Satisfiability Checking - WS 2016/2017

Series 8

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Exercise 1

Consider the first-order logical formula over the reals with addition:

$$\begin{array}{rlllll} \varphi^{LRA} := & 2x_1 + x_3 \leq 2 & \wedge & (-x_1 + 2x_3 \leq 3 & \vee & x_3 \leq 0) \\ \wedge & x_2 + x_3 \leq -5 & \wedge & -x_1 + 4x_2 + x_3 \leq 0 & \wedge & -x_2 - 5x_3 \leq 1 \end{array}$$

The Boolean abstraction of this formula is

$$a_1 \wedge (a_2 \vee a_3) \wedge a_4 \wedge a_5 \wedge a_6.$$

Simulate how a less-lazy SMT solver solves φ^{LRA} for satisfiability as presented in the lecture. Show the progress in the SAT solver and the theory solver, which implements the incremental and infeasible subset generating version of the Fourier-Motzkin method, which was presented in the lecture. If the SAT solver makes a decision, it chooses the unassigned variable a_i with the lowest index and assigns it to false. If the Fourier-Motzkin method chooses a variable to eliminate, it chooses the variable x_i with the lowest index. Show how the theory solver benefits from its incrementality support, both when adding and removing constraints, and show how the infeasible subset(s) are computed.

Compared to Exercise 1, Series 7, here, the SMT formula is in negation normal form as it contains no negations. Therefore, a constraint is only added to the theory solver if the corresponding Boolean abstraction variable is assigned to true.

Exercise 2

Consider the first-order logical formula over the reals with addition:

$$\begin{array}{rcl} \varphi^{LRA} & := & 2x_1 + x_3 \leq 2 \quad \wedge \quad (\, -x_1 + 2x_3 \leq 3 \qquad \vee \qquad x_3 \leq 0 \,) \\ & & \wedge \quad x_2 + x_3 \leq -5 \quad \wedge \quad -x_1 + 4x_2 + x_3 \leq 0 \quad \wedge \quad -x_2 - 5x_3 \leq 1 \end{array}$$

The Boolean abstraction of this formula is

$$a_1 \wedge (a_2 \vee a_3) \wedge a_4 \wedge a_5 \wedge a_6.$$

Simulate how a less-lazy SMT solver solves φ^{LRA} for satisfiability as presented in the lecture. Show the progress in the SAT solver and the theory solver, which implements the incremental and infeasible subset generating version of the Simplex method, which was presented in the lecture. If the SAT solver makes a decision, it chooses the unassigned variable a_i with the lowest index and assigns it to false. Use the variable order

$$x_1 < x_2 < x_3 < s_1 < \ldots < s_6$$

for the Simplex method, where s_i corresponds to the slack variable introduced for the constraint abstracted by a_i ($1 \leq i \leq 6$). Show how the theory solver benefits from its incrementality support, both when adding and removing constraints, and show how the infeasible subset(s) are computed.

Compared to Exercise 1, Series 7, here, the SMT formula is in negation normal form as it contains no negations. Therefore, a constraint is only added to the theory solver if the corresponding Boolean abstraction variable is assigned to true.