$$\int_0^{2\pi}\left[\int_0^{\pi} r^2 \sin\theta \, d\theta\right] d\phi = 4\pi r^2$$

$$\boxed{\begin{matrix} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{matrix}}$$

Amplitude Radiation Pattern

1. Isotropic, Directional, Omnidirectional
2. Principal patterns
3. Pattern lobes

III. Far-Field

A. Fields exhibit spherical wavefront (e^{-jkr}/r); thus the pattern, ideally, does not vary with distance
B. Electric & magnetic fields are in-phase
C. Wave impedance is, ideally, real
D. Power predominantly real; propagating energy

$$\underline{W}\big|_{ave} = \frac{1}{2}\mathrm{Re}\left[\underline{E}\times\underline{H}^*\right]$$

$$P_{rad} = P_{ave} = \int_0^{2\pi}\int_0^{\pi}\left(\frac{1}{2}\mathrm{Re}\left[\underline{E}\times\underline{H}^*\right]\right)\cdot\hat{a}_r r^2 \sin\theta \, d\theta \, d\phi \quad \text{(2-9)}$$

Far-Field

3. $\underline{W}_{rad} \simeq \hat{a}_r \frac{1}{2\eta r^2}\left[\left|E_\theta^\circ(\theta,\phi)\right|^2 + \left|E_\phi^\circ(\theta,\phi)\right|^2\right]$

4. $U = r^2 W_{rad} \simeq \frac{1}{2\eta}\left[\left|E_\theta^\circ(\theta,\phi)\right|^2 + \left|E_\phi^\circ(\theta,\phi)\right|^2\right]$

$$U(\theta,\phi) = B_o F(\theta,\phi)$$

$$D = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$

$$D_o = \frac{4\pi U_{max}}{P_{rad}}$$

$$U_{max} = U(\theta,\phi)\big|_{max} = B_o F_{max}(\theta_m,\phi_m)$$

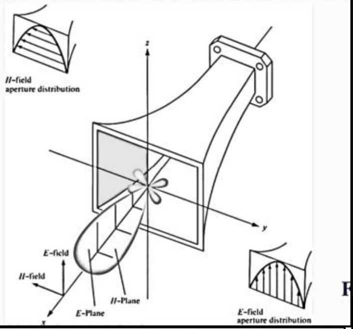
Tai & Pereira

$$\frac{1}{D_0} = \frac{1}{2}\left(\frac{1}{D_1}+\frac{1}{D_2}\right) \quad \text{Arithmetic mean}$$

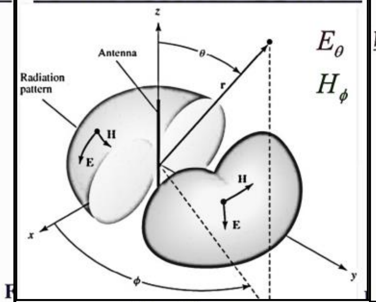
$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{32\ln(2)}{\Theta_{1r}^2+\Theta_{2r}^2} = \frac{22.181}{\Theta_{1r}^2+\Theta_{2r}^2}$$

$$D_0 = \frac{72,815}{\Theta_{1d}^2+\Theta_{2d}^2}$$

Directional Pattern of a Horn



Omnidirectional Pattern



E_θ

H_ϕ

Example 2.2

$$\underline{W}_{ave} = \underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin\theta}{r^2}$$

$$P_{ave} = P_{rad} = \oiint_S \underline{W}_{ave}\cdot d\underline{s}$$

$$= \int_0^{2\pi}\int_0^{\pi}\left[\hat{a}_r A_0 \frac{\sin\theta}{r^2}\right]\cdot\left[\hat{a}_r r^2 \sin\theta \, d\theta \, d\phi\right]$$

$$P_{ave} = P_{rad} = A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta \, d\phi = 2\pi A_0 \int_0^{\pi} \sin^2\theta \, d\theta$$

$$P_{rad} = A_0 \pi^2$$

Summary

1. $U = A_o \Rightarrow P_{rad} = 4\pi A_o$ (isotropic)
2. $U = A_o \sin\theta \Rightarrow P_{rad} = \pi^2 A_o$ (no specific one)
3. $U = A_o \sin^2\theta \Rightarrow P_{rad} = \frac{8\pi}{3} A_o$ (infinitesimal dipole)
4. $U = A_o \sin^3\theta \Rightarrow P_{rad} = \frac{3\pi^2}{4} A_o$ ($\lambda/2$ dipole)

$$D_0 = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} \underbrace{\frac{F(\theta,\phi)}{F_{max}(\theta_m,\phi_m)}}_{F_n(\theta,\phi)} \sin\theta \, d\theta \, d\phi}$$

$$D_0 = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi) \sin\theta \, d\theta \, d\phi} = \frac{4\pi}{\Omega_A} \quad \text{(2-23)}$$

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi) \sin\theta \, d\theta \, d\phi = \text{Beam solid angle} \quad \text{(2-24)}$$

Kraus

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} F_n(\theta,\phi) \sin\theta \, d\theta \, d\phi \simeq \Theta_{1r}\Theta_{2r}$$

$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{41,253}{\Theta_{1d}\Theta_{2d}} \quad \text{(2-26)}$$

$$D_0 \simeq \frac{4\pi}{\Theta_{1r}\Theta_{2r}} = \frac{4\pi(180/\pi)^2}{\Theta_{1d}\Theta_{2d}} = \frac{41,253}{\Theta_{1d}\Theta_{2d}} \quad \text{(2-27)}$$

$$V_T = V_{oc} = \int \underline{E}^i \cdot d\underline{\ell} = \underline{E}^i \cdot \underline{\ell}_e(\theta,\phi)$$

Field Regions

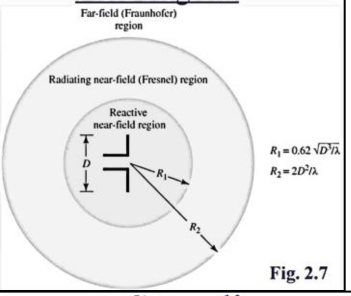
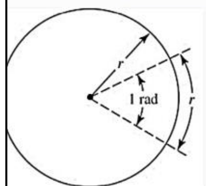


Fig. 2.7

Radian



$$C = 2\pi r$$

$$\text{Rads} = \frac{C}{r} = \frac{2\pi r}{r}$$

$$\text{Rads} = 2\pi$$

Isotropic Source

$$W_o = \frac{P}{4\pi r^2}\left(W/m^2\right)$$

$$U_o = \frac{P}{4\pi}\left(W/Sr\right)$$

1. $D = \frac{4\pi U}{P_{rad}} = 1 = D_o$ (isotropic)
2. $D = \frac{4}{\pi}\sin\theta = 1.27 \sin\theta = D_o \sin\theta$ (no specific one)
3. $D = \frac{3}{2}\sin^2\theta = 1.5\sin^2\theta = D_o \sin^2\theta$ (infinitesimal dipole)
4. $D = 1.67 \sin^3\theta = D_o \sin^3\theta$ ($\lambda/2$ dipole)

Omnidirectional Patterns

$$U = \left|\sin^n(\theta)\right| \begin{cases} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{cases} \quad \text{(2-32)}$$

Directivity:

McDonald

$$D_0 = \frac{101}{HPBW(\text{degrees}) - 0.0027[HPBW(\text{degrees})]^2} \quad \text{(2-33a)}$$

Pozar

$$D_0 = -172.4 + 191\sqrt{0.818 + 1/HPBW(\text{degrees})} \quad \text{(2-33b)}$$

Absolute Gain G_{abs}

$$G_{abs}(\theta,\phi) = e_o D(\theta,\phi) = e_r e_{cd} D(\theta,\phi)$$

$$= (1-|\Gamma_{in}|^2) e_{cd} D(\theta,\phi)$$

e_o = antenna total efficiency

$e_r = (1-|\Gamma_{in}|^2)$ = Reflection efficiency

$e_{cd} = e_c e_d$ = Radiation efficiency

e_c = Conduction efficiency

e_d = Dielectric efficiency

Equating the two

$$\left.\begin{matrix} D_t A_r = D_r A_t \\ \frac{D_t}{A_t} = \frac{D_r}{A_r} \end{matrix}\right\} \Rightarrow \text{Maximum} \quad \frac{D_{ot}}{A_{tm}} = \frac{D_{or}}{A_{rm}} \quad \text{(2-106)}$$

If $D_t = D_{ot} = 1$ (transmitter) & receiver is infinitesimal dipole

$$A_{tm} = \frac{A_{rm}}{D_{or}} = \frac{3\lambda^2/8\pi}{1.5} = \frac{\lambda^2}{4\pi} \quad \text{(2-10}$$

Intensity U (Far-field)

$$\boxed{U = r^2 W_{rad} = r^2 W_{ave}}$$

$$d\Omega = \frac{dA}{r^2} = \frac{r^2 \sin\theta \, d\theta \, d\phi}{r^2}$$

$$\boxed{d\Omega = \sin\theta \, d\theta \, d\phi}$$

$$D = \frac{U(\theta,\phi)}{U_o} = \frac{4\pi U(\theta,\phi)}{P_{rad}} \quad \text{(2-16)}$$

$$D_{max} = D_o = \frac{U_{max}}{U_o} = \frac{4\pi U_{max}}{P_{rad}} \quad \text{(2-16a)}$$

$$\boxed{D(dB) = 10\log_{10}\left[D(\text{dimensionless})\right]}$$

$$BE = \frac{\int_0^{2\pi}\int_0^{\theta_1} U(\theta,\phi)\sin\theta \, d\theta \, d\phi}{\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta \, d\theta \, d\phi}$$

II. Circular

$E_{xo} = E_{yo}$ (2-59)

$\Delta\phi = \pm\left(\frac{1}{2}+2n\right)\pi, \quad n = 0,1,2,\ldots$ (2-60, -

\+ : clockwise (RH)

\- : counterclockwise (LH)

Antenna Radiation Efficiency

$$e_o = e_r e_c e_d = e_r e_{cd} = \underbrace{\left(1-|\Gamma|^2\right)}_{e_r} e_{cd} \quad \text{(2-44, -45)}$$

where

e_{cd} = Radiation Efficiency

$$e_{cd} = \frac{\text{Power Radiated by Antenna } (P_r)}{\text{Power Delivered to Antenna } (P_r + P_L)}$$

$$e_{cd} = \frac{\frac{1}{2}|I_g|^2 R_r}{\frac{1}{2}|I_g|^2 R_r + \frac{1}{2}|I_g|^2 R_L} = \frac{R_r}{R_r + R_L} \quad \text{(2-90)}$$

Polarization Efficiency (Polarization mismatch or loss factor)

$$p_e = \frac{|\underline{\ell}_e \cdot \underline{E}^{inc}|^2}{|\underline{\ell}_e|^2 |\underline{E}^{inc}|^2} \quad \text{(2-71a)}$$

p_e = polarization efficiency

$\underline{\ell}_e$ = vector effective length

$\underline{E}^i$ = incident electric field

Transmitter: Isotropic

Receiver: Infinitesimal Dipole

$$A_{em} = A_{tm} = \frac{\lambda^2}{4\pi} = \frac{\lambda^2}{4\pi} \quad (1)$$

$$\boxed{A_{em} = A_{tm} = \frac{\lambda^2}{4\pi} D_o}$$

Steradian

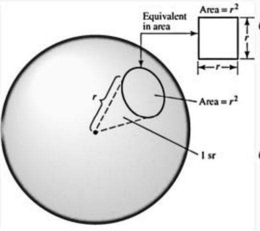


Fig. 2.10(b)

$$d\Omega = \frac{dA}{r^2} \quad \text{(2-1)}$$

$$= \frac{r^2 \sin\theta \, d\theta \, d\phi}{r^2}$$

$$d\Omega = \sin\theta \, d\theta \, d\phi \quad \text{(2-2)}$$

$$\boxed{\begin{matrix} 0 \le \theta \le \pi \\ 0 \le \phi \le 2\pi \end{matrix}}$$

W = Power Density $= P/A\left(W/m^2\right)$

U = Radiation Intensity $= P/\Omega\left(W/Sr\right)$

D = directivity (dimensionless)

D_0 = maximum directivity (dimen.)

U = radiation intensity (W/unit solid angle)

U_{max} = maximum radiation intensity

U_o = radiation intensity of isotropic

P_{rad} = radiated power (W)

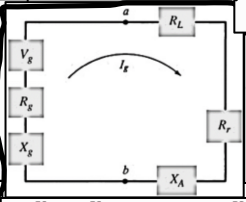
I. Linear

A. $E_{xo} \ne 0, \quad E_{yo} = 0$

B. $E_{xo} = 0, \quad E_{yo} \ne 0$

C. $E_{xo} \ne 0, \quad E_{yo} \ne 0$

$\Delta\phi = \pm n\pi, \quad n = 0,1,2,$



d-c applications: $R_L = R_{dc} = \frac{1}{\sigma}\frac{\ell}{A}$ (2-90a)

h-f applications: $R_L = R_{hf} = \frac{\ell}{P}R_s = \frac{\ell}{P}\sqrt{\frac{\omega\mu_o}{2\sigma}}$

Effective Aperture

$$A_e = \text{Effective Aperture} = \frac{P_T}{W_i} = \frac{(1/2)|I_T|^2 R_T}{W_i}$$

$$A_e = \frac{|V_T|^2}{W_i}\left[\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}\right]$$

A_{em} = Maximum Effective Area $= A_e|_{max}$

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_r + R_L)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{R_L + R_r}{(R_L + R_r)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_L + R_r}\right]$$

and occurs under conjugate matching $\begin{pmatrix} R_T = R_L + R_r \\ X_T = -X_A \end{pmatrix}$

Correct Polarization Loss Factor

$$PLF = |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = |\cos\psi_p|^2$$

Using this

CW-CW, CCW-CCW } Maximum; CW-CCW, CCW-CW } Minimum

$$I_g = \frac{V_g}{Z_t} = \frac{V_g}{Z_A + Z_g} = \frac{V_g}{(R_g + R_L + R_r) + j(X_A + X_g)} \quad \text{(2-7}$$

$$|I_g| = \frac{|V_g|}{\sqrt{(R_g + R_L + R_r)^2 + (X_A + X_g)^2}} \quad \text{(2-7}$$

$$P_r + P_L = P_g = \frac{|V_g|^2}{8}\left[\frac{1}{(R_r + R_L)}\right]$$

$$R_g = R_r + R_L; \quad X_g = -X_A$$

SOLUTION:

a. Exact: Using (2-2), we can write that

$$\Omega_A = \int_0^{360^\circ}\int_0^{30^\circ} d\Omega = \int_0^{2\pi}\int_0^{\pi/6} \sin\theta \, d\theta \, d\phi = \int_0^{2\pi} d\phi \int_0^{\pi/6} \sin\theta \, d\theta$$

$$= 2\pi\left[-\cos\theta\right]_0^{\pi/6} = 2\pi\left[-0.867 + 1\right]$$

$$\Omega_A = 2\pi(0.133) = 0.83566 \text{ sterads}$$

b. Approximate:

$$\Omega_A \approx \Delta\Theta_1 \cdot \Delta\Theta_2 \overset{\Delta\Theta_1 = \Delta\Theta_2}{=} (\Delta\Theta_1)^2 = \frac{\pi}{3}\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9} = 1.09662 \text{ sterads}$$

It is apparent that the approximate beam solid angle is about 31.23% in error.

Example 2.3

$$U = r^2 W_{rad} = r^2\left(A_0 \frac{\sin\theta}{r^2}\right) = A_0 \sin\theta$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \, d\Omega = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi$$

$$= A_0 \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \, d\theta \, d\phi = A_0 \pi^2$$

$$P_{rad} = A_0 \pi^2$$

Partial Directivities

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta \, d\theta \, d\phi = \int_0^{2\pi}\int_0^{\pi}\left[U_\theta + U_\phi\right]\sin\theta \, d\theta \, d\phi$$

$$D = \frac{4\pi U(\theta,\phi)}{P_{rad}} = \frac{4\pi(U_\theta + U_\phi)}{P_{rad}}$$

$$= \frac{4\pi U_\theta}{P_{rad}} + \frac{4\pi U_\phi}{P_{rad}}$$

$$D = D_\theta + D_\phi$$

D_θ, D_ϕ = are the partial directivities

$$U = U_\theta + U_\phi$$

$$D_0 = D_\theta + D_\phi$$

$$D_\theta = \frac{4\pi U_\theta}{(P_{rad})_\theta + (P_{rad})_\phi}$$

$$D_\phi = \frac{4\pi U_\phi}{(P_{rad})_\theta + (P_{rad})_\phi}$$

Example 2.5:

$$\underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin\theta}{r^2}$$

Solution:

$$P_{rad} = \pi^2 A_0$$

$$U = r^2 W_{rad} = A_0 \sin\theta$$

$$U_{max} = U\big|_{max} = A_0 \sin\theta\big|_{\theta=\pi/2} = A_0$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(1)A_0}{\pi^2 A_0} = 1.27 = \underline{1.038 \text{ dB}}$$

$$D = D_0 \sin\theta = 1.27\sin\theta$$

Example 2.6:

$$\underline{W}_{rad} = \hat{a}_r A_0 \frac{\sin^2\theta}{r^2}$$

Solution:

$$P_{rad} = \frac{8\pi}{3}$$

$$U = r^2 W_{rad} = A_0 \sin^2\theta$$

$$U_{max} = U\big|_{max} = A_0 \sin^2\theta\big|_{\theta=\pi/2} = A_0$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi A_0}{8\pi/2} = 1.5 = \underline{1.761 \text{ dB}}$$

$$D = D_0 \sin^2\theta = 1.5\sin^2\theta$$

Very Maximum (no losses) A_{em}

$$A_{em} = \frac{\lambda^2}{4\pi} D_o$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 \underbrace{e_{cd}\left(1-|\Gamma|^2\right)}_{e_o} \underbrace{\left|\hat{\rho}_w \cdot \hat{\rho}_a\right|^2}_{e_p = PLF}$$

$$\varepsilon_{ap} = \frac{A_{em}}{A_p} = \text{aperture efficiency}$$

$$\boxed{0 \le \varepsilon_{ap} \le 1}$$

$$\mathbf{E}_a = \hat{\mathbf{a}}_\theta E_\theta + \hat{\mathbf{a}}_\phi E_\phi = -j\eta \frac{kI_{in}}{4\pi r} \ell_e e^{-jkr}$$

III. Elliptical

A. $E_{xo} \ne E_{yo}, \quad \Delta\phi \ne \pm n\pi, \quad n = 0,1,2,\ldots$

B. $E_{xo} = E_{yo}, \quad \Delta\phi \ne \pm\left(\frac{1}{2}+2n\right)\pi, \quad n = 0,1,2,$

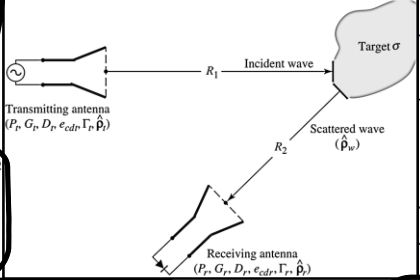
Friis Transmission Equation

$$\frac{P_r}{P_t}=e_t e_r \frac{\lambda^2 D_t(\theta_t,\phi_t) D_r(\theta_r,\phi_r)}{(4\pi R)^2}$$

$$\frac{P_r}{P_t}=e_{cdt}e_{cdr}\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)\left(\frac{\lambda}{4\pi R}\right)^2 D_t D_r$$

$$\frac{P_r}{P_t}=e_{cdt}e_{cdr}\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)\left(\frac{\lambda}{4\pi R}\right)^2 D_t D_r|\hat{\rho}_t\cdot\hat{\rho}_r|^2$$

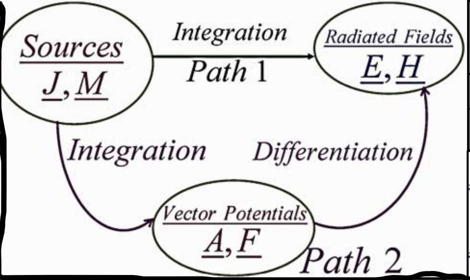
$$\boxed{\frac{P_r}{P_t}=\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)\left(\frac{\lambda}{4\pi R}\right)^2 G_t G_r|\hat{\rho}_t\cdot\hat{\rho}_r|^2}$$



$$\frac{P_r}{P_t}=e_{cdt}e_{cdr}\sigma\frac{D_t(\theta_t,\phi_t)D_r(\theta_r,\phi_r)}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2 \quad (2\text{-}124)$$

$$\frac{P_r}{P_t}=e_{cdt}e_{cdr}\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)\cdot\sigma\frac{D_t(\theta_t,\phi_t)D_r(\theta_r,\phi_r)}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2|\hat{\rho}_w\cdot\hat{\rho}_r|^2 \quad (2\text{-}125)$$

$$\frac{P_r}{P_t}=\sigma\frac{G_{ot}G_{or}}{4\pi}\left(\frac{\lambda}{4\pi R_1 R_2}\right)^2|\hat{\rho}_w\cdot\hat{\rho}_r|^2 \quad (2\text{-}126)$$



$$\underline{E}=\underline{E}_A+\underline{E}_F=\underbrace{-j\omega\underline{A}-j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\underline{A})}_{E_A}\underbrace{-\frac{1}{\varepsilon}\nabla\times\underline{F}}_{E_F}$$

$$\underline{H}=\underline{H}_A+\underline{H}_F=\underbrace{\frac{1}{\mu}\nabla\times\underline{A}}_{H_A}\underbrace{-j\omega\underline{F}-j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\underline{F})}_{H_F}$$

$$\underline{A}=\frac{\mu}{4\pi}\iiint_V \underline{J}\frac{e^{-jkR}}{R}dv' \qquad \underline{F}=\frac{\varepsilon}{4\pi}\iiint_V \underline{M}\frac{e^{-jkR}}{R}dv'$$

$$\underline{A}=\frac{\mu}{4\pi}\iint_S \underline{J}_s\frac{e^{-jkR}}{R}ds' \qquad \underline{F}=\frac{\varepsilon}{4\pi}\iint_S \underline{M}_s\frac{e^{-jkR}}{R}ds'$$

$$\underline{A}=\frac{\mu}{4\pi}\int_C \underline{I}_e\frac{e^{-jkR}}{R}dl' \qquad \underline{F}=\frac{\varepsilon}{4\pi}\int_C \underline{I}_m\frac{e^{-jkR}}{R}dl'$$

$$E_r=\eta\frac{I_o\ell}{2\pi r^2}\cos\theta\left[1+\frac{1}{jkr}\right]e^{-jkr} \quad (4\text{-}10a)$$

$$E_\theta=j\eta\frac{kI_o\ell}{4\pi r}\sin\theta\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr} \quad (4\text{-}10b)$$

$$E_\phi=H_r=H_\theta=0 \quad (4\text{-}8a),(4\text{-}10c)$$

$$H_\phi=j\frac{kI_o\ell}{4\pi r}\left[1+\frac{1}{jkr}\right]\sin\theta\, e^{-jkr} \quad (4\text{-}8b)$$

$$\underline{W}=\hat{a}_r W_r+\hat{a}_\theta W_\theta$$

$$W_r=\frac{1}{2}E_\theta H_\phi^*=\frac{\eta}{8}\left|\frac{I_o\ell}{\lambda}\right|^2\frac{\sin^2\theta}{r^2}\left[1-j\frac{1}{(kr)^3}\right] \quad (4\text{-}12a)$$

$$W_\theta=-\frac{1}{2}E_r H_\phi^*=j\eta\frac{k|I_o\ell|^2\cos\theta\sin\theta}{16\pi^2r^3}\left[1+\frac{1}{(kr)^2}\right] \quad (4\text{-}12b)$$

Assuming No Losses:

$$\boxed{\frac{P_r}{P_t}=D_{ot}D_{or}\left(\frac{\lambda}{4\pi R}\right)^2}$$

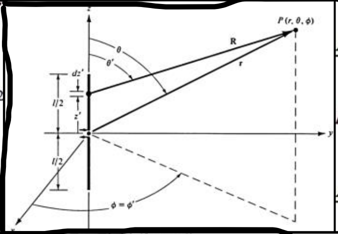
More General: Includes Losses:

$$\frac{P_r}{P_t}=G_{ot}G_{or}\underbrace{\left(\frac{\lambda}{4\pi R}\right)^2}_{\text{Free space loss factor}}|\hat{\rho}_t\cdot\hat{\rho}_r|^2 \quad (2\text{-}119)$$

Radar Cross Section (RCS) σ
(Using Power Densities)

$$\lim_{R\to\infty}\left(\frac{\sigma W_i}{4\pi R^2}\right)=W_s \quad (2\text{-}12$$

$$\sigma=\lim_{R\to\infty}\left[4\pi R^2\frac{W_s}{W_i}\right] \quad (2\text{-}$$



$$\underline{A}=\frac{\mu}{4\pi}\int_{-\ell/2}^{\ell/2}\underline{I}_e(z')\frac{e^{-jkR}}{R}d\ell'$$

Approximation: $R\approx r$

$$\underline{A}=\frac{\mu}{4\pi}\int_{-\ell/2}^{\ell/2}\hat{a}_z I_o\frac{e^{-jkr}}{r}dz'$$

$$\begin{pmatrix}A_r\\A_\theta\\A_\phi\end{pmatrix}=\begin{pmatrix}\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta\\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta\\ -\sin\phi & \cos\phi & 0\end{pmatrix}\begin{pmatrix}A_x\\A_y\\A_z\end{pmatrix}$$

$$R_r=80\pi^2\left(\frac{\ell}{\lambda}\right)^2$$

$$\ell\le\lambda/50$$

$$P=P_{rad}+j2\omega(\widetilde{W}_m-\widetilde{W}_e)$$

$$P_{rad}=\boxed{\eta\left(\frac{\pi}{3}\right)\left|\frac{I_o l}{\lambda}\right|^2} \quad \text{remains constant as } r \text{ increases} \quad (4\text{-}$$

$$2\omega(\widetilde{W}_m-\widetilde{W}_e)=\boxed{-\eta\left(\frac{\pi}{3}\right)\left|\frac{I_o l}{\lambda}\right|^2\frac{1}{(kr)^3}} \quad \text{decreases rapidly as } r \text{ increases} \quad (4\text{-}$$

$$W_{rad}=\frac{\eta}{8}\left|\frac{kI_o}{\lambda}\right|^2\frac{\sin^2\theta}{r^2}$$

$$U=r^2W_{rad}=\frac{\eta}{8}\left|\frac{kI_o}{\lambda}\right|^2\sin^2\theta$$

$$R_r=\frac{2P_{rad}}{|I_o|^2};\qquad R_{in}=\frac{R_r}{\sin^2\left(\frac{kl}{2}\right)}$$

If $kr \gg 1$ (Far-Field)

$$E_r=\eta\frac{I_o\ell e^{-jkr}}{2\pi r^2}\cos\theta\approx 0$$

$$E_\theta\simeq j\eta\frac{kI_o\ell e^{-jkr}}{4\pi r}\sin\theta \quad (4\text{-}26a)$$

$$H_\phi\simeq j\frac{kI_o\ell e^{-jkr}}{4\pi r}\sin\theta \quad (4\text{-}26c)$$

$$\frac{E_\theta}{H_\phi}=Z_w=\eta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$E_r\simeq E_\phi=H_r=H_\theta=0 \quad (4\text{-}26b)$$

$$D_o=\frac{U_{max}}{U_o}=\frac{4\pi U_{max}}{P_{rad}}=\frac{4\pi\frac{\eta}{8}\left|\frac{kI_o}{\lambda}\right|^2}{\eta\left(\frac{\pi}{3}\right)\left|\frac{I_o l}{\lambda}\right|^2}=\frac{3}{2}$$

$$R_{in}=\frac{R_r}{\sin^2\left(\frac{k\ell}{2}\right)}$$

$$\underline{I}_e(x',y',z')=\begin{cases}\hat{a}_z I_o\sin\left[k\left(\frac{l}{2}-z'\right)\right] & 0\le z'\le l/2\\ \hat{a}_z I_o\sin\left[k\left(\frac{l}{2}+z'\right)\right] & -l/2\le z'\le 0\end{cases} \quad (4\text{-}56)$$

any length

Because $\sin(x)\overset{x\to 0}{=}x$

$$\underline{I}_e(x',y',z')=\underbrace{\begin{cases}\hat{a}_z I_o\left(1-\frac{2}{l}z'\right) & 0\le z'\le l/2\\ \hat{a}_z I_o\left(1+\frac{2}{l}z'\right) & -l/2\le z'\le 0\end{cases}}_{\lambda/50\le l\le\lambda/10} \quad (4\text{-}33)$$

$$\underline{A}(x,y,z)=\frac{\mu}{4\pi}\left[\hat{a}_z\int_{-l/2}^{0}I_o\left(1+\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}dz'+\hat{a}_z\int_{0}^{l/2}I_o\left(1-\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}dz'\right] \quad (4$$

$$\underline{A}(x,y,z)=\hat{a}_z\frac{1}{2}\left[\frac{l\mu I_o e^{-jkr}}{4\pi r}\right]=\hat{a}_z\frac{1}{2}\left[A_z'\right] \quad (4$$

$$A_z'(\text{uniform current})=\frac{\mu I_o l e^{-jkr}}{4\pi r}$$

$$=2A_z(\text{triangular current})$$

$$(\lambda/50\le l\le\lambda/10)$$

Far-Field ($kr \gg 1$)

$$E_\theta=\frac{1}{2}E_\theta'=j\eta\frac{kI_o l e^{-jkr}}{8\pi r}\sin\theta \quad (4\text{-}36a)$$

$$H_\phi=\frac{1}{2}H_\phi'=j\frac{kI_o l e^{-jkr}}{8\pi r}\sin\theta \quad (4\text{-}36c)$$

$$E_r\simeq E_\phi=H_r=H_\theta=0 \quad (4\text{-}36b)$$

$$W_{rad}=\frac{1}{4}W_{rad}',\quad P_{rad}=\frac{1}{4}P_{rad}',\quad U_{rad}=\frac{1}{4}U_{rad}'$$

Infinitesimal Dipole ($\ell\le\lambda/50$)

$$\underline{E}=\hat{a}_\theta E_\theta=\hat{a}_\theta j\eta\frac{kI_o\ell}{4\pi r}e^{-jkr}\sin\theta$$

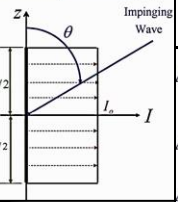
$$=\left(-j\frac{\omega\mu I_o\ell e^{-jkr}}{4\pi r}\right)\underbrace{\left(-\hat{a}_\theta\ell\sin\theta\right)}_{\underline{\ell}_e(\theta)}$$

$$\underline{\ell}_e(\theta)=-\hat{a}_\theta\ell\sin\theta \quad (4-26a)$$

$$|\underline{\ell}_e(\theta)=-\hat{a}_\theta\ell\sin\theta|_{max}$$

$=\ell$ when $\theta=90°$

which is 100% of physical length (4 – 26a)



$$A_r=A_z\cos\theta=\frac{\mu I_o\ell}{4\pi r}\cos\theta\, e^{-jkr}$$

$$A_\theta=-A_z\sin\theta=-\frac{\mu I_o\ell}{4\pi r}\sin\theta\, e^{-jkr}$$

$$A_\phi=0$$

$$W_{av}=\hat{a}_r\eta\frac{|I_0|^2}{8\pi^2r^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2$$

Current Distribution of a Linear Wire Antenna When Current Maximum Does not Occur at the Input Terminals

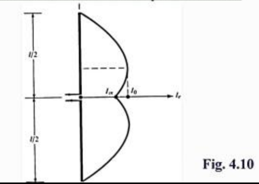


Fig. 4.10

- The assumed sinusoidal current distribution predicts zero current at the input terminals (and hence infinite input resistance) for dipoles which are of length equal to integer multiples of the wavelength.
- In reality, the current is not zero in these cases due to the finite feed gap and the finite radius of the wire.

Finite Dipole

$$\underline{I}_e(z')=\hat{a}_z\begin{cases}I_o\sin\left[k\left(\frac{\ell}{2}-z'\right)\right] & 0\le z'\le+\ell/2\\ I_o\sin\left[k\left(\frac{\ell}{2}+z'\right)\right] & -\ell/2\le z'\le 0\end{cases}$$

$$A_z=\frac{\mu}{4\pi}\left\{I_o\int_{-\ell/2}^{0}\sin\left[\left(\frac{\ell}{2}+z'\right)\right]\frac{e^{-jkR}}{R}dz'+I_o\int_{0}^{+\ell/2}\sin\left[\left(\frac{\ell}{2}-z'\right)\right]\frac{e^{-jkR}}{R}dz'\right\}$$

$$R=r\sqrt{1+\left[\frac{(-2zz'+z'^2)}{r^2}\right]}$$

$$dE_\theta\simeq j\eta\frac{k\,I_e(z')e^{-jk(r-z'\cos\theta)}}{4\pi r}\sin\theta\,dz'$$

$$\simeq j\eta\frac{k\,I_e(z')e^{-jk\cancel{(r-z'\cos\theta)}}}{4\pi r}\sin\theta\,e^{jkz'\cos\theta}dz' \quad (4\text{-}58)$$

$$E_\theta=\int_{-l/2}^{+l/2}dE_\theta=\underbrace{j\eta\frac{ke^{-jkr}}{4\pi r}\sin\theta}_{\text{Element Factor}}\underbrace{\left[\int_{-l/2}^{+l/2}I_e(z')e^{jkz'\cos\theta}dz'\right]}_{\text{Space Factor}} \quad (4\text{-}58a)$$

Pattern Multiplication Rule

Total Field = *Element Factor* × *Space Factor*

$$E_\theta=j\eta\frac{ke^{-jkr}}{4\pi r}\sin\theta\left\{\int_{-l/2}^{0}I_o\sin\left[k\left(\frac{l}{2}+z'\right)\right]e^{+jkz'\cos\theta}dz'+\int_{0}^{+l/2}I_o\sin\left[k\left(\frac{l}{2}-z'\right)\right]e^{-jkz'\cos\theta}dz'\right\} \quad (4\text{-}60)$$

$$\int e^{\alpha x}\sin[\beta x+\gamma]dx=\frac{e^{\alpha x}}{\alpha^2+\beta^2}\left[\alpha\sin(\beta x+\gamma)-\beta\cos(\beta x+\gamma)\right] \quad (4\text{-}61)$$

$$\alpha=\pm jk\cos\theta,\qquad \beta=\pm k,\qquad \gamma=\frac{kl}{2} \quad (4\text{-}61a,b,c)$$

$$E_\theta=j\eta\frac{I_o e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right)-\cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right]$$

$$E_\theta\cong C\underbrace{\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right)-\cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right]}_{\text{Field Pattern}}$$

$$H_\phi\cong\frac{E_\theta}{\eta},\qquad C=j\eta\frac{I_o e^{-jkr}}{2\pi r}$$

$$U_{rad}=r^2W_{av}=\underbrace{\eta\frac{|I_0|^2}{8\pi^2}}_{B_0}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2$$

$$U_{rad}=B_0\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2$$

$$P_{rad}=\frac{1}{2}|I_o|^2R_r=\frac{1}{2}|I_{in}|^2R_{in}$$

$$R_{in}=R_r\frac{|I_o|^2}{|I_{in}|^2}=R_r\left|\frac{I_o}{I_{in}}\right|^2$$

$$\underline{I}_e\Big|_{z'=0}=I_o\sin\left[k\left(\frac{\ell}{2}\pm z'\right)\right]\Bigg|_{z'=0}=I_{in}$$

$$I_{in}=I_o\sin\left(\frac{k\ell}{2}\right)$$

$$R=r\left[1+\underbrace{\left(\frac{-2rz'\cos\theta+z'^2}{r^2}\right)}_{x}\right]^{1/2}=r[1+x]^{1/2}$$

$$R=r\left[1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3\ldots\ldots\right]$$

$$R=r-z'\cos\theta+\frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right)+\frac{1}{r^2}\left(\frac{z'^3}{2}\cos\theta\sin^2\theta\right)$$

Maximum Phase Error $=k\dfrac{l^2}{8r}\le\dfrac{\pi}{8}$ radians (22.5°) (4-44)

$$\frac{kl^2}{8r}\le\frac{\pi}{8}\quad\Rightarrow\quad\boxed{r\ge\frac{2l^2}{\lambda}} \quad (4\text{-}45)$$

Far Field Conditions : $r>\dfrac{2D^2}{\lambda}$, $r\gg D$, $r\gg\lambda$

Chapter 4 Linear Wire Antennas

Far-Field Region

$$A_z=\frac{\mu}{4\pi}\int_{-l/2}^{+l/2}I_e(z')\frac{e^{-jkR}}{R}dz'$$

$$A_z\simeq\frac{\mu}{4\pi}\int_{-l/2}^{+l/2}I_e(z')\frac{e^{-jk(r-z'\cos\theta)}}{r}dz'$$

$$A_z\simeq\frac{\mu e^{-jkR}}{4\pi r}\int_{-l/2}^{+l/2}I_e(z')e^{jkz'\cos\theta}dz'$$

Ideal Sinusoidal Current Distribution

$$\underline{I}_e=\begin{cases}\hat{a}_z I_0\sin\left[k\left(\frac{l}{2}-z'\right)\right] & 0\le z'\le l/2\\ \hat{a}_z I_0\sin\left[k\left(\frac{l}{2}+z'\right)\right] & -l/2\le z'\le 0\end{cases}$$

Half-Wavelength Dipole

$$U_n(\theta,\phi)=\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)}{\left[1-\cos\left(\frac{kl}{2}\right)\right]\sin\theta}\right]^2=\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2\approx\sin^3\theta$$

Finite Feed Gap

- The Schelkunoff/Friis current distribution is given for $-l/2\le z'\le l/2$ by

$$I_e(z)=I_0\sin\left[k\left(\frac{l}{2}\right)-|z'|\right]+jpI_0\left[\cos(kz')-\cos\left(\frac{kl}{2}\right)\right]$$

$$Q(kl)=\begin{cases}C+\ln(k\ell)-C_i(k\ell)+\frac{1}{2}\sin(k\ell)\left[S_i(2k\ell)-2S_i(k\ell)\right]\\+\frac{1}{2}\cos(k\ell)\left[C+\ln\left(\frac{k\ell}{2}\right)+C_i(2k\ell)-2C_i(k\ell)\right]\end{cases} \quad (4\text{-}75a)$$

Far-Field Approximations

$$R\simeq\begin{cases}r-z'\cos\theta & \text{For phase terms}\\ r & \text{For amplitude terms}\end{cases} \quad (4\text{-}46)$$

Maximum Phase Error $:\ \dfrac{kl^3}{8(3\sqrt{3})r^2}=\dfrac{\pi l^3}{12\sqrt{3}r^2}\le\dfrac{\pi}{8}$

$$\frac{\pi l^3}{12\sqrt{3}r^2}\le\frac{\pi}{8}\Rightarrow\boxed{r^2\ge\frac{2}{3\sqrt{3}}\left(\frac{l^3}{\lambda}\right)}$$

Fresnel Region

$$(2\ell^2/\lambda>r\ge 0.62\sqrt{\ell^3/\lambda})$$

$$R\simeq\begin{cases}r-z'\cos\theta+\frac{1}{r}\left(\frac{z'^2}{2}\sin^2\theta\right) & \text{For phase terms}\\ r & \text{For amplitude terms}\end{cases}$$

$$r\ge 0.62\sqrt{\ell^3/\lambda}$$

Half-Wavelength Dipole

dipoles of about one-half wavelength are very common in practice.

$$kl=\pi$$

$$Q(\pi)=1.219$$

Half-Wavelength Dipole

$$R_r=R_{in}=\frac{\eta}{2\pi}Q(kl)=73.1\ \Omega$$

$$D_0=\frac{2\left[1-\cos\left(\frac{kl}{2}\right)\right]^2}{Q(kl)}=\frac{2}{Q(\pi)}=1.64=2.15\text{ dB}$$

Half-Wavelength Dipole

The input impedance of the half-wavelength dipole may be obtained as

$$Z_{in}=73+j42.5\,\Omega$$

To eliminate the imaginary part of the input impedance, we can reduce the antenna length slightly. Depending on the radius of the wire, "resonance" occurs when the length is about 0.47λ to 0.48λ.

Radiation Resistance and Directivity of Finite Length Dipole

$$R_r=\frac{2P_{rad}}{|I_o|^2}=\frac{\eta}{2\pi}Q(kl) \quad (4\text{-}7$$

$$D_0=\frac{4\pi U_{max}}{P_{rad}}=\frac{2U_{max}}{B_0 Q}=\frac{2\left[1-\cos\left(\frac{kl}{2}\right)\right]^2}{Q(k\ell)} \quad (4\text{-}7$$

$$P_{rad}=\eta\frac{|I_o|^2}{4\pi}\begin{cases}C+\ln(k\ell)-C_i(k\ell)+\frac{1}{2}\sin(k\ell)\left[S_i(2k\ell)-2S_i(k\ell)\right]\\+\frac{1}{2}\cos(k\ell)\left[C+\ln\left(\frac{k\ell}{2}\right)+C_i(2k\ell)-2C_i(k\ell)\right]\end{cases}$$

Dipole Summary (Gain G_0 Vs. Absolute Gain G_{0abs})

(Resonant ⇒ X_A=0; f = 100 MHz; σ = 5.7 x 10^7 S/m; Z_c = 50; b = 3x10$^{-4}\lambda$)

	$\ell=\lambda/50$	$\ell=\lambda/10$	$\ell=\lambda/2$	$\ell=\lambda$
R_{hf}	0.0279	0.2792	0.698	1.3962
R_L	0.0279	0.1396	0.349	0.6981
R_r	0.3158	1.9739	73	199
R_{in}	0.3158	1.9739	73	∞
e_{cd}	0.9188 (-0.368 dB)	0.9339 (-0.296 dB)	0.9952 (-0.021 dB)	0.9965 (-0.015 dB)
D_0	1.5 (1.761 dB)	1.5 (1.761 dB)	1.6409 (2.151 dB)	2.411 (3.822 dB)
G_0	1.3782 (1.393 dB)	1.4009 (1.464 dB)	1.6331 (2.13 dB)	2.4026 (3.807 dB)
Γ	-0.9863	-0.9189	0.18929	1
e_r	0.0271 (-15.67 dB)	0.1556 (-8.08 dB)	0.9642 (-0.158 dB)	0 (-∞ dB)
G_{0abs}	0.0374 (-14.27 dB)	0.2181 (-6.613 dB)	1.5746 (1.972 dB)	0 (-∞ dB)

$$E_\theta = \underbrace{j\eta\frac{kI_o\ell e^{-jkr}}{4\pi r}\sin\theta}_{Element\ \ Factor}\ \underbrace{\{2\cos(kh\cos\theta)\}}_{Array\ \ Factor} \qquad z\ge 0$$

$$E_\theta = 0 \qquad z<0$$

Vertical Electric Dipole (VED)

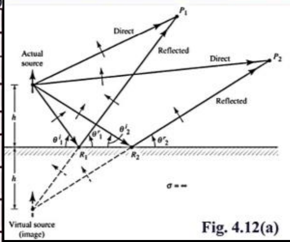
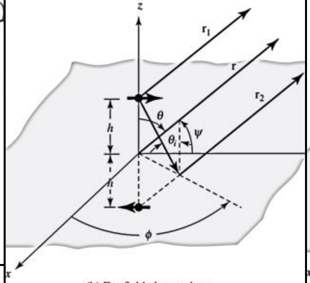
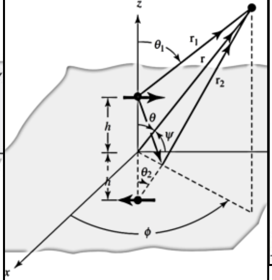


Fig. 4.12(a)



(b) Far-field observations



(a) Horizontal electric dipole above ground plane

$$E_\theta^d = j\eta\frac{kI_o\ell e^{-jkr_1}}{4\pi r_1}\sin\theta_1$$

$$E_\theta^r = R_v\left\{j\eta\frac{kI_o\ell e^{-jkr_2}}{4\pi r_2}\sin\theta_2\right\} = +1\left\{j\eta\frac{kI_o\ell e^{-jkr_2}}{4\pi r_2}\sin\theta_2\right\}$$

$$E_\theta^t = E_\theta^d + E_\theta^r$$

$$r_1 = \left[r^2+h^2-2rh\cos\theta\right]^{1/2} \quad (4\text{-}96a)$$

$$\overset{r>>h}{\simeq}\left[r^2-2rh\cos\theta\right]^{1/2} \overset{r>>h}{\simeq} r\left[1-\frac{1}{r}2h\cos\theta\right]^{1/2} \overset{r>>h}{\simeq} r\left[1-\frac{h}{r}\cos\theta\right]$$

$$r_1 \overset{r>>h}{\simeq} r-h\cos\theta \quad (4\text{-}97a)$$

Far-Field Approximations

$$\left.\begin{aligned} r_1 &= r-h\cos\theta \\ r_2 &= r+h\cos\theta \end{aligned}\right\} \text{ for phase terms} \quad (4\text{-}$$

$$r_1 \cong r_2 \cong r \;\} \text{ for amplitude terms}$$

$$\theta_1 \cong \theta_2 \cong \theta$$

$$P_{rad} = \frac{1}{2\eta}\int_0^{2\pi}\int_0^{\pi/2}|E_\theta|^2 r^2\sin\theta\, d\theta\, d\phi \quad (4\text{-}$$

$$P_{rad} = \pi\eta\left|\frac{I_o\ell}{\lambda}\right|^2\left\{\frac{1}{3}-\frac{\cos(2kh)}{(2kh)^2}+\frac{\sin(2kh)}{(2kh)^3}\right\} \quad (4\text{-}$$

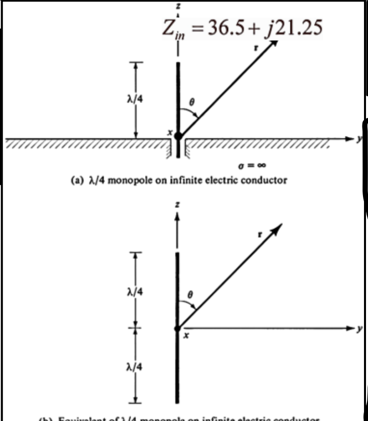
$$U = r^2W_{av} = r^2\frac{1}{2\eta}|E_\theta|^2 = \frac{\eta}{2}\left|\frac{I_o\ell}{\lambda}\right|^2\sin^2\theta\cos^2(kh\cos\theta) \quad (4\text{-}$$

$$U_{\max} = U|_{\theta=\pi/2} = \frac{\eta}{2}\left|\frac{I_o\ell}{2}\right|^2$$

$$D_o = \frac{4\pi U_{\max}}{P_{rad}} = \frac{2}{\left[\frac{1}{3}-\frac{\cos(2kh)}{(2kh)^2}+\frac{\sin(2kh)}{(2kh)^3}\right]}$$

$$R_r = \frac{2P_{rad}}{|I_o|^2} = 2\pi\eta\left(\frac{\ell}{\lambda}\right)^2\cdot\left[\frac{1}{3}-\frac{\cos(2kh)}{(2kh)^2}+\frac{\sin(2kh)}{(2kh)^3}\right]$$

$$Z_{in} = 36.5 + j21.25$$



(a) λ/4 monopole on infinite electric conductor

(b) Equivalent of λ/4 monopole on infinite electric conductor

$$Z_{in}\ \text{(monopole)} = \frac{1}{2}Z_{in}\ \text{(dipole)}$$

$$D_0\ \text{(monopole)} = 2D_0\ \text{(dipole)}$$

Vertical Dipole

$$\text{Number of Lobes} \cong 2\left(\frac{h}{\lambda}\right)+1 \quad (4\text{-}100$$

$$h >> \lambda$$

Input Resistance Dipole/Monopole

$$G = k\ell/2:\ \text{Dipole} \quad (4\text{-}10$$

$$G = k\ell:\ \text{Monopole} \quad (4\text{-}10$$

I. $0 < G < \pi/4$ ($R_{in} < 12.337$ ohms)

$$R_{in}(dipole) = 20G^2 \qquad 0<\ell<\lambda/4 \quad (4\text{-}10$$

$$R_{in}(monopole) = 10G^2 \qquad 0<\ell<\lambda/8 \quad (4\text{-}10$$

II. $\pi/4 \le G < \pi/2$ ($R_{in} < 76.383$ ohms)

$$R_{in}(dipole) = 24.7G^{2.5} \qquad \lambda/4\le\ell<\lambda/2 \quad (4\text{-}109a)$$

$$R_{in}(monopole) = 12.35G^{2.5} \qquad \lambda/8\le\ell<\lambda/4 \quad (4\text{-}109b)$$

III. $\pi/2 \le G < 2$ ($R_{in} < 200.53$)

$$R_{in}(dipole) = 11.14G^{4.17} \qquad \lambda/2\le\ell<0.6366\lambda \quad (4\text{-}110a)$$

$$R_{in}(monopole) = 5.57G^{4.17} \qquad \lambda/4\le\ell<0.3183\lambda$$

Far-Field Approximations

$$\left.\begin{aligned} r_1 &\cong r-h\cos\theta \\ r_2 &\cong r+h\cos\theta \end{aligned}\right\} \text{ for phase terms} \quad (4\text{-}115a)$$

$$r_1 \cong r_2 \cong r\} \quad \text{for amplitude terms} \quad (4\text{-}115b)$$

$$\theta_1 \cong \theta_2 \cong \theta$$

$$E_\psi^t = E_\psi^d + E_\psi^r = \underbrace{j\eta\frac{kI_o\ell e^{-jkr}}{4\pi r}\sqrt{1-\sin^2\theta\sin^2\phi}}_{Element\ \ Pattern}\ \underbrace{[2j\sin(kh\cos\theta]}_{Array\ \ Factor} \quad (4\text{-}116)$$

Example 4.5

$$\underline{A} = \hat{a}_y\frac{\mu I_o\ell e^{-jkr}}{4\pi r}$$

$$A_\theta = A_y\cos\theta\sin\phi = \frac{\mu I_o\ell e^{-jkr}}{4\pi r}\cos\theta\sin\phi$$

$$A_\phi = A_y\cos\phi = \frac{\mu I_o\ell e^{-jkr}}{4\pi r}\cos\phi$$

$$E_\theta \cong -j\omega A_\theta = -j\frac{\omega\mu I_o\ell e^{-jkr}}{4\pi r}\cos\theta\sin\phi$$

$$E_\phi \cong -j\omega A_\phi = -j\frac{\omega\mu I_o\ell e^{-jkr}}{4\pi r}\cos\phi$$

Radiation Intensity of Horizontal Element above a Ground Plane

$$U(\theta,\phi) = \frac{\eta}{2}|I_0|^2\left(\frac{l}{\lambda}\right)^2\{\cos^2\theta\sin^2\phi+\cos^2\phi\} \times \sin^2(kh\cos\theta),\ 0\le\theta\le\frac{\pi}{2} \quad (4\text{-}121)$$

- The maximum radiation intensity and the direction in which it occurs is a function of kh

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi}U(\theta,\phi)\sin\theta\, d\theta\, d\phi = \eta\frac{\pi}{2}|I_0|^2\left(\frac{l}{\lambda}\right)^2\left[\frac{2}{3}-\frac{\sin(2kh)}{2kh}-\frac{\cos(2kh)}{(2kh)^2}+\frac{\sin(2kh)}{(2kh)^3}\right]$$

$$R_r = \frac{2P_{rad}}{|I_0|^2} = \eta\pi\left(\frac{l}{\lambda}\right)^2\left[\frac{2}{3}-\frac{\sin(2kh)}{2kh}-\frac{\cos(2kh)}{(2kh)^2}+\frac{\sin(2kh)}{(2kh)^3}\right]$$

$$U_n(\theta,\phi) = \begin{cases}\dfrac{(1-\sin^2\theta\ \sin^2\phi)\ \sin^2(kh\cos\theta)}{\sin^2(kh)}, & h\le\dfrac{\lambda}{4}\\[2ex] (1-\sin^2\theta\ \sin^2\phi)\ \sin^2(kh\cos\theta), & h>\dfrac{\lambda}{4}\end{cases}$$

$$D_0 = \frac{4\pi U_{\max}}{P_{rad}} = \begin{cases}\dfrac{4\sin^2(kh)}{\left[\frac{2}{3}-\frac{\sin(2kh)}{2kh}-\frac{\cos(2kh)}{(2kh)^2}+\frac{\sin(2kh)}{(2kh)^3}\right]}, & h\le\dfrac{\lambda}{4}\\[2ex] \dfrac{4}{\left[\frac{2}{3}-\frac{\sin(2kh)}{2kh}-\frac{\cos(2kh)}{(2kh)^2}+\frac{\sin(2kh)}{(2kh)^3}\right]}, & h>\dfrac{\lambda}{4}\end{cases}$$

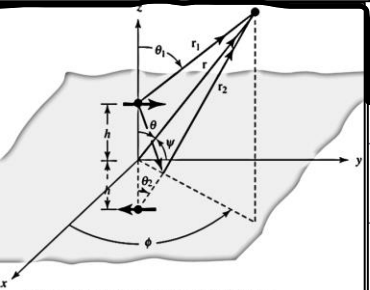
Radiation Intensity of Horizontal Element above a Ground Plane

- For $kh\le\pi/2$ ($h\le\lambda/4$):

$$U_{\max} = \frac{\eta}{2}|I_0|^2\left(\frac{l}{\lambda}\right)^2\sin^2(kh),\ (\theta_{\max},\phi_{\max}) = (0,\phi) \quad (4\text{-}122a)$$

- For $kh>\pi/2$ ($h>\lambda/4$):

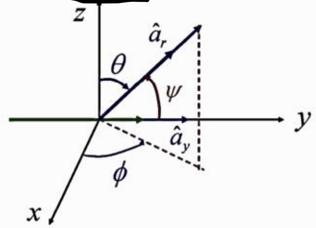
$$U_{\max} = \frac{\eta}{2}|I_0|^2\left(\frac{l}{\lambda}\right)^2,\ (\theta_{\max},\phi_{\max}) = \left[\cos^{-1}\left(\frac{\pi}{2kh}\right),0\right]$$



(a) Horizontal electric dipole above ground plane

$$E_\psi^d = j\eta\frac{kI_o\ell e^{-jkr_1}}{4\pi r_1}\sin\psi$$

$$E_\psi^r = jR_h\eta\frac{kI_o\ell e^{-jkr_2}}{4\pi r_2}\sin\psi = -j\eta\frac{kI_o\ell e^{-jkr_2}}{4\pi r_2}\sin\psi$$



$$\hat{a}_r = \hat{a}_x\sin\theta\cos\phi + \hat{a}_y\sin\theta\sin\phi + \hat{a}_z\cos\theta$$

$$\hat{a}_y\cdot\hat{a}_r = (1)(1)\cos\psi = \hat{a}_y\cdot(\hat{a}_x\sin\theta\cos\phi+\hat{a}_y\sin\theta\sin\phi+\hat{a}_z\cos\theta)$$

$$\cos\psi = \sin\theta\sin\phi \quad (4\text{-}11$$

$$\sin\psi = \pm\sqrt{1-\cos^2\psi} = \pm\sqrt{1-\sin^2\theta\sin^2\phi} \quad (4\text{-}11$$

Horizontal Dipole

$$\text{Number of Lobes} \cong 2\left(\frac{h}{\lambda}\right)$$

$$h >> \lambda$$

Directivity of Horizontal Element above a Ground Plane

- For $kh\to 0$, the directivity is given approximately by

$$D_0 \overset{kh\to 0}{\cong} \frac{4\sin^2(kh)}{\left[\frac{2}{3}-\frac{2}{3}+\frac{8}{15}(kh)^2\right]} = 7.5\left(\frac{\sin(kh)}{kh}\right)^2 \quad (4\text{-}124$$

- For $h = 0$, the element is shorted and does not radiate.
- For $kh\to\infty$, the directivity is 6.

$$U \simeq \frac{r^2}{2\eta}\left[\underbrace{\left|E^\circ_\theta(\theta,\phi)\right|^2}+\underbrace{\left|E^\circ_\phi(\theta,\phi)\right|^2}\right]$$

$$U = \quad U_\theta \quad + \quad U_\phi$$

$$\underline{W}_{rad}(r,\theta,\phi) = \hat{a}_r \frac{1}{2}\left[\frac{|E_\theta|^2+|E_\phi|^2}{\eta}\right] = \hat{a}_r \frac{1}{r^2}\left|f(\theta,\phi)\right|^2$$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\sin\theta d\theta d\phi = \int_0^{2\pi}\int_0^{\pi}\left[U_\theta+U_\phi\right]\sin\theta d\theta d\phi$$

$$D = \frac{4\pi U(\theta,\phi)}{P_{rad}} = \frac{4\pi\left(U_\theta+U_\phi\right)}{P_{rad}}$$

$$= \underbrace{\frac{4\pi U_\theta}{P_{rad}}} + \underbrace{\frac{4\pi U_\phi}{P_{rad}}}$$

$$D = \quad D_\theta \quad + \quad D_\phi$$

D_θ, D_ϕ = are the partial directivities

$$\text{Gain} = G = 4\pi\frac{\text{Radiation intensity}}{\text{Total input (accepted) power}}$$

$$G = 4\pi\frac{U(\theta,\phi)}{P_{in}}$$

$$P_{rad} = e_{cd}P_{in} \Rightarrow P_{in} = \frac{P_{rad}}{e_{cd}}$$

$$G = 4\pi\frac{U(\theta,\phi)}{P_{rad}/e_{cd}} = e_{cd}\underbrace{\left[4\pi\frac{U(\theta,\phi)}{P_{rad}}\right]}_{D}$$

$$G = e_{cd}D$$

$$\boxed{G_o = e_{cd}D_o} \qquad \text{(2-49a)}$$

$e_{cd} = e_c e_d = \textit{Radiation efficiency}$

$e_c = \textit{Conduction efficiency}$

$e_d = \textit{Dielectric efficiency}$

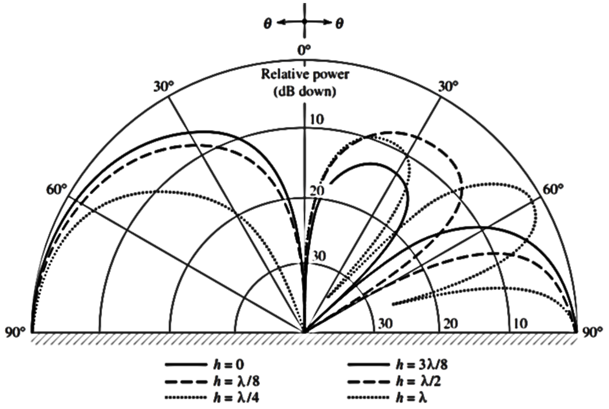


Figure 4.18 Elevation plane amplitude patterns of a vertical infinitesimal electric dipole for different heights above an infinite perfect electric conductor.

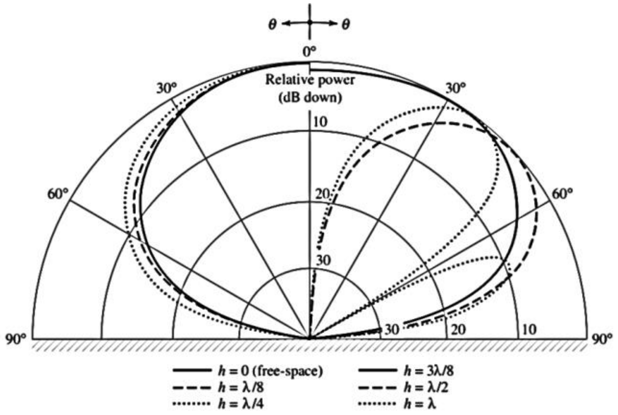


Figure 4.29 Elevation plane ($\phi = 90°$) amplitude patterns of a horizontal infinitesimal electric dipole for different heights above an infinite perfect electric conductor.