Ans no 1

$\delta = 1.1 \times 10^{-11}$

$a_0 = 5$

$b_0 = 27$

minimum iteration needed $= \dfrac{\log|b_0 - a_0| - \log(\delta)}{\log(2)} - 1$

$= \dfrac{\log|22| - \log(1.1 \times 10^{-11})}{\log(2)} - 1$

$= 40.86 - 1$

$= 39.86$

≈ 40

$\therefore$ Ans 40

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Ans no 2

(a) $f(x) = x^4 + 2x^2 - x - 3$

$f(x) = x^4 + 2x^2 - x - 3$

So, $x^4 + 2x^2 - x - 3 = 0$

$$\Rightarrow 2x^2 = \frac{x+3-x^4}{2}$$

$$\Rightarrow x = \sqrt{\left(\frac{x+3-x^4}{2}\right)}$$

$$\therefore g_1(x) = \left(\frac{x+3-x^4}{2}\right)^{\frac{1}{2}}$$

Again,

$$x^4 + 2x^2 - x - 3 = 0$$

$$\Rightarrow x^4 + 2x^2 = x + 3$$

$$\Rightarrow x^2(x^2+2) = x + 3$$

$$\Rightarrow x^2 = \frac{x+3}{x^2+2}$$

$$\therefore x = \left(\frac{x+3}{x^2+2}\right)^{\frac{1}{2}}$$

$$\therefore g_2(x) = \left(\frac{x+3}{x^2+2}\right)^{\frac{1}{2}}$$

Again,

$$x^4+2x^2-x-3 = 0$$

$$\Rightarrow \cancel{x^4+}$$

$$\Rightarrow \cancel{x^4 = x = 2}$$

$$\Rightarrow x^4+2x^2-x-3+2x^2+3 = 2x^2+3$$

$$\Rightarrow x^4+4x^2-x = 2x^2+3$$

$$\Rightarrow x^4+4x^2-x+3x^4 = 3x^4+2x^2+3$$

$$\Rightarrow 4x^4+4x^2-x = 3x^4+2x^2+3$$

$$\Rightarrow \cancel{x =}$$

$$\Rightarrow x(4x^3+4x-1) = 3x^4+2x^2+3$$

$$\Rightarrow x = \frac{3x^4+2x^2+3}{4x^3+4x-1}$$

$$\boxed{\therefore\ g_3(x) = \frac{3x^4+2x^2+3}{4x^3+4x-1}}$$

[showed]

$\therefore g_1'(x) =$

b

For
$$g_1(x) = \left(\frac{x+3-x^4}{2}\right)^{\frac{1}{2}}$$

$x_0 = 1$

Iteration	x	$g(x)$	δ (Error)
1	1	1.22474	0.22474
2	1.22474	0.99367	0.188675
3	0.99367	1.22857	0.2363998
4	1.22857	0.98751	~~18~~ 0.1962139

Here, we saw after 3rd iteration error increased and after 4th iteration the error was 0.1962139

So, ~~it is~~ $g_1(x)$ divergence very rapidly

For $g_2(x) = \left(\frac{x+3}{x^4+2}\right)^{\frac{1}{2}}$

$x_0 = 1$

Iteration	x	$g(x)$	δ (Error)
1	1	1.15470	0.15470
2	1.15470	1.11643	0.0331458
3	1.11643	1.12605	0.0086211
4	1.12605	1.12364	0.0021432

We saw the error was decreasing as the iteration goes on. After 4th iteration the error was 0.0021432.

$g_2(x)$ convergence to a root 1.124

$g_3 \cancel{(x)} =$

For

$$g_3(x) = \frac{3x^4 + 2x^2 + 3}{4x^3 + 4x - 1}$$

$x_0 = 1$

Iteration	x	$g(x)$	δ (Error)
1	1	1.4286	0.142857
2	1.14286	1.12448	0.0160785
3	1.12448	1.12412	0.0003188
4	1.12412	1.12412	0.0000001

After 4th iteration the error ≈ 0

$\therefore$ $g_3(x)$ converges to a root 1.124

c

~~g_3~~ $g_3(x)$ gives the best approximation.
Because $g_3(x)$ ~~given~~ very rapidly convergence to a fixed point.

$g_1(x)$ ~~diverges~~ divergence very rapidly and $g_2(x)$ & $g_3(x)$ convergence to fixed points.

$$g_1(x) = \left(\frac{x+3-x^4}{-2}\right)^{\frac{1}{2}}$$

$$\therefore g'(1.12412) = 1.09 > 1 \quad \text{[Non linear]}$$

For, $$g_2(x) = \left(\frac{x+3}{x^2+2}\right)^{\frac{1}{2}}$$

$$\therefore \lambda = g_2'(x) = 0.25$$

which

$0.25 < 1$

For,

$$g_3(x) = \frac{3x^4 + 2x^2 + 3}{4x^3 + 4x - 1}$$

$$\therefore \lambda = g'(1.12412) = 0 \text{ [superlinear]}$$

If $\lambda = 0$, the convergence is very fast, and this is called the superlinear convergence.

So, for $g_3(x)$, $\lambda = 0$.

Therefore $g_3(x)$ gives the best approximation. It convergence to the point 1.12412.