$$\hat{y}_{k+1} = -A_1(q^{-1})\, y_k + B(q^{-1})\, u_{k+1-d} + C_1(q^{-1})\,[y_k - \hat{y}_k]$$
$$= -\sum_{m=1}^{n_a} a_m y_{k-m+1} + \sum_{p=0}^{n_b} b_p u_{k+1-d-p} + \sum_{n=1}^{n_c} c_n (y_{k-n} - \hat{y}_{k-n})$$

$$y(k) = -A_1(q^{-1})\, y(k-1) + B(q^{-1})\, u(k-d) + C(q^{-1})\, e(k)$$
$$= \hat{y}_k + e_k$$
$$\rightarrow e_k = \frac{A(q^{-1})}{C(q^{-1})} y_k - \frac{B(q^{-1})}{C(q^{-1})} u_k$$

→ part. Ableitung nach a_n:

$$\partial/\partial a_n\, \hat{y}_k = \partial/\partial a_n\, B(q^{-1}) u_{k-d} - \partial/\partial a_n \sum_{m=1}^{n_a} a_m y_{k-m} + \partial/\partial a_n \sum_{t=1}^{n_c} c_t (y_{k-t} - \hat{y}_{k-t})$$

$$\frac{\partial \hat{y}_k}{\partial a_n} = 0 - y_{k-n} - \sum_{m=1}^{n_a} a_m \frac{\partial y_{k-m}}{\partial a_n} + \sum_{t=1}^{n_c} c_t \left[\frac{\partial y_{k-t}}{\partial a_n} - \frac{\partial \hat{y}_{k-t}}{\partial a_n}\right]$$

– rekursiv Teil

$$\frac{\partial y_k}{\partial a_n} = \frac{\partial \hat{y}_k}{\partial a_n} + \frac{\partial e_k}{\partial a_n} \quad \text{mit} \quad \frac{\partial e_k}{\partial a_n} = \partial/\partial a_n \left(\frac{A(q^{-1})}{C(q^{-1})} y_k + \overset{=0}{\cancel{\frac{B(q^{-1})}{C(q^{-1})}}} u_{k-d} \right)$$
$$= \partial/\partial a_n \frac{1 + \sum_{m=1}^{n_a} a_m q^{-m}}{1 + \sum_{t=1}^{n_c} c_t q^{-t}} y_k$$
$$\vdots$$
$$= \text{lange Rechnung} \ldots$$

→ harte Annahme:

$\frac{\partial y_k}{\partial \Theta_e} = 0 = \frac{\partial u_k}{\partial \Theta_e}$, da Ein- und Ausgang von $\hat{\Theta}$ unabhängig

$$\Rightarrow \frac{\partial \hat{y}_k}{\partial \hat{a}_n} = -y_{k-n} - \sum_{t=1}^{n_c} \hat{c}_t \frac{\partial \hat{y}_{k-t}}{\partial \hat{a}_n}$$

→ analog zu $\partial \hat{b}_n$

$$\frac{\partial \hat{y}_k}{\partial \hat{b}_n} = \partial/\partial \hat{b}_n (B(q^{-1}) u_{k-d}) - \partial/\partial \hat{b}_n (A_1(q^{-1}) y_{k-1}) + \partial/\partial \hat{b}_n (C_1(q^{-1}) e_{k-1})$$
$$= u_{k-d-n} + \sum_{r=0}^{n_b} \hat{b}_r \cancel{\frac{\partial u_{k-d-r}}{\partial \hat{b}_n}} - \sum_{m=1}^{n_a} \hat{a}_m \cancel{\frac{\partial y_{k-m}}{\partial \hat{b}_n}} + \sum_{t=1}^{n_c} \hat{c}_t \left(\cancel{\frac{\partial y_{k-t}}{\partial \hat{b}_n}} - \frac{\partial \hat{y}_{k-t}}{\partial \hat{b}_n} \right)$$

$$\frac{\partial \hat{y}_k}{\partial \hat{b}_n} = u_{k-d-n} - \sum_{t=1}^{n_c} \hat{c}_t \frac{\partial \hat{y}_{k-t}}{\partial \hat{b}_n}$$

$\rightarrow$ part. Ableitung nach $\partial \hat{c}_n$

$$\frac{\partial \hat{y}_k}{\partial \hat{c}_n} = \frac{\partial}{\partial \hat{c}_n}\left(B(q^{-1})u_{k-d}\right) - \frac{\partial}{\partial c_n}\left(A_1(q^{-1})y_{k-1}\right) + \frac{\partial}{\partial c_n}\left(C_1(q^{-1})e_{k-1}\right)$$

$$= 0 - 0 + \sum_{i=1, i\neq n}^{n_c} \hat{c}_i \left[\cancel{\frac{\partial y_{k-i}}{\partial \hat{c}_n}} - \frac{\partial \hat{y}_{k-i}}{\partial \hat{c}_n}\right] + (y_{k-n} - \hat{y}_{k-n}) + \hat{c}_n\left[\cancel{\frac{\partial y_{k-n}}{\partial \hat{c}_n}} - \frac{\partial \hat{y}_{k-n}}{\partial \hat{c}_n}\right]$$

$$= (y_{k-n} - \hat{y}_{k-n}) + \sum_{i=1}^{n_c} \hat{c}_i \frac{\partial \hat{y}_{k-i}}{\partial \hat{c}_n}$$

$$\frac{\partial \hat{y}_k}{\partial \hat{c}_n} = e_{k-n} + \sum_{i=1}^{n_c} \hat{c}_i \frac{\partial y_{k-i}}{\partial \hat{c}_n}$$

Zusammenfassung:

$$\frac{\partial \hat{y}(k+1)}{\partial \hat{a}_n} = \hat{\Psi}_n^a(k+1) = -y(k+1-n) - \sum_{m=1}^{n_c} \hat{c}_m \hat{\Psi}_n^a(k+1-m)$$

$$\frac{\partial \hat{y}(k+1)}{\partial \hat{b}_n} = \hat{\Psi}_n^b(k+1) = u(k+1-d-n) - \sum_{m=1}^{n_c} \hat{c}_m \hat{\Psi}_n^b(k+1-m)$$

$$\frac{\partial \hat{y}(k+1)}{\partial \hat{c}_n} = \hat{\Psi}_n^c(k+1) = e(k+1-n) - \sum_{m=1}^{n_c} \hat{c}_m \hat{\Psi}_n^c(k+1-m)$$