EXERCISE 4 — SOLUTION

1. Projection I

Suppose you want to render a WUXGA image (1920×1200) with square pixels. Your camera has a vertical field of view of $\theta = 60°$ and a symmetric view frustum.

(a) Write down a perspective projection matrix for

- distance to near clipping plane: 3,
- distance to far clipping plane: 6.

Solution

- WUXGA, square pixels → aspect ratio a:

$$a = \frac{n_x}{n_y} = \frac{1920}{1200} = \frac{16}{10} = \frac{8}{5} = \frac{r}{t} \quad \Longleftrightarrow \quad r = \frac{8}{5} \cdot t$$

- symmetric frustum:

$$t = -b \quad , \quad r = -l$$

- vertical field of view:

$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{t}{|n|} \quad \Longleftrightarrow \quad t = |n| \cdot \frac{1}{\sqrt{3}}$$

- clipping planes (looking along negative z!):

$$n = -3 \quad , \quad f = -6$$

- matrix:

$$\mathbf{P} = \begin{bmatrix} -\frac{5\sqrt{3}}{8} & 0 & 0 & 0 \\ 0 & -\sqrt{3} & 0 & 0 \\ 0 & 0 & -3 & -12 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) Check the matrix. Use at least 4 points for which you know the projection in advance.

Solution

- $\mathbf{P} \cdot \begin{bmatrix} 0 & 0 & -3 & 1 \end{bmatrix}^\top = \begin{bmatrix} 0 & 0 & -3 & -3 \end{bmatrix}^\top \equiv \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}^\top$
- $\mathbf{P} \cdot \begin{bmatrix} 0 & 0 & -6 & 1 \end{bmatrix}^\top = \begin{bmatrix} 0 & 0 & 6 & -6 \end{bmatrix}^\top \equiv \begin{bmatrix} 0 & 0 & -1 & 1 \end{bmatrix}^\top$
- $\mathbf{P} \cdot \begin{bmatrix} -\frac{8}{5}\sqrt{3} & -\sqrt{3} & -3 & 1 \end{bmatrix}^\top = \begin{bmatrix} 3 & 3 & -3 & -3 \end{bmatrix}^\top \equiv \begin{bmatrix} -1 & -1 & 1 & 1 \end{bmatrix}^\top$
- $\mathbf{P} \cdot \begin{bmatrix} \frac{8}{5}\sqrt{3} & \sqrt{3} & -3 & 1 \end{bmatrix}^\top = \begin{bmatrix} 3 & 3 & -3 & -3 \end{bmatrix}^\top \equiv \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^\top$

2. Projection II

Determine how a symmetric perspective projection matrix' maps z-values. Draw a graph.

Solution

- use the above matrix for this task

Solution cont.

- we only require the matrix' / vector's sub-block:

$$\begin{bmatrix} -3 & -12 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} p_z \\ 1 \end{bmatrix}$$

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p_z	-3.0	-3.25	-3.5	-3.75	-4.0	-4.5	-5.0	-5.5	-6.0
p'_z	1.00	0.69	0.43	0.20	0.00	-0.33	-0.60	-0.82	-1.00

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