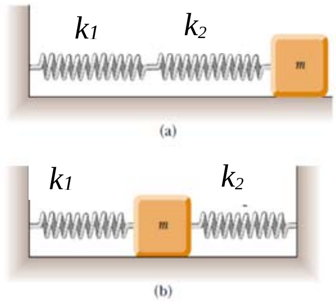
SERIE # 1 : Simple Harmonic Motion

Exercise # 1

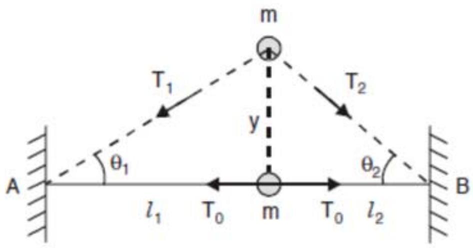


A block of mass m is connected to two springs of force constants k_1 and k_2 as shown in Figures. In each case, the block moves on a frictionless table after it is displaced from equilibrium and released. Show that in the two cases the block exhibits simple harmonic motion with periods

$$(a) \qquad T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}} \quad , \qquad (b) \qquad T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$$

Exercise # 2

A ball of mass m is connected to rigid walls by means of two massless wires of lengths l_1 and l_2 see figure. At equilibrium the tension in each wire is T_0. The mass m is displaced slightly from equilibrium in the vertical direction and released. Determine the frequency for small oscillations.



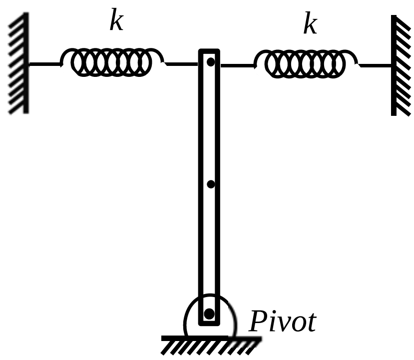
Exercise # 3

Consider the system of uniform rod of length ℓ connected between two springs as shown on the figure.

a) Write the equation of motion in terms of the angle, θ, the bar makes with the vertical.

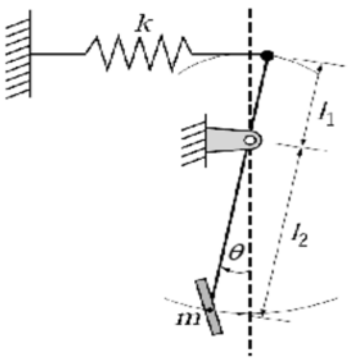
b) Discuss the stability of the system's solutions in terms of the physical constants; m, k, and ℓ .

The moment of inertia about pivot axis rod is $I_{/p} = m\ell^2/3$.

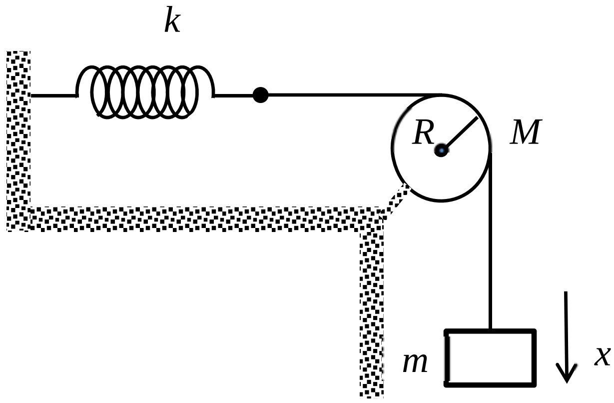


Exercise # 4

A control pedal of an aircraft can be modeled as the system of Figure. Consider the lever as a massless rod and the pedal as a lumped mass at the end of the rod. Set up the differential equation of motion in θ and calculate the natural frequency of the system. Assume the spring to be unstretched at θ = 0.

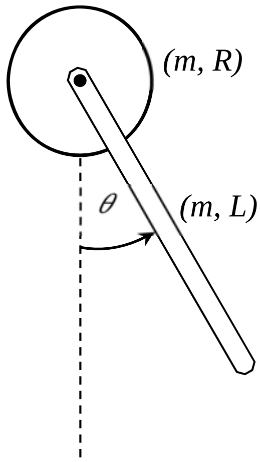


Exercise # 5



One end of a light spring with spring constant k is attached to a vertical wall. A light string is tied to the other end of the horizontal spring. The string changes from horizontal to vertical as it passes over a solid pulley of radius R and mass M. The pulley is free to turn on a fixed smooth axle. The vertical section of the string supports an object of mass m. The string does not slip at its contact with the pulley. Find the frequency of oscillation of the object if the mass of the pulley is (a) negligible, (b) M.
The moment of inertia of pulley is $MR^2/2$

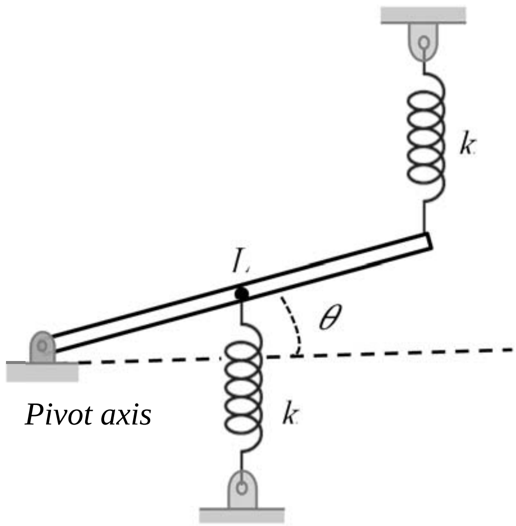
Exercise # 6



A rigid body is composed of a uniform disk (mass m, radius R) and a uniform rod (mass m, length L) that is rigidly fixed to the center of the disk. This body is pivoted about the center of the disk around a horizontal axis that is perpendicular to the plane of the page. Assume that there is no friction,

1) Find the moment of inertia of the system $I_{/pivot}$ about the pivot axis.
2) For small oscillations set up the differential equation of motion and find the angular frequency.
Given, the inertia of the rod about its center of mass $I_{/com} = mL^2/12$ and the inertia of disk about its center of mass is $I_{/com} = mR^2/2$.

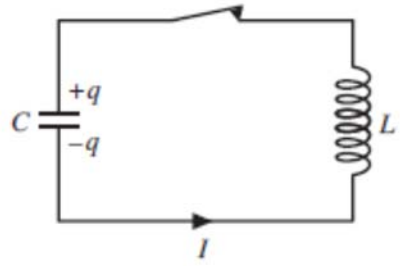
Exercise # 7



The 5-kg uniform rod of length L and moment of inertia about pivot axis of $I = ML^2/3$, is attached to two springs of spring constant k = 500 N/m, which can act in tension or compression. The first spring is placed at the middle of rod and the second one at the end as shown in the figure. If the end of the rod is slightly pulled up and released, determine the angular frequency of small oscillations.

Exercise # 8

We would like to make an LC circuit that oscillates at 440 Hz. If we have a 2 H inductor, what value of capacitance should we use?. If the capacitor is initially charged to 5 V, what will be the peak charge on the capacitor and the peak current? What is the total energy in the circuit?



Exercise # 9

A solid cylinder (M, R) is rolling without slipping on a horizontal plane when attached at the top of the cylinder to a linear spring of constant k_1 and at a distance R from the top through a rigid rod of negligible mass to a 2nd spring of constant k_2. The moment of inertia of cylinder with respect to its centre of mass is $MR^2/2$. For small displacement set up the differential equation of motion and find the angular frequency.