Computer Vision – Lecture 7

Segmentation as Energy Minimization

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Announcements

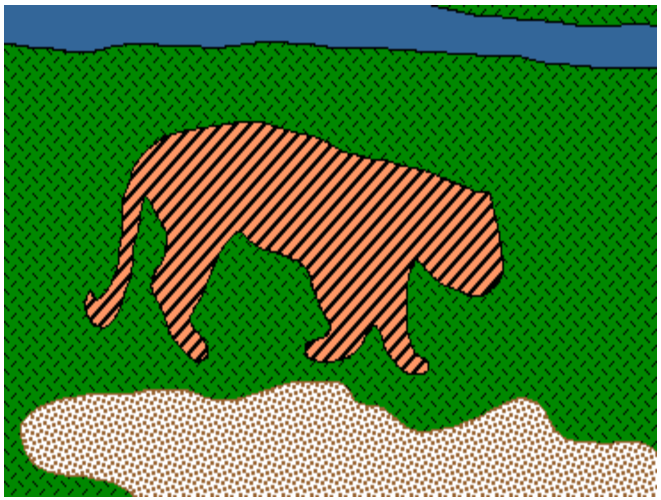
- **Please don't forget to register for the exam!**
  - On the Campus system

Course Outline

- Image Processing Basics
- Segmentation
  - Segmentation and Grouping
  - Segmentation as Energy Minimization
- Recognition
  - Global Representations
  - Subspace representations
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
- Motion and Tracking

Recap: Image Segmentation

- **Goal: identify groups of pixels that go together**



Slide credit: Steve Seitz, Kristen Grauman

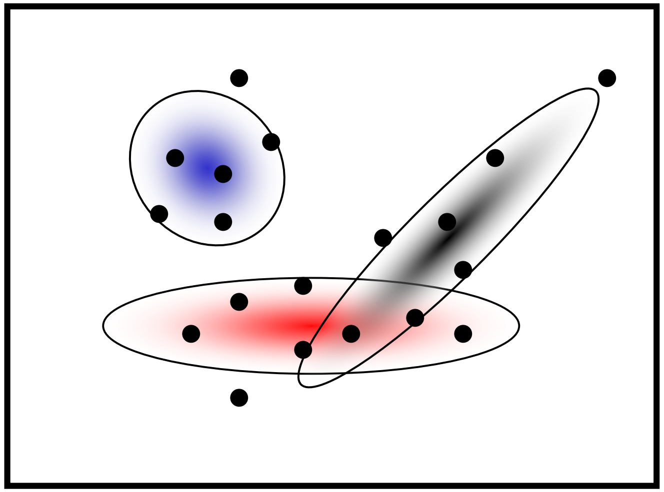
Recap: K-Means Clustering

- **Basic idea: randomly initialize the *k* cluster centers, and iterate between the two steps we just saw.**
  1. Randomly initialize the cluster centers, $c_1, \ldots, c_K$
  2. Given cluster centers, determine points in each cluster
     - For each point p, find the closest c_i. Put p into cluster i
  3. Given points in each cluster, solve for c_i
     - Set c_i to be the mean of points in cluster i
  4. If c_i have changed, repeat Step 2

- **Properties**
  - Will always converge to *some* solution
  - Can be a "local minimum"
    - Does not always find the global minimum of objective function:

$$\sum_{\text{clusters } i} \sum_{\text{points p in cluster } i} ||p - c_i||^2$$

Recap: Expectation Maximization (EM)



- **Goal**
  - Find blob parameters θ that maximize the likelihood function:

$$p(data|\theta) = \prod_{n=1}^{N} p(\mathbf{x}_n|\theta)$$

- **Approach:**
  1. E-step: given current guess of blobs, compute ownership of each point
  2. M-step: given ownership probabilities, update blobs to maximize likelihood function
  3. Repeat until convergence

Recap: EM Algorithm

see lecture Machine Learning!

- **Expectation-Maximization (EM) Algorithm**
  - **E-Step**: softly assign samples to mixture components

$$\gamma_j(\mathbf{x}_n) \leftarrow \frac{\pi_j \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)}{\sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)} \qquad \forall j = 1,\ldots,K, \quad n = 1,\ldots,N$$

  - **M-Step**: re-estimate the parameters (separately for each mixture component) based on the soft assignments

$$\hat{N}_j \leftarrow \sum_{n=1}^{N} \gamma_j(\mathbf{x}_n)$$ = soft number of samples labeled j

$$\hat{\pi}_j^{\text{new}} \leftarrow \frac{\hat{N}_j}{N}$$

$$\hat{\boldsymbol{\mu}}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^{N} \gamma_j(\mathbf{x}_n)\mathbf{x}_n$$

$$\hat{\boldsymbol{\Sigma}}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^{N} \gamma_j(\mathbf{x}_n)(\mathbf{x}_n - \hat{\boldsymbol{\mu}}_j^{\text{new}})(\mathbf{x}_n - \hat{\boldsymbol{\mu}}_j^{\text{new}})^{\mathrm{T}}$$

Slide adapted from Bernt Schiele

MoG Color Models for Image Segmentation

(a) input image (b) user input (c) inferred segmentation

- User assisted image segmentation
  - User marks two regions for foreground and background.
  - Learn a MoG model for the color values in each region.
  - Use those models to classify all other pixels.

  ⇒ Simple segmentation procedure
  (building block for more complex applications)

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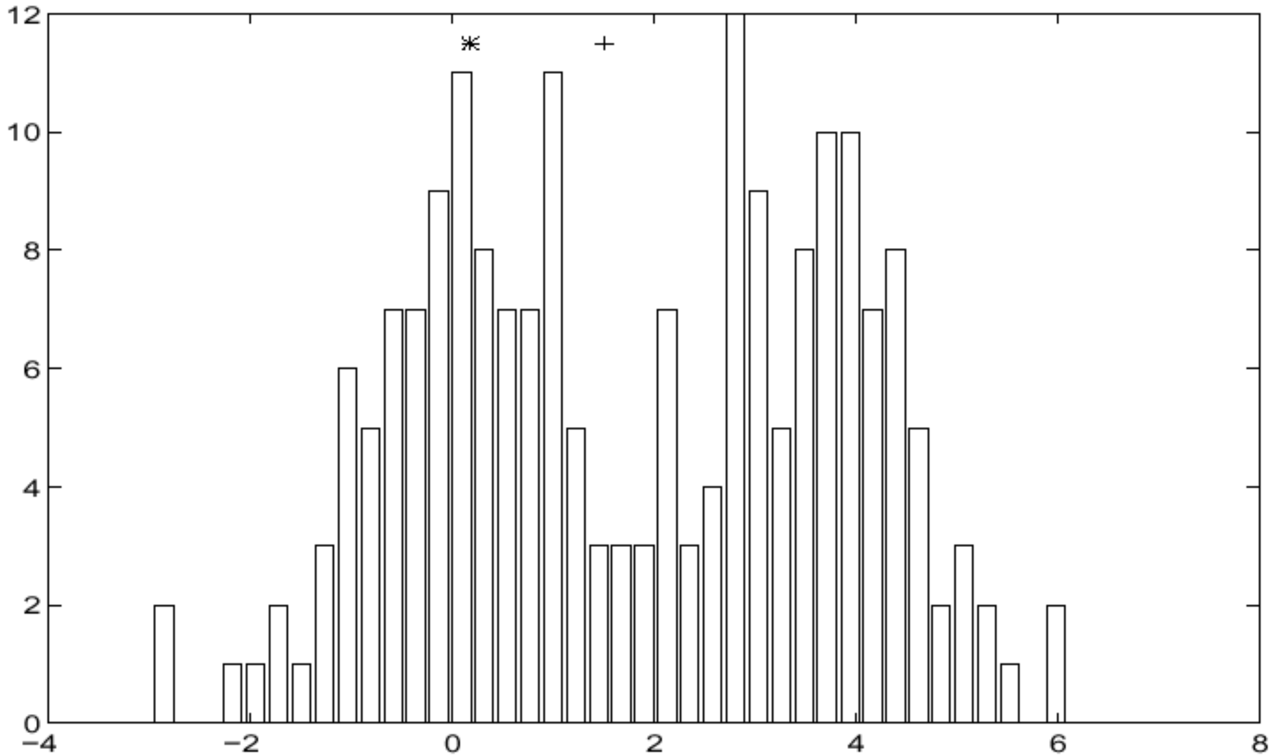
Recap: Mean-Shift Algorithm

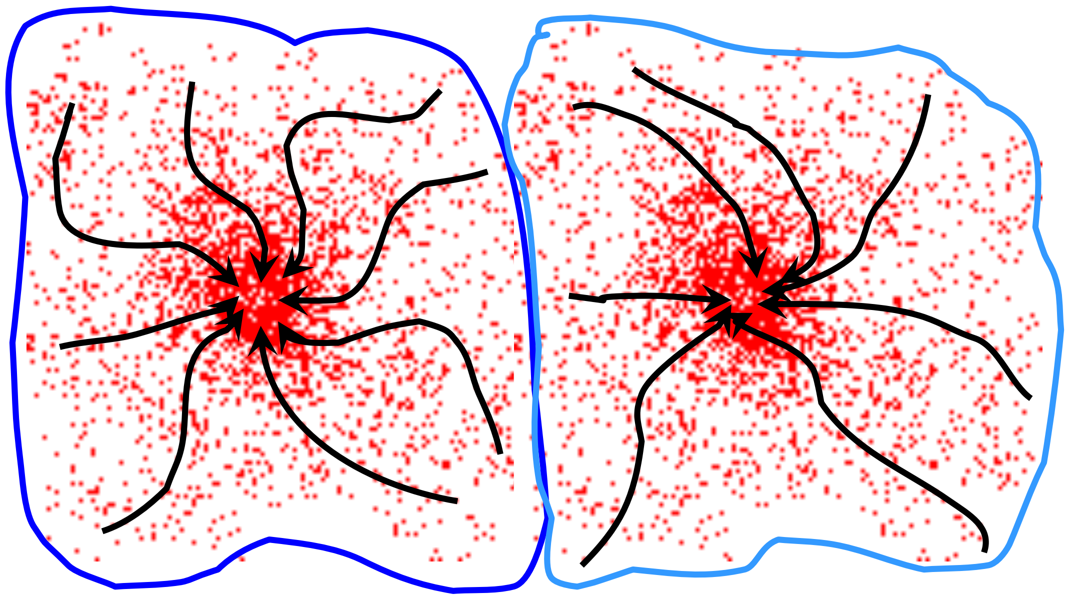


- **Iterative Mode Search**
  1. **Initialize random seed, and window W**
  2. **Calculate center of gravity (the "mean") of W:** $\sum_{x \in W} x H(x)$
  3. **Shift the search window to the mean**
  4. **Repeat Step 2 until convergence**

Slide credit: Steve Seitz

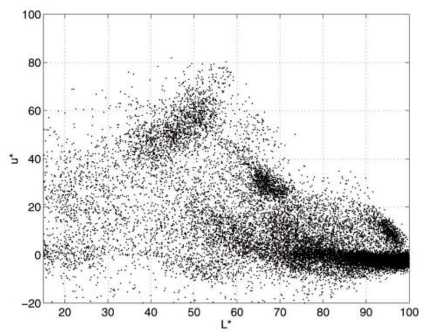
Recap: Mean-Shift Clustering

- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode

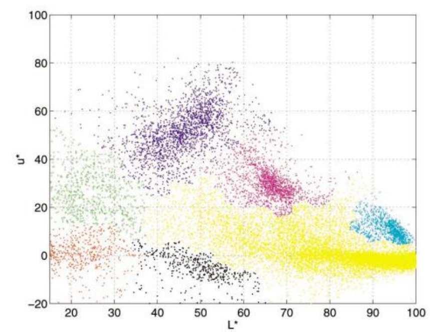


Recap: Mean-Shift Segmentation

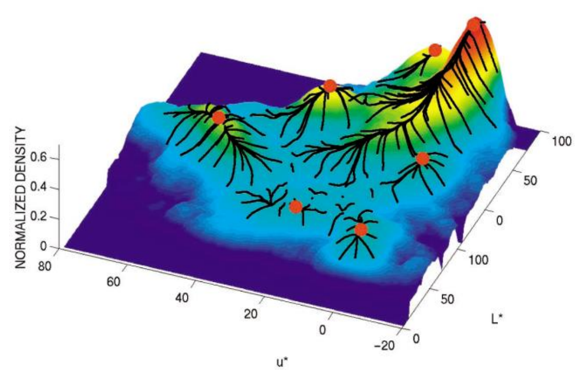
- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same “peak” or mode



(a)

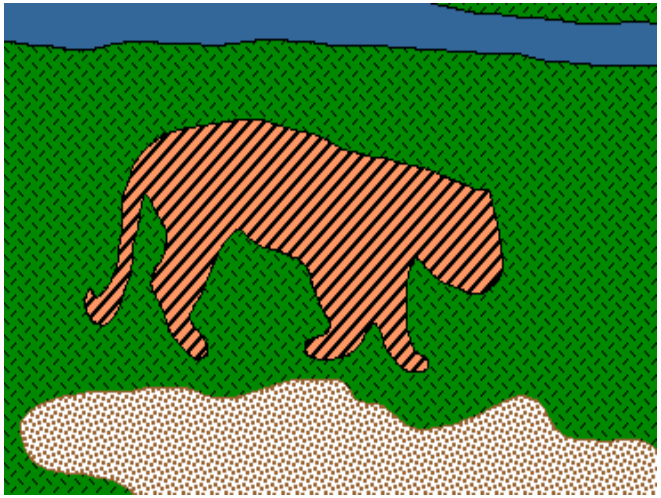


(b)



Back to the Image Segmentation Problem…

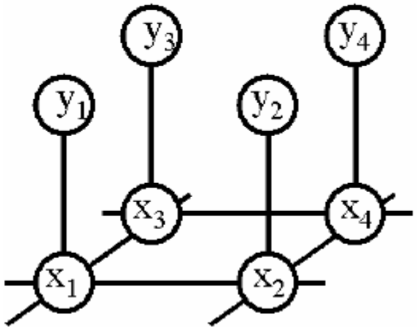
- Goal: identify groups of pixels that go together



- Up to now, we have focused on ways to group pixels into image segments based on their appearance…
  - Segmentation as clustering.
- We also want to enforce region constraints.
  - Spatial consistency
  - Smooth borders

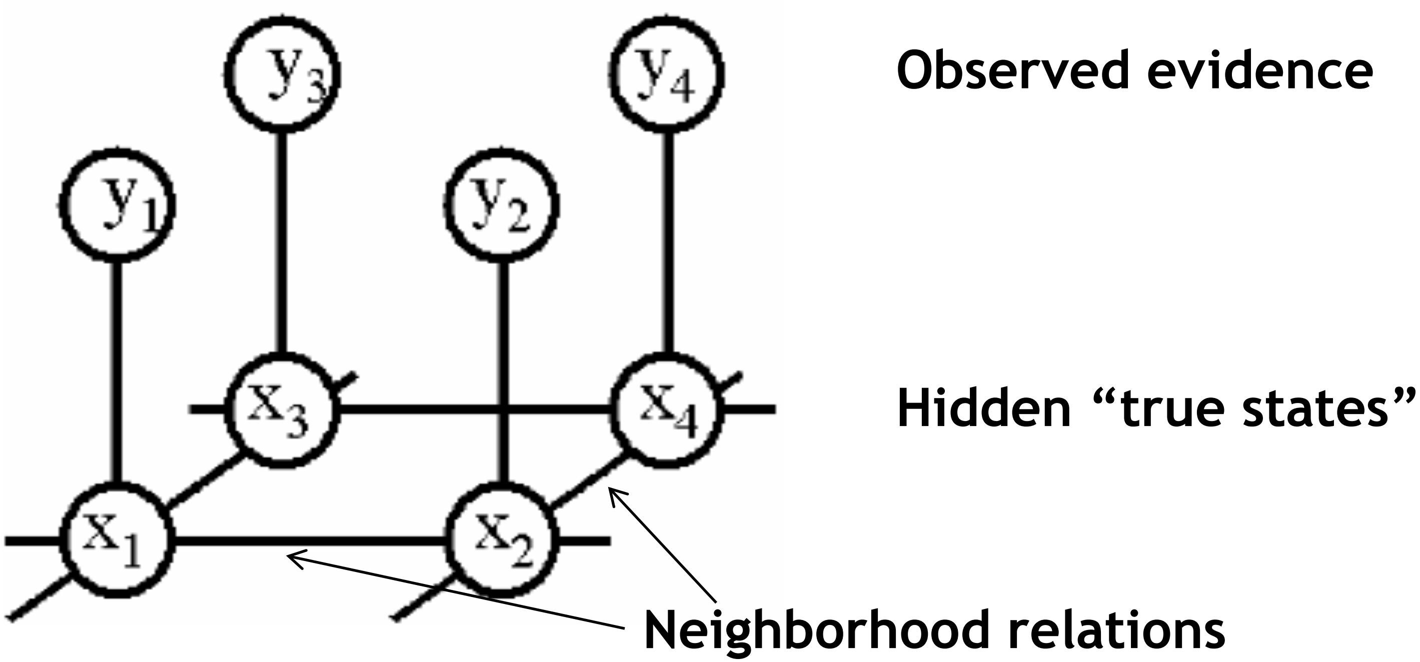
Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case
- Applications
  - Interactive segmentation



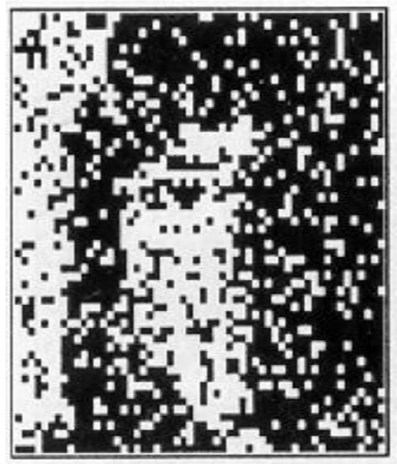
Markov Random Fields

- Allow rich probabilistic models for images
- But built in a local, modular way
  - Learn local effects, get global effects out



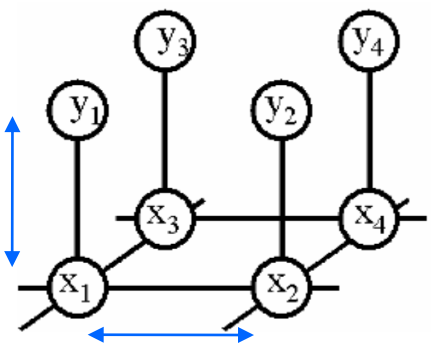
MRF Nodes as Pixels

Original image



Degraded image

Reconstruction from MRF modeling pixel neighborhood statistics



$\Phi(x_i, y_i)$

$\Psi(x_i, x_j)$

Network Joint Probability

$$p(\mathbf{x},\mathbf{y}) = \prod_i \Phi(x_i, y_i) \prod_{i,j} \Psi(\underbrace{x_i, x_j}_{\text{Neighboring scene nodes}})$$

Scene

Image

Image-scene compatibility function

Local observations

Scene-scene compatibility function

Neighboring scene nodes

Energy Formulation

- **Joint probability**

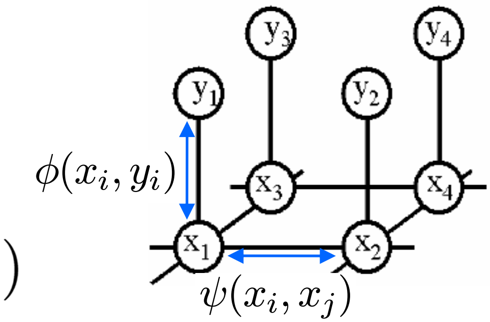
$$p(\mathbf{x},\mathbf{y}) = \prod_i \Phi(x_i, y_i) \prod_{i,j} \Psi(x_i, x_j)$$

- **Maximizing the joint probability is the same as minimizing the negative log**

$$\begin{aligned} -\log p(\mathbf{x},\mathbf{y}) &= -\sum_i \log \Phi(x_i, y_i) - \sum_{i,j} \log \Psi(x_i, x_j) \\ E(\mathbf{x},\mathbf{y}) &= \sum_i \phi(x_i, y_i) + \sum_{i,j} \psi(x_i, x_j) \end{aligned}$$

- **This is similar to free-energy problems in statistical mechanics (spin glass theory). We therefore draw the analogy and call E an *energy function*.**
- **ϕ and ψ are called *potentials*.**

Energy Formulation

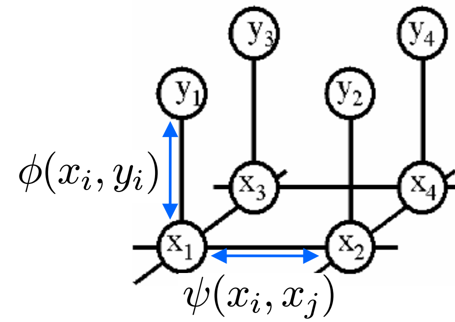


- Energy function

$$E(\mathbf{x}, \mathbf{y}) = \sum_i \underbrace{\phi(x_i, y_i)}_{\text{Single-node potentials}} + \sum_{i,j} \underbrace{\psi(x_i, x_j)}_{\text{Pairwise potentials}}$$

- Single-node potentials ϕ ("unary potentials")
  - Encode local information about the given pixel/patch
  - How likely is a pixel/patch to belong to a certain class (e.g. foreground/background)?
- Pairwise potentials ψ
  - Encode neighborhood information
  - How different is a pixel/patch's label from that of its neighbor? (e.g. based on intensity/color/texture difference, edges)

Energy Minimization



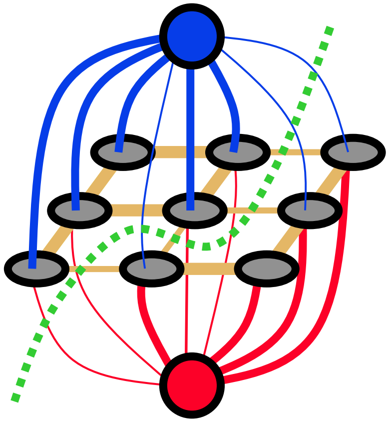
- **Goal:**
  - Infer the optimal labeling of the MRF.
- **Many inference algorithms are available, e.g.**
  - Gibbs sampling, simulated annealing
  - Iterated conditional modes (ICM)
  - Variational methods
  - Belief propagation
  - Graph cuts

see lecture Machine Learning!

- **Recently, Graph Cuts have become a popular tool**
  - Only suitable for a certain class of energy functions
  - But the solution can be obtained very fast for typical vision problems (~1MPixel/sec).

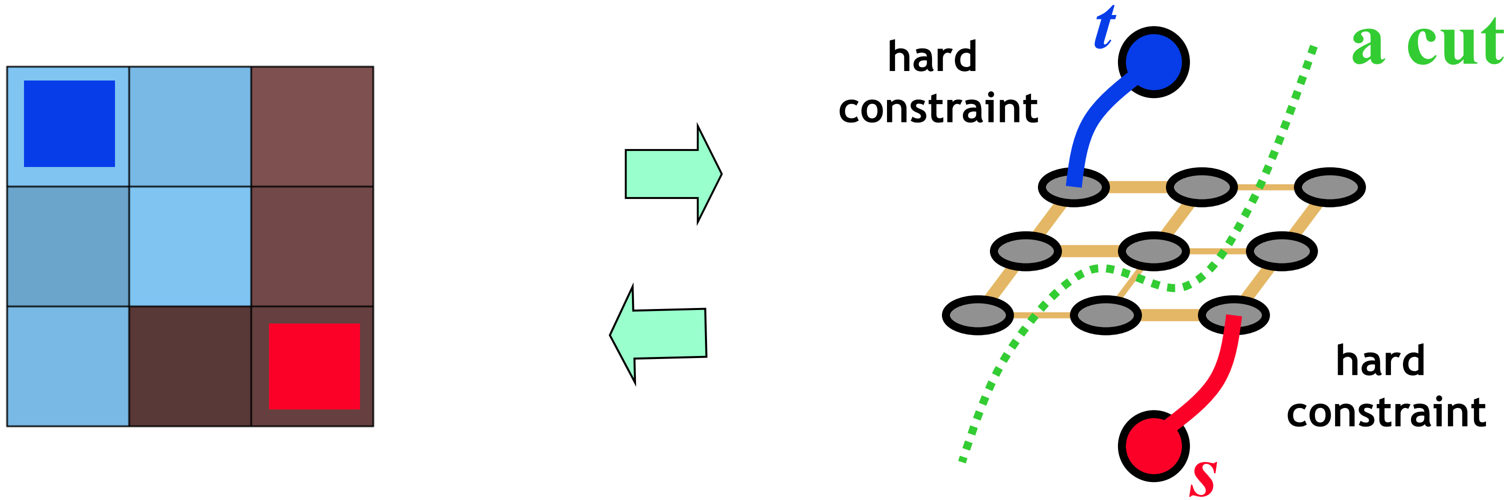
Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- **Graph cuts for image segmentation**
  - **Basic idea**
  - **s-t Mincut algorithm**
  - **Extension to non-binary case**
- Applications
  - Interactive segmentation



Graph Cuts for Optimal Boundary Detection

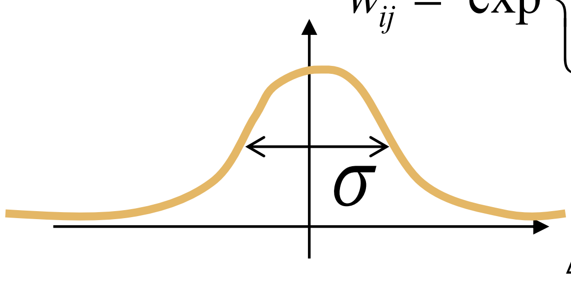
- Idea: convert MRF into source-sink graph



Minimum cost cut can be computed in polynomial time

(max-flow/min-cut algorithms)

$$w_{ij} = \exp\left\{-\frac{\Delta I_{ij}}{2\sigma^2}\right\}$$

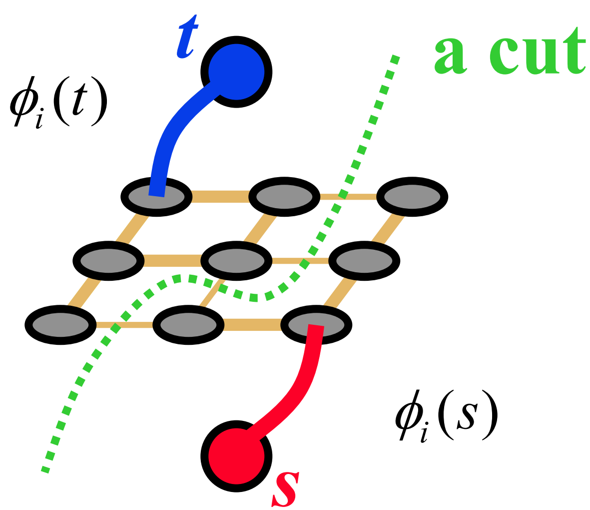


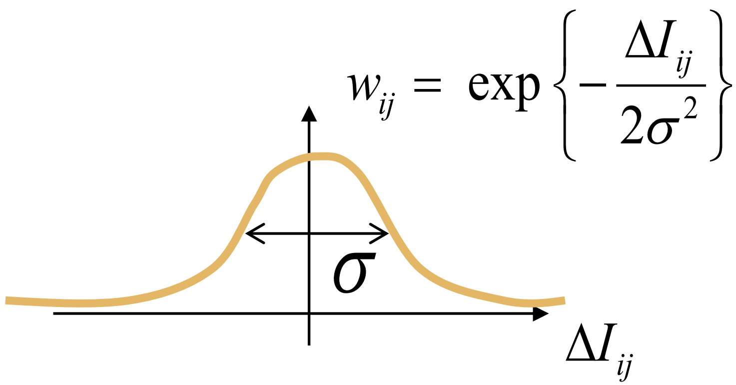
[Boykov & Jolly, ICCV'01]

Simple Example of Energy

$$E(\mathbf{x},\mathbf{y}) = \sum_i \phi_i(x_i) + \sum_{i,j} w_{ij} \cdot \delta(x_i \neq x_j)$$

Unary terms **Pairwise terms**



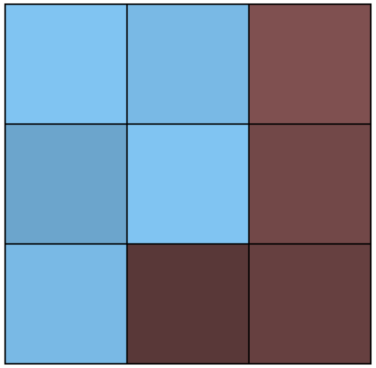


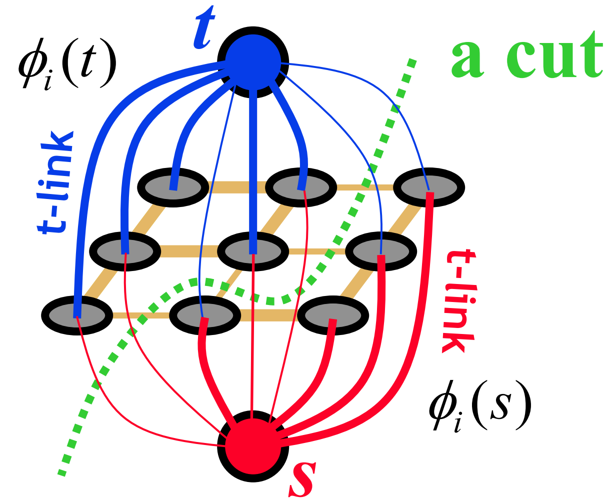
$$x \in \{s, t\}$$

(binary object segmentation)

Slide credit: Yuri Boykov

Adding Regional Properties





Regional bias example

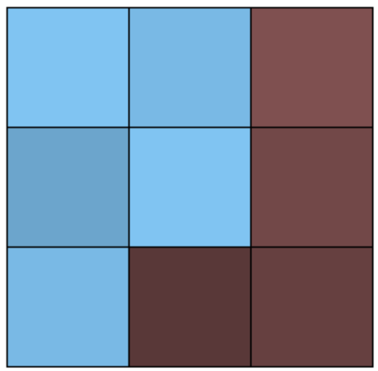
Suppose I^s and I^t are given "expected" intensities of object and background

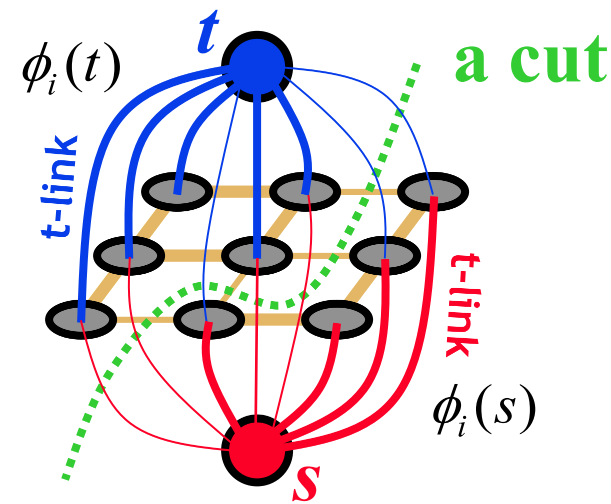
$$\phi_i(s) \propto \exp\left(-\| I_i - I^s \|^2 / 2\sigma^2\right)$$

$$\phi_i(t) \propto \exp\left(-\| I_i - I^t \|^2 / 2\sigma^2\right)$$

NOTE: hard constrains are not required, in general.

Adding Regional Properties





"expected" intensities of object and background
I^s and I^t
can be re-estimated

$$\phi_i(s) \propto \exp\left(-\| I_i - I^s \|^2 / 2\sigma^2\right)$$

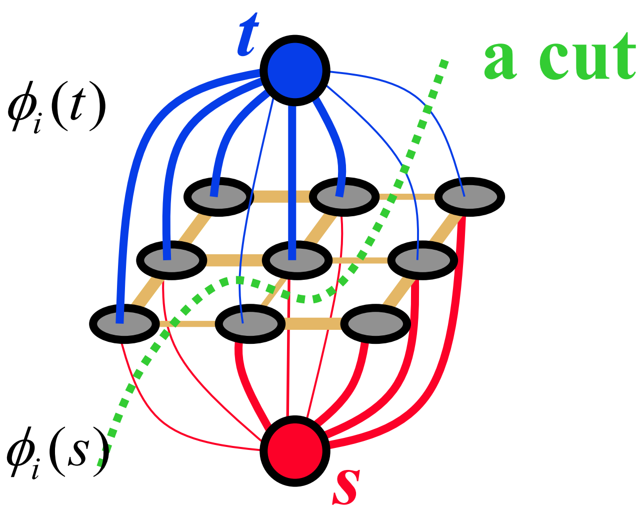
$$\phi_i(t) \propto \exp\left(-\| I_i - I^t \|^2 / 2\sigma^2\right)$$

EM-style optimization

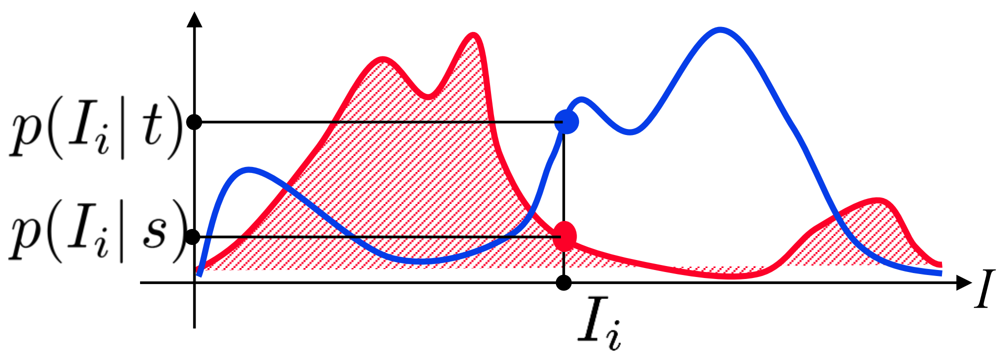
[Boykov & Jolly, ICCV'01]

Adding Regional Properties

- More generally, regional bias can be based on any intensity models of object and background



$$\phi_i(L_i) = -\log p(I_i | L_i)$$



given object and background intensity histograms

How to Set the Potentials? Some Examples

- Color potentials
  - e.g., modeled with a Mixture of Gaussians

$$\phi(x_i, y_i; \theta_\phi) = \log \sum_k \theta_\phi(x_i, k) p(k|x_i) \mathcal{N}(y_i; \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

- Edge potentials
  - E.g., a “contrast sensitive Potts model”

$$\psi(x_i, x_j, g_{ij}(\mathbf{y}); \theta_\psi) = -\theta_\psi g_{ij}(\mathbf{y}) \delta(x_i \neq x_j)$$

where

$$g_{ij}(\mathbf{y}) = e^{-\beta \|y_i - y_j\|^2} \qquad \beta = \frac{1}{2}\left(\mathrm{avg}\left(\|y_i - y_j\|^2\right)\right)^{-1}$$

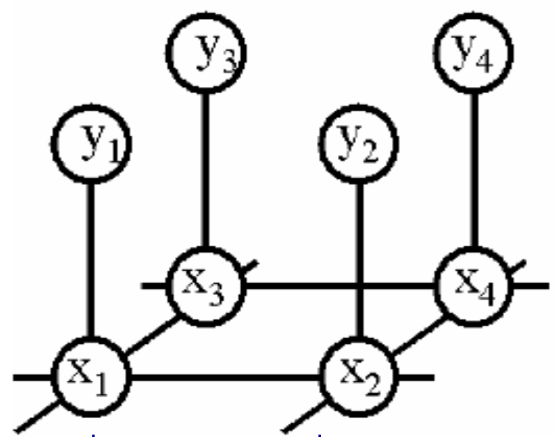
- Parameters θ_ϕ, θ_ψ need to be learned, too!

[Shotton & Winn, ECCV’06]

Example: MRF for Image Segmentation

- **MRF structure**

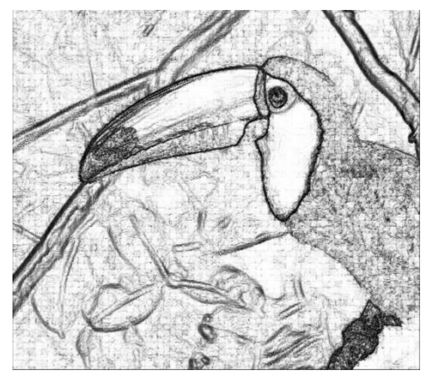
unary potentials



pairwise potentials

Data (D)

Unary likelihood



Pair-wise Terms

MAP Solution

Slide adapted from Phil Torr

Topics of This Lecture

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  - Basic idea
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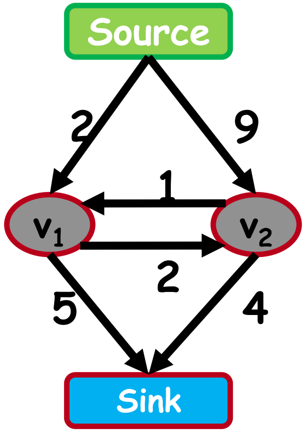
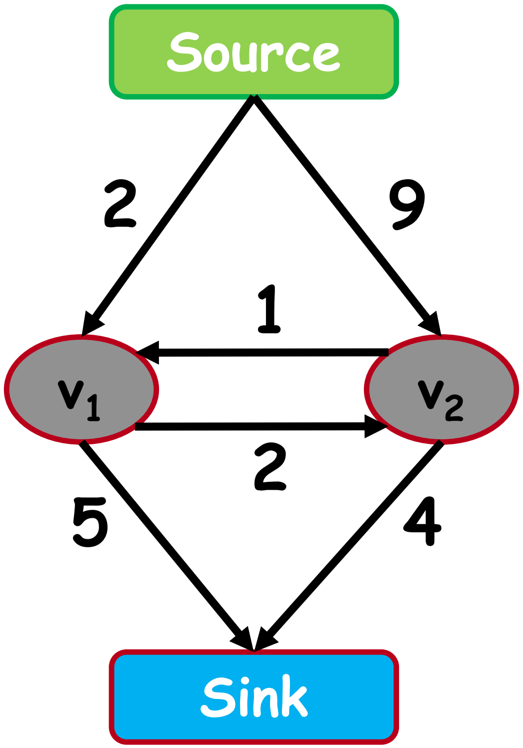
  - Extension to non-binary case
- Applications
  - Interactive segmentation



How Does it Work? The s-t-Mincut Problem



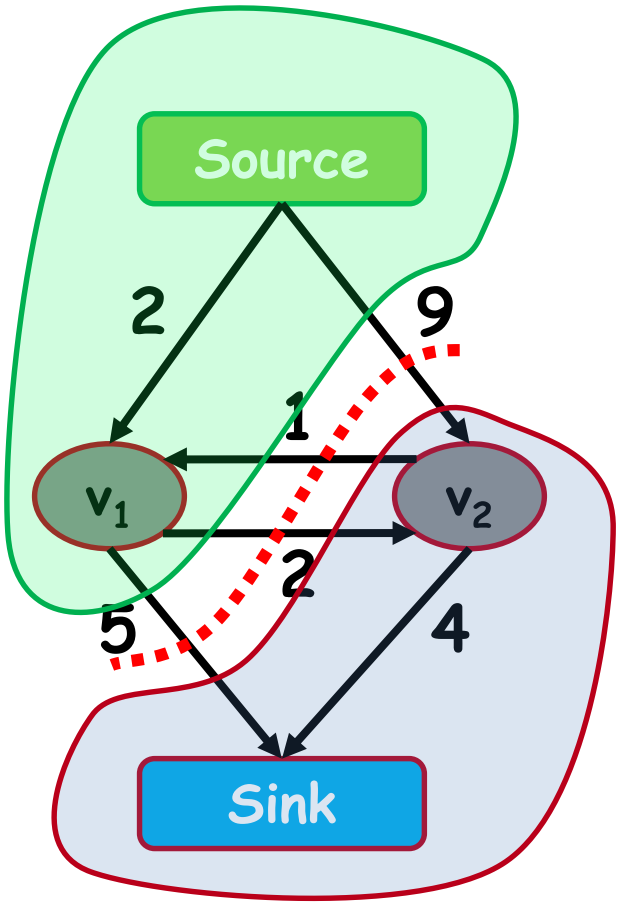
Graph (V, E, C)

Vertices $V = \{v_1, v_2 \ldots v_n\}$

Edges $E = \{(v_1, v_2) \ldots\}$

Costs $C = \{c_{(1, 2)} \ldots\}$

The s-t-Mincut Problem



5 + 2 + 9 = 16

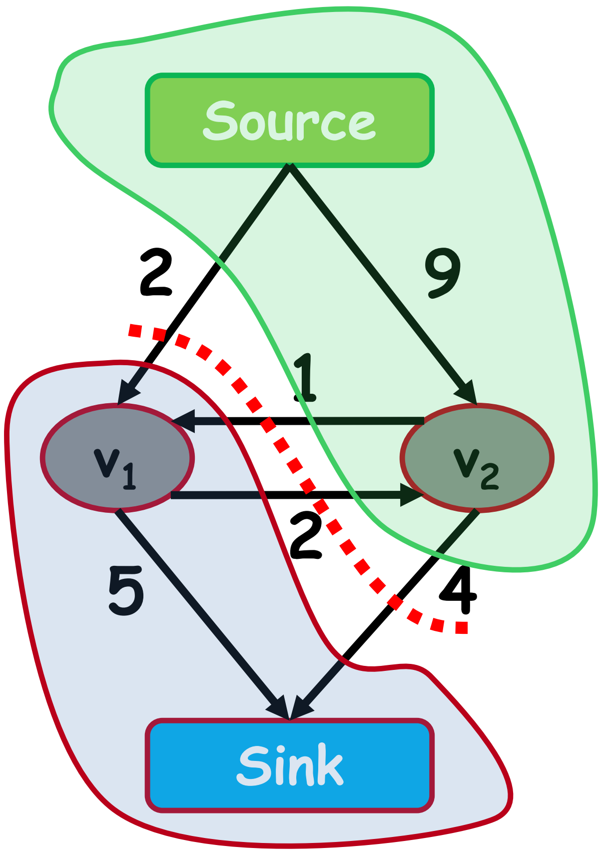
What is an st-cut?

An st-cut (S,T) divides the nodes between source and sink.

What is the cost of a st-cut?

Sum of cost of all edges going from S to T

The s-t-Mincut Problem



2 + 1 + 4 = 7

What is an st-cut?

An st-cut (S,T) divides the nodes between source and sink.

What is the cost of a st-cut?

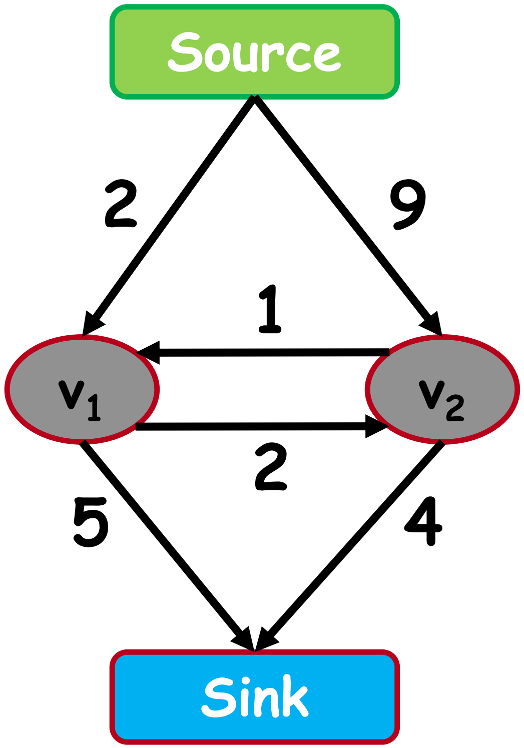
Sum of cost of all edges going from S to T

What is the st-mincut?

st-cut with the minimum cost

Slide credit: Pushmeet Kohli

How to Compute the s-t-Mincut?



Solve the dual maximum flow problem

Compute the maximum flow between Source and Sink

Constraints

Edges: Flow < Capacity

Nodes: Flow in = Flow out

Min-cut/Max-flow Theorem

In every network, the maximum flow equals the cost of the st-mincut

History of Maxflow Algorithms

Augmenting Path and Push-Relabel

year	discoverer(s)	bound
1951	Dantzig	$O(n^2mU)$
1955	Ford & Fulkerson	$O(m^2U)$
1970	Dinitz	$O(n^2m)$
1972	Edmonds & Karp	$O(m^2 \log U)$
1973	Dinitz	$O(nm \log U)$
1974	Karzanov	$O(n^3)$
1977	Cherkassky	$O(n^2m^{1/2})$
1980	Galil & Naamad	$O(nm \log^2 n)$
1983	Sleator & Tarjan	$O(nm \log n)$
1986	Goldberg & Tarjan	$O(nm \log(n^2/m))$
1987	Ahuja & Orlin	$O(nm + n^2 \log U)$
1987	Ahuja et al.	$O(nm \log(n\sqrt{\log U}/m))$
1989	Cheriyan & Hagerup	$E(nm + n^2 \log^2 n)$
1990	Cheriyan et al.	$O(n^3/\log n)$
1990	Alon	$O(nm + n^{8/3} \log n)$
1992	King et al.	$O(nm + n^{2+\epsilon})$
1993	Phillips & Westbrook	$O(nm(\log_{m/n} n + \log^{2+\epsilon} n))$
1994	King et al.	$O(nm \log_{m/(n \log n)} n)$
1997	Goldberg & Rao	$O(m^{3/2} \log(n^2/m) \log U)$ $O(n^{2/3}m \log(n^2/m) \log U)$

n: #nodes

m: #edges

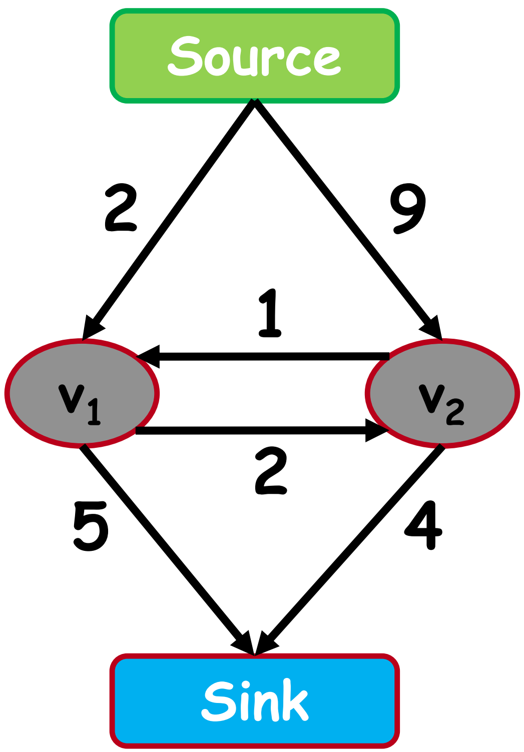
U: maximum edge weight

Algorithms assume non-negative edge weights

Slide credit: Andrew Goldberg

Maxflow Algorithms

Flow = 0



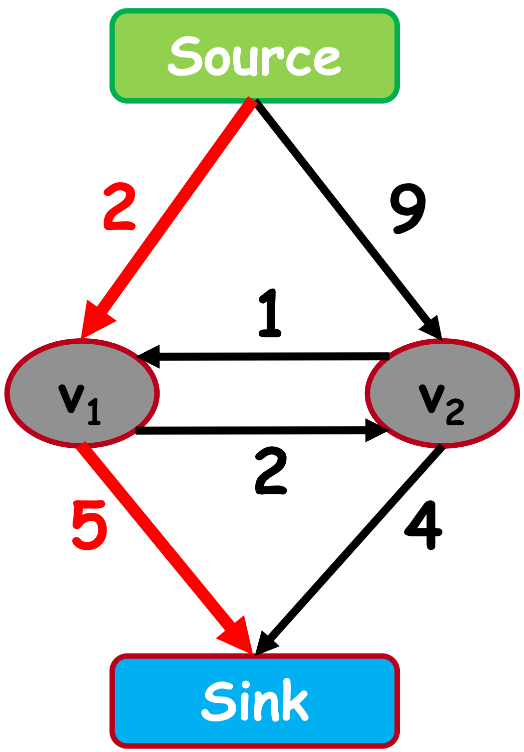
Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

Maxflow Algorithms

Flow = 0



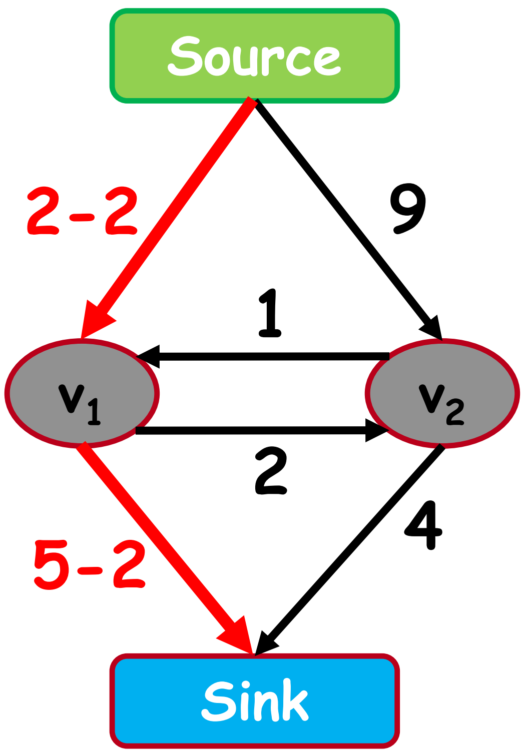
Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

Maxflow Algorithms

Flow = 0 + 2



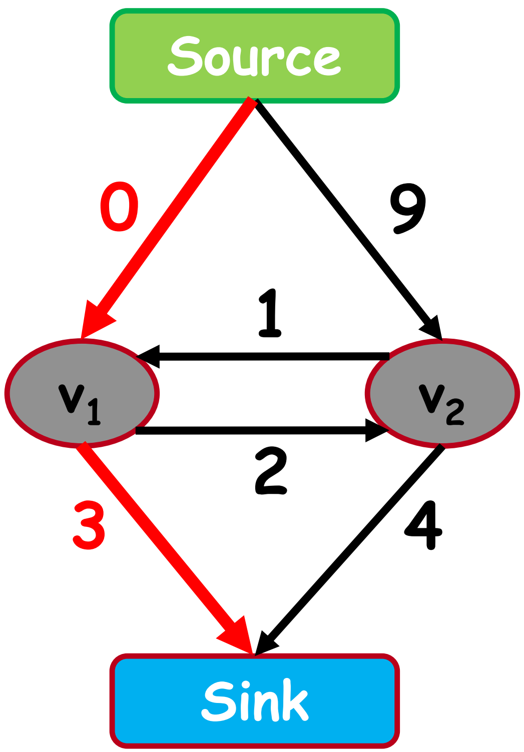
Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

Maxflow Algorithms

Flow = 2



Augmenting Path Based Algorithms

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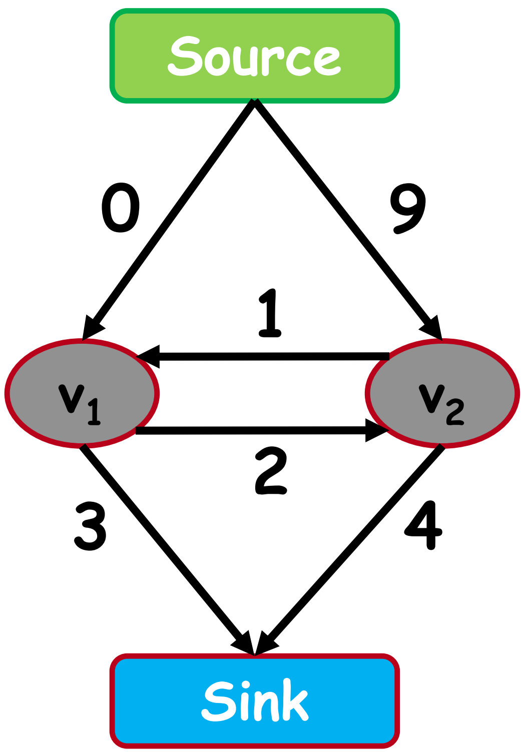
Algorithms assume non-negative capacity

Maxflow Algorithms

Flow = 2



Augmenting Path Based Algorithms

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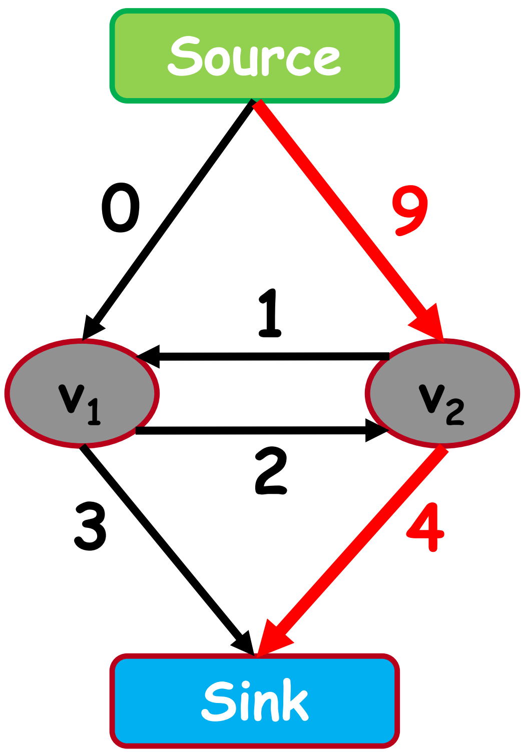
Algorithms assume non-negative capacity

Slide credit: Pushmeet Kohli

Maxflow Algorithms

Flow = 2



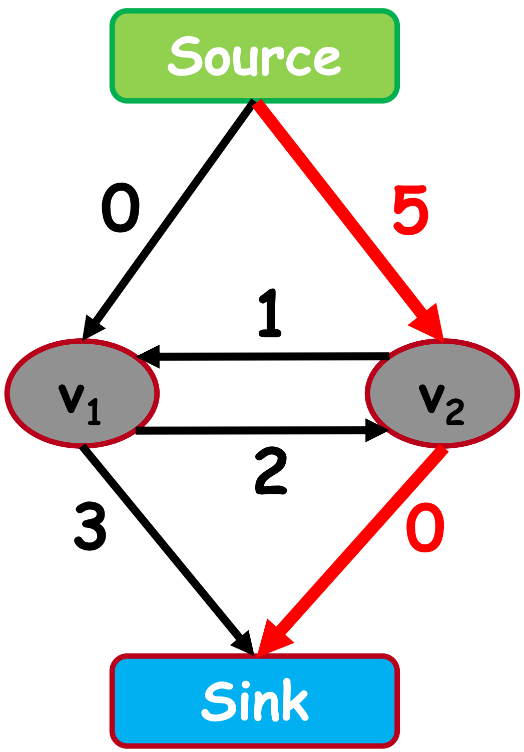
Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

Maxflow Algorithms

Flow = 2 + 4



Augmenting Path Based Algorithms

1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

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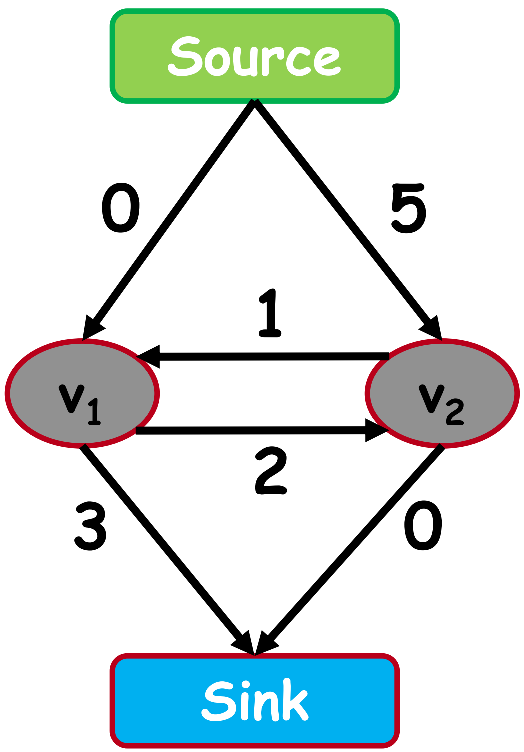
Maxflow Algorithms

Flow = 6



Augmenting Path Based Algorithms

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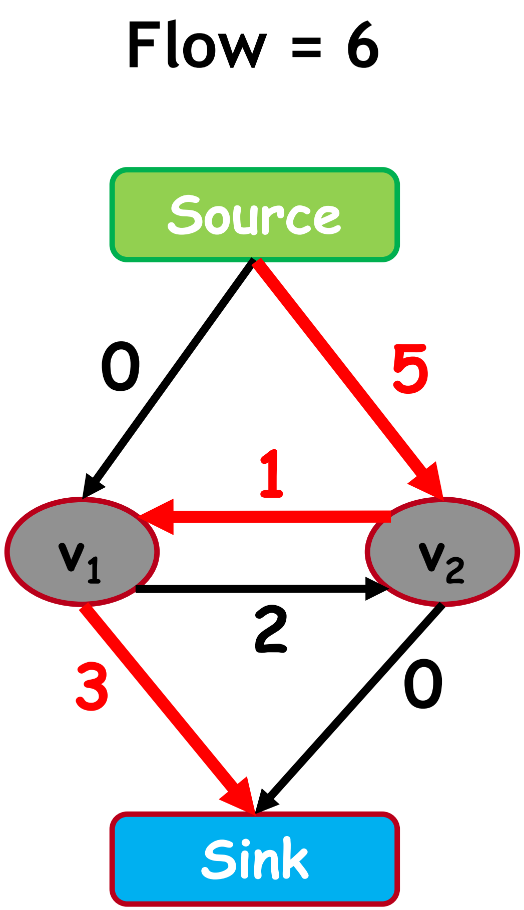
Algorithms assume non-negative capacity

Maxflow Algorithms



Augmenting Path Based Algorithms

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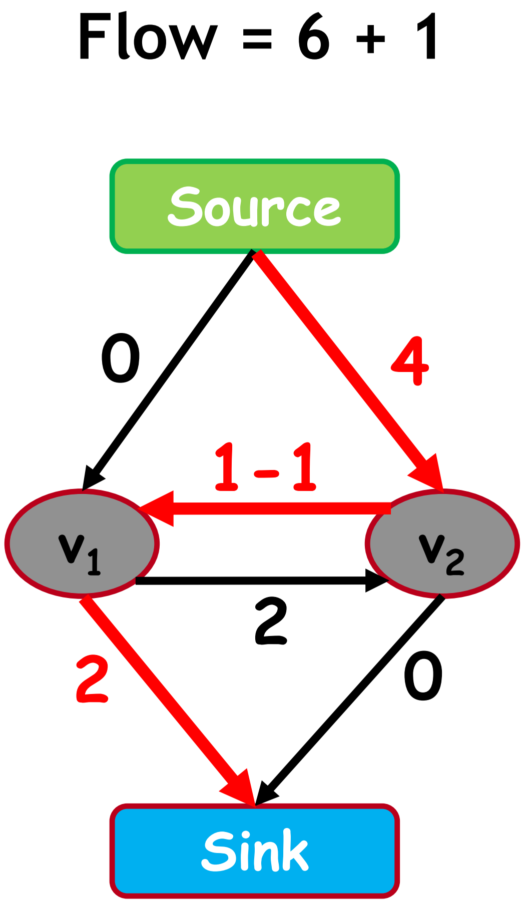
Algorithms assume non-negative capacity

Maxflow Algorithms



Augmenting Path Based Algorithms

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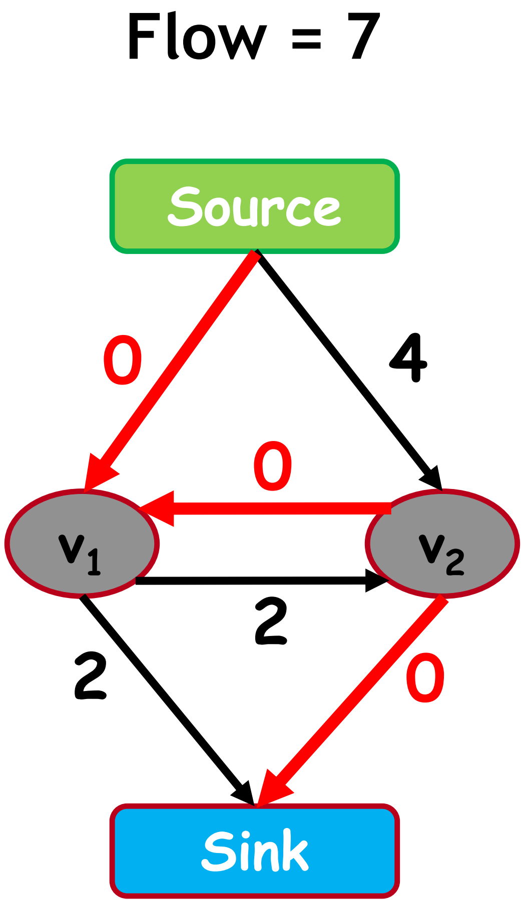
Algorithms assume non-negative capacity

Maxflow Algorithms



Augmenting Path Based Algorithms

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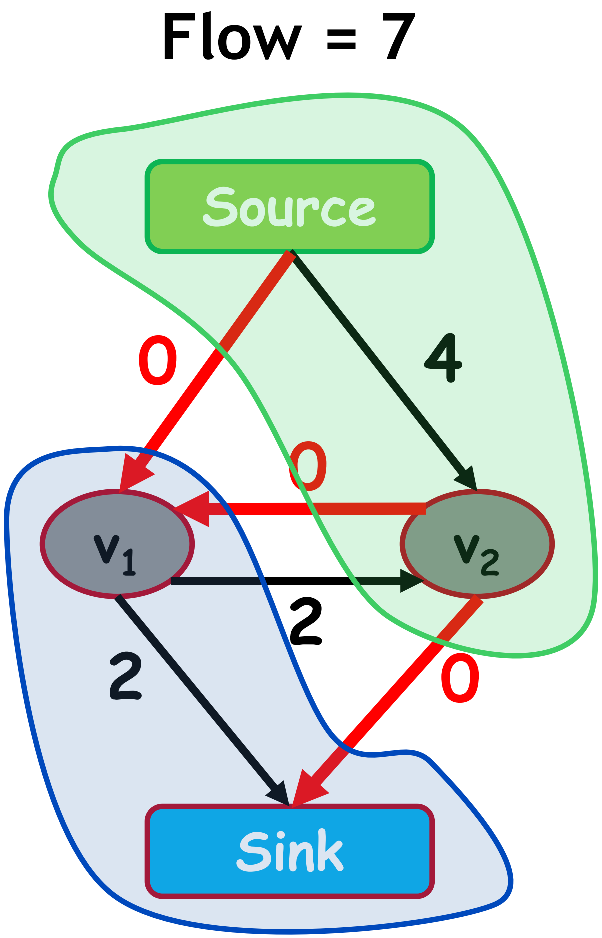
Maxflow Algorithms



Augmenting Path Based Algorithms

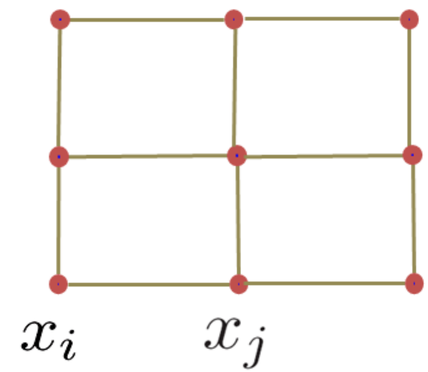
1. Find path from source to sink with positive capacity
2. Push maximum possible flow through this path
3. Repeat until no path can be found

Algorithms assume non-negative capacity

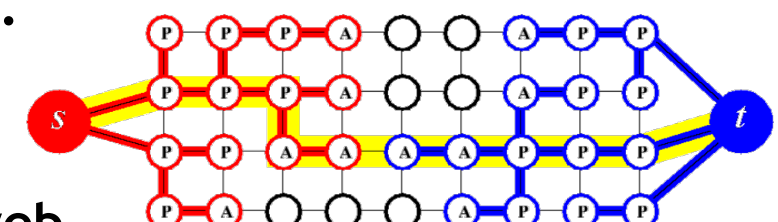
Slide credit: Pushmeet Kohli

Applications: Maxflow in Computer Vision

- Specialized algorithms for vision problems
  - Grid graphs
  - Low connectivity (m ~ O(n))



- Dual search tree augmenting path algorithm [Boykov and Kolmogorov PAMI 2004]
  - Finds approximate shortest augmenting paths efficiently.
  - High worst-case time complexity.
  - Empirically outperforms other algorithms on vision problems.
  - Efficient code available on the web http://www.cs.ucl.ac.uk/staff/V.Kolmogorov/software.html



When Can s-t Graph Cuts Be Applied?

Unary potentials Pairwise potentials

$$E(L) = \sum_{p} E_p(L_p) + \sum_{pq \in N} E(L_p, L_q)$$

t-links n-links

$$L_p \in \{s, t\}$$

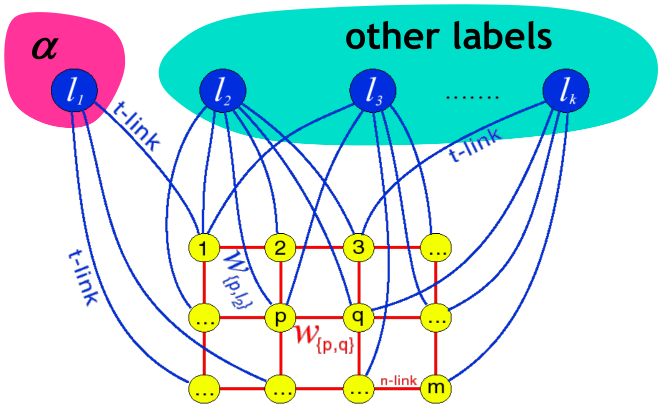
- s-t graph cuts can only globally minimize binary energies that are submodular. [Boros & Hummer, 2002, Kolmogorov & Zabih, 2004]

E(L) can be minimized by *s-t* graph cuts	$\Leftrightarrow$	$E(s,s) + E(t,t) \leq E(s,t) + E(t,s)$
		Submodularity ("convexity")

- Submodularity is the discrete equivalent to convexity.
  - Implies that every local energy minimum is a global minimum.
  - $\Rightarrow$ Solution will be globally optimal.

Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case



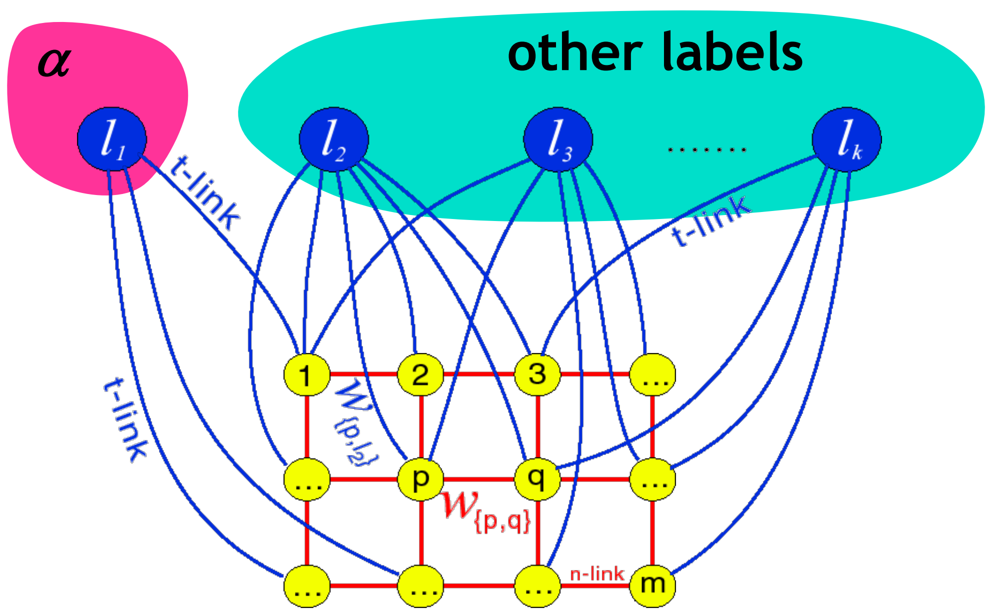
- Applications
  - Interactive segmentation

Dealing with Non-Binary Cases

- Limitation to binary energies is often a nuisance.
  $\Rightarrow$ E.g. binary segmentation only…
- We would like to solve also multi-label problems.
  - The bad news: Problem is NP-hard with 3 or more labels!
- There exist some approximation algorithms which extend graph cuts to the multi-label case:
  - α-Expansion
  - $\alpha\beta$-Swap
- They are no longer guaranteed to return the globally optimal result.
  - But α-Expansion has a guaranteed approximation quality (2-approx) and converges in a few iterations.

α-Expansion Move

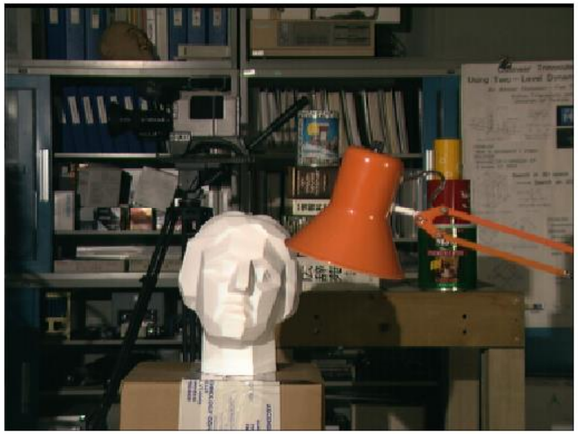
- **Basic idea:**
  - Break multi-way cut computation into a sequence of binary s-t cuts.

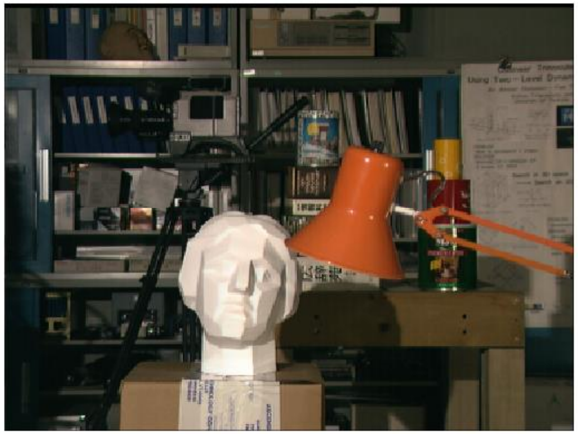


α-Expansion Algorithm

1. Start with any initial solution
2. For each label "α" in any (e.g. random) order:
    1. Compute optimal α-expansion move (s-t graph cuts).
    2. Decline the move if there is no energy decrease.
3. Stop when no expansion move would decrease energy.

Example: Stereo Vision





Original pair of “stereo” images

Depth map

α-Expansion Moves

- In each α-expansion a given label "α" grabs space from other labels

initial solution

-expansion

-expansion

-expansion

-expansion

-expansion

-expansion

-expansion

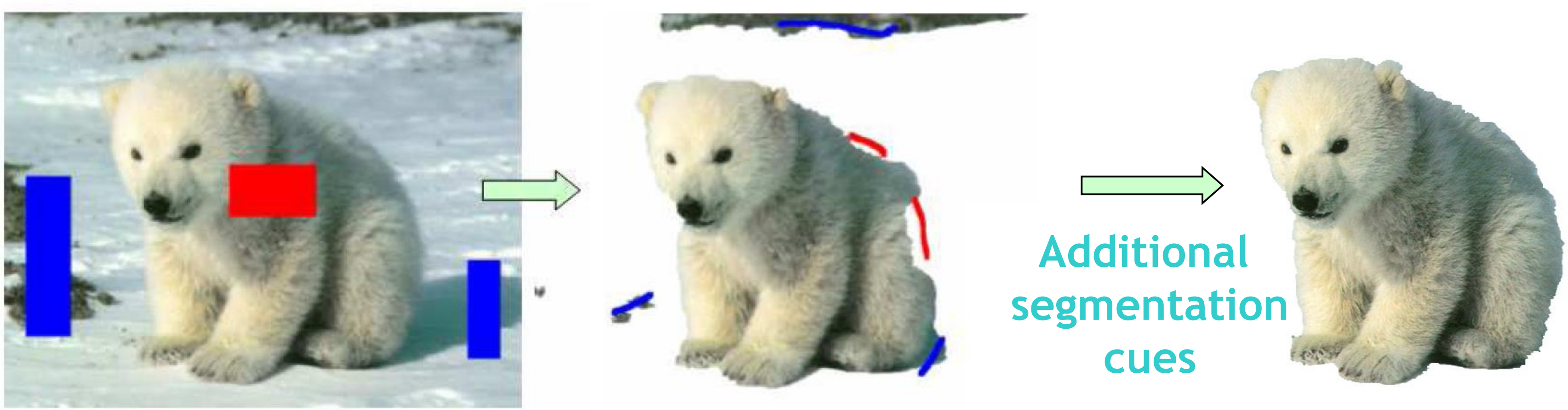
For each move, we choose the expansion that gives the largest decrease in the energy: ⇒ binary optimization problem

Topics of This Lecture

- Segmentation as Energy Minimization
  - Markov Random Fields
  - Energy formulation
- Graph cuts for image segmentation
  - Basic idea
  - s-t Mincut algorithm
  - Extension to non-binary case
- **Applications**
  - **Interactive segmentation**

GraphCut Applications: “GrabCut”

- Interactive Image Segmentation [Boykov & Jolly, ICCV’01]
  - Rough region cues sufficient
  - Segmentation boundary can be extracted from edges
- Procedure
  - User marks foreground and background regions with a brush.
  - This is used to create an initial segmentation
    which can then be corrected by additional brush strokes.

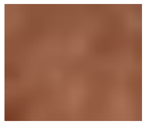


User segmentation cues

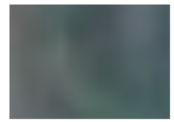
Slide credit: Matthieu Bray

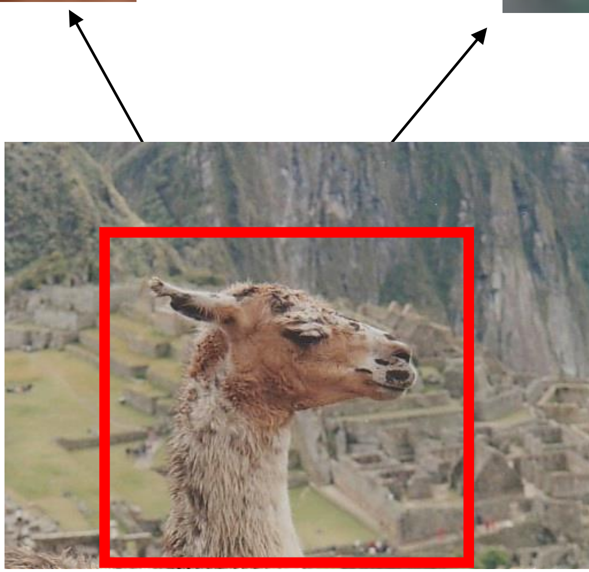
GrabCut: Data Model

Foreground color



Background color





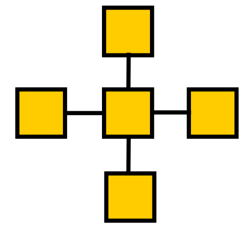
Global optimum of the energy

- **Obtained from interactive user input**
  - User marks foreground and background regions with a brush
  - Alternatively, user can specify a bounding box

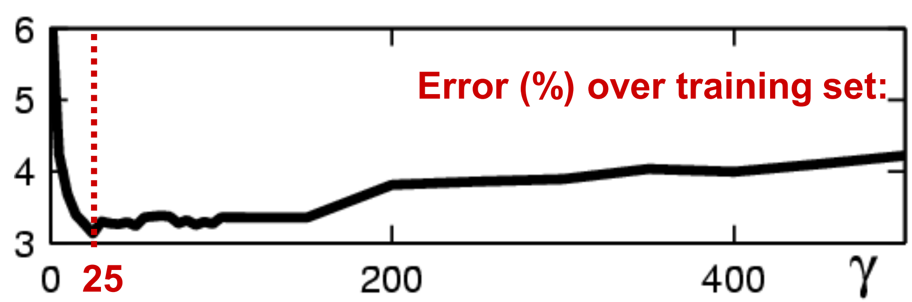
GrabCut: Coherence Model

- **An object is a coherent set of pixels:**

$$\psi(x,y) = \gamma \sum_{(m,n)\in C} \delta[x_n \neq x_m] e^{-\beta \|y_m - y_n\|^2}$$

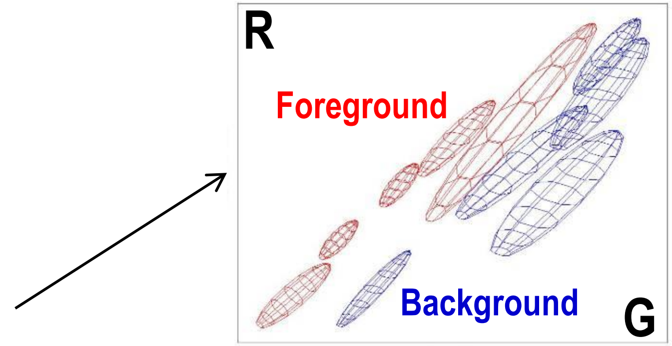


How to choose γ?

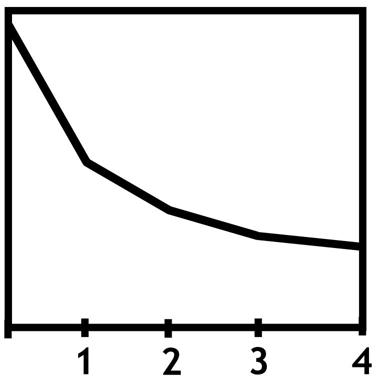


Iterated Graph Cuts

Result



Color model
(Mixture of Gaussians)



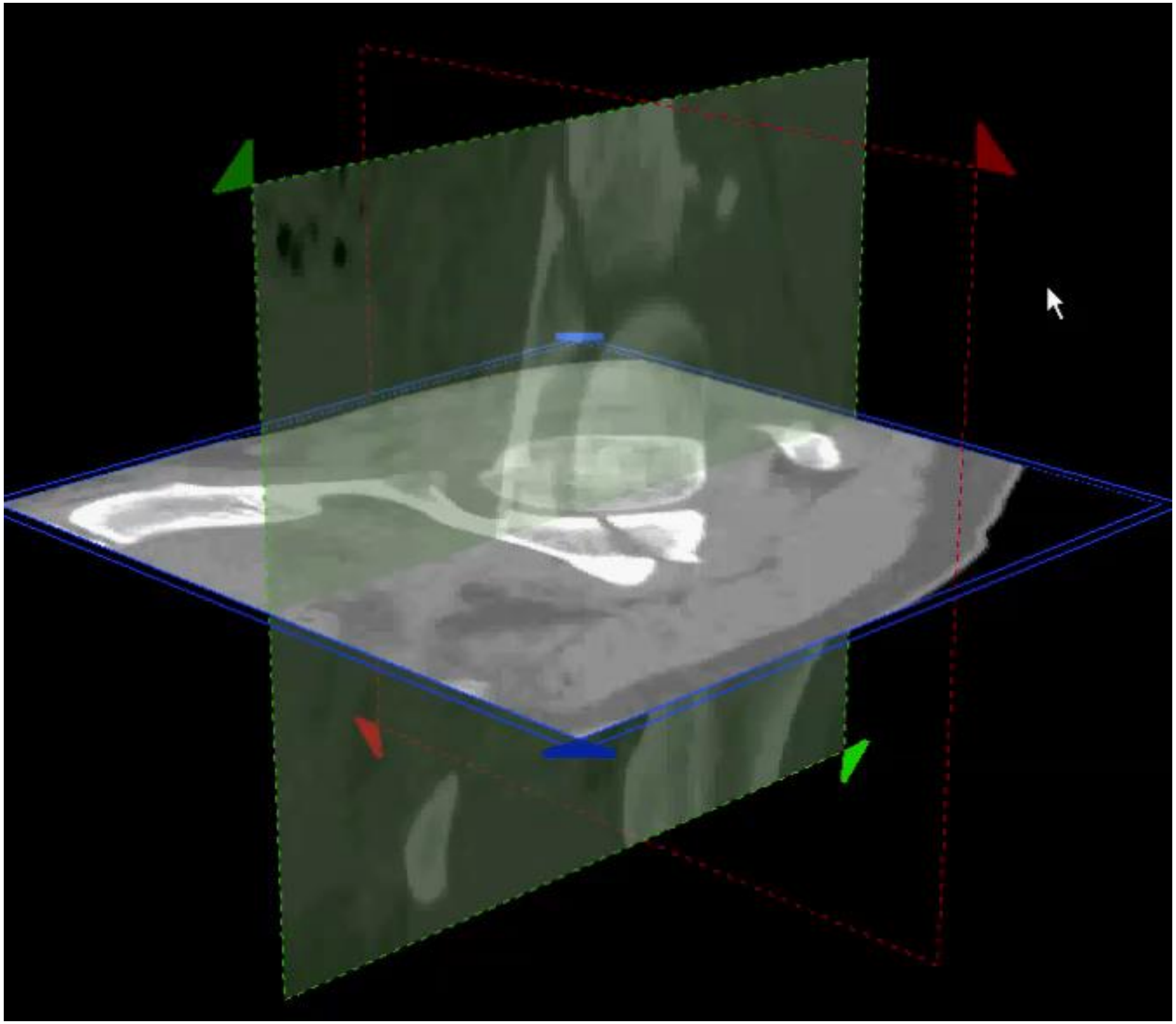
Energy after
each iteration

GrabCut: Example Results

- ***This is included in the newest version of MS Office!***

Image source: Carsten Rother

Applications: Interactive 3D Segmentation



Slide credit: Yuri Boykov

[Y. Boykov, V. Kolmogorov, ICCV'03]

Summary: Graph Cuts Segmentation

- **Pros**
  - Powerful technique, based on probabilistic model (MRF).
  - Applicable for a wide range of problems.
  - Very efficient algorithms available for vision problems.
  - Becoming a de-facto standard for many segmentation tasks.
- **Cons/Issues**
  - Graph cuts can only solve a limited class of models
    - Submodular energy functions
    - Can capture only part of the expressiveness of MRFs
  - Only approximate algorithms available for multi-label case

References and Further Reading

- A gentle introduction to Graph Cuts can be found in the following paper:
  - Y. Boykov, O. Veksler, Graph Cuts in Vision and Graphics: Theories and Applications. In *Handbook of Mathematical Models in Computer Vision*, edited by N. Paragios, Y. Chen and O. Faugeras, Springer, 2006.
- Read how the interactive segmentation is realized in MS Office 2010
  - C. Rother, V. Kolmogorov, Y. Boykov, A. Blake, Interactive Foreground Extraction using Graph Cut, Microsoft Research Tech Report MSR-TR-2011-46, March 2011
- Try the GraphCut implementation at

  http://www.cs.ucl.ac.uk/staff/V.Kolmogorov/software.html