EXERCISE 9 — SOLUTION

1. Bounding Volumes

(a) Discuss the advantages and disadvantages of the bounding volume types

- sphere,
- AABB,
- OBB, and
- k-DOP

with regard to how well they are suited for

- translation and
- rotation.

Solution

	Translation	Rotation
Bounding Sphere	+	++
AABB	+	–
OBB	+	+

(b) Sketch one algorithm each for constructing a

- bounding sphere,
- AABB, and
- OBB

for objects that are represented as triangle meshes.

Solution

- Sphere
  - $\mathbf{c} = \frac{1}{2}\begin{bmatrix} \min \mathbf{p}_x + \max \mathbf{p}_x \\ \min \mathbf{p}_y + \max \mathbf{p}_y \\ \min \mathbf{p}_z + \max \mathbf{p}_z \end{bmatrix}$
  - $r = \max \|\mathbf{p} - \mathbf{c}\|$
- AABB: $\begin{bmatrix} \min \mathbf{p}_x \\ \min \mathbf{p}_y \\ \min \mathbf{p}_z \end{bmatrix} \dots \begin{bmatrix} \max \mathbf{p}_x \\ \max \mathbf{p}_y \\ \max \mathbf{p}_z \end{bmatrix}$
- OBB:
  - Point sample the convex hull of the geometry to be bound: n vertices $\mathbf{v}_i$
  - Find the mean and the covariance matrix of the samples:

$$\mu = \frac{1}{n}\sum_{i=1}^{n} \mathbf{v}_i \quad , \quad C_{j,k} = \frac{1}{n}\sum_{i=1}^{n} \overline{\mathbf{v}}_{i,j} \cdot \overline{\mathbf{v}}_{i,k} \quad , \quad \overline{\mathbf{v}}_i = \mathbf{v}_i - \mu \quad , \quad j,k = 1,2,3$$

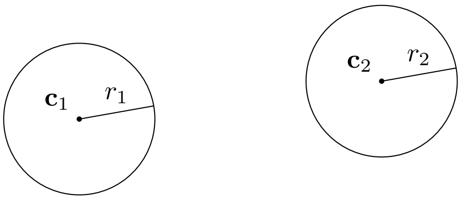
  - Mean: Center of the box
  - Eigenvectors of covariance matrix: Axes of box

$$\text{Eigenvalues } \lambda\text{:} \quad |C - \lambda \cdot I| = 0 \quad , \quad \text{Eigenvectors } a\text{:} \quad Ca = \lambda a$$

2. Overlap Test

Sketch an algorithm that checks whether the two bounding volumes overlap.

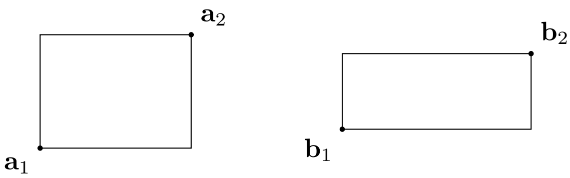
(a) Bounding spheres:



Solution

overlap iff $\|\mathbf{c}_1 - \mathbf{c}_2\| \leq r_1 + r_2$

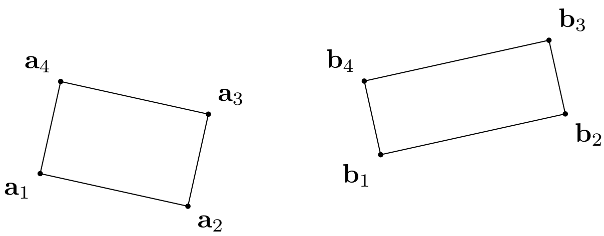
(b) AABB:



Solution

overlap iff both ranges on x- and y-axis overlap

(c) OBB:



Solution

- test for
  - overlap along each face normal,
  - overlap along each edge-pair normal
- overlap iff all of the above exists

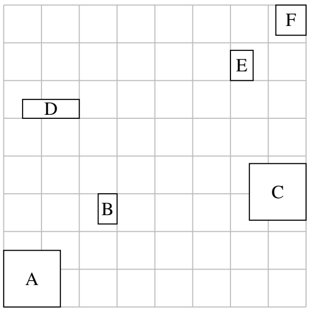
General:

- Given two generic polytopes, each with E edges and F faces, number of candidate axes to test is: $2F + E^2$
- OBBs have only $E = 3$ distinct edge directions, and only $F = 3$ distinct face normals. OBBs need at most 15 axis tests.
- AABBs need at most 3 axis tests.

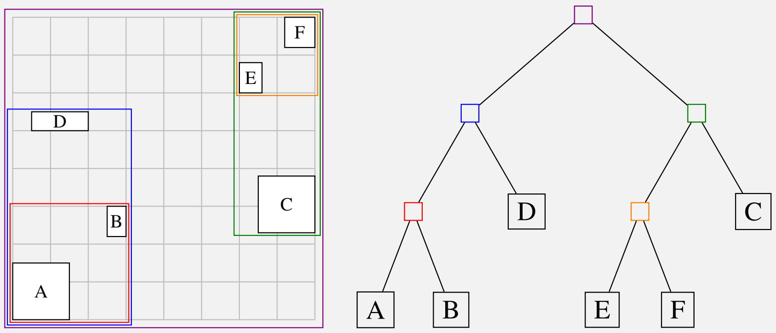
3. Accelerating Structures

Construct an acceleration structure for the following objects that are already organized in separate AABBs.

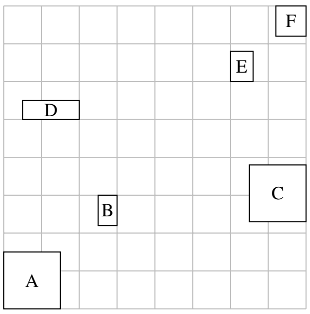
(a) Create a *sensible* BVH consisting of AABBs using a binary tree.



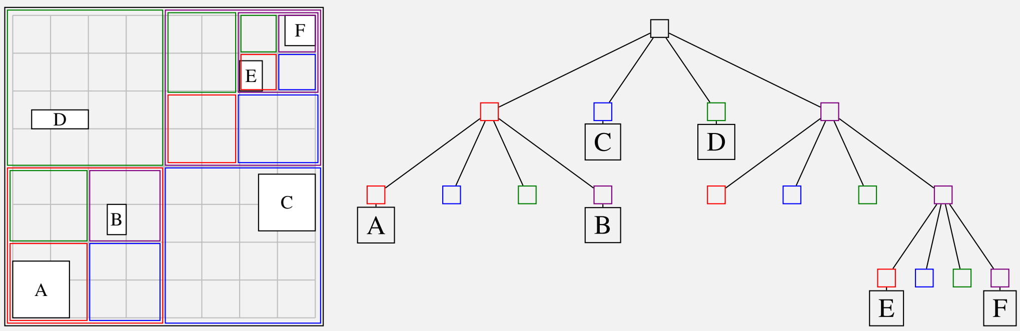
Solution



(b) Create a Quadtree with each leaf node containing at most one of the AABBs.



Solution



4. Computational Costs

Consider the following ways of organizing multiple objects, i.e., triangle meshes, in a scene:

1. one single array of individual triangles,
2. one AABB per object, and
3. AABBs organized in a BVH, one AABB per object as the leafs.

Discuss briefly their impact on the computational cost for collision detection. Do not consider update costs.

Solution

1. simple test, quadratic in number of triangles
2. approx. equally simple test, quadratic in number of AABBs + quadratic in number of triangles in overlapping boxes
3. approx. equally simple test, log-squared in number of AABBs + quadratic in number of triangles in overlapping boxes