I. EDGE LENGTH UPDATES

In this section we want to derive an expression describing how changes in the preferred lengths of edges $\partial_t \ell_{ij}^2(t)$ produce changes in the positions of the vertices $\partial_t r_i^\alpha(t)$. We assume the edge lengths are changing adiabatically with fast elastic response of the vertex positions. Specifically, assume the elastic energy has the form $E[\{r_i^\alpha\},\{\ell_{k\ell}\}]$, with the $\{\ell_{k\ell}\}$ appearing as adiabatic parameters. In an infinitesimal time interval we have $\delta \ell_{ij}^2 = \partial_t \ell_{ij}^2\ dt$. We want to calculate the corresponding change δr_i^α. To accomplish this we start from the assumption that the equilibrium configuration at time $t+dt$ is close the configurations at time t. In other words we make the following Taylor expansion

$$E[r_i^\alpha, \bar{\ell}_{k\ell}; t+dt] = E[r_i^\alpha, \bar{\ell}_{k\ell}; t] + \frac{\partial E}{\partial \ell_{k\ell}^2}\ \delta \bar{\ell}_{ij}^2$$
$$\frac{\partial^2 E}{\partial \bar{\ell}_{k\ell}^2\ \partial r_i^\alpha}\ \delta \bar{\ell}_{ij}^2\ \delta r_i^\alpha + \frac{\partial^2 E}{\partial r_i^\alpha\ \partial r_j^\beta}\ \delta r_i^\alpha\ \delta r_j^\beta + \cdots, \qquad (1)$$

where we've neglected other terms because they are higher order in time or they do not depend on δr_i^α. Our aim is to minimize this function with respect to δr_i^α, which can easily be done since the Euler-Lagrange equations have been linearized.

In order to find the displacement δr_i^α, we will need to compute the second derivatives of the energy $M \equiv -\frac{\partial^2 E}{\partial \ell_{k\ell}^2\ \partial r_i^\alpha}$ and $D \equiv \frac{\partial^2 E}{\partial r_i^\alpha\ \partial r_j^\beta}$. Recall that the energy is given by

$$E = \sum_{k\ \ell} \frac{A_{k\ell}\ K_{k\ell}}{2\times 8}\left[(\mathbf{r}_k - \mathbf{r}_\ell)^2 - \bar{\ell}_{k\ell}^2\right]^2. \qquad (2)$$

Where the sum runs over all vertices and $A_{k\ell}$ is the adjacency matrix which is equal to zero unless k and ℓ are connected by an edge, in which case it is equal to 1. Note that we divided by an extra factor of 2 to avoid double counting the edges. Given this definition we find the second derivatives to be

$$M_{k\ell,i\alpha} = -\frac{K_{k\ell}}{2}(r_k^\alpha - r_\ell^\alpha)\ (\delta_{ik} - \delta_{i\ell}) \equiv -\frac{1}{2} K \cdot R$$
$$D_{k\ell}^{i\alpha} = d_{k\ell}^{i\alpha} + \delta^{\alpha\beta} \sum_k (\delta_{ij} - \delta_{jk}) \frac{A_{ik}\ K_{ik}}{2}\left[(\mathbf{r}_i - \mathbf{r}_k)^2 - \bar{\ell}_{ik}^2\right]. \qquad (3)$$

Where we've defined the usual rigidity matrix ($R \equiv (r_k^\alpha - r_\ell^\alpha)\ (\delta_{ik} - \delta_{i\ell})$), K (the diagonal matrix with spring constants for entries) and dynamical matrices ($d \equiv R^T K R$).

Now we can easily find the resulting displacement by minimizing the energy to get

$$\delta \mathbf{r} = D^{-1} \cdot M^T \cdot \delta \bar{\ell}. \qquad (4)$$

Which we implement and test in Mathematica and Python. The way we test this equation is by finding the actual global minimum of the energy when the lengths are changed, then we compare that to the displacement generated from this equation we get the expected agreement with the error scaling as $O(dt)$.