Consider the problem from a previous assignment.

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

1. Solve the problem using lpsolve, or any other equivalent library in R.

   See hw3_weigelt_sensitivity.R

2. Identify the shadow prices, dual solution, and reduced costs

   Shadow prices:
   0.00 0.00 0.00 12.00 20.00 60.00 0.00 0.00 0.00 -0.08 0.56

   Dual solution:
   0.00 0.00 0.00 12.00 20.00 60.00 0.00 0.00 0.00 -0.08 0.56

   Reduced cost:
   0 0 -24 -40 0 0 -360 -120 0

3. Further, identify the sensitivity of the above prices and costs. That is, specify the range of shadow prices and reduced cost within which the optimal solution will not change.

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> cbind(get.sensitivity.rhs(lprec)$duals[1:11], get.sensitivity.rhs(lprec)$dualsfrom[1:11],
get.sensitivity.rhs(lprec)$dualstill[1:11])
      price         lower        upper
 [1,]  0.00 -1.000000e+30 1.000000e+30
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 [2,]  0.00 -1.000000e+30 1.000000e+30
 [3,]  0.00 -1.000000e+30 1.000000e+30
 [4,] 12.00  1.122222e+04 1.388889e+04
 [5,] 20.00  1.150000e+04 1.250000e+04
 [6,] 60.00  4.800000e+03 5.181818e+03
 [7,]  0.00 -1.000000e+30 1.000000e+30
 [8,]  0.00 -1.000000e+30 1.000000e+30
 [9,]  0.00 -1.000000e+30 1.000000e+30
[10,] -0.08 -2.500000e+04 2.500000e+04
[11,]  0.56 -1.250000e+04 1.250000e+04

> cbind(get.sensitivity.rhs(lprec)$duals[12:20], get.sensitivity.rhs(lprec)$dualsfrom[12:20],
get.sensitivity.rhs(lprec)$dualstill[12:20])
      cost          lower         upper
 [1,]    0 -1.000000e+30 1.000000e+30
 [2,]    0 -1.000000e+30 1.000000e+30
 [3,]  -24 -2.222222e+02 1.111111e+02
 [4,]  -40 -1.000000e+02 1.000000e+02
 [5,]    0 -1.000000e+30 1.000000e+30
 [6,]    0 -1.000000e+30 1.000000e+30
 [7,] -360 -2.000000e+01 2.500000e+01
 [8,] -120 -4.444444e+01 6.666667e+01
 [9,]    0 -1.000000e+30 1.000000e+30
```

4. Formulate the dual of the above problem and solve it. Does the solution agree with what you observed for the primal problem?

   See weigelt_dual.lp for the dual formulation and weigelt_sensitivity.R for solving the dual LP problem.

   The solution of the dual is the same as the shadow price in the primal problem. The optimal objective value is the same as that of the primal problem.

Objective Function:

Max. $\mathbf{Z = 420}\ \text{Capac}_1 + \mathbf{360}\ \text{Storage}_1 \mathbf{+ 300}\ \text{Sal}_1 + \mathbf{420}\ \text{Capac}_2 + \mathbf{360}\ \text{Storage}_{2\,+}\ \mathbf{300}\ \text{Sal}_2 + \mathbf{420}\ \text{Capac}_3 + \mathbf{360}\ \text{Storage}_3 + \mathbf{300}\ \text{Sal}_3)$

R1	Capac1	+ Storage1	+ Sal1							≤ 750
B1				Capac2	+ Storage2	+ Sal2				≤ 900
C1							Capac3	+ Storage3	+ Sal3	≤ 450
R2	20 Capac1	+ 15 Storage1	+ 12 Sal1							≤ 13000
B2				20 Capac2	+ 15 Storage2	+ 12 Sal2				≤ 12000
C2							20 Capac3	+ 15 Storage3	+12 Sal3	≤ 5000
R3	Capac1			+ Capac2			+ Capac3			≤ 900
B3		Storage1			+ Storage2			+ Storage3		≤ 1200
C3			Sal1			+ Sal2			+ Sal3	≤ 750

R4 900 Capac1 +900 Storage1 + 900 Sal1 -750 Capac2 + 450Storage2 +750Sal2 =
0
B4 450 Capac1 +450 Storage1 + 450 Sal1 - 750Capac3 - 750Storage3 - 750Sal3 =
0

Capac1,Capac2,Capac3,Storage1,Storage2,Storage3, Sal-L,Sal-M,Sal-S ≥ 0

Becision VCriCbles:
V: objective vCriCble to minimize cost

Objective Function:

Min. V = 750 S1 + 900 K1 + 450 W1 + 13000 S2 + 12000 K2 + 5000 W2 + 900 S3 +
1200 K3 + 750 W3 + (0) S4 + (0) K4)

S.T

S1 + 20 S2 + S3 +900 S4 +450 K4 $\geq$ 420 (1)
S1 + 15 S2 + K3 +900 S4 +450 K4 $\geq$ 360 (2)
S1 + 12 S2 + W3 +900 S4 + 450 K $\geq$ 300 (3)
K1 + 20 K2 + S3 -750 S4 $\geq$ 420 (4)
K1 + 15 K2 + K3 -750 S4 $\geq$ 360 (5)
K1 + 12 K2 + K3 -750 S4 $\geq$ 300 (6)
W1 + 20 W2 + S3 -750 K4 $\geq$ 420 (7)
W1 + 15 W2 + K3 -750 K4 $\geq$ 360 (8)
W1 + 12 W2 + W3 -750 K4 $\geq$ 300 (9)

S1, S2, S3, K1, K2, K3, W1, W2, W3 ≥ 0