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Date : .... Promotion : .... Course name : ....

Recitation #2 : D.H.O

EXO #1 :

$m = 90\,g$, $b = 0{,}18\,kg\,s^{-1}$, $\frac{\Delta E}{E_0} = 0{,}95$

1) Period:

$E = E_0 e^{-\gamma t}$ so for one period $(t = T)$

$\Rightarrow E = E_0 e^{-\gamma T}$ and $\Rightarrow E_0 - E = \Delta E = E_0 (1 - e^{-\gamma T})$

$\Rightarrow \frac{\Delta E}{E_0} = 1 - e^{-\gamma T} = 0{,}95 \Rightarrow$ AN: $\boxed{T = 1{,}5\,s}$

2) ω_0 ?: $\omega_D^2 = \omega_0^2 - \frac{\gamma^2}{4} \Rightarrow \omega_0 = \left[\omega_D^2 + \frac{\gamma^2}{4}\right]^{1/2}$

when $\omega_D = \frac{2\pi}{T}$ we replace we get: $\boxed{\omega_0 = 4{,}3\,rd/s}$

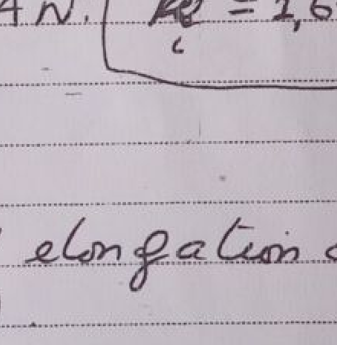
3) $k = ?$: $\omega_0^2 = \frac{k}{m} \Rightarrow k = m\omega_0^2 \Rightarrow$ AN: $\boxed{k = 1{,}67\,N.m^{-1}}$

EXO #2

$m = 1{,}5\,kg$, $x_0 = 0{,}4\,m$ (initial elongation of spring).

$A = 1\,m.$, $b = 15\,kg\,s^{-1}$

a) The diff. equ:



$\sum \vec{F} = m\ddot{\vec{x}} \Rightarrow \vec{F}_D + \vec{F}_s = m\ddot{\vec{x}}$

$-kx - b\dot{x} = m\ddot{x} \Rightarrow$

$\boxed{\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0}$ / where $\gamma = \frac{b}{m}$, $\omega_0^2 = \frac{k}{m}$

(2)

we must find γ and ω_0 to complet the answer.

so;

- $\gamma = \frac{b}{m} = \frac{15}{1,5} \Rightarrow \boxed{\gamma = 10\, s^{-1}}$

- $\omega_0 = \sqrt{\frac{k}{m}}$ we need k : ?? , we know that

at equilibrium , $k \Delta x_0 = mg \Rightarrow k = \frac{mg}{\Delta x} = 36,75\ N.m^{-1}$

$\Rightarrow$ N.A: $\boxed{\omega_0 = 4.95\, rd/s}$

So the diff equ of oscillators $\boxed{\ddot{x} + 10\dot{x} + 24.5x = 0}$

b) $\frac{\gamma}{2} = 5\, s^{-1}$ and $\omega_0 = 4,95\, rd/s \Rightarrow \gamma/2 > \omega_0 \Rightarrow$

The system is overdamped $\Rightarrow$

$$x(t) = e^{-\frac{\gamma}{2}t}\left[A e^{-\sqrt{\frac{\gamma^2}{4} - \omega_0^2}\, t} + B e^{+\sqrt{\frac{\gamma^2}{4}t - \omega_0^2 t}}\right]$$

$$\begin{cases} -\frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} - \omega_0^2} = -5,71\, s^{-1} \\ -\frac{\gamma}{2} + \sqrt{\frac{\gamma^2}{4} - \omega_0^2} = -4,29\, s^{-1} \end{cases} \Rightarrow$$

$$x(t) = A e^{-5,71t} + B e^{-4,29t}$$

$$\dot{x}(t) = -5,71 A e^{-5,71t} - 4.29 B e^{-4,29t}$$

at $t=0$ $\begin{cases} x(t=0) = 1m = A + B \Rightarrow \boxed{A = 1 - B} \\ \dot{x}(t=0) = 0 = -5,71A - 4,29B \end{cases}$

$\Rightarrow \boxed{A = -3,03\, m}$, $\boxed{B = 4,03\, m}$

$$x(t) = -3,03e^{-5,71t} + 4,03e^{-4,29t} \quad (m)$$

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(3)

Exo # 3

$T = 2,1s$, $\frac{\Delta A}{A_0} = 4\% = 0,04$

a) $\frac{\Delta E}{E_0} = ?$ / $A = A_0 e^{-\frac{\gamma}{2}t} \Rightarrow$

$\Delta A = A_0 - A = A_0(1 - e^{-\frac{\gamma}{2}t})$ for one cycle ($t = T$) $\Rightarrow$

$\frac{\Delta A}{A_0} = 1 - e^{-\frac{\gamma}{2}T} = 0,04 \Rightarrow e^{-\frac{\gamma}{2}T} = 1 - \frac{\Delta A}{A_0} \Rightarrow$

$\left(e^{-\frac{\gamma}{2}T}\right)^2 = \left(1 - \frac{\Delta A}{A_0}\right)^2$ we have $\frac{\Delta A}{A_0} \ll 1 \Rightarrow$

we use the mathematical approximation

$\left((1 - x)^n = 1 - nx \text{ if } x \ll 1\right) \Rightarrow$

$e^{-\gamma T} = 1 - 2\frac{\Delta A}{A_0} \Rightarrow \frac{\Delta E}{E_0} = 1 - e^{-\gamma T} = 1 - \left(1 - \frac{2\Delta A}{A_0}\right)$

$\Rightarrow \frac{\Delta E}{E_0} = \frac{2\Delta A}{A_0}$ $\Rightarrow \frac{\Delta E}{E_0} = 8\%$

b) τ ?? $\tau = \frac{1}{\gamma}$, we have $\frac{\Delta E}{E_0} = 0,08 = 1 - e^{-\gamma T}$

$\Rightarrow e^{-\gamma T} = 0,92 \Rightarrow \gamma T = -\ln(0,92)$, NA $\boxed{\tau = 20,5s}$

c) The Quality factor Q ?

$Q = \frac{\omega_0}{\gamma}$ / $\omega_0 = ?$ we have $\omega_D^2 = \omega_0^2 - \frac{\gamma^2}{4} \Rightarrow$

$\omega_0^2 = \omega_D^2 + \frac{\gamma^2}{4} = \frac{4\pi^2}{T^2} + \frac{\gamma^2}{4} \Rightarrow \boxed{\omega_0 = 2,99 \text{ rd/s}}$

$Q = 62,29 \Rightarrow \boxed{Q \simeq 62}$

EX 04 4

$\omega_0 = 20\,rd/s$.

a) Relaxation time τ.

$E = E_0 e^{-\gamma t} = E_0 e^{-t/\tau}$ wher $\tau = \frac{1}{\gamma}$

from the graph $E_0 = 14,00\ J$ (at $t = 0s$)

and for $t = 11s \Rightarrow E(11s) = 1,5\ J \Rightarrow$

$1,5 = 14\, e^{-11/\tau} \Rightarrow \boxed{\tau = 5,0\ s}$

b) The factor of damping: γ:

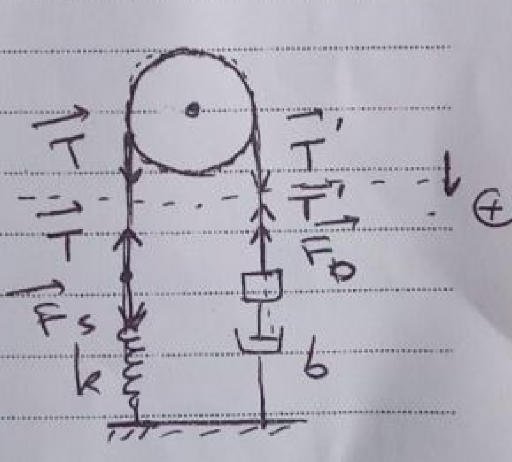
$\tau = \frac{1}{\gamma} \Rightarrow \boxed{\gamma = 0,20\ s^{-1}}$

c) The Quality factor Q:

$\boxed{Q = \frac{\omega_0}{\gamma} = 100}$

•) EX 04 5

•) The frequency of oscillation:



(we do it as we have done it for exercise in Recitation 4)

So: spring. we apply 3d N.L:

$\Rightarrow \sum \vec{F} = 0 \Rightarrow \vec{F_s} + \vec{T} = 0 \Rightarrow -kx + T = 0 \Rightarrow$

$\boxed{kx = T}$ ... (1)

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Suite exo #1 5

pulley: $\sum \vec{\tau} = I_{/\Delta} \ddot{\vec{\theta}} \Rightarrow \vec{\tau}_T + \vec{\tau}_{T'} = I_{/\Delta} \ddot{\vec{\theta}}$

$\Rightarrow RT - RT' = -I_{/\Delta} \ddot{\theta} \Rightarrow T' = T + \frac{I}{R} \ddot{\theta}$

$$\boxed{T' = kx + \frac{I}{R} \ddot{\theta}} \quad \ldots (2) \ldots$$

•) For mass 1 2^{d} NL: $\sum \vec{F} = m \ddot{\vec{x}} \Rightarrow$

$\vec{F}_D + \vec{T'} = m \ddot{\vec{x}} \Rightarrow -b\dot{x} - T' = m\ddot{x}$

we replace T' we get: $m\ddot{x} + b\dot{x} + kx + \frac{I}{R}\ddot{\theta} = 0$

$x = R\theta \Rightarrow \ddot{x} = R\ddot{\theta} \Rightarrow$

$m\ddot{x} + b\dot{x} + kx + \frac{I}{R^2}\ddot{x} = 0 \Rightarrow$

$(m + \frac{I}{R^2})\ddot{x} + b\dot{x} + kx = 0$

$$\ddot{x} + \frac{b}{(m + \frac{I}{R^2})}\dot{x} + \frac{k}{(m + \frac{I}{R^2})} x = 0$$

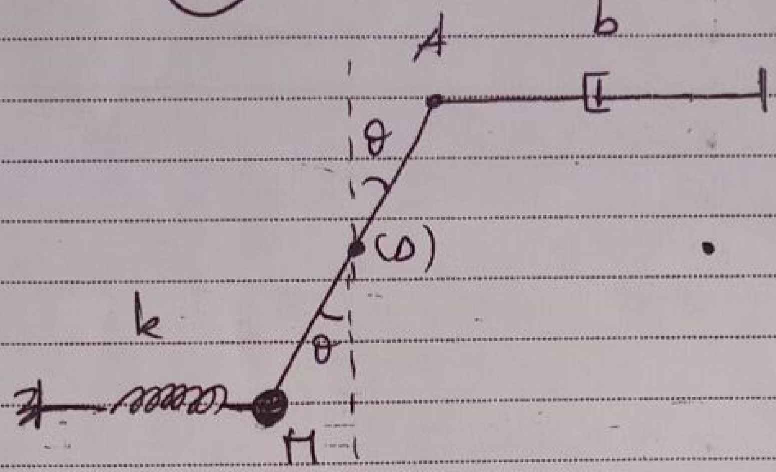
$\Rightarrow \omega_D^2 = \omega_0^2 - \frac{\gamma^2}{4}$ / where $\begin{cases} \gamma = \frac{b}{(m + I/R^2)} \\ \omega_0^2 = \frac{k}{(m + I/R^2)} \end{cases}$

N.A: $\begin{cases} \gamma = 4 s^{-2} \\ \omega_0 = 3 \text{ rd/s} \end{cases}$

$\Rightarrow \omega_D = 2,24 \text{ rd/s} \Rightarrow \boxed{f_D = 0,36 \text{ Hz}}$

EXO #6

(6)



a) Kinetic energy: the motion is rotational $\Rightarrow$

$K^{rot} = \frac{1}{2} I_{/\Delta} \dot{\theta}^2$ / $I_{/\Delta}$ is the inertia of pendulum.

$\Rightarrow I_{/\Delta} = M\left(\frac{L}{2}\right)^2 = \frac{1}{4} M L^2$.

$K^{rot} = \frac{1}{8} M L^2 \dot{\theta}^2$

b) Potential energy:

$U = U_{sp} + U_M$ where sp for spring and M for mass.

.) $U_s = \frac{1}{2} k x^2$ (x here is the same as displacement)

$x = \frac{L}{2}\theta \Rightarrow \dot{x} = \frac{L}{2}\dot{\theta}$, we replace we get:

$U_{sp} = \frac{1}{2} k \left(\frac{L}{2}\theta\right)^2 \Rightarrow \boxed{U_{sp} = \frac{1}{8} k L^2 \theta^2}$

.) $U_M = +Mgh$ (the potential encreases for this position)

$U_M = Mg\frac{L}{2}(1 - \cos(\theta))$ / where $h = \frac{L}{2}(1 - \cos(\theta))$ (done in previous exercice)

and $\cos(\theta) = 1 - \frac{\theta^2}{2!}$

$\Rightarrow \boxed{U_M = \frac{M}{4} L \theta^2}$

Suite exo#6:

so $U = \left(\frac{1}{8}kL^2 + \frac{1}{4}ML\right)\theta^2$

c) The diff - equ:

the total energy of the system is $E = K + U$

$\Rightarrow E = \frac{1}{8}ML^2\dot{\theta}^2 + \left(\frac{1}{8}kL^2 + \frac{1}{4}ML\right)\theta^2$

our system is D.H.O $\Rightarrow$ E is not conserved $\Rightarrow$

$\frac{dE}{dt} = -b\dot{x}^2 \neq 0 \Rightarrow$

$\frac{dE}{dt} = 2\left(\frac{1}{8}kL^2 + \frac{1}{4}MgL\right)\dot{\theta}\theta + \frac{2}{8}ML^2\dot{\theta}\ddot{\theta} = -b\dot{x}^2$

where $\dot{x} = \frac{L}{2}\dot{\theta}$ we replace and simplify.

$\Rightarrow$ we get:

$\frac{1}{4}ML^2\ddot{\theta} + \left(\frac{1}{4}kL^2 + \frac{1}{2}MgL\right)\theta = -\frac{1}{4}b\dot{\theta}$

$\Rightarrow \ddot{\theta} + \frac{b}{M}\dot{\theta} + \left(\frac{k}{M} + \frac{2g}{L}\right)\theta = 0$

$\ddot{\theta} + \gamma\dot{\theta} + \omega_0^2\theta = 0$ / $\gamma = \frac{b}{M}$, $\omega_0 = \left(\frac{k}{M} + \frac{2g}{L}\right)^{1/2}$

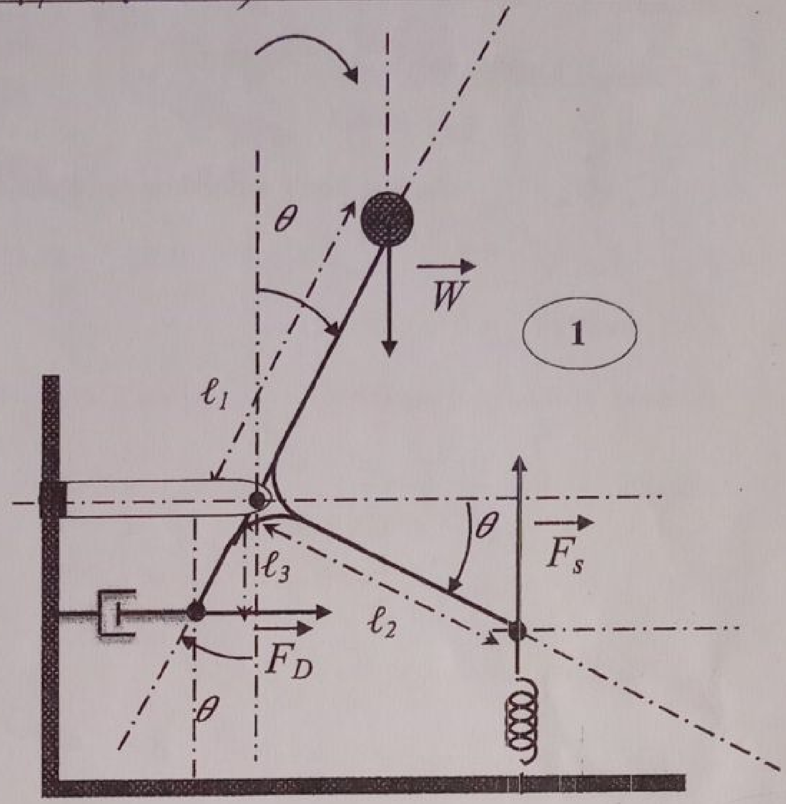
e) the constant of damping b:

$A = A_0 e^{-\frac{\gamma}{2}t}$, so after 5 periods $A = \frac{A_0}{2} = A_0 e^{-\frac{\gamma}{2}(5T)} \Rightarrow$

$\frac{1}{2} = e^{-\frac{5}{2}\gamma T} \Rightarrow \ln(0,5) = -\frac{5}{2}\gamma T = -\frac{5}{2}\frac{b}{M}.T$

N.A gives: $b = 1,39\ kg\ s^{-1}$

Exercise # 7 (final exam 2017/2018, 6 marks)



a) Free body diagram:
done on the figure.

b) The natural frequency of the system:

1st Method: Torque:

We use torque, so;

$$\sum \vec{\tau} = I_{IO}\ddot{\vec{\theta}} \Rightarrow \vec{\tau}_W + \vec{\tau}_s + \vec{\tau}_D = I_{IO}\ddot{\vec{\theta}}$$

where W, s and D reference to weight, spring and damping respectively.

$$\begin{cases} \tau_W = -\ell_1 .m.g.\sin(\pi - \theta) & 0.5 \\ \tau_s = +\ell_2 .k.x'.\sin(\frac{\pi}{2} + \theta) & 0.5 \\ \tau_D = +\ell_3 .b.\dot{x}''.\sin(\frac{\pi}{2} + \theta) & 0.5 \end{cases}$$

for small oscillations $\sin(\theta) \approx \theta = \frac{x}{\ell_1} = \frac{x'}{\ell_2} = \frac{x''}{\ell_3} = \Rightarrow$

$\dot{x}'' = \ell_3 .\theta$ and $\cos(\theta) \approx 1$ 0.5

we replace in our equations, thus

$$\Rightarrow \begin{cases} \tau_W = -\ell_1 .m.g.\theta \\ \tau_s = +\ell_2 .k.(\ell_2 \theta) \\ \tau_D = +\ell_3 .b.(\ell_3 \dot{\theta}) \end{cases}$$

$$\Rightarrow -\ell_1 .m.g.\theta + \ell_2^2 .k.\theta + \ell_3^2 .b.\dot{\theta} = -m\ell_1^2 \ddot{\theta} \quad 0.5$$

$$\Rightarrow m\ell_1^2 \ddot{\theta} + \ell_3^2 .b.\dot{\theta} + \ell_2^2 .k.\theta - \ell_1 .m.g.\theta = 0$$

$$\Rightarrow \boxed{\ddot{\theta} + \frac{\ell_3^2}{\ell_1^2}\frac{b}{m}\dot{\theta} + \left(\frac{\ell_2^2}{\ell_1^2}\frac{k}{m} - \frac{g}{\ell_1}\right)\theta = 0} \quad 0.5$$

it's the differential equation of DSHM, $\ddot{\theta} + \gamma\dot{\theta} + \omega_0^2 \theta = 0$, where

$$\boxed{\omega_0 = \sqrt{\frac{\ell_2^2}{\ell_1^2}\frac{k}{m} - \frac{g}{\ell_1}}} \quad 1$$

is the natural frequency of the system and

$$\boxed{\gamma = \frac{\ell_3^2}{\ell_1^2}\frac{b}{m}} \quad 1$$

is the factor of damping.

Suite exo # 7

2d method: Energy Method:

$E = K + U$

$E = K_m + U_m + U_s$ where m and s are reference of mass and spring.

$K_m = \frac{1}{2} I_{/O}\dot{\theta}^2$, 0.5 $U_m = m.g.h$ 0.5 *and* $U_s = \frac{1}{2}kx'^2$ 0.5

$h = \ell_1\cos(\theta) - \ell_1 = \ell_1(\cos(\theta) - 1)$

for small oscillations, $\cos(\theta) \approx 1 - \frac{\theta^2}{2!} \Rightarrow h = \ell_1 \times (-\frac{\theta^2}{2}) = -\ell_1\frac{\theta^2}{2}$ 0.5

and $\sin(\theta) \approx \theta \approx \frac{x}{\ell_1} = \frac{x'}{\ell_2} = \frac{x''}{\ell_3}$ *and* $I_{/O} = m\ell_1^2 \Rightarrow$ 0.5

$E = \frac{1}{2} I_{/O}\dot{\theta}^2 - m.g\ell_1\frac{\theta^2}{2} + \frac{1}{2}kx'^2$

$\Rightarrow E = \frac{1}{2}(m\ell_1^2)\dot{\theta}^2 - m.g\ell_1\frac{\theta^2}{2} + \frac{1}{2}k(\ell_2\theta)^2$

$\Rightarrow E = \frac{1}{2}m\ell_1^2\dot{\theta}^2 - \frac{1}{2}mg\ell_1\theta^2 + \frac{1}{2}k\ell_2^2\theta^2$ 0.5

the system is damped so it's DSHM, so the total energy is dissipated and thus,

$\frac{dE}{dt} = -b(\dot{x}'')^2 \neq 0$ 0.5

$\Rightarrow \frac{dE}{dt} = m\ell_1^2\ddot{\theta}\dot{\theta} - mg\ell_1\dot{\theta}\theta + \ell_2^2k\dot{\theta}\theta = -b(\ell_3\dot{\theta})^2$

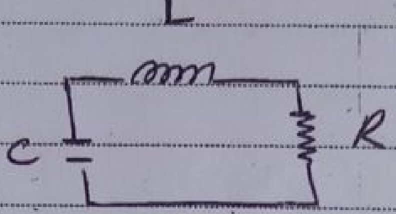
$\Rightarrow m\ell_1^2\ddot{\theta} + b\ell_3^2\dot{\theta} - mg\ell_1\theta + \ell_2^2k\theta = 0$

$\Rightarrow \boxed{\ddot{\theta} + \frac{\ell_3^2}{\ell_1^2}\frac{b}{m}\dot{\theta} + (\frac{\ell_2^2}{\ell_1^2}\frac{k}{m} - \frac{g}{\ell_1})\theta = 0}$ 0.5 differential equation of DSHM, where

the natural frequency is $\boxed{\omega_0 = \sqrt{\frac{\ell_2^2}{\ell_1^2}\frac{k}{m} - \frac{g}{\ell_1}}}$ 1 and the factor of damping is $\boxed{\gamma = \frac{\ell_3^2}{\ell_1^2}\frac{b}{m}}$ 1

EXO# 8

(8)



a) Using Kirchhoff's Law

$V_L + V_R + V_C = 0$

$L\frac{dI}{dt} + RI + \frac{1}{C}q = 0 \qquad (I = \frac{dq}{dt})$

$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = 0 \qquad / \quad \ddot{q} + \gamma\dot{q} + \omega_0^2 q = 0$

$\gamma = R/L \,, \quad \omega_0 = \frac{1}{\sqrt{LC}}$

b) the system is critically damped $\Rightarrow$

$\frac{\gamma}{2} = \omega_0 \Rightarrow \frac{R_{cit}}{2L} = \frac{1}{\sqrt{LC}} \Rightarrow R_{cit} = \frac{2L}{\sqrt{LC}}$

$R_{cit} = 2\sqrt{\frac{L}{C}}$

c) $\omega_D = \left[\omega_0^2 - \frac{\gamma^2}{4}\right]^{1/2} = \left(\omega_0^2 - \frac{R^2}{4L^2}\right)^{1/2}$

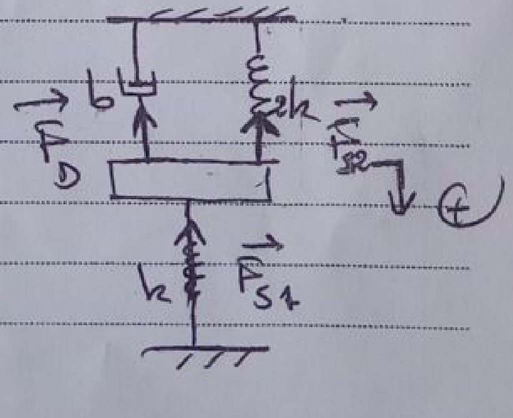
$= \omega_0\left(1 - \frac{R^2}{4L^2} \cdot \frac{1}{\omega_0^2}\right)^{1/2}$

$\omega_D = \omega_0\left(1 - \frac{R^2}{4L^2} \cdot LC\right)^{1/2} = \omega_0\left(1 - \frac{R^2}{4} \cdot \frac{C}{L}\right)^{1/2}$

$\omega_D = \omega_0\left(1 - \frac{R^2}{R_{cit}^2}\right)^{1/2}$

EXO# 9

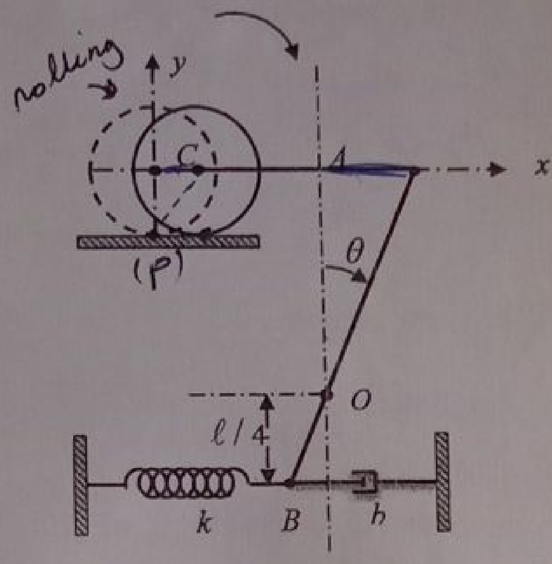
a) The diff. equ.



$\sum \vec{F} = m\vec{\ddot{x}} \Rightarrow \vec{F}_D + \vec{F}_{S1} + \vec{F}_{S2} = m\vec{\ddot{x}}$

$-b\dot{x} - kx - 2kx = m\ddot{x}$

Exercise # 10 (Make up exam 2017/2018, 5 marks).



a) The Kinetic energy:

$$K_{system} = K_{m_1} + K_{m_2} \quad \text{where} \begin{cases} K_{m_1} = \frac{1}{2} I_{/p}^{m_1} \dot{\alpha}^2 \\ K_{m_2} = \frac{1}{2} I_{/o}^{m_2} \dot{\theta}^2 \end{cases}$$

where α is the angular displacement done by the disk and θ the one done by the bar with respect to the vertical.

so $K_{system} = \frac{1}{2} I_{/p}^{m_1} \dot{\alpha}^2 + \frac{1}{2} I_{/o}^{m_2} \dot{\theta}^2$ 0.5

$$and \begin{cases} I_{/p}^{m_1} = I_{/com}^{m_1} + m_1 R^2 = \frac{1}{2} m_1 R^2 + m_1 R^2 = \frac{3}{2} m_1 R^2 & 0.5 \\ I_{/o}^{m_2} = I_{/com}^{m_2} + m_2 (\frac{\ell}{4})^2 = \frac{1}{12} m_2 \ell^2 + \frac{1}{16} m_2 \ell^2 = \frac{7}{48} m_2 \ell^2 & 0.5 \end{cases}$$

we replace we get

$$K_{system} = \frac{1}{2} \times \frac{3}{2} m_1 R^2 \dot{\alpha}^2 + \frac{1}{2} \times \frac{7}{48} m_2 \ell^2 \dot{\theta}^2 \qquad (1)$$

- finding the relation between $\dot{\alpha}$ and $\dot{\theta}$.

Let x_C be the displacement of the *com* of the disk and x_A the diplacement of the point A of the bar.

$$\text{we know that : } x_C = x_A \text{ where } \begin{cases} x_C = R\alpha \\ x_A = (\frac{\ell}{2} + \frac{\ell}{4})\theta \end{cases} \Rightarrow R\alpha = \frac{3}{4} \ell \theta$$

$$\Rightarrow \boxed{R\dot{\alpha} = \frac{3}{4} \ell \dot{\theta}} \quad 0.5$$

we replace in equation (1) we get

$$K_{system} = \frac{1}{2} \left[\frac{3}{2} m_1 (\frac{3}{4} \ell \dot{\theta})^2 + \frac{7}{48} m_2 \ell^2 \dot{\theta}^2 \right] \quad \Rightarrow$$

$$\boxed{K_{system} = \frac{1}{2} \left[\frac{27}{32} m_1 + \frac{7}{48} m_2 \right] \ell^2 \dot{\theta}^2} \quad 1$$

Suite exo# 10

b) The potential energy.

$U_{system} = U_{m_2} + U_{spring}$, where

$$\begin{cases} U_{m_2} = m_2 g h \\ U_{spring} = \frac{1}{2} k x'^2 \end{cases} \Rightarrow \text{ where } \begin{cases} h = \frac{\ell}{4}\cos(\theta) - \frac{\ell}{4} = \frac{\ell}{4}(\cos(\theta) - 1) \\ x' = \frac{\ell}{4}\sin(\theta) \end{cases}$$

0.5

For small oscillations;

$$\begin{cases} (\cos(\theta) - 1) \approx -\frac{\theta^2}{2!} \\ \sin(\theta) \approx \theta \end{cases}$$

we replace in equation U_{system} we get:

$$U_{system} = -m_2 g\left(\frac{\ell}{4}.\frac{\theta^2}{2}\right) + \frac{1}{2}k\left(\frac{\ell}{4}\theta\right)^2$$

$$\boxed{U_{system} = \frac{1}{2}\left[\frac{1}{16}k - \frac{m_2 g}{4\ell}\right]\ell^2\theta^2}$$

0.5

c) The differential equation:

The total energy of the system is

$$E = K_{system} + U_{system} \Rightarrow$$

$$E = \frac{1}{2}.\left[\frac{27}{32}m_1 + \frac{7}{48}m_2\right]\ell^2\dot{\theta}^2 + \frac{1}{2}.\left[\frac{1}{16}k - \frac{m_2 g}{4\ell}\right]\ell^2\theta^2$$

We have a dumped system so the total energy is not conserved and thus,

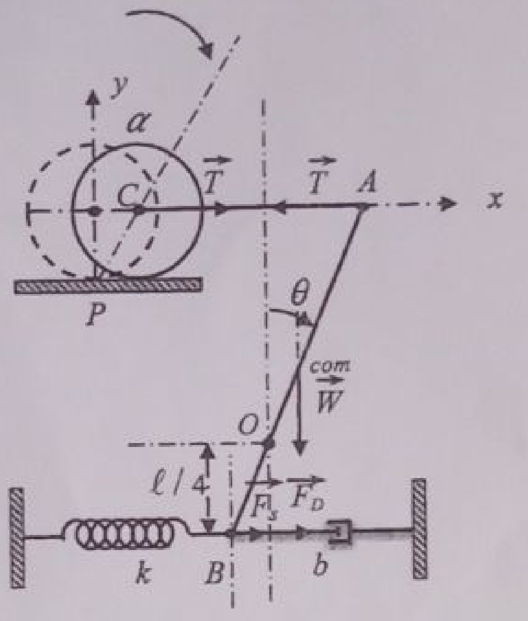
$$\frac{dE}{dt} = -b\dot{x}''^2 \quad \text{where } x'' = \frac{\ell}{4}\theta \Rightarrow \dot{x}'' = \frac{\ell}{4}\dot{\theta}$$

$$\frac{dE}{dt} = -b\frac{\ell^2}{16}\dot{\theta}^2 = \left[\frac{27}{32}m_1 + \frac{7}{48}m_2\right]\ell^2\dot{\theta}\ddot{\theta} + \left[\frac{1}{16}k - \frac{m_2 g}{4\ell}\right]\ell^2\dot{\theta}\theta$$

$$\left[\frac{27}{32}m_1 + \frac{7}{48}m_2\right]\ddot{\theta} + \frac{b}{16}\dot{\theta} + \left[\frac{1}{16}k - \frac{m_2 g}{4\ell}\right]\theta = 0$$

$$\Rightarrow \boxed{\ddot{\theta} + \left[\frac{b}{\left(\frac{27}{2}m_1 + \frac{7}{3}m_2\right)}\right]\dot{\theta} + \left[\frac{k - \frac{4m_2 g}{\ell}}{\left(\frac{27}{2}m_1 + \frac{7}{3}m_2\right)}\right]\theta = 0}$$ the differential equation of the system

1



Using torque.

As we have done it in previous calculus,

We have the relation between the two angles α and θ there for the one between the two accelerations.

finding the relation between α and θ.

Let x_C be the displacement of the *com* of the disk and x_A the diplacement of the point A of the bar.

we know that : $x_C = x_A$ where $\begin{cases} x_C = R\alpha \\ x_A = (\frac{\ell}{2}+\frac{\ell}{4})\theta \end{cases} \Rightarrow R\alpha = \frac{3}{4}\ell\theta$

$\Rightarrow R\dot{\alpha} = \frac{3}{4}\ell\dot{\theta}$ *and so* $R\ddot{\alpha} = \frac{3}{4}\ell\ddot{\theta}$

Now, calculate the inertia for each mass about their axis of rotations P and O.

$$and \begin{cases} I^{m_1}_{/p} = I^{m_1}_{/com} + m_1R^2 = \frac{1}{2}m_1R^2 + m_1R^2 = \frac{3}{2}m_1R^2 \\ I^{m_2}_{/o} = I^{m_2}_{/com} + m_2(\frac{\ell}{4})^2 = \frac{1}{12}m_2\ell^2 + \frac{1}{16}m_2\ell^2 = \frac{7}{48}m_2\ell^2 \end{cases}$$

We apply newton's la for torque we get:

- For disk: $\sum\bar{\tau} = I^{m_1}_{/p}\bar{\ddot{\alpha}} \Rightarrow \bar{\tau}_T = I^{m_1}_{/p}\bar{\ddot{\alpha}} \Rightarrow -RT\sin(\frac{\pi}{2}-\alpha) = -I^{m_1}_{/p}\ddot{\alpha}$

$\Rightarrow RT\cos(\alpha) = \frac{3}{2}m_1R^2\ddot{\alpha} \qquad \Rightarrow \quad T = \frac{3}{2}m_1R\ddot{\alpha} = \frac{3}{2}m_1\frac{3}{4}\ell\ddot{\theta}$ ...(1)...

- For rod: $\sum\bar{\tau} = I^{m_2}_{/O}\bar{\ddot{\theta}} \Rightarrow \bar{\tau}_s + \bar{\tau}_D + \bar{\tau}_T = I^{m_2}_{/O}\bar{\ddot{\theta}} \Rightarrow$

$+\frac{\ell}{4}kx\sin(\frac{\pi}{2}+\theta) + \frac{\ell}{4}b\dot{x}\sin(\frac{\pi}{2}+\theta) - \frac{\ell}{4}m_2g\sin(\pi-\theta) + \frac{3\ell}{4}T = -\frac{7}{48}m_2\ell^2\ddot{\theta}$ ...(2)...

For small oscillations, $\cos(\alpha)=1$ *and* $\sin(\theta) = \frac{x'}{(\ell/4)} = \frac{x''}{(\ell/4)} = \theta$, we replace in equ (2) we get,

$\frac{7}{48}m_2\ell^2\ddot{\theta} + \frac{\ell}{4}k(\frac{\ell}{4}\theta) + \frac{\ell}{4}b(\frac{\ell}{4}\dot{\theta}) - \frac{\ell}{4}m_2g\theta + \frac{3\ell}{4}T = 0$ ...(2)...

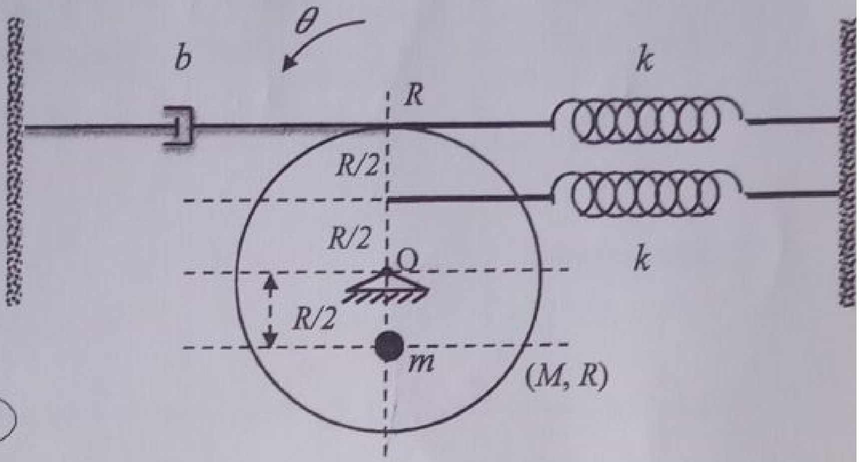
$\Rightarrow \frac{7}{48}m_2\ell^2\ddot{\theta} + \frac{\ell}{4}k(\frac{\ell}{4}\theta) + \frac{\ell}{4}b(\frac{\ell}{4}\dot{\theta}) - \frac{\ell}{4}m_2g\theta + \frac{3\ell}{4}T = 0$, we replace T by its expression in equa (1), we get

$\Rightarrow \frac{7}{48}m_2\ell^2\ddot{\theta} + \frac{\ell^2}{16}k\theta + \frac{\ell^2}{16}b\dot{\theta} - \frac{\ell}{4}m_2g\theta + \frac{3\ell}{4}(\frac{9}{8}m_1\ell\ddot{\theta}) = 0$

$\Rightarrow \left(\frac{27}{32}m_1 + \frac{7}{48}m_2\right)\ddot{\theta} + \frac{1}{16}k\theta + \frac{1}{16}b\dot{\theta} - \frac{1}{4}m_2\frac{g}{\ell}\theta = 0$

$$\Rightarrow \boxed{\ddot{\theta} + \frac{b}{\left(\frac{27}{2}m_1 + \frac{7}{3}m_2\right)}\dot{\theta} + \frac{\left(k - \frac{4m_2g}{\ell}\right)}{\left(\frac{27}{2}m_1 + \frac{7}{3}m_2\right)}\theta = 0}$$

Exercise # 11 (final exam 2018/2019, 5 marks).



a) Kinetic energy:
$K^{syst} = K^m + K^M$ where, K^{syst} is the kinetic energy of the system, K^m is the kinetic of the mass m and K^M is the kinetic energy of the cylinder.

So, $K^M = \frac{1}{2} I_{/O}^M \dot{\theta}^2$ (0.25) and $K^m = \frac{1}{2} m v^2$ (0.25)

$v = \frac{R}{2}\dot{\theta}$ we replace in the total kinetic (0.25) energy we get,

$$K^{syst} = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\dot{\theta}^2 + \frac{1}{2}m\frac{R^2}{4}\dot{\theta}^2$$

$$\Rightarrow \boxed{K^{syst} = \frac{1}{4}R^2\dot{\theta}^2\left[M + \frac{1}{2}m\right]} \quad (1)$$

b) Potential energy.
$U^{syst} = U^m + U^M + U^{springs}$ where, U^{syst} is the potential energy of the system, U^m is the potential energy of the mass m and U^M the one of the cylinder.

So, $U^m = mgh$, where $h = \frac{R}{2}(1-\cos(\theta)) = \frac{R}{2}\frac{\theta^2}{2} = \frac{R}{4}\theta^2$ (0.25)

U^M is zero (no potential energy for cylinder in rotation)

$U^{springs} = \frac{1}{2}kx'^2 + \frac{1}{2}kx''^2$ where x' is the elongation of spring (1) and x'' is the elongation of spring (2). (0.25)

$$\begin{cases} x' = R\theta \\ x'' = \frac{R}{2}\theta \end{cases} \Rightarrow U^{syst} = mg\frac{R}{4}\theta^2 + \frac{1}{2}k\left(R^2 + \frac{R^2}{4}\right)\theta^2 \quad (0.25)$$

$$\Rightarrow \boxed{U^{syst} = \frac{1}{4}\left[\frac{mg}{R} + \frac{5k}{2}\right]R^2\theta^2} \quad (1)$$

c) Differential equation:

$$E = U + K = \frac{1}{4}\left[\frac{2M+m}{2}\right]R^2\dot{\theta}^2 + \frac{1}{4}\left[\frac{mg}{R} + \frac{5k}{2}\right]R^2\theta^2$$

Damped oscillator $\Rightarrow \frac{dE}{dt} = -b\dot{x}^2$ where $\dot{x} = R\dot{\theta}$ (0.25)

$$\Rightarrow \frac{1}{2}\left[\frac{2M+m}{2}\right]R^2\dot{\theta}\ddot{\theta} + \frac{1}{2}\left[\frac{mg}{R} + \frac{5k}{2}\right]R^2\dot{\theta}\theta = -bR^2\dot{\theta}^2 \quad (0.25)$$

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$$\Rightarrow \boxed{\ddot{\theta}+\left[\frac{4b}{2M+m}\right]\dot{\theta}+\left[\frac{2m\frac{g}{R}+5k}{2M+m}\right]\theta=0}$$ The differential equation of the oscillator

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Second method

$$\sum\vec{\tau}=I_{/O}\ddot{\vec{\theta}}=\vec{\tau}_{s1}+\vec{\tau}_{s2}+\vec{\tau}_{damp}+\vec{\tau}_{m}$$

$$-Rkx_1\sin(\frac{\pi}{2}+\theta)-\frac{R}{2}kx_2(\frac{\pi}{2}+\theta)-Rb\dot{x}_1(\frac{\pi}{2}+\theta)-\frac{R}{2}mg\sin(\theta)=I_{/0}^{syst}\ddot{\theta}$$

where $I_{/0}^{syst}=I_{/0}^{M}+I_{/0}^{m}=\frac{1}{2}MR^2+m(\frac{R}{2})^2=\frac{1}{2}MR^2+\frac{1}{4}mR^2$

and $\begin{cases}x_1=R\theta \;\Rightarrow\; \dot{x}_1=R\dot{\theta}\\ x_2=\frac{R}{2}\theta\end{cases}$

$$\Rightarrow R^2k\theta+\frac{R^2}{4}k\theta+R^2b\dot{\theta}+mg\frac{R}{2}\theta=\left(\frac{1}{2}MR^2+\frac{1}{4}mR^2\right)\ddot{\theta}$$

$$\frac{1}{4}(2M+m)\ddot{\theta}+b\dot{\theta}+\left(\frac{5}{4}k+\frac{mg}{2R}\right)\theta=0$$

$$\Rightarrow \boxed{\ddot{\theta}+\left[\frac{4b}{2M+m}\right]\dot{\theta}+\left[\frac{2m\frac{g}{R}+5k}{2M+m}\right]\theta=0}$$