Theory of Hybrid Systems Informatik 2
RWTH AACHEN UNIVERSITY

Satisfiability Checking - WS 2016/2017

Series 3

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Exercise 1

Consider the following formula:

$$\begin{array}{llllll} c_1 : (\neg b \vee c \vee \neg d) & \wedge & c_2 : (\neg b \vee d) & \wedge & c_3 : (\neg c \vee \neg d) & \wedge \\ c_4 : (\neg a \vee c) & \wedge & c_5 : (a \vee b) & \wedge & c_6 : (\neg a \vee \neg c) & \wedge \\ c_7 : (a \vee \neg c \vee \neg d) & & & & & \end{array}$$

a) Give valid initial watch lists for each literal occurring in the formula. Note that there are several possibilities to do so.

b) Draw the implication graph for the initial decision using the variable state independent decaying sum (VSIDS) decision heuristic In the case that the activities of the literals are the same, choose the alphabetically smallest variable first (a<b<c<d) and assign it by default to false. Is there a conflict? If so, apply conflict resolution to the conflict which occurred, i.e., use resolution on the conflict clause and the clauses that implied the current assignments, in reverse order back until the first unique implication point is reached. Furthermore, answer the following questions:

   i) Which asserting clause is generated?

   ii) Which assignments would be undone using conflict-driven non-chronological backtracking?

   iii) What does the DPLL algorithm do after backtracking?

c) Give the watch lists resulting from the initial decision, and also the lists resulting from the propagation of the first implication drawn in part b).

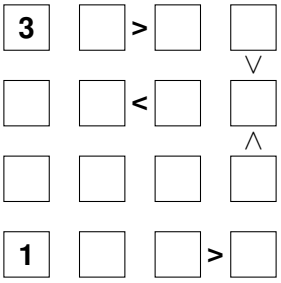
d) Is the formula satisfiable? If so, give a satisfying assignment. Otherwise, give an unsatisfiable core, i.e., a preferably small subset of the original set of clauses which is unsatisfiable.

Exercise 2

Consider the game **Unequal** having the following rules:
You have an $n \times n$ square grid; each square may contain a number from 1 to n, and some squares have clue signs between them. Your aim is to fully populate the grid with numbers such that

- each row contains only one occurrence of each number,
- each column contains only one occurrence of each number,
- and all the clue signs are satisfied.

Abbildung 1: An example starting instance of Unequal.



a) Formulate the rules of Unequal for an arbitrary square grid in propositional logic. (With no clue signs between the grids and no square fixed to a value as in the example.)

b) Formulate the starting instance given by Figure 1 in propositional logic such that the resulting formula is satisfiable iff this instance has a solution according to the game's rules.

c) If the formula is not yet in CNF, convert it into CNF.

d) Transfer the formula into the standard SAT input format (DIMACS) and let MiniSat solve it. Is the formula satisfiable?