EXERCISE 7 — SOLUTION

1. Visible Light

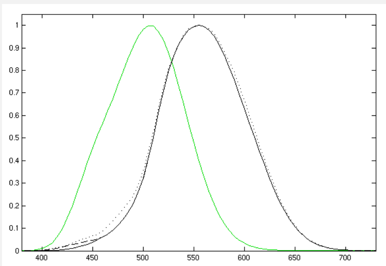
(a) Describe the spectrum of visible light.

Solution

$380\,\mathrm{nm}$ (violet) to $780\,\mathrm{nm}$ (red)

(b) Characterize the average spectral sensitivity of human visual perception of brightness.

Solution

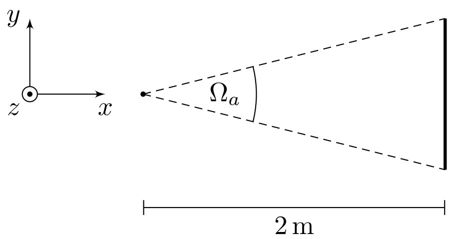


black: photopic (well-lit) conditions,
green: scotopic (low light) conditions.

2. Photometric Quantities, Solid Angle

Assume there is a quad with dimensions $1\,\mathrm{m}$ by $1\,\mathrm{m}$ positioned according to the following illustrations.

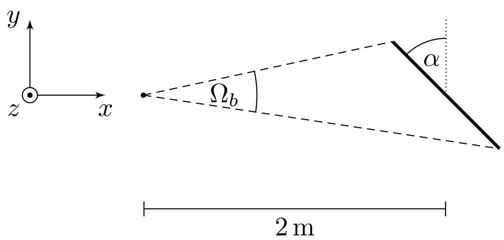
(a) Compute an approximation of the solid angle Ω_a that the quad spans.



Solution

$\Omega_a = \frac{A_V}{r^2} = \frac{1\,\mathrm{m}^2}{4\,\mathrm{m}^2} = \frac{1}{4}\mathrm{sr}$

(b) Compute an approximation of the solid angle Ω_b that the quad spans.



Solution

- Quad is rotated by α around z
- Thus, one dimension appears shortened: $l = 1\,\mathrm{m} \cdot \cos\alpha$
- Consequently $\Omega_b = \frac{A_V}{r^2} = \frac{1\,\mathrm{m} \cdot 1\,\mathrm{m} \cdot \cos\alpha}{4\,\mathrm{m}^2} = \frac{1}{4}\mathrm{sr}\cos\alpha$

Prof. Dr. Torsten W. Kuhlen <kuhlen@vr.rwth-aachen.de>
Dr. Tom Vierjahn <vierjahn@vr.rwth-aachen.de>

(c) Fill in the spaces
Assume, a light source emits a ___________ of $I_V = 1\,\mathrm{cd}$, then we can compute the ___________ (E_V) in lx (Lux) at the quad.

Solution

- luminous intensity
- illuminance

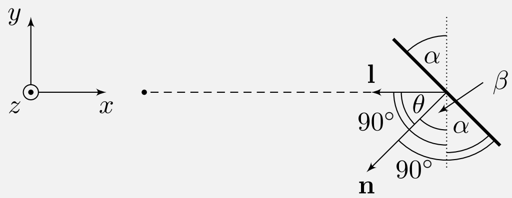
(d) Compute these measurements, for which you filled the spaces. Compare them for both quads.

Solution

- surface area: $A = 1\,\mathrm{m}^2$
- $E_{V,a} = \frac{I_V \Omega_a}{A} = \frac{1\,\mathrm{cd} \cdot 1\,\mathrm{sr}}{4 \cdot 1\,\mathrm{m}^2} = \frac{1}{4}\mathrm{lx}$
- $E_{V,b} = \frac{I_V \Omega_b}{A} = \frac{1\,\mathrm{cd} \cdot 1\,\mathrm{sr} \cdot \cos\alpha}{4 \cdot 1\,\mathrm{m}^2} = \frac{1}{4}\cos\alpha\mathrm{lx}$

(e) Compare this to the Lambert reflection model. Prove your finding.

Solution

- $E_{V,b} = \frac{I_V \Omega_b}{A} = \frac{1\,\mathrm{cd} \cdot 1\,\mathrm{sr} \cdot \cos\alpha}{4 \cdot 1\,\mathrm{m}^2} = \frac{1}{4}\cos\alpha\mathrm{lx}$ resembles the $I \cdot \cos\theta = I \cdot (\mathbf{n}^\top \mathbf{l})$ term in the Lambert reflection model.
- 

- $\beta = 90° - \alpha$
- $\theta = 90° - \beta = 90° - 90° + \alpha = \alpha$ q.e.d.