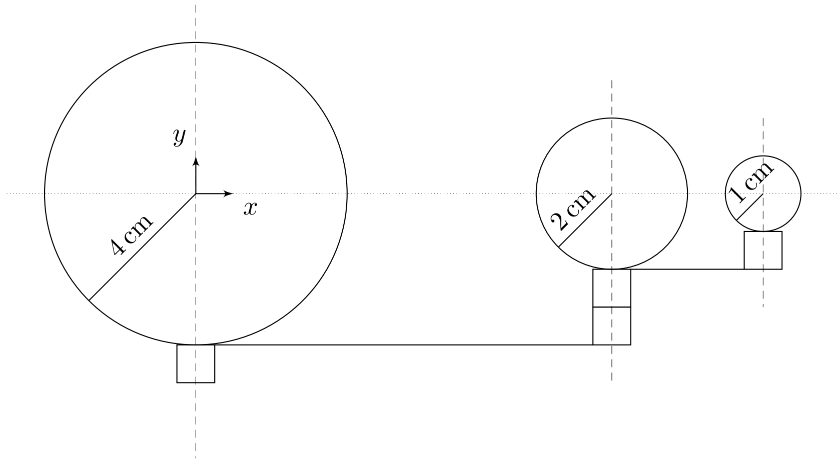
EXERCISE 2 — SOLUTION

1. Advanced Scene Graph

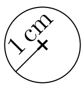
Consider the following scene:



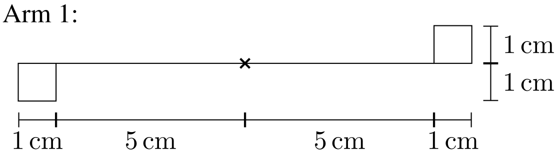
The spheres and arms can rotate around the dashed axes.

Use the following parts as your building blocks:

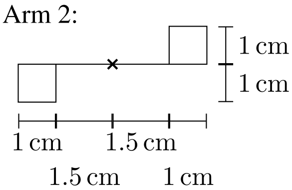
- Sphere:



- Arm 1:



- Arm 2:



The cross indicates the origin of the local coordinate frame.

(a) Create the individual transformation matrices in order to create the above scene. Do not scale the arms.

Solution

- Sun:
  - rotate in order to create days

$$\mathbf{R}_S = \cancel{\begin{bmatrix} \cos\omega_S t & -\sin\omega_S t & 0 & 0 \\ \sin\omega_S t & \cos\omega_S t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} \cos\omega_S t & 0 & \sin\omega_S t & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\omega_S t & 0 & \cos\omega_S t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Solution cont.

- scale to the correct size

$$\mathbf{S}_S = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- concatenate:

$$\mathbf{C} = \mathbf{S}_S \cdot \mathbf{R}_S$$

- Arm 1:
  - translate according to rotation pivot

$$\mathbf{T}_{A1} = \begin{bmatrix} 1 & 0 & 0 & -5.5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - rotate

$$\mathbf{R}_{A1} = \xcancel{\begin{bmatrix} \cos\omega_{A1}t & -\sin\omega_{A1}t & 0 & 0 \\ \sin\omega_{A1}t & \cos\omega_{A1}t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} \cos\omega_{A1}t & 0 & \sin\omega_{A1}t & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\omega_{A1}t & 0 & \cos\omega_{A1}t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - concatenate:

$$\mathbf{A}_1 = \mathbf{R}_{A1} \cdot \mathbf{T}_{A1}$$

- Earth: scale and then “glue to the end of arm 1”
  - rotate in order to create days

$$\mathbf{R}_E = \xcancel{\begin{bmatrix} \cos\omega_E t & -\sin\omega_E t & 0 & 0 \\ \sin\omega_E t & \cos\omega_E t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} \cos\omega_E t & 0 & \sin\omega_E t & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\omega_E t & 0 & \cos\omega_E t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - scale to the correct size

$$\mathbf{S}_E = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - translate to the end of arm 1, leave room for arm 2

$$\mathbf{T}_E = \begin{bmatrix} 1 & 0 & 0 & 5.5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - concatenate (we need to keep scale and rotation earth-local):

$$\mathbf{E}_G = \mathbf{A}_1 \cdot \mathbf{T}_E$$
$$\mathbf{E}_L = \mathbf{E}_G \cdot \mathbf{S}_E \cdot \mathbf{R}_E$$

- Arm 2: “glue to earth's south pole”

Solution cont.

- translate according to rotation pivot and earth's south pole

$$\mathbf{T}_{A2} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- rotate

$$\mathbf{R}_{A2} = \xcancel{\begin{bmatrix} \cos\omega_{A2}t & -\sin\omega_{A2}t & 0 & 0 \\ \sin\omega_{A2}t & \cos\omega_{A2}t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} \cos\omega_{A2}t & 0 & \sin\omega_{A2}t & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\omega_{A2}t & 0 & \cos\omega_{A2}t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- concatenate:

$$\mathbf{A}_2 = \mathbf{E}_G \cdot \mathbf{R}_{A2} \cdot \mathbf{T}_{A2}$$

- Moon: "glue to the end of arm 2", see earth
  - rotate in order to create days

$$\mathbf{R}_M = \xcancel{\begin{bmatrix} \cos\omega_M t & -\sin\omega_M t & 0 & 0 \\ \sin\omega_M t & \cos\omega_M t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} \cos\omega_M t & 0 & \sin\omega_M t & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\omega_M t & 0 & \cos\omega_M t & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - translate to the end of arm 2

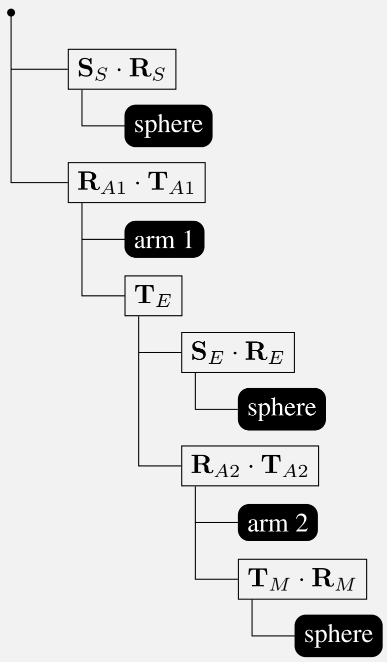
$$\mathbf{T}_M = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - concatenate:

$$\mathbf{M} = \mathbf{A}_2 \cdot \mathbf{T}_M \cdot \mathbf{R}_M$$

(b) Create a scene graph for the above scene. Include information on how to compute the transformation matrices.

Solution



Compute the required matrices for each shape by multiplying along a path from the root to the respective shape. E.g., for the earth:

$$\mathbf{E}_L = \mathbf{R}_{A1} \cdot \mathbf{T}_{A1} \cdot \mathbf{T}_E \cdot \mathbf{S}_E \cdot \mathbf{R}_E$$