Satisfiability Checking SAT-Solving Example II

Prof. Dr. Erika Ábrahám

RWTH Aachen University Informatik 2 LuFG Theory of Hybrid Systems

WS 16/17

SMT-solving example

- We use enumeration and propagation in DPLL-style for the search, and CDCL-style conflict resolution for backtracking.
- We use watched literals to speed up propagation.
- We use VSIDS as variable ordering heuristics and assign the value false to decision variables.
- In VSIDS, to order variables with the same activity value, we use the lexicographic order; in our example this will be a < b < c < d.

$$c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_5: (a \lor \neg c \lor d) \land c_6: (\neg c \lor \neg d)$$

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_5: (a \lor \neg c \lor d) \land c_6: (\neg c \lor \neg d)
```

- a :
- $\neg a$:
 - b :
- $\neg b$:
 - *c* :
- $\neg c$:
 - d :
- ¬d :

```
c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (b \vee c) \wedge c_3: (\neg a \vee b \vee \neg c) \wedge c_4: (a \vee \neg b \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
```

- $a: c_1$
- $\neg a$:
 - $b: c_1$
- $\neg b$:
 - *c* :
- $\neg c$:
- d :
- $\neg d$:

```
c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee b \vee \neg c) \wedge c_4: (a \vee \neg b \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
```

Watch lists:

 $a: c_1$ $\neg a:$ $b: c_1, c_2$ $\neg b:$ $c: c_2$ $\neg c:$ d: $\neg d:$

```
c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \neg b \vee c) \wedge c_5: (\underline{a} \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
```

```
a: c_1
\neg a: c_3
b: c_1, c_2, c_3
\neg b:
c: c_2
\neg c:
d:
\neg d:
```

```
c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
```

```
a: c_1, c_4
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c:
d:
\neg d:
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\neg c \vee \neg d)$$

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5
d: \neg d: c_1, c_2, c_3
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5, c_6
d:
\neg d: c_6
```

Example – Activities

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

$$a: c_1, c_4, c_5$$
 $\neg a: c_3$
 $b: c_1, c_2, c_3$
 $\neg b: c_4$
 $c: c_2$

 $\neg c: c_5, c_6$

d : ¬d : c₆ Activities (increment=1):

a 0 b 0 c 0

d 0

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

$$a: c_1, c_4, c_5$$

 $\neg a: c_3$

$$b: c_1, c_2, c_3$$

$$\neg b$$
: c_4

$$c: c_2$$

$$\neg c$$
: c_5, c_6

d :

$$\neg d$$
: c_6

Activities Trail: (increment=1):

a 0

b (

c (

d 0

```
c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

Watch lists:

Decide $\neg a$

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5, c_6
d:
\neg d: c_6
```

```
Activities Trail: (increment=1): DL1: \neg a: NULL a 0 b 0 c 0 d 0
```

```
c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

Watch lists:

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5, c_6
d:
\neg d: c_6
```

Propagate $\neg a$ in

```
Activities Trail: (increment=1): DL1: \neg a: NULL a 0 b 0 c 0 d 0
```

```
c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_4
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                     Activities
                                                                                   Trail:
                                                     (increment=1):
            a: c_1, c_4, c_5
                                                                                           DL1: \neg a: NULL
          \neg a: c_3
                                                                     0
                                                               а
            b: c_1, c_2, c_3
          \neg b: c_4
           c: c_2
          \neg c: c_5, c_6
          d :
          \neg d: c_6
  Propagate \neg a in c_1: (a \lor b \lor \neg c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                              Activities
                                                                        Trail:
                                              (increment=1):
          a: \mathcal{C}_1, \mathcal{C}_4, \mathcal{C}_5
                                                                               DL1: \neg a: NULL
        \neg a: c_3
                                                       a 0
          b: c_1, c_2, c_3
        \neg b: c_4
         C: C2
        \neg c: c_5, c_6, c_1
         d :
        \neg d: c_6
 Propagate \neg a in c_1: (a \lor b \lor \neg c) \rightarrow (a \lor b \lor \neg c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                             Activities
                                                                       Trail:
                                             (increment=1):
          a: \mathcal{C}_1, \mathcal{C}_4, \mathcal{C}_5
                                                                             DL1: \neg a: NULL
        \neg a: c_3
                                                      a 0
          b: c_1, c_2, c_3
        \neg b: c_4
         C: C_2
        \neg c: c_5, c_6, c_1
        d :
        \neg d: c_6
 Propagate \neg a in c_4: (a \lor \neg b \lor c)
```

 $\neg d$: c_6

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                              Activities
                                                                         Trail:
                                              (increment=1):
          a: \mathcal{G}_1, \mathcal{G}_4, \mathcal{C}_5
                                                                               DL1: \neg a: NULL
        \neg a: c_3
                                                       a 0
          b: c_1, c_2, c_3
        \neg b: c_4
         c: c_2, c_4
        \neg c: c_5, c_6, c_1
         d :
```

Propagate $\neg a$ in $c_4: (a \lor \neg b \lor c) \rightarrow (a \lor \neg b \lor c)$

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                             Activities
                                                                       Trail:
                                             (increment=1):
          a: \mathcal{G}_1, \mathcal{G}_4, \mathcal{C}_5
                                                                             DL1: \neg a: NULL
        \neg a: c_3
                                                      a 0
          b: c_1, c_2, c_3
        \neg b: c_4
         C: C_2, C_4
        \neg c: c_5, c_6, c_1
         d :
        \neg d: c_6
 Propagate \neg a in c_5: (a \lor \neg c \lor d)
```

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:
                                    Activities
                                                        Trail:
                                    (increment=1):
        a: 91,94,95
                                                              DL1: \neg a: NULL
      \neg a: c_3
                                           a 0
        b: c_1, c_2, c_3
      \neg b: c_4
       c: c_2, c_4
      \neg c: c_5, c_6, c_1
       d: c5
      \neg d: c_6
 Propagate \neg a in c_5: (a \lor \neg c \lor d) \rightarrow (a \lor \neg c \lor d)
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:

a:
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6

Decide $\neg b$

```
Activities
(increment=1):

a 0
b 0
c 0
d 0
```

```
Trail:

DL1: \neg a: NULL

DL2: \neg b: NULL
```

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

```
a:
\neg a:
c_3
b:
c_1, c_2, c_3
\neg b:
c_4
c:
c_2, c_4
\neg c:
c_5, c_6, c_1
```

 $d: c_5$ $\neg d: c_6$

Propagate $\neg b$ in

```
Activities
(increment=1):

a 0
b 0
c 0
d 0
```

DL1: ¬a: NULL

Trail:

DL2: $\neg b$: NULL

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg \underline{c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_4
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                          Activities
                                                                                           Trail:
                                                          (increment=1):
                                                                                                   DL1: \neg a: NULL
             a :
                                                                                                   DL2: \neg b: NULL
           \neg a: c_3
                                                                     а
            b: c_1, c_2, c_3
          \neg b: c_4
             c: c_2, c_4
          \neg c: c_5, c_6, c_1
           d: c5
          \neg d: c_6
 Propagate \neg b in c_1: (\underbrace{a} \lor \underline{b}, \lor \underline{\neg c})
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                          Activities
                                                                  Trail:
                                          (increment=1):
                                                                        DL1: \neg a: NULL
          a :
                                                                        DL2: \neg b: NULL
        \neg a: c_3
                                                  a 0
         b: c_1, c_2, c_3
                                                  b 0
                                                                                  \neg c : c_1
        \neg b: c_4
         c: c_2, c_4
        \neg c: c_5, c_6, c_1
        d: c5
        \neg d: c_6
  Propagate \neg b in c_1: (a \lor b \lor \neg c) \rightarrow assign \neg c
```

 $d: c_5$ $\neg d: c_6$

Propagate $\neg b$ in $c_2:(b \lor c)$

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg \underline{c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_4
  c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                       Activities
                                                                                       Trail:
                                                        (increment=1):
                                                                                               DL1: \neg a: NULL
             a :
                                                                                               DL2: \neg b: NULL
          \neg a: c_3
                                                                        0
                                                                  a
            b: c_1, c_2, c_3
                                                                  b 0
                                                                                                           \neg c : c_1
          \neg b: c_4
                                                                  c 0
            c: c_2, c_4
          \neg c: c_5, c_6, c_1
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                          Activities
                                                                  Trail:
                                          (increment=1):
                                                                        DL1: \neg a: NULL
          a :
                                                                        DL2: \neg b: NULL
        \neg a: c_3
                                                  a 0
         b: c_1, c_2, c_3
                                                                                 \neg c : c_1
        \neg b: c_4
         c: c_2, c_4
        \neg c: c_5, c_6, c_1
        d: c5
        \neg d: c_6
```

Propagate $\neg b$ in $c_2: (\underline{b} \lor \underline{c}) \to \text{$\rlap/$$} conflict!$

```
c_1: (a \vee \underline{b} \vee \overline{c} \land c_2: (\underline{b} \vee \underline{c}) \land c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \land c_4: (a \vee \underline{\neg b} \vee \underline{c}) \land c_5: (a \vee \underline{\neg c} \vee \underline{d}) \land c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:
                                    Activities
                                                        Trail:
                                    (increment=1):
        a :
      \neg a: c_3
        b: c_1, c_2, c_3
      \neg b: c_4
        c: c_2, c_4
      \neg c: c_5, c_6, c_1
       d: c5
      \neg d: c_6
 Conflict resolution:
```

DL1: $\neg a$: NULL DL2: $\neg b$: NULL $\neg c$: c_1

```
c_{1}: (a \lor \underline{b} \lor \neg \underline{c}) \land c_{2}: (\underline{b} \lor \underline{c}) \land c_{3}: (\neg \underline{a} \lor \underline{b} \lor \neg c) \land c_{4}: (a \lor \neg \underline{b} \lor \underline{c}) \land c_{5}: (a \lor \neg \underline{c} \lor \underline{d}) \land c_{6}: (\neg \underline{c} \lor \neg \underline{d})
\text{Watch lists:} \qquad \text{Activities} \qquad \text{Trail:} \\ (\text{increment} = 2):
```

```
a:
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6

a Ø 1 b Ø 1 c Ø 1 d 0 DL1: $\neg a : \text{NULL}$ DL2: $\neg b : \text{NULL}$ $\neg c : c_1$

Conflict resolution: $\frac{c_2:(b\lor c) \ c_1:(a\lor b\lor \neg c)}{c_7:(a\lor b)}$

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Add conflict clause

```
Activities Trail: (increment=2): \qquad \qquad DL1: \quad \neg a: NULL \\ a \quad 1 \qquad \qquad DL2: \quad \neg b: NULL \\ b \quad 1 \qquad \qquad \neg c: c_1 \\ c \quad 1 \\ d \quad 0
```

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:

a: c_7

\neg a: c_3

b: c_1, c_2, c_3, c_7

\neg b: c_4

c: c_2, c_4

\neg c: c_5, c_6, c_1

d: c_5
```

 $\neg d$: c_6 Backtrack to DL1

```
Activities (increment=2):

a 1
b 1
c 1
d 0
```

```
DL1: ¬a:NULL
DL2: ¬b:NULL
¬c:-cī
```

Trail:

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:

a: c_7

\neg a: c_3

b: c_1, c_2, c_3, c_7

\neg b: c_4

c: c_2, c_4

\neg c: c_5, c_6, c_1

d: c_5
```

 $\neg d$: c_6 Assign **b** at DL1 by c_7

```
Activities Trail: (increment=2): DL1: \neg a: NULL b: c_7
c: 1
d: 0
```

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Propagate b in

```
Activities Trail: (increment=2): DL1: \neg a: NULL b: c_7 c: 1 d: 0
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
Watch lists:
                                          Activities
                                                                 Trail:
                                          (increment=2):
                                                                        DL1: \neg a: NULL
          a: C7
                                                                                   b : c7
        \neg a: c_3
                                                  a 1
         b: c_1, c_2, c_3, c_7
        \neg b: c_4
         c: c_2, c_4
        \neg c: c_5, c_6, c_1
        d: c5
        \neg d: c_6
  Propagate b in c_4: (a \vee \neg b \vee c)
```

 $\neg d$: c_6

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
Watch lists:
                                           Activities
                                                                   Trail:
                                           (increment=2):
                                                                         DL1: \neg a: NULL
          a: C7
        \neg a: c_3
                                                                                     b : c<sub>7</sub>
                                                   a 1
          b: c_1, c_2, c_3, c_7
                                                                                     C:C_4
        \neg b: c_4
                                                   c 1
         c: c_2, c_4
        \neg c: c_5, c_6, c_1
        d: c5
```

Propagate b in $c_4: (a \vee \neg b \vee c) \rightarrow assign c$

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Propagate c in

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                  Activities
                                                     Trail:
                                  (increment=2):
                                                          DL1: \neg a: NULL
        a: C7
      \neg a: c_3
                                                                    b : c7
                                        a 1
       b: c_1, c_2, c_3, c_7
                                                                    C:C_4
      \neg b: c_4
       c: c_2, c_4
      \neg c: c_5, c_6, c_1
      d: c5
      \neg d: c_6
 Propagate c in c_5: (a \lor \neg c \lor d)
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                 Activities
                                                     Trail:
                                  (increment=2):
                                                          DL1: \neg a: NULL
       a: C7
      \neg a: c_3
                                                                   b : c7
                                        a 1
       b: c_1, c_2, c_3, c_7
                                                                   C:C_{4}
      \neg b: c_4
                                                                   d:c_5
                                        c 1
       c: c_2, c_4
      \neg c: c_5, c_6, c_1
      d: c5
      \neg d: c_6
 Propagate c in c_5: (a \lor \neg c \lor d) \to assign d
```

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                 Activities
                                                    Trail:
                                 (increment=2):
                                                         DL1: \neg a: NULL
       a: C7
      \neg a: c_3
                                                                   b : c7
                                        a 1
       b: c_1, c_2, c_3, c_7
                                                                   C:C4
      \neg b: c_4
                                                                   d:c_5
                                        c 1
       c: c_2, c_4
      \neg c: c_5, c_6, c_1
      d: c5
      \neg d: c_6
 Propagate c in c_6: (\neg c \lor \neg d)
```

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                 Activities
                                                     Trail:
                                  (increment=2):
                                                          DL1: \neg a: NULL
       a: C7
      \neg a: c_3
                                                                   b : c7
                                        a 1
       b: c_1, c_2, c_3, c_7
                                                                   C:C4
      \neg b: c_4
                                                                   d:c_5
                                        c 1
       c: c_2, c_4
      \neg c: c_5, c_6, c_1
      d: c5
      \neg d: c_6
 Propagate c in c_6: (\neg c \lor \neg d) \to f conflict!
```

$$c_1: (a \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{a}) \wedge c_6: (\underline{\neg c} \vee \underline{a}) \wedge c_7: (\underline{a} \vee \underline{b})$$

```
Watch lists:
                                   Activities
                                                       Trail:
                                   (increment=2):
                                                            DL1: \neg a: NULL
        a :
             C7
      \neg a: c_3
        b: c_1, c_2, c_3, c_7
      \neg b: c_4
        c: c_2, c_4
      \neg c: c_5, c_6, c_1
      d: c5
      \neg d: c_6
 Conflict resolution:
```

(a)

b : c₇

d : c₅

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                    Activities
                                                         Trail:
                                    (increment=3):
                                                              DL1: \neg a: NULL
        a :
              C7
      \neg a: c_3
                                                                        b : c<sub>7</sub>
                                           a 1/3
        b: c_1, c_2, c_3, c_7
                                           b 1/3
                                                                        C:C4
      \neg b: c_4
                                                                        d:c_5
                                           c 1/3
        c: c_2, c_4
                                           d Ø 2
      \neg c: c_5, c_6, c_1
       d: c5
      \neg d: c<sub>6</sub>
```

Conflict resolution: $\frac{c_6: (\neg c \lor \neg d) \ c_5: (}{(a \lor \neg c) \ c_4: (a}$

 $\frac{(a \vee \neg c) \ c_4 : (a \vee \neg b \vee c)}{(a \vee \neg b) \ c_7 : (a \vee b)}$

Satisfiability Checking — Prof. Dr. Erika Ábrahám (RWTH Aachen University)

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (\underline{a})
```

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Add conflict clause

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6 Backtrack to DL0

Activities (increment=3):

a 3
b 3
c 3
d 2

Trail:

DLT: Ja: NUTE

b: c7

c:e4

d:c5

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

```
Watch lists:

a: c_7

\neg a: c_3

b: c_1, c_2, c_3, c_7

\neg b: c_4

c: c_2, c_4

\neg c: c_5, c_6, c_1
```

 $d: c_5$ $\neg d: c_6$ Assign a at DL0 by c_8

```
Activities (increment=3):

a 3
b 3
c 3
d 2
```

```
Trail:

DL0: a: c<sub>8</sub>
```

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $d: c_5$ $\neg d: c_6$ Propagate a in

Activities Trail: (increment=3):

a 3b 3c 3

DL0: a:c₈

```
c_{1}: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_{2}: (\underline{b} \vee \underline{c}) \wedge c_{3}: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_{4}: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_{5}: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_{6}: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_{7}: (\underline{a} \vee \underline{b}) \wedge c_{8}: (a)
\text{Watch lists:} \qquad \text{Activities} \qquad \text{Trail:} 
(\text{increment=3}): \qquad \qquad \text{DL0:} \quad a: c_{8}
```

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

a 3 b 3 c 3 d 2

Propagate a in $c_3: (\underline{\neg a} \lor \underline{b} \lor \neg c)$

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

```
Watch lists:
                                    Activities
                                                         Trail:
                                     (increment=3):
        a: C7
                                                               DL0: a:c_8
      \neg a: \emptyset
                                           a 3
        b: c_1, c_2, c_3, c_7
      \neg b: c_4
                                            c 3
       c: c_2, c_4
      \neg c: c_5, c_6, c_1, c_3
       d: c5
      \neg d: c_6
 Propagate a in c_3: (\neg a \lor b \lor \neg c) \to (\neg a \lor b \lor \neg c)
```

```
c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

Watch lists:

```
a: c_7
\neg a:
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6

Decide \neg b
```

```
Activities (increment=3):

a 3
b 3
c 3
d 2
```

```
DL0: a: c_8
DL1: \neg b: \text{NULL}
```

Trail:

$$c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

Watch lists:

```
a: c_7
\neg a:
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Propagate $\neg b$ in

```
Activities
               Trail:
(increment=3):
     a 3
     c 3
```

```
DL0: a: c_8
DL1: \neg b: \text{NULL}
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b) \wedge c_8: (a)
Watch lists:
                                          Activities
                                                                  Trail:
                                          (increment=3):
                                                                        DL0: a:c_8
          a :
              C7
                                                                        DL1: \neg b: NULL
        \neg a :
                                                  a 3
         b: c_1, c_2, c_3, c_7
        \neg b: c_4
                                                  c 3
         c: c_2, c_4
        \neg c: c_5, c_6, c_1, c_3
        d: c5
        \neg d: c_6
```

Propagate $\neg b$ in $c_1:(a \lor b \lor \neg c)$

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land
 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b) \wedge c_8: (a)
Watch lists:
                                          Activities
                                                                   Trail:
                                           (increment=3):
                                                                         DL0: a:c_8
         a: c_7, c_1
                                                                         DL1: \neg b: NULL
        \neg a:
                                                   a 3
         b: \varsigma_1, c_2, c_3, c_7
        \neg b: c_4
                                                   c 3
         c: c_2, c_4
        \neg c: c_5, c_6, c_1, c_3
        d: c5
        \neg d: c_6
```

Propagate $\neg b$ in $c_1: (a \lor b \lor \neg c) \to (a \lor b \lor \neg c)$

```
c_{1}: (\underline{a} \lor \underline{b} \lor \underline{\neg c}) \land c_{2}: (\underline{b} \lor \underline{c}) \land c_{3}: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_{4}: (\underline{a} \lor \underline{\neg b} \lor \underline{c}) \land c_{5}: (\underline{a} \lor \underline{\neg c} \lor \underline{d}) \land c_{6}: (\underline{\neg c} \lor \underline{\neg d}) \land c_{7}: (\underline{a} \lor \underline{b}) \land c_{8}: (\underline{a})
Watch lists:

Activities \qquad Trail: (increment=3):
a : c_{7} c_{1} c_{1}
Dl 0: a : c_{9}
```

```
a: c_7, c_1
\neg a:
b: \varphi_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Propagate $\neg b$ in $c_2 : (\underline{b} \lor \underline{c})$

a 3b 3c 3d 2

DL0: $a: c_8$ DL1: $\neg b: \text{NULL}$

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \underline{\neg b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Watch lists:
                                 Activities
                                                    Trail:
                                 (increment=3):
                                                         DL0: a:c_8
       a: c_7, c_1
                                                         DL1: \neg b: NULL
      \neg a:
                                       a 3
       b: g_1, c_2, c_3, c_7
                                                                  C:C_2
      \neg b: c_4
                                       c 3
       C: C_2, C_4
      \neg c: c_5, c_6, c_1, c_3
      d: c5
      \neg d: c_6
 Propagate \neg b in c_2:(b \lor c) \to assign c
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Watch lists:
                                   Activities
                                                        Trail:
                                   (increment=3):
                                                             DL0: a:c_8
        a: c_7, c_1
                                                             DL1: \neg b: NULL
       \neg a:
                                          a 3
       b: \varsigma_1, c_2, c_3, c_7
                                                                       C:C_2
      \neg b: c_4
                                          c 3
       C: C_2, C_4
      \neg c: c_5, c_6, c_1, c_3
       d: c5
      \neg d: c_6
 Propagate \neg b in c_3: (\neg a \lor b \lor \neg c)
```

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a)
```

```
Watch lists:
                                      Activities
                                                           Trail:
                                      (increment=3):
                                                                 DL0: a:c_8
         a: c_7, c_1
                                                                 DL1: \neg b: NULL
       \neg a:
                                             a 3
        b: \varsigma_1, c_2, c_3, c_7
                                                                            C:C_2
       \neg b: c_4
                                             c 3
        C: C_2, C_4
       \neg c: c_5, c_6, c_1, c_3
       d: c5
       \neg d: c_6
 Propagate \neg b in c_3: (\neg a \lor b \lor \neg c) \to \text{$\frac{1}{2}$ conflict!}
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a)$$

Activities

(increment=3):

Watch lists:

```
a: c_7, c_1
\neg a:
 b: \varsigma_1, c_2, c_3, c_7
\neg b: c_4
 c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c5
```

 $\neg d$: c_6

a 3 c 3 Conflict resolution:

DL0: $a:c_8$ DL1: $\neg b$: NULL $C:C_2$

Trail:

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \underline{\neg b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Watch lists:
                                     Activities
                                                          Trail:
                                     (increment=4):
                                                                DL0: a:c_8
        a: c_7, c_1
                                                                DL1: \neg b: NULL
       \neg a:
                                            a 36
        b: c_1, c_2, c_3, c_7
                                            b 36
                                                                          C:C_2
      \neg b: c_4
                                            c 36
        c: c_2, c_4
                                            d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c5
      \neg d: c<sub>6</sub>
                           c_3:(\neg a \lor b \lor \neg c) \ c_2:(b \lor c)
 Conflict resolution:
```

 $c_0:(\neg a \lor b)$

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

```
a: c_7, c_1
\neg a: c_9
b: c_1, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
Add conflict clause
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

 $\neg d$: c_6 Backtrack to DL0

Activities Trail:
(increment=4):

a 6
b 6
c 6
d 2

DLO: a: c₈
DLT: >b:NUTT

c:e₂

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

Assign b by c_0

```
a: c_7, c_1
\neg a: c_9
b: c_1, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

$$a: c_7, c_1$$
 $\neg a: c_9$
 $b: c_2, c_3, c_7, c_9$
 $\neg b: c_4$
 $c: c_2, c_4$
 $\neg c: c_5, c_6, c_1, c_3$
 $d: c_5$

 $\neg d : c_6$ Propagate b in

```
Activities Trail:
(increment=4):

a \quad 6
b \quad 6
c \quad 6
d \quad 2

Trail:

DL0: a: c_8
b: c_9
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Watch lists:
                                  Activities
                                                     Trail:
                                  (increment=4):
                                                          DL0: a:c_8
       a: c_7, c_1
                                                                   b:c_9
      \neg a: c_9
                                        a 6
       b: c_2, c_3, c_7, c_9
      \neg b: c_4
                                        c 6
       C: C_2, C_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c5
      \neg d: c_6
 Propagate b in c_4: (a \vee \neg b \vee c)
```

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})
```

```
Watch lists:
                                     Activities
                                                           Trail:
                                     (increment=4):
                                                                 DL0: a:c_8
        a: c_7, c_1, c_4
                                                                           b:c_0
       \neg a: c_9
                                            a 6
        b: c_2, c_3, c_7, c_9
       \neg b: \varsigma_4
                                             c 6
        c: c_2, c_4
       \neg c: c_5, c_6, c_1, c_3
       d: c5
       \neg d: c_6
 Propagate b in c_4: (a \vee \neg b \vee c) \rightarrow (a \vee \neg b \vee c)
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

$$a: c_7, c_1, c_4$$
 $\neg a: c_9$
 $b: c_2, c_3, c_7, c_9$
 $\neg b:$
 $c: c_2, c_4$
 $\neg c: c_5, c_6, c_1, c_3$
 $d: c_5$
 $\neg d: c_6$

Decide $\neg c$

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

Watch lists:

$$a: c_7, c_1, c_4$$
 $\neg a: c_9$
 $b: c_2, c_3, c_7, c_9$
 $\neg b:$
 $c: c_2, c_4$
 $\neg c: c_5, c_6, c_1, c_3$
 $d: c_5$
 $\neg d: c_6$

Propagate $\neg c$ in

Activities Trail: (increment=4): DL0: $a:c_8$ a 6 DL1: $\neg c : NULL$ c 6 d 2

b : c9

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Watch lists:
                                 Activities
                                                     Trail:
                                 (increment=4):
                                                          DL0: a:c_8
       a: c_7, c_1, c_4
                                                                  b : c9
      \neg a: c_9
                                        a 6
                                                          DL1: \neg c : NULL
       b: c_2, c_3, c_7, c_9
      \neg b:
                                        c 6
       C: C_2, C_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c5
      \neg d: c_6
 Propagate \neg c in c_2:(b \lor c)
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Watch lists:
                                   Activities
                                                       Trail:
                                   (increment=4):
                                                            DL0: a:c_8
        a: c_7, c_1, c_4
                                                                     b : c9
      \neg a: c_9
                                         a 6
                                                            DL1: \neg c : NULL
        b: c_2, c_3, c_7, c_9
      \neg b:
                                          c 6
       c: c_2, c_4
      \neg c: c_5, c_6, c_1, c_3
      d: c5
      \neg d: c_6
 Propagate \neg c in c_{4}: (a \lor \neg b \lor c)
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

Decide $\neg d$

$$a: c_7, c_1, c_4$$
 $\neg a: c_9$
 $b: c_2, c_3, c_7, c_9$
 $\neg b:$
 $c: c_2, c_4$
 $\neg c: c_5, c_6, c_1, c_3$
 $d: c_5$
 $\neg d: c_6$

```
Activities Trail: (increment=4):

a 6
b 6
c 6
d 2
```

```
DL0: a: c_8

b: c_9

DL1: \neg c: NULL

DL2: \neg d: NULL
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

$$a: c_7, c_1, c_4$$
 $\neg a: c_9$
 $b: c_2, c_3, c_7, c_9$
 $\neg b:$
 $c: c_2, c_4$
 $\neg c: c_5, c_6, c_1, c_3$
 $d: c_5$
 $\neg d: c_6$

Propagate $\neg d$ in

Activities (increment=4):

a 6
b 6
c 6
d 2

DL0: $a: c_8$ $b: c_9$ DL1: $\neg c: NULL$ DL2: $\neg d: NULL$

Trail:

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})
```

```
Watch lists:
                                   Activities
                                                      Trail:
                                   (increment=4):
                                                            DL0: a:c_8
        a: c_7, c_1, c_4
      \neg a: c_9
                                                                     b : c9
                                         a 6
        b: c_2, c_3, c_7, c_9
                                                            DL1: \neg c : NULL
                                                            DI 2: \neg d: NULL
      \neg b:
                                         c 6
       c: c_2, c_4
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg d in c_5: (a \lor \neg c \lor d)
```

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

Watch lists:

$$a: c_7, c_1, c_4$$

$$\neg a: c_9$$

$$b: c_2, c_3, c_7, c_9$$

$$\neg b:$$

$$c: c_2, c_4$$

$$\neg c: c_5, c_6, c_1, c_3$$

$$d: c_5$$

$$\neg d: c_6$$

$$\Rightarrow SAT \quad a \quad b \quad \neg c \quad \neg d$$

Activities (increment=4):

a 6
b 6
c 6
d 2

DL0: $a: c_8$ $b: c_9$ DL1: $\neg c: NULL$ DL2: $\neg d: NULL$

Trail: