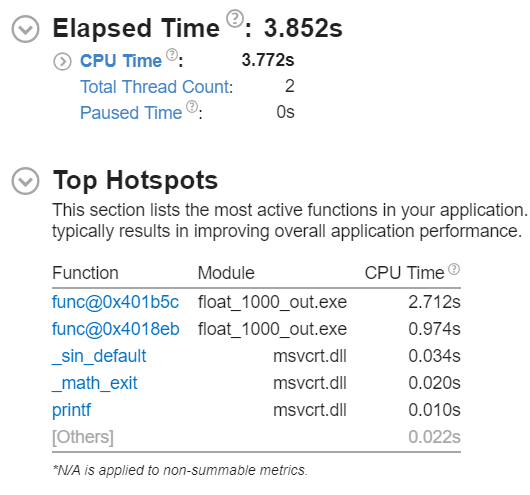


**double data type:**

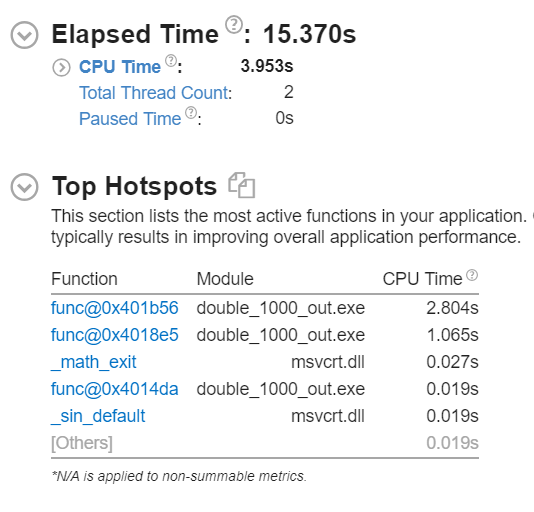


(1000 \* 1000) Matrix

**float data type:**

****

**double data type:**



* Execution analysis:

Program has two main method as:

* ***LUPdecompose:***

With time complexity as O(n2) based on code reviewing and space complexity as O(n+n2) =O(n2).

But according to the algorithm documents, LU decomposition can be computed in time O(M(n)). M(n) ≥ na where a > 2. It means O (n2.376)

* ***LUPinverse:***

With time complexity as O(n3) and space complexity as O(3n+2n2) =O(n2)

And according to code result on the used machine, size (in bytes) and precision (in number of decimal digits) of

float: 4 and 6,

double: 8 and 15

* Proposed improvement:

As matrix size and comparison accuracy increases the execution times growth.

Matrix space always is a good application for parallelism due to high ILP property of matrix-based problems.

As the result shows ***LUPinverse*** takes the most of execution time.

This method solving mathematical equationiteratively over each column.

We can divide this work over multi thread tasks that independently solve equation for specific column vector and in this way make the code much faster.

For another method ***LUPdecompose*** that take O(n2) time we can split the outer loop in specific sizes and pass them to some thread and make it faster.in other word the partial pivoting process that is comparison-based process across each column could be done in parallel manner.