(1)

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Date : Promotion : Course name :

# Solution of Recitation 1:

## EXO # 1:

$$x(t) = 5.\cos(2t + \frac{\pi}{6}) \equiv A.\cos(\omega_0 t + \phi)$$

a) $T = ?$ from expression $x(t) = A.\cos(\omega_0 t + \phi)$ we can guess that

$$\begin{cases} A = 5\ cm \rightarrow \text{Amplitude} \\ \omega_0 = 2\ rd/s \rightarrow \text{frequency} \\ \phi = \frac{\pi}{6}\ rd \rightarrow \text{phase constant} \end{cases}$$

So, $T = \frac{2\pi}{\omega_0} \Rightarrow \boxed{T = \pi\ s}$

•) $t = 0 \rightarrow x(t=0) = ?\ ,\ \dot{x}(t=0) = ?\ ,\ \ddot{x}(t=0) = ?$

$$x(t=0) = 5.\cos(\frac{\pi}{6}) = 0{,}43\ cm.$$

$$\Rightarrow \boxed{x(t=0) = 0{,}43\ cm}$$

•) $\dot{x}(t=0) = -A\omega_0.\sin(\phi) = -10.\sin(\frac{\pi}{6}) = -5\ cm/s$

$$\boxed{\dot{x}(t=0) = -5.0\ cm/s}$$

•) $\ddot{x}(t=0) = -A\omega_0^2.\cos(\phi) = -20.0\cos(\frac{\pi}{6}) = -17{,}32\ cm/s^2$

$$\boxed{\ddot{x}(t=0) = -17.32\ cm/s^2}$$

②

# Exo # 2

SHM $\Rightarrow x(t) = A.\cos(\omega_0 t + \phi)$ /

$k = 6,5\,N.m^{-1}$, $A = 10,0\,cm$,

$\dot{x} = 30,0\,cm/s$ when $x = 5,0\,cm$

a) $m = ?$ you can use 2 methodes for that;

1^rst one: we use the expression of $A = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}$

(done in lecture) so, where $x_0$ and $v_0$ are position and speed of the block at $t = 0$, or special time

So; for our problem, $x_0 = \frac{A}{2}$ and $v_0 = 30,0\,cm/s$

we replace we get $\omega_0 = \sqrt{\frac{v_0^2}{A^2 - x_0^2}} \Rightarrow$

$\omega_0 = 3,46\,rd/s$ and

$\omega_0 = \sqrt{\frac{k}{m}} \Rightarrow m = \frac{k}{\omega_0^2} = 0,54\,kg \Rightarrow \underline{m = 0,54\,kg}$

b) 2^d method:

$$\begin{cases} x(t_0) = \frac{A}{2} = 5 = 10.\cos(\omega_0 t_0 + \phi) \;..\; ① \\ \dot{x}(t_0) = 30.0 = -10.\omega_0.\sin(\omega_0 t_0 + \phi) \;.\; ② \end{cases}$$

$$\begin{cases} ①^2 \Rightarrow 25,0 = 100,0\cos^2(\omega_0 t_0 + \phi) \\ ②^2 \Rightarrow 900 = 100.\omega_0^2 \sin^2(\omega_0 t_0 + \phi) \end{cases}$$

Now, $①^2 \times \omega_0^2 + ②^2 \Rightarrow 100\,\omega_0^2 = 25\,\omega_0^2 + 900 \Rightarrow$

$\omega_0 = 3,46\,rd/s \Rightarrow m = \frac{k}{\omega_0^2} = 0,54\,kg$ |

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Date : Promotion : Course name :

suite exo # 2:

b) The Period:

$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{3,46} = 1,82 s \Rightarrow \boxed{T = 1,82 s}$

• c) maximum acceleration of the block:

$\ddot{x}(t) = -A\omega_0^2 . \cos(\omega_0 t + \phi) = \ddot{x}_{max} \cos(\omega_0 t + \phi + \pi)$

where $\ddot{x}_{max} = A\omega_0^2 = 120 \ cm/s^2 \Rightarrow \boxed{\ddot{x}_{max} = 120 cm/s^2}$

•) EXO # 3

$T = 4s$, $x = 4 cm$, starts from rest $\Rightarrow$.

at $t = 0$, $\boxed{x = 4 cm = A}$ and $\boxed{\phi = 0}$

$\Rightarrow x(t) = 4. \cos(\omega_0 t) = 2 cm \Rightarrow$ / $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$

$2 = 4.\cos(\frac{\pi}{2} t) \Rightarrow t = \frac{2}{\pi} \arccos(\frac{1}{2}) = 0,67 s$

$\boxed{t = 0,67 s}$

•) velocity? $\dot{x}(t = 0,67s) = -4. \cos(\frac{\pi}{2} . 0,67) = -5,46 cm/s$

$\boxed{\dot{x}(t = 0,67) = -5,46 cm/s}$

• time to reach centre : $x(t = ?) = 4. \cos(\omega_0 t) = 0 \Rightarrow$

$\boxed{t = 1 s}$

4

## EXO# 4

$x(t=0) = -8{,}6\,cm$ , $\dot{x}(t=0) = 0{,}93\,m/s$, $\ddot{x}(t=0) = 4$

a) $\omega_0 = ?$ and $f = ?$

$$\begin{cases} x(t) = A.\cos(\omega_0 t + \phi) \Rightarrow x(t=0) = A.\cos(\phi) = -0{,}086\,m \quad (1) \\ \dot{x}(t) = -A\omega_0 \sin(\omega_0 t + \phi) \Rightarrow \dot{x}(t=0) = -A\omega_0 \sin(\phi) = 0{,}93\,m/s \quad (2) \\ \ddot{x}(t) = -A\omega_0^2 \cos(\omega_0 t + \phi) \Rightarrow \ddot{x}(t=0) = -A\omega_0^2 \cos(\phi) = 48\,m/s^2 \quad (3) \end{cases}$$

$$\text{equ } \frac{(3)}{(1)} \Rightarrow \frac{48}{-0{,}086} = -\omega_0^2 \Rightarrow \boxed{\omega_0 = 23{,}6\,rd/s}$$

$$\Rightarrow f = \frac{\omega_0}{2\pi} \Rightarrow \boxed{f = 3{,}76\,Hz}$$

b) $\phi = ?$ $\Rightarrow$ equ $\frac{(2)}{(1)} \Rightarrow \frac{-0{,}93}{-0{,}086} = -\omega_0 \tan\phi \Rightarrow \phi = \tan^{-1}\left(\frac{0{,}93}{0{,}086 \times 23{,}6}\right)$

$\phi = 155{,}4°$ or $335{,}4°$, but we know that $A > 0 \Rightarrow$

from equ (1) $\Rightarrow \cos(\phi)$ must be $< 0 \Rightarrow \boxed{\phi = 155{,}4°}$

c) $A = ??$ from equ (1) $\Rightarrow$

$$\boxed{A = \frac{-0{,}086}{\cos(\phi)} = 0{,}09\,m}$$

## EXO# 5

$T = \pi s$ , $x = 0 \Rightarrow \dot{x}_{max} = 0{,}1\,m/s = A\omega_0$

$$\boxed{\omega_0 = \frac{2\pi}{T} = 2\,rd/s} \Rightarrow \boxed{A = \frac{\dot{x}_{max}}{\omega_0} = 0{,}05\,m.}$$

we can use $A^2 = x_0^2 + \frac{v_0^2}{\omega_0^2}$ / $x_0$ and $v_0$ are given for the same time

First Name : Last Name : Group / Option :
Date : Promotion : Course name :

suite exo 5 :

$\Rightarrow v_0 = \dot{x}(x = 0{,}03m) = \sqrt{\omega_0^2 (A^2 - x_0^2)} = 0{,}08 m/s$

$\boxed{\dot{x}(x = 0{,}03m) = 0{,}08 m/s}$

EXO # 6

$m = 1 gr$ , $A = 1mm$ , $f = 500 s^{-1}$

$\boxed{\omega_0 = 2\pi f = 1000\pi \ rd/s}$ , $\omega_0 = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega_0^2$

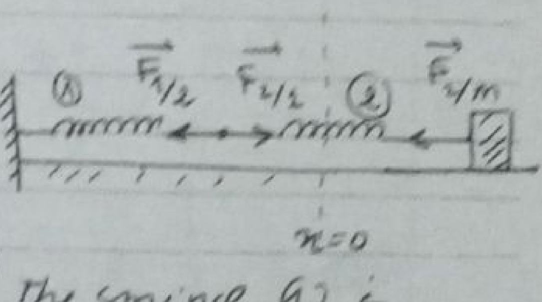
$\boxed{\Rightarrow k = 9869{,}6 \ N.m^{-1}}$

maximum force $\Rightarrow F_{max} = |k x_{max}| = |k.A|$

$\boxed{\Rightarrow F_{max} = 9{,}87 N}$

EXO # 7

case (a)




When, the block is displaced by a distance "$x$" from its equi, the spring ① is stretched by "$x_1$" and the spring ② by "$x_2$".

by Newton's 3$^d$ Law . $|\vec{F}_{1/2}| = |\vec{F}_{2/1}| \Rightarrow$

$\boxed{k_1 x_1 = k_2 x_2}$ .... ① ...

where / $\vec{F}_{1/2}$ is the force applied by spring ① on ②

6

$\vec{F}_{2/1}$ is the force applied by spring (2) on (1)

and $\vec{F}_{2/m}$ is the force applied by spring (2) on "m"

so, the total displacement of the mass is "x" where

$$\boxed{x = x_1 + x_2}$$

and $\boxed{F_{2/m} = -k_2 x_2}$ .... (2) ....

from equ (1) $\Rightarrow x_1 = \frac{k_2}{k_1} x_2 \Rightarrow x = \frac{k_2}{k_1} x_2 + x_2$

$\Rightarrow \boxed{x = \left(\frac{k_1 + k_2}{k_1}\right) x_2} \Rightarrow x_2 = \left(\frac{k_1}{k_1 + k_2}\right) x$

. Now, Newton's 2d Law $\Rightarrow$

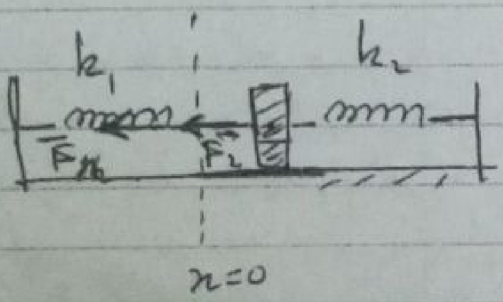
$\Sigma \vec{F} = m\ddot{\vec{x}} \Rightarrow \vec{F}_{2/m} = m\ddot{\vec{x}} \Rightarrow$

$-k_2 x_2 = m\ddot{x} \Rightarrow m\ddot{x} + k_2 x_2 = 0 \Rightarrow$

$m\ddot{x} + \frac{k_2 \cdot k_1}{k_1 + k_2} x = 0 \Rightarrow \ddot{x} + \frac{1}{m} \cdot \frac{k_1 \cdot k_2}{[k_1 + k_2]} \cdot x = 0$

$$\Rightarrow T = 2\pi \cdot \left[m \cdot \frac{(k_1 + k_2)}{k_1 k_2}\right]^{1/2}$$

<u>case b</u>




here $\vec{F}_1$ is the force applied by spring (1) on "m"

and $\vec{F}_2$ the force applied by spring (2) on "m"

$\Rightarrow \Sigma \vec{F} = m\ddot{\vec{x}} \Rightarrow \vec{F}_1 + \vec{F}_2 = m\ddot{\vec{x}} \Rightarrow$

First Name : Last Name : Group / Option :

Date : Promotion : Course name :

suite exo #7

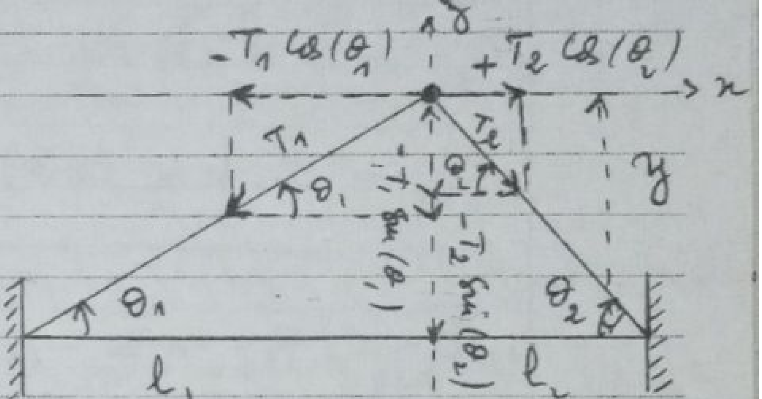
$$-k_1 x_1 - k_2 x_2 = m\ddot{x}$$

but $x_1 = x_2 = x \Rightarrow$

$$m\ddot{x} + (k_1 + k_2)\,x = 0 \Rightarrow \ddot{x} + \left(\frac{k_1 + k_2}{m}\right)x = 0$$

$$\Rightarrow \boxed{T = 2\pi \cdot [m/(k_1 + k_2)]^{1/2}}$$

EXO # 8




$$\sum \vec{F} = m\ddot{\vec{y}}$$

As you can see no motion on "x" axis $\Rightarrow$

$$\begin{cases} \text{on "x" axis} \Rightarrow -T_1\cos(\theta_1) + T_2\cos(\theta_2) = 0 \quad \dots(1)\dots \\ \text{on } y \text{ axis} \Rightarrow -T_1\sin(\theta_1) - T_2\sin(\theta_2) = m\ddot{y} \quad \dots(2)\dots \end{cases}$$

for small oscillations $\theta \ll \Rightarrow \cos(\theta) \simeq 1$ and $\sin(\theta) \simeq \theta$

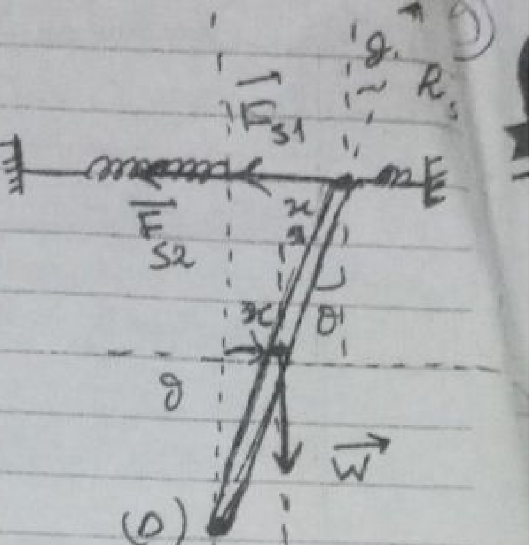
from (1) $\quad T_1 = T_2 = T$ and $\sin(\theta_1) = \frac{y}{\ell_1}$, $\sin(\theta_2) = \frac{y}{\ell_2}$

from (2) $\Rightarrow m\ddot{y} + T\left[\frac{y}{\ell_1} + \frac{y}{\ell_2}\right] = 0$

$$\Rightarrow \ddot{y} + \frac{T}{m}\left(\frac{\ell_1 + \ell_2}{\ell_1 \ell_2}\right)y = 0 \quad \rightarrow \text{diff - equ of SHM.}$$

$$\Rightarrow \ddot{y} + \omega_0^2\, y = 0 \quad / \quad \omega_0 = \sqrt{\frac{T}{m}\left(\frac{\ell_1 + \ell_2}{\ell_1 \ell_2}\right)}$$

## EXO # 9




a) Equation of motion:

We use torque:

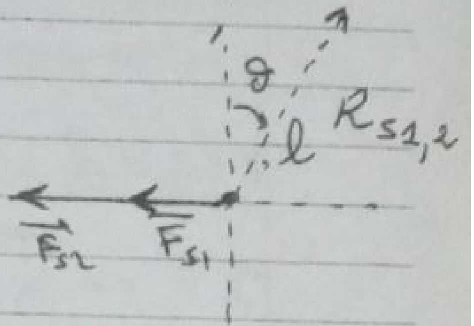
$\sum \vec{\tau} = I_{/O}\,\ddot{\vec{\theta}}$

$\vec\tau_{F_{s1}} + \vec\tau_{F_{s2}} + \vec\tau_{W} = I_{/O}\,\ddot{\vec\theta}$

$\vec\tau_{F_{s1}} = ?$ / $\vec\tau_{F_{s1}} = \vec R_{s1} \wedge \vec F_{s1}$

$\tau_{F_{s1}} = +\,l.k.x_1.\sin(\theta + \pi/2)$

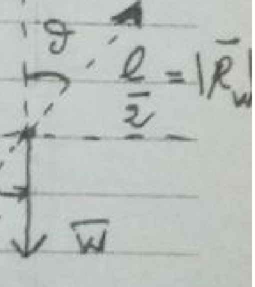
$\boxed{\tau_{F_{s1}} = l.k x_1 \cos(\theta)}$




•) $\vec\tau_{F_{s2}} = \vec R_{s2} \wedge \vec F_{s2} \Rightarrow \tau_{F_{s2}} = l.k.x_2 \sin(\theta + \frac{\pi}{2})$

$\boxed{\tau_{F_{s2}} = l.k.x_2 \cos(\theta)}$

•) $\vec\tau_W = \vec R_W \wedge \vec W \Rightarrow \tau_W = -\frac{l}{2}.m.g.\sin(\pi - \theta)$




$\boxed{\tau_W = -\frac{l}{2}.m.g.\sin(\theta)}$

We have SHM → $\theta <<$ ⇒ $\sin(\theta) = \frac{x_1}{l} = \frac{x_2}{l} = \frac{x}{l/2} =$

We know $x_1 = x_2 \Rightarrow \boxed{x_1 = x_2 = l.\theta}$ and

$\boxed{x = \frac{l}{2}\theta}$ we replace we get:

$2l^2 k$ [illegible] $- \frac{l}{2}.m.g.\theta = -I_{/O}\ddot\theta$ / $I_{/O} = \frac{1}{3} m l^2$

First Name : ____ Last Name : ____ Group / Option : ____

Date : ____ Promotion : ____ Course name : ____

## Suite exo #9

$$\Rightarrow \frac{1}{3} m l^2 \ddot{\theta} + 2 l^2 k \cancel{\theta} - \frac{l}{2} m.g\theta = 0$$

$$\Rightarrow \ddot{\theta} + \left(6\frac{k}{m} - \frac{3}{2}\frac{g}{l}\right)\theta = 0$$

$$\ddot{\theta} + \omega_0^2 \theta = 0 \quad \Big| \quad \omega_0 = \left[\frac{6k}{m} - \frac{3}{2}\frac{g}{l}\right]^{1/2}$$

## 2d Method : Energy :

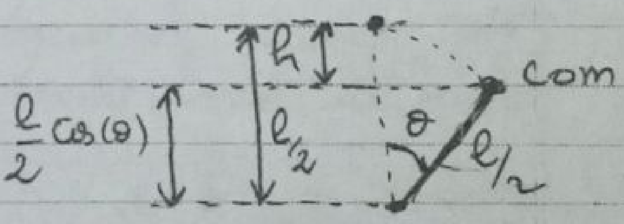
$$E = K + U = K(\text{rod}) + U(\text{rod}) + U(\text{spr } 1) + U(\text{spr } 2)$$

$$K^{\text{rod}} = \frac{1}{2} I_{/O} \dot{\theta}^2, \quad U(\text{rod}) = mgh, \quad U(sp1) = \frac{1}{2} k x_1^2$$

and $U(sp1) = U(sp2) \Rightarrow$

$$E = \frac{1}{2} I_{/O} \dot{\theta}^2 + \frac{1}{2} k x_1^2 + \frac{1}{2} k x_1^2 + mgh$$

$x_1 = l\theta$ and $h =$




$$h = \frac{l}{2}\cos(\theta) - \frac{l}{2}$$

$$h = \frac{l}{2}(\cos(\theta) - 1)$$

$$\theta << \Rightarrow \cos(\theta) \simeq 1 - \frac{\theta^2}{2!} \Rightarrow \cos(\theta) - 1 \simeq -\frac{\theta^2}{2}$$

$$\Rightarrow E = \frac{1}{2} \cdot \frac{1}{3} m l^2 \dot{\theta}^2 + k l^2 \theta^2 - mg \cdot \frac{l}{4}\theta^2$$

$E$ is constant $\Rightarrow \frac{dE}{dt} = 0 \Rightarrow$

Suite ex # 9

$$\frac{dE}{dt} = \frac{1}{3} m l^2 \dot{\theta}\ddot{\theta} + 2 k l^2 \theta\dot{\theta} - \frac{1}{2} m g . l . \theta \dot{\theta}$$

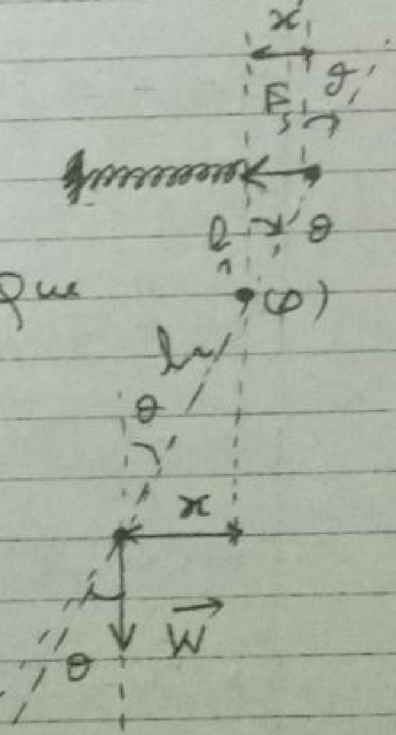
$$\Rightarrow \frac{1}{3} m l^2 \ddot{\theta} + 2 k l^2 \theta - \frac{1}{2} m g l . \theta = 0$$

$$\Rightarrow \ddot{\theta} + \left(\frac{6k}{m} - \frac{3}{2}\frac{g}{l}\right)\theta = 0 \quad / \omega_0 = \left[\frac{6k}{m} - \frac{3}{2}\frac{g}{l}\right]$$

b) stability of the system

Since the leading coefficient is positive, sign of the coefficient of determines the stability.

- if $\frac{6k}{m} - \frac{3}{2}\frac{g}{l} > 0 \Rightarrow 4k > \frac{mg}{l} \Rightarrow$ system is stable
- if $\frac{6k}{m} - \frac{3g}{2l} = 0 \Rightarrow 4k = \frac{mg}{l} \Rightarrow \theta(t) = at + b \Rightarrow$ unstable
- if $\frac{6k}{m} - \frac{3g}{2l} < 0 \Rightarrow 4k < \frac{mg}{l} \Rightarrow$ unstable




- Exo # 10

The applied forces don't start from the same point $\Rightarrow$ use torque

$$\Sigma \vec{\tau} = I_{/O} \vec{\ddot{\theta}} \Rightarrow$$

$$\vec{\tau}_s + \vec{\tau}_w = I_{/O} \vec{\ddot{\theta}}$$

- $\tau_s = l_1 . k . x' . \sin(\theta + \frac{\pi}{2})$

$$\boxed{\tau_s = l_1 . k . x' \cos(\theta)}$$

First Name : Last Name : Group / Option :

Date : Promotion : Course name :

Suite exo ~~#~~10

$$\bar{\tau}_W = \bar{R}_W \wedge \bar{W} \Rightarrow \tau_W = +\ell_2 \cdot m \cdot g \cdot \sin(\theta)$$

$$\theta \ll \Rightarrow \sin(\theta) \simeq \theta = \frac{x'}{\ell_1} = \frac{x}{\ell_2} \Rightarrow x' = \frac{\ell_1}{\ell_2} x$$

$$\Rightarrow \frac{\ell_1^2}{\ell_2} k x + \ell_2 \cdot m \cdot g \cdot \frac{x}{\ell_2} = -I_{/O} \ddot{\theta} \quad / \quad I_{/O} = m \ell_2^2$$

and $\ddot{x} = \ell_2 \cdot \ddot{\theta} \Rightarrow \ddot{\theta} = \frac{\ddot{x}}{\ell_2}$ we replace, we get

$$\ddot{x} + \left(\frac{\ell_1^2}{\ell_2^2} \frac{k}{m} + \frac{g}{\ell_2}\right) x = 0$$

$$\ddot{x} + \omega_0^2 x = 0 \quad /$$

$$\omega_0 = \left[\frac{\ell_1^2}{\ell_2^2} \frac{k}{m} + \frac{g}{\ell_2}\right]^{1/2}$$

•) Energy Method:

$$E = K + U = K(m) + U(m) + U(\text{spring}).$$

$$E = \frac{1}{2} I_{/O} \dot{\theta}^2 + \frac{1}{2} k x'^2 + m g h \quad /$$

$$h = \ell_2 - \ell_2 \cos(\theta) \quad / \quad U \nearrow \text{ increases.}$$

$$h = \ell_2 (1 - \cos(\theta)) \quad / \quad \theta \ll \Rightarrow \cos(\theta) = 1 - \frac{\theta^2}{2}$$

$$\Rightarrow h = \ell_2 \cdot \frac{\theta^2}{2} \quad | \quad \text{and } \sin(\theta) = \theta = \frac{x'}{\ell_1}$$

$\Rightarrow x' = \ell_1 \theta$. we replace in $E$, we get

$$E = \frac{1}{2} m \ell_2^2 \dot{\theta}^2 + \frac{1}{2} k \ell_1^2 \theta^2 + m \cdot g \cdot \ell_2 \cdot \frac{\theta^2}{2}$$

## suite exo # 10

$E = \text{constant} \Rightarrow \dfrac{dE}{dt} = 0 \Rightarrow$

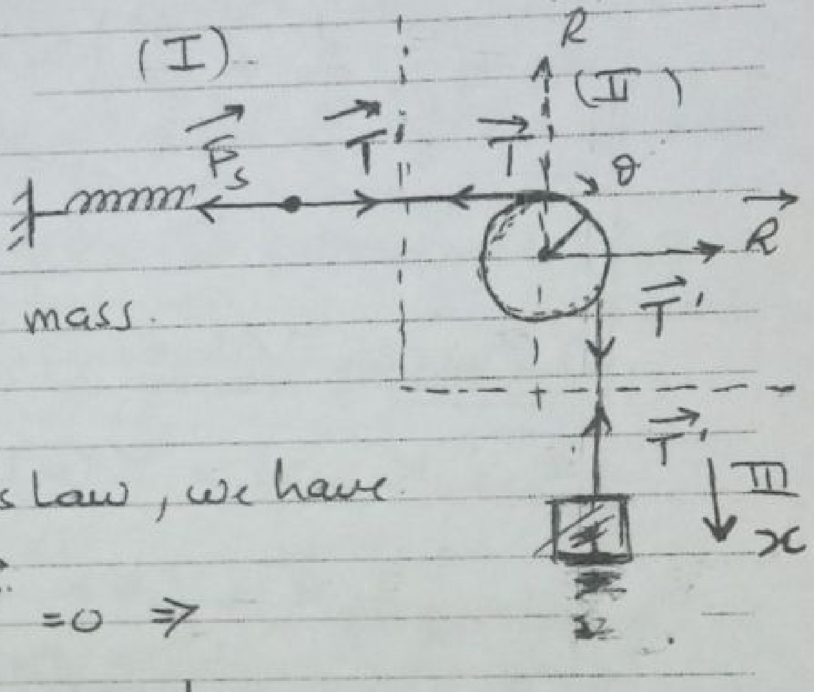
$$\frac{dE}{dt} = m\ell_2^2\dot{\theta}\ddot{\theta} + k\ell_1^2\theta\dot{\theta} + mg\frac{\ell}{2}\theta\dot{\theta} = 0$$

$$\Rightarrow \ddot{\theta} + \left[\frac{\ell_1^2}{\ell_2^2}\frac{k}{m} + \frac{g}{\ell_2}\right]\theta = 0 \quad / \ddot{\theta} + \omega_0^2\theta = 0$$

$$/ \omega_0 = \left[\frac{\ell_1^2}{\ell_2^2}\frac{k}{m} + \frac{g}{\ell_2}\right]^{1/2}$$

## Ex # 11 :

we divide the system on 3 parts: spring, pulley and mass.




•) Spring: from 3^d Newton's Law, we have

$$\sum \vec{F} = 0 \Rightarrow \vec{F}_s + \vec{T} = 0 \Rightarrow$$

$$-kx + T = 0 \Rightarrow \underline{kx = T}$$

•) Pulley: here we use torque $\Rightarrow$

$$\sum \vec{\tau} = I_{/D}\ddot{\vec{\theta}} \Rightarrow \vec{\tau}_T + \vec{\tau}_{T'} = I_{/D}\ddot{\vec{\theta}} \Rightarrow$$

$$+RT - RT' = -I_{/D}\ddot{\theta} \Rightarrow RT - RT' = -\frac{1}{2}MR^2\ddot{\theta}$$

$$\Rightarrow T' = T + \frac{1}{2}MR\ddot{\theta} \Rightarrow T' = kx + \frac{1}{2}MR\ddot{\theta}$$

(4)

## Suite exo#12

$I^{d}_{/\Delta} = I^{d}_{/com} = \frac{1}{2} m R^2$ because the axis of rotation passes through the com of disk.

Now for rod: $I^{rod}_{/\Delta} = I^{rod}_{/com} + m D^2$ / D is the distance between com of rod and axis of rotation.

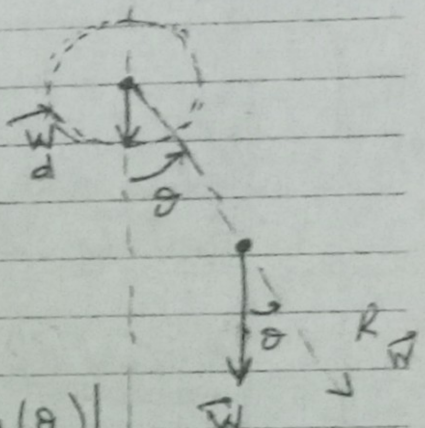
$$I^{rod}_{/\Delta} = \frac{1}{12} m L^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{12} mL^2 + \frac{1}{4} mL^2 = \frac{1}{3} mL^2$$

$$\Rightarrow I^{system}_{/\Delta} = \frac{1}{2} mR^2 + \frac{1}{3} mL^2 \Rightarrow$$

$$I^{S}_{/\Delta} = m\left[\frac{R^2}{2} + \frac{L^2}{3}\right]$$

•) b) the diff-equ and angular frequency:

We use TorQue: $\sum \vec{\tau} = I_{/\Delta} \ddot{\vec{\theta}}$




$$\vec{\tau}_{disk} + \vec{\tau}_{rod} = I_{/\Delta} \ddot{\theta} \Rightarrow$$

$\vec{\tau}_{disk} = 0$ / the weight of the disk exerts no torque ($R_W = 0$)

$$\vec{\tau}_{rod} = \vec{R}_{rod} \wedge \vec{W}_{rod} \Rightarrow \tau_{rod} = -\frac{L}{2} m g \sin(\theta)$$

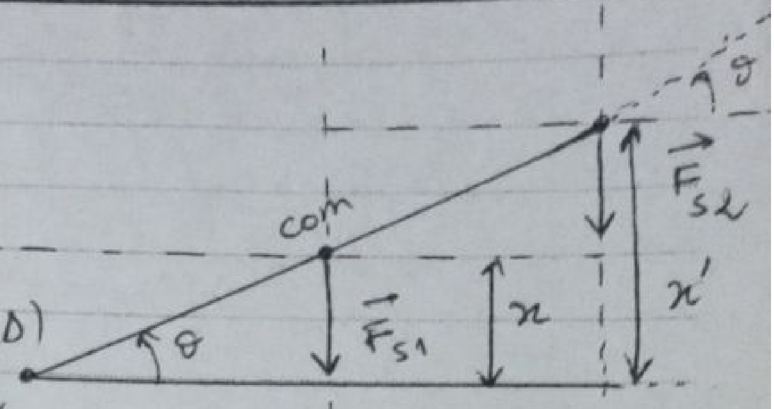
$$\Rightarrow -\frac{L}{2} m.g \sin(\theta) = I^{S}_{/\Delta} \ddot{\theta} \Rightarrow$$ / $\theta << \Rightarrow \sin(\theta) \simeq \theta$

$$I_{/\Delta} \ddot{\theta} + \frac{L}{2} m.g\,\theta = 0 \Rightarrow \ddot{\theta} + \left[\frac{Lg}{\left[\frac{R^2}{2} + \frac{L^2}{3}\right]}\right] \theta = 0$$

$$\omega_0 = \left[\frac{Lg}{\left(\frac{R^2}{2} + \frac{L^2}{3}\right)}\right]^{1/2}$$

| First Name : | Last Name : | Group / Option : |
| --- | --- | --- |
| Date : | Promotion : | Course name : |

<u>Exo # 13:</u>




$$\Sigma \vec{\tau} = I_{/\Delta}\ddot{\vec{\theta}}$$

$$\vec{\tau}_{s1} + \vec{\tau}_{s2} = I_{/\Delta}\ddot{\vec{\theta}}$$

we don't speak about weight, because, we have the case of vertical spring-block system and so, the initial elongations of spring will be canceled with weight of rod.

• $\vec{\tau}_{s1} = ?$ : $\vec{\tau}_{s1} = \vec{R}_1 \wedge \vec{F}_{s1}$ / where $|\vec{R}_1| = \frac{L}{2}$

$\Rightarrow \tau_{s1} = -\frac{L}{2} k . x_1 . \sin(\theta + \frac{\pi}{2})$ / $x_1 = x$.

$\Rightarrow \boxed{\tau_{s1} = -\frac{L}{2} k x . \cos(\theta)}$ / $\cos(\theta) \simeq 1$ ($\theta$<<)

•) $\vec{\tau}_{s2} = ?$ : $\vec{\tau}_{s2} = \vec{R}_2 \wedge \vec{F}_{s2}$ / $|\vec{R}_2| = L$

$\Rightarrow \tau_{s2} = - L . k . x_2 . \sin(\theta + \frac{\pi}{2})$

$\sin(\theta) = \frac{x}{L/2} = \frac{x_2}{L} \Rightarrow 2x = x_2 \Rightarrow$

$\boxed{\tau_{s2} = -2L k . x . \cos(\theta)}$ / $\cos(\theta) \simeq 1$ ($\theta$<<)

$\Rightarrow -\frac{L}{2} k x - 2L k x = \frac{1}{3} m L^2 \ddot{\theta}$ / where

$\ddot{x} = \frac{L}{2}\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{2}{L}\ddot{x}$ we replace we get

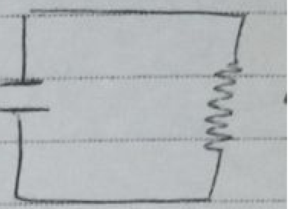
$$\frac{2}{3} ML\ddot{x} + \frac{5}{2} Lk.x = 0 \Rightarrow$$

$$\ddot{x} + \frac{15}{4}\frac{k}{M} x = 0 \Rightarrow \ddot{x} + \omega_0^2 x$$

$$\Rightarrow \omega_0 = \sqrt{\frac{15}{4}\frac{k}{M}}$$

EXO N 14

$f = 440 Hz$ , $L = 2H$




$\omega = 2\pi f \Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow$

$\omega_0^2 = 4\pi^2 f^2 = \frac{1}{LC} \Rightarrow C = 0,065 \mu C$

•) $V_c = 5V$ is the initial value of voltage at $t = 0 \Rightarrow$

$V_c = \frac{q_0}{C} \Rightarrow q_0 = V_c \cdot C = 0,33$

) Peak current

$q(t) = q_0 . \cos(\omega_0 t + \phi)$

$i(t) = \frac{dq(t)}{dt} = -\omega_0 q_0 . \sin(\omega_0 t + \phi)$

$i(t) = \omega_0 q_0 . \cos(\omega_0 t + \phi + \frac{\pi}{2})$

$i(t) = I_0 . \cos(\omega_0 t + \phi + \frac{\pi}{2})$ / when $I_0 = \omega_0 q_0$

is the peak or maximum current $\Rightarrow$

$I_0 = 0,9 mA$

**Recitatio 1, Exercise # 15 (Final exam 2018) (5 marks).**

The differential equation:

**1st Methode: Pure rotation / P:**

We study the system as pure rotation around axis (P) and we use torque.

$$\sum \vec{\tau} = I_{/P}\ddot{\vec{\theta}} \Rightarrow \vec{\tau}_{s1} + \vec{\tau}_{s2} = I_{/P}\ddot{\vec{\theta}}$$

$$\begin{cases} \vec{\tau}_{s1} = 2R.k_1.x_1.\sin(\frac{\pi}{2}+\theta) & 1 \\ \vec{\tau}_{s2} = 3R.k_2.x_2.\sin(\frac{\pi}{2}+\theta) & 1 \end{cases}$$

$$\Rightarrow 2R.k_1.x_1\cos(\theta) + 3R.k_2.x_2\cos(\theta) = -I_{/P}\ddot{\theta}$$

where $x_1$ is the elongation of spring $k_1$ and $x_2$ is the elongation of spring $k_2$

and $I_{/P} = I_{/com} + MR^2 = \frac{1}{2}MR^2 + MR^2 \Rightarrow I_{/P} = \frac{3}{2}MR^2$ 0.5

For small oscillations $\sin(\theta) \approx \theta = \frac{x}{R} = \frac{x_1}{2R} = \frac{x_2}{3R}$ and $\cos(\theta) \approx 1$,

we replace in the differential equation we get

$$2R.k_1.(2R\theta) + 3R.k_2.(3R\theta) = -\frac{3}{2}MR^2\ddot{\theta} \quad 0.5$$

$$\frac{3}{2}M\ddot{\theta} + (4k_1 + 9k_2)\theta = 0 \quad \Rightarrow$$

$\boxed{\ddot{\theta} + \left(\frac{8k_1 + 18k_2}{3M}\right)\theta = 0}$ 0.5 which is the differential equation of SHM, ,

where the angular frequency is $\boxed{\omega_0 = \sqrt{\frac{8k_1 + 18k_2}{3M}}}$ 1.5

**2d Method: Energy method:**

$$E = K + U = K_M + U_{s1} + U_{s2}$$

$$E = \frac{1}{2}I_{/P}\dot{\theta}^2 + \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 \quad 2$$

$$= \frac{1}{2}(\frac{3}{2}MR^2).\dot{\theta}^2 + \frac{1}{2}k_1(2R\theta)^2 + \frac{1}{2}k_2(3R\theta)^2$$

$$E = \frac{3}{4}MR^2\dot{\theta}^2 + 2R^2k_1\theta^2 + \frac{9}{2}R^2k_2\theta^2 \quad 1$$

we are in the case where no damping so the total energy is conserved, thus

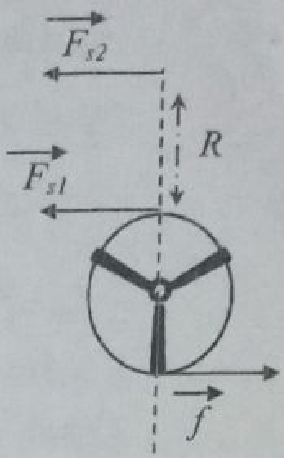
$$\frac{dE}{dt} = 0 \Rightarrow \frac{3}{2}MR^2\ddot{\theta}\dot{\theta} + 4R^2k_1\dot{\theta}\theta + 9R^2k_2\dot{\theta}\theta = 0$$

$\Rightarrow \boxed{\ddot{\theta} + \left(\frac{8k_1 + 18k_2}{3M}\right)\theta = 0}$ 0.5 thus, $\boxed{\omega_0 = \sqrt{\frac{8k_1 + 18k_2}{3M}}}$ 1.5

## 3ᵈ Method: rotation + translation; Rolling.

In this case we study the system as it is, means rotation about its com + translation,
For translation we have,

$$\sum \vec{F} = M\ddot{\vec{x}} \Rightarrow \vec{F}_{s1} + \vec{F}_{s2} + \vec{f} = M\ddot{\vec{x}}$$




$$\Rightarrow -k_1.x_1 - k_2.x_2 + f = M\ddot{x} \quad ...(1).... \quad 1$$

and for rotation about centre of mass we have;

$$\sum \vec{\tau} = I_{/com}\ddot{\vec{\theta}} \Rightarrow \vec{\tau}_{s1} + \vec{\tau}_{s2} + \vec{\tau}_f = I_{/com}\ddot{\vec{\theta}}$$

$$\Rightarrow Rk_1.x_1.\sin(\frac{\pi}{2}+\theta) + 2Rk_2.x_2.\sin(\frac{\pi}{2}+\theta) + R.f = -\frac{1}{2}MR^2\ddot{\theta} \quad 0.5$$

for small oscillations $\sin(\theta) \approx \theta = \frac{x}{R} = \frac{x_1}{2R} = \frac{x_2}{3R}$ we replace we get,

$$Rk_1(2R\theta) + 2Rk_2(3R\theta) + Rf = -\frac{1}{2}MR^2\ddot{\theta} \Rightarrow \boxed{f = -\frac{1}{2}MR\ddot{\theta} - 2Rk_1\theta - 6Rk_2\theta} \quad 1$$

we replace in equation (1) thus,

$$-k_1.(2R\theta) - k_2.(3R\theta) - \frac{1}{2}MR\ddot{\theta} - 2Rk_1\theta - 6Rk_2\theta = M(R\ddot{\theta}) \quad 0.5$$

$$\frac{3}{2}M\ddot{\theta} + 4k_1\theta + 9k_2\theta = 0 \Rightarrow \boxed{\ddot{\theta} + \left(\frac{8k_1 + 18k_2}{3M}\right)\theta = 0} \quad 0.5$$

and thus the angular frequency is

$$\boxed{\omega_0 = \sqrt{\frac{8k_1 + 18k_2}{3M}}} \quad 1.5$$