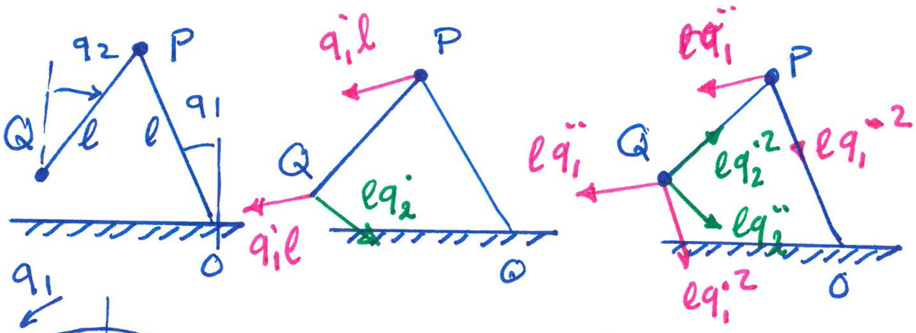
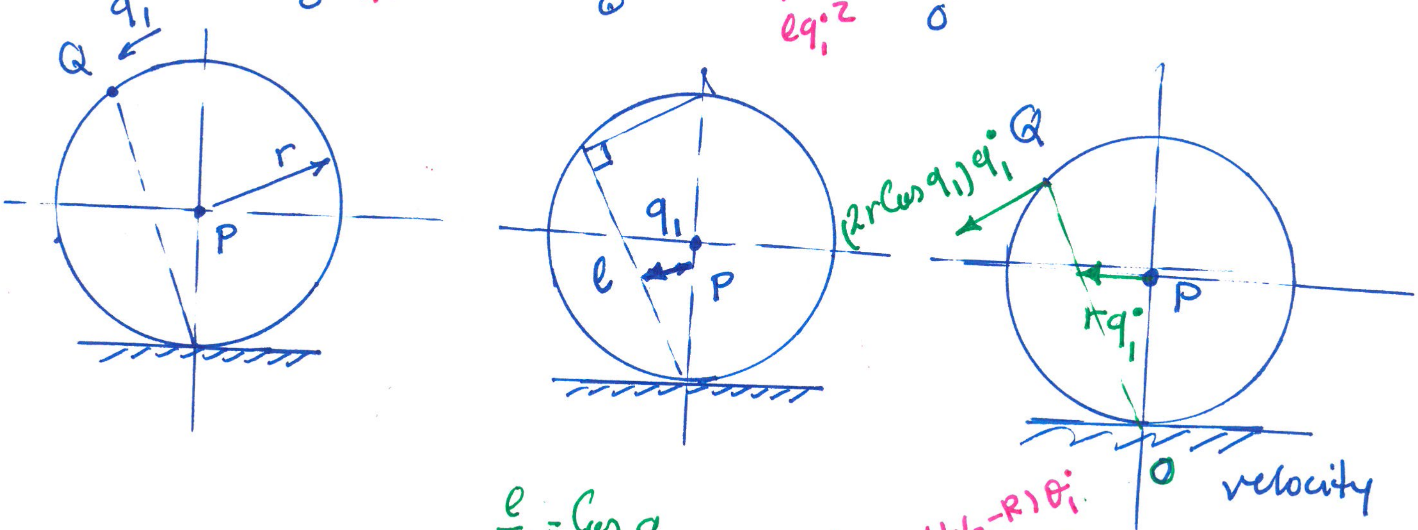
Assignment 1

Sec 1

velocity

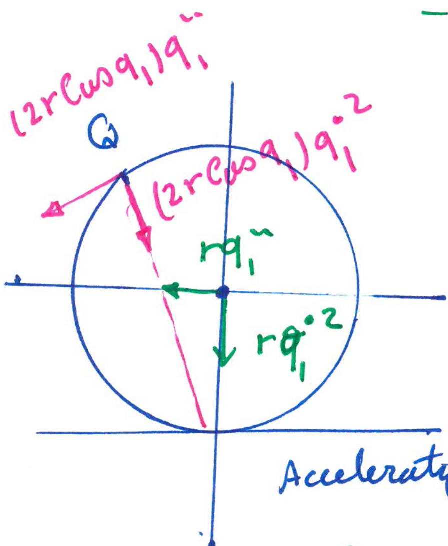
Acceleration



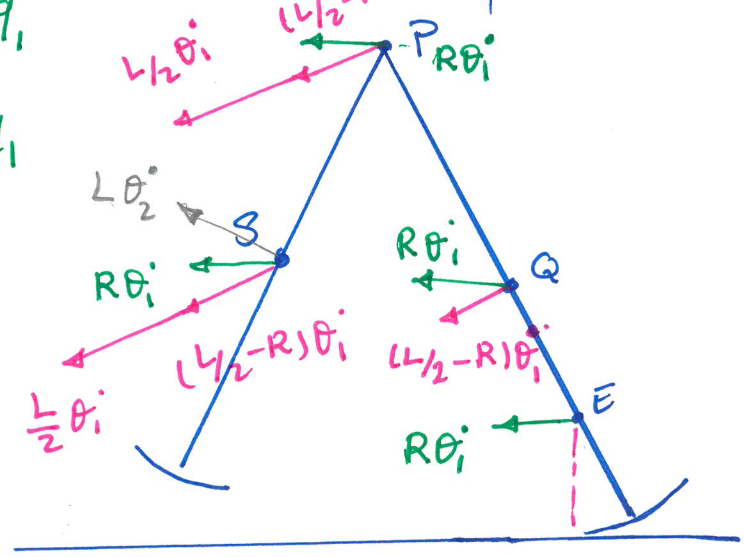


$$\frac{l}{2r} = \cos q_1$$

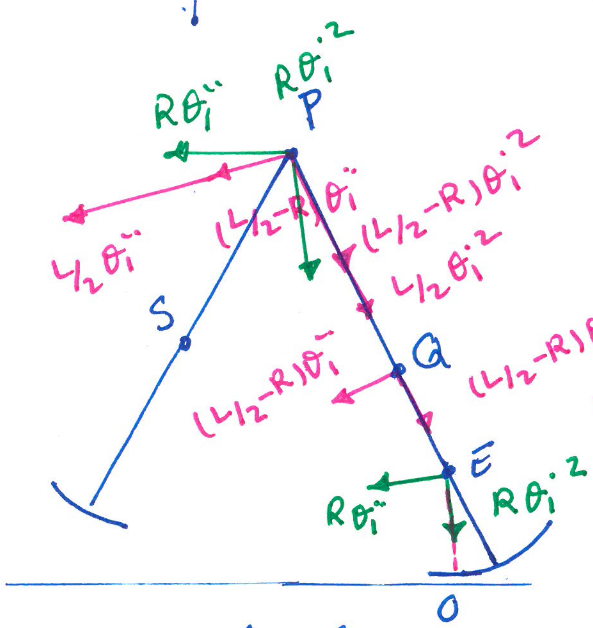
$$\Rightarrow l = 2r\cos q_1$$

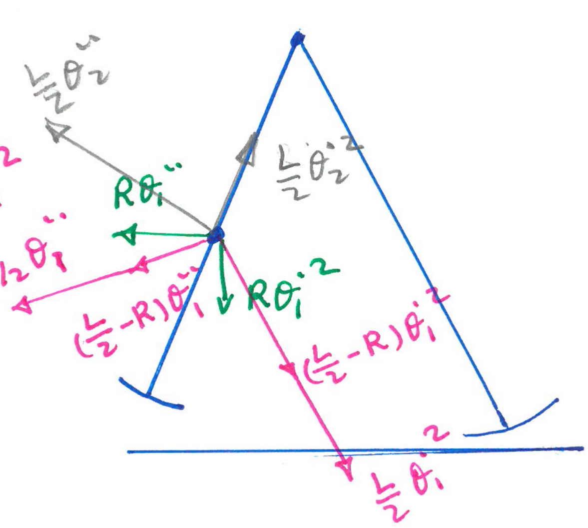


Acceleration



Velocity

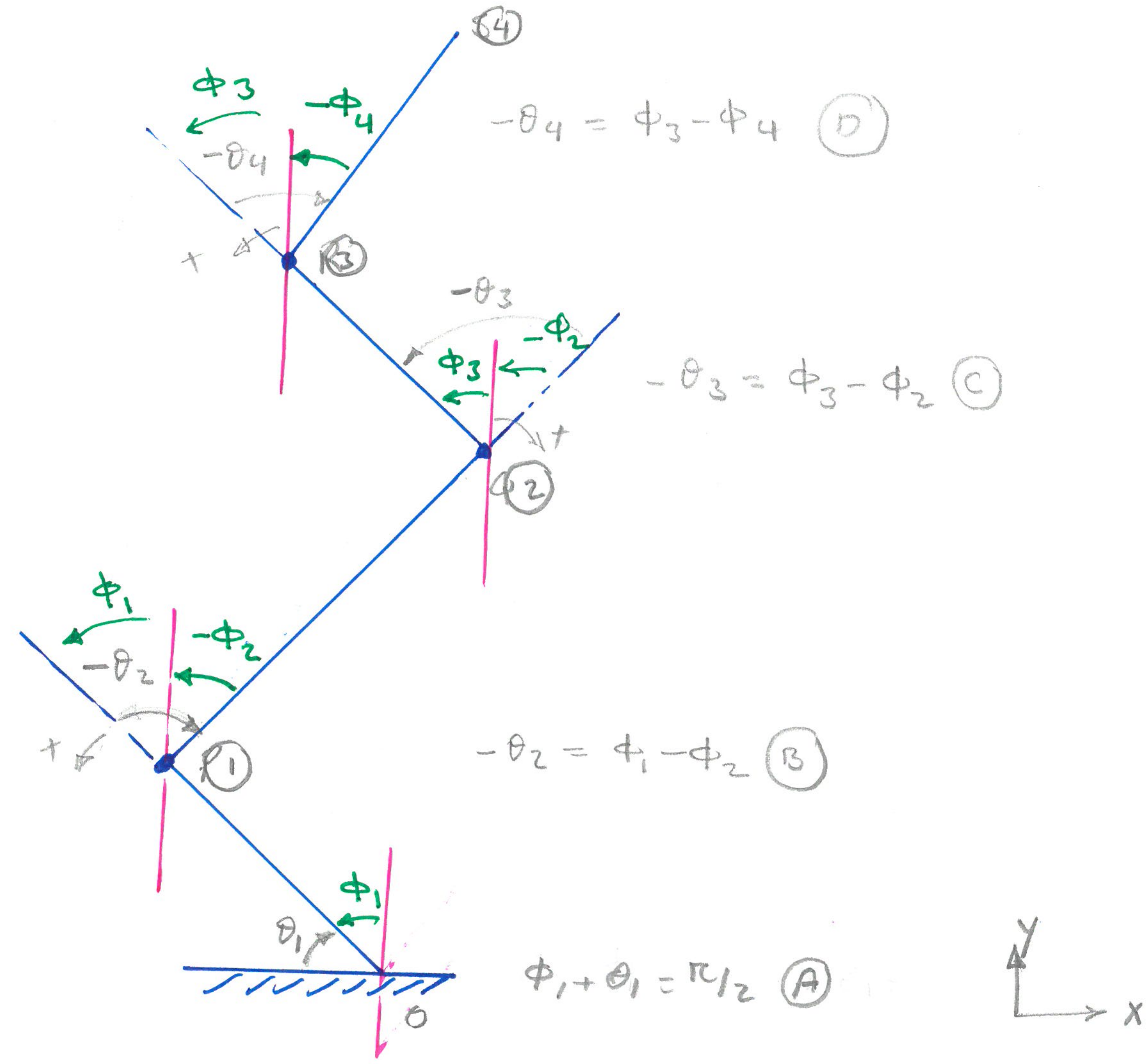




Acceleration

2)

A)



$-\theta_4 = \phi_3 - \phi_4$ (D)

$-\theta_3 = \phi_3 - \phi_2$ (C)

$-\theta_2 = \phi_1 - \phi_2$ (B)

$\phi_1 + \theta_1 = \pi/2$ (A)

A, B, C, D :

$$\begin{cases} \phi_1 + \theta_1 = \pi/2 \rightarrow \theta_1 = \pi/2 - \phi_1 \\ -\theta_2 = \phi_1 - \phi_2 \Rightarrow \theta_2 = \phi_2 - \phi_1 \\ -\theta_3 = \phi_3 - \phi_2 \rightarrow \theta_3 = \phi_2 - \phi_3 \\ -\theta_4 = \phi_3 - \phi_4 \rightarrow \theta_4 = \phi_4 - \phi_3 \end{cases} \rightarrow \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = F \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} \pi/2 - \phi_1 \\ \phi_2 - \phi_1 \\ \phi_2 - \phi_3 \\ \phi_4 - \phi_3 \end{bmatrix} = \begin{bmatrix} f_1(\phi) \\ f_2(\phi) \\ f_3(\phi) \\ f_4(\phi) \end{bmatrix}$$

$$\rightarrow J_{\theta,\phi} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

The constant offset is $\pi/2$ between θ_1 and ϕ_1. Due to the fact that the offset is constant calculating the Jacobean ($\partial F / \partial \phi_1$) the offset disappears calculating the derivating a constant value with respect to ϕ_i (in this case ϕ_1)

A, B, C

$$\begin{cases} \pi/2 - \phi_1 = \theta_1 \rightarrow \phi_1 = \pi/2 - \theta_1 \\ -\theta_2 = \phi_1 - \phi_2 \rightarrow \phi_2 = \theta_2 + \phi_1 = \pi/2 - \theta_1 + \theta_2 \\ -\theta_3 = \phi_3 - \phi_2 \rightarrow \phi_3 = -\theta_3 + \phi_2 = \pi/2 - (\theta_1 + \theta_3) + \theta_2 \\ -\theta_4 = \phi_3 - \phi_4 \rightarrow \phi_4 = \phi_3 + \theta_4 = \frac{\pi}{2} - (\theta_1 + \theta_3) + (\theta_2 + \theta_4) \end{cases}$$

$$\Rightarrow \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = F \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} \pi/2 - \theta_1 \\ \pi/2 - \theta_1 + \theta_2 \\ \pi/2 - (\theta_1 + \theta_3) + \theta_2 \\ \frac{\pi}{2} - (\theta_1 + \theta_3) + (\theta_2 + \theta_4) \end{bmatrix} = \begin{bmatrix} f_1(\theta) \\ f_2(\theta) \\ f_3(\theta) \\ f_4(\theta) \end{bmatrix}$$

$$\Rightarrow J(\phi, \theta) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

For Section B:

$x_1 = -l_1 S\phi_1 \quad = f_1$

$y_1 = l_1 C\phi_1 \quad = f_2$

$\phi_1 = \phi_1 \quad = f_3$

$x_2 = -l_1 S_1 + l_2 S_2 \quad = f_4$

$y_2 = l_1 C_1 + l_2 C_2 \quad = f_5$

$\phi_2 = \phi_2 \quad = f_6$

$\vdots \qquad \vdots$

B)

$$\begin{bmatrix} x_1 \\ y_1 \\ \phi_1 \\ x_2 \\ y_2 \\ \phi_2 \\ x_3 \\ y_3 \\ \phi_3 \\ x_4 \\ y_4 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} -l_1 C_1 & 0 & 0 & 0 \\ -l S_1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -l_1 C_1 & l_2 C_2 & 0 & 0 \\ -l S_1 & -l_2 C_2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix}$$

C) $\dot{X} = J_{x,\phi} \dot{\phi}$

$\dot{\theta} = J_{\theta,\phi} \dot{\phi} \rightarrow \dot{\phi} = J_{\theta,\phi}^{-1} \dot{\theta}$

$\rightarrow \dot{X} = J_{x,\phi} J_{\theta,\phi}^{-1} \dot{\theta}$

Having $\dot{X} = J_{x,\theta} \dot{\theta}$

$\Rightarrow J_{x,\theta} = J_{x,\phi} J_{\theta,\phi}^{-1}$