Some analysis for lightning strike project

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Suppose we have some initial condition $f(x)$ on a 1d system such as a displacement along a string. Then the FFT gives us the $c_n(0)$ in:

$$f(x) = \sum_n c_n(0) e^{i(2\pi k_1 n)x}. \tag{1}$$

$k_1 n$ is the nth "linear frequency." k_1 is the minimum frequency, $1/n\Delta$, where Δ = spacing between sample points. In other words, k_1 is one divided by the total length of the system.

We generalize Eq. (1) by replacing $c_n(0)$ with $c_n(t)$. Recall the wave equation,

$$\frac{\partial^2 f}{\partial t^2} = v^2 \left(\frac{\partial^2 f}{\partial x^2} \right). \tag{2}$$

Substituting the time-dependent analogue of Eq. (1) into (2) results in:

$$\sum_n \ddot{c}_n e^{i(2\pi k_1 n)x} = v^2 \sum_n -c_n (4\pi^2 k_1^2 n^2) e^{i(2\pi k_1 n)x}. \tag{3}$$

Blah, blah, blah, linearity, blah, and we get:

$$\ddot{c}_n + (2v\pi k_1 n)^2 c = 0, \tag{4}$$

which has solutions of the form

$$c_n(t) = A \sin \omega t + B \cos \omega t, \tag{5}$$

in which for zero initial velocity we can take $A = 0,\ B = c_n(0)$. We then have the a time-dependent set of fourier coefficients. Applying the inverse FFT to these coefficients for any time t_1 gives us the solution of the wave equation at that time t_1.