Comparison of the performance of Simplex method *vs* Interior point method

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Abstract

The purpose of this paper is to compare the efficiency of two methods (Simplex and interior point) in solving a linear programming problem. The efficiency is measured by calculating the CPU run time for both of these method. MATLAB is used here to develop the routine of these two methods. Data have been collected from so called NETLIB suites. The performance of the methods are completely depended on problem size and complexity. Moreover, Computer processor strength is also a vital issue for CPU run time measurement. Simplex method is more faster than interior point method for small and relatively less complex problems.

Keywords
Simplex method, Interior point method, NETLIB

1 Introduction

Linear programming is a mathematical technique to optimize (maximize or minimize) a linear function generally known as objective function with respect to some constraints formulated by some variables [1]. Simplex method and Interior point method are most popular among all other techniques to solve a LP. Simplex method is proposed by George Dantzig in 1947 to solve linear programming problem related to planning and decision making in vast-scale enterprises [2]. This technique is now using in operational research in most of the industries. But interior point method is an updated method to solve the linear programming problems. Idea of implementing central path rather finding all possible solution of an LP is initiate the development of interior point method. Now a days to solve most of the LP and NLP problems, interior point technique is replacing simplex method. However, This technique was known as barrier method in early 1960 but got popularity after publishing on paper name *"A new polynomial-time algorithm for linear programming"* by Narandra Karmakar in 1984 [3]. He has showed this technique takes less step to find the optimal solution than other methods hence less time and space consuming. However, In this project we have examined this claim to see if the performance of interior point method is better than well established simplex method by implementing both of the techniques in solving some LP problems collected from NETLIB. The coding scripts were written by using MATLAB.

2 Methodology

Following is a problem of LP

$$Maximize \qquad \sum_{j=1}^{n} c_j x_j \tag{1}$$

$$Subject \quad to \qquad \sum_{j=1}^{n} a_{ij} xj \leq b_i \qquad i = 1, 2, ..., m \tag{2}$$

$$x_j \geq 0 \qquad j = 1, 2, ..., n \tag{3}$$

2.1 Primal simplex method

We can describe the simplex method in brief like following:

- We need to check first for optimality of objective function. If the objective function is greater than 0 we can stop else select an entering variable.
- Entering variable will be selected according to the sign of variable in objective function. We need to pick the variable which coefficient is most negative.
- We need to calculate the primal step direction and length to select the leaving variable.
- Then we need to update the dual step direction and dual step length to update primal and dual solutions.
- Then we need to update our basis. These are the steps of one iteration to the primal simplex method. we will continue this step until we get our optimal solution.

2.2 Interior point method

To implement interior point method to solve a linear problem we have used predictor corrector algorithm. We have developed a MATLAB code to implement on NETLIB problems. This algorithm alters in between two steps In each iteration the algorithm performs a predictor step. Then this algorithm calculates step direction calculated with no centering. The calculation of step length is restricted within a certain boundary so that they remain close to each other.
In every even step this algorithm calculate a corrector step. Then the step directions are computed by using pure centering. The step length parameter set to 1.

3 Data

Data set are collected from NETLIB suites. Eight data set have been used in this project. The data are arranged in matrix form.

4 Result

The following table will display the required time to solve the NETLIB problems by both primal simplex method and interior point method.

Problem Name	Time required to solve the problem (Second)	
	Simplex Method	Interior Point Method
Forplan.mat	3.26	0.89
israel.mat	2.68	0.24
afiro.mat	0.04	0.02
25fv47.mat	123.48	4.83
agg.mat	3.67	0.62
agg2.mat	3.54	0.77
fffff800.mat	17.01	2.62
share2b.mat	0.37	0.07

Table 1: CPU run time chart of Simplex and Interior point method

5 Conclusion

The result of the time requirement clearly demonstrate the efficiency of Interior point method over Simplex method. But it is not a certain thing as they are dependent of several factors such as bug in code, Processor speed etc.

References

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