

# A Pseudo-Value Approach to Causal Deep Learning of Semi-Competing Risks

Stephen Salerno
Public Health Science, Biostatistics
Fred Hutchinson Cancer Center
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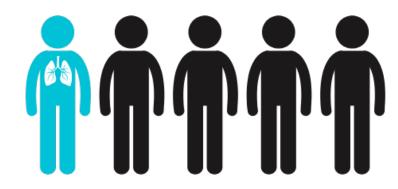




### **Clinical Motivation**

- Lung cancer has a 5-year survival rate of **20%** [3]
- Patients can experience recurrence, remission, metastasis prior to death [8]
- Cancer recurrence is an important endpoint in patients who have undergone curative treatment
- Further understanding **patient-specific** treatment efficacy is crucial for **individualized** care [5, 2, 7]

Approximately 1 in 5 cancer deaths are attributed to lung cancer.



Source: WHO International Agency for Research on Cancer

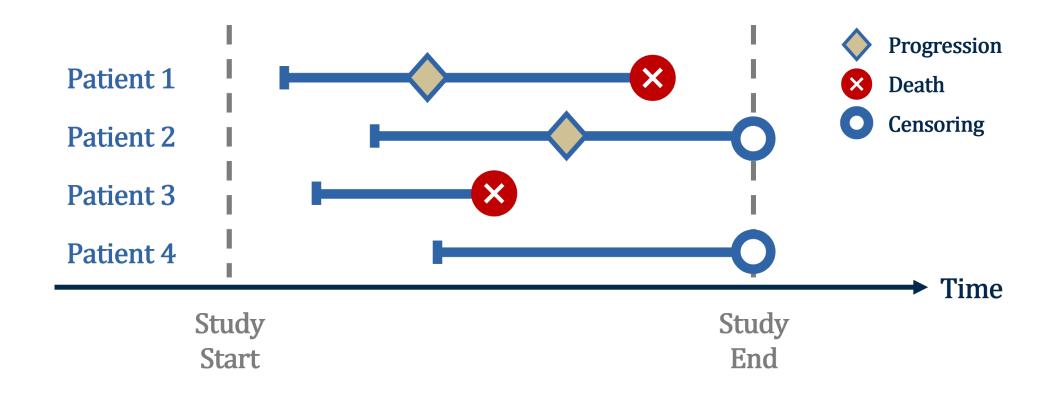
### **Statistical Motivation**

- While non-fatal health events impact treatment decisions and disease management:
  - Overall survival is often studied without considering competing events
  - Composite endpoints such as recurrence-free survival are used [9]
  - The effects of treatments or risk factors may differ across disease states [1, 6]
- Data from the **Boston Lung Cancer Study**, a large cancer epidemiology cohort:
  - Observational studies provide a wealth of information on individualized risk factors
  - However, observational data suffer from confounding and covariate imbalance

## **Our Proposal**

We propose a **deep learning** approach for estimating the **causal effect** of treatment on **disease recurrence** in the presence of **complex covariate relationships** 

## **Semi-Competing Risks**



### **Outcome Notation**

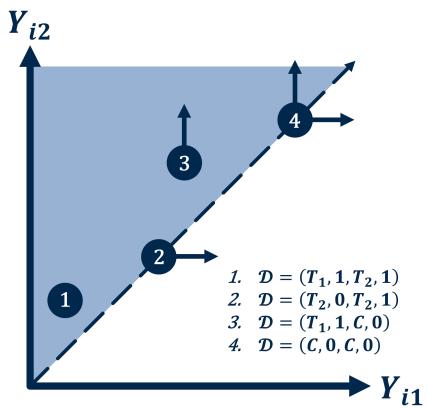
#### **Event Times:**

- *T<sub>i1</sub>*: Time to Recurrence
- $T_{i2}$ : Time to Death
- *C<sub>i</sub>*: Censoring Time

#### **Observable Outcomes:**

- $Y_{i1} = \min(T_{i1}, T_{i2}, C_i)$
- $\bullet \ Y_{i2} = \min(T_{i2}, C_i)$
- $\bullet \ \delta_{i1} = I[T_{i1} \leq \min(T_{i2}, C_i)]$
- $\delta_{i2} = I(T_{i2} \leq C_i)$

Outcomes observable only on upper wedge:



### **Potential Outcomes Framework**

- $Z_i$  = Causal variable of interest ( $Z_i$  = 1 for surgical resection and  $Z_i$  = 0 for other first-line treatment options)
- $X_i = p$ -vector of additional confounding variables
- $T_{i1}^z$ : Potential time to recurrence had *i*th patient received treatment  $z \in \{0, 1\}$
- Seek to estimate an average treatment effect (ATE; i.e., expected difference in potential outcomes)
- For time-to-recurrence, consider the average causal **risk difference** at time *t*:

$$ATE = \mathbb{E}\left[I\left(T_{i1}^{1} > t\right) - I\left(T_{i1}^{0} > t\right)\right]$$
(1)

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### **Proposed Three-Stage Approach**

- **Step 1.** Estimate **survival function** for time-to-recurrence
- Step 2. Calculate pseudo-survival probabilities at fixed times
- **Step 3.** Train **deep neural network** to estimate **causal target** (ATE)

### **Step 1: Estimate the Recurrence Survival Function**

• Use a Clayton copula to jointly model the survival times for disease recurrence and death:

$$S(t_1, t_2) = [S_1(t_1)^{-\theta} + S_2(t_2)^{-\theta} - 1]^{-1/\theta},$$
(2)

• Yields an expression for the recurrence survival function that is always estimable:

$$S_1(t) = [S_*(t)^{-\theta} - S_2(t)^{-\theta} + 1]^{-\frac{1}{\theta}}$$
(3)

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### Step 2: Calculate Pseudo-Survival Probabilities

• Use the estimated  $S_1(t)$  to jackknife **probability of no recurrence** at fixed time points (e.g., 1, 5 years):

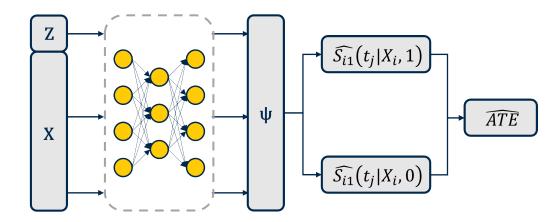
$$\hat{S}_{i1}(t_j) = n \times \hat{S}_1(t_j) - (n-1) \times \hat{S}_1^{-i}(t_j)$$
 for times  $t_j$ ;  $j = 1, ..., J$ 

- Represents **contribution** of *i*th individual in estimating  $\mathbb{E}[S_1(t_i)]$  in the sample of *n* subjects
- $S_{i1}(t_j)$  is approximately independent of  $S_{i'1}(t_j)$  for  $i \neq i'$  as  $n \to \infty$
- $\lim_{n\to\infty} E[S_{i1}(t_j) \mid Z_i, X_i] = S_1(t_j \mid Z, X)$  (important for this method)

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## **Step 3: Train Causal S-Learner**

- 3a. Fit model for  $S_{i1}(t \mid X_i, Z_i)$  non-parametrically with a feed-forward, fully-connected S-Learner
- 3b. Predict potential outcomes  $\hat{S}_{i1}(t \mid X_i, Z_i = z); z \in \{0, 1\}$
- 3c. Calculate  $\hat{ATE} = n^{-1} \sum_{i=1}^{n} \{\hat{S}_{i1}(t \mid X_i, 1) \hat{S}_{i1}(t \mid X_i, 0)\}$



Encodes features into **lower representative space**,  $\Psi$ , w/ covariates **decorrelated** from treatment

### Rationale for DNN in Causal Estimation

- Use the pseudo-survival probabilities as **targets** in a deep learning model
  - Similar to logistic model fit to  $I(T_{i1} > t_i)$  if the data were **fully observed**
  - More natural to interpret than hazards ratios
  - No proportional hazards assumption
- Binary cross-entropy loss function, optimizing predictions for survival probabilities
- Faster convergence than MSE due to steeper gradient when prediction is far from truth

## **Simulation Settings**

Setting 1: Similar Performance Expected

 $T_{i1}$ ,  $T_{i2}$  follow PH models with linear risks, independent covariates, and correlated errors

Risk function of 3 covariates,  $Z_i \sim Bern(0.5)$ ,  $X_{i1}$ ,  $X_{i12} \sim TN(1, 0.5, 0, 2)$ 

Setting 2: Proposed Method Expected to Perform Better

 $T_{i1}$ ,  $T_{i2}$  follow PH models with non-linear risks, correlated covariates, and correlated errors

Risk function of 3 covariates,  $X \sim TN_3(\mathbf{0}, \Sigma, -\mathbf{1}, \mathbf{1})$  where  $\Sigma$  is AR(1) and  $Z = I(X_1 \ge 0)$ ,  $X_2$  and  $X_3$  squared

Compared bias and MSE in estimating ATE to parametric GEE with complementary log-log link across 50 independent datasets, also varying n (500 and 1,000),  $\theta$  (0.5 and 2.0), and censoring rates (0% and 50%)

## **Simulation Setting 1**

Methods Perform Similarly as Parametric Model is Correctly Specified

Average bias and MSE for estimated vs. true ATE (Setting 1)

Simulation Settings		Bia	as	MSE		
n	$\theta$	Censoring	Parametric	Proposed	Parametric	Proposed
500	0.5	50%	0.0025	0.0060	0.0020	0.0063
500	0.5	0%	0.0025	0.0045	0.0022	0.0042
500	2.0	50%	0.0025	0.0057	0.0022	0.0053
500	2.0	0%	0.0018	0.0069	0.0019	0.0011
1000	0.5	50%	0.0018	0.0025	0.0013	0.0028
1000	0.5	0%	0.0023	0.0035	0.0014	0.0028
1000	2.0	50%	0.0019	0.0048	0.0014	0.0037
1000	2.0	0%	0.0018	0.0030	0.0012	0.0021

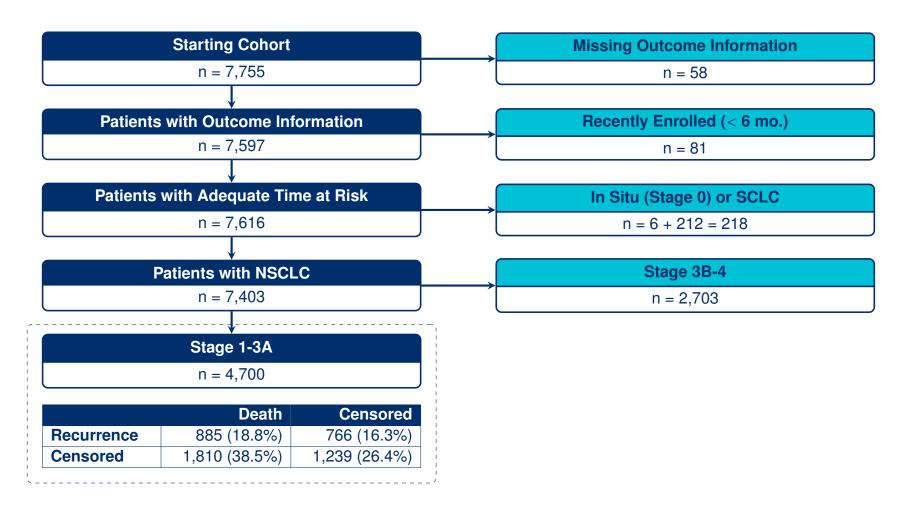
## **Simulation Setting 2**

Proposed Outperforms Current Approach as Risk Function is Complex

Average bias and MSE for estimated vs. true ATE (Setting 2)

Simulation Settings		Bia	as	MSE		
n	$\theta$	Censoring	Parametric	Proposed	Parametric	Proposed
500	0.5	50%	0.0483	0.0043	0.0076	0.0032
500	0.5	0%	0.0520	0.0030	0.0078	0.0031
500	2.0	50%	0.0444	-0.0083	0.0081	0.0045
500	2.0	0%	0.0476	-0.0030	0.0079	0.0046
1000	0.5	50%	0.0485	-0.0043	0.0036	0.0028
1000	0.5	0%	0.0518	-0.0034	0.0038	0.0024
1000	2.0	50%	0.0444	-0.0040	0.0046	0.0032
1000	2.0	0%	0.0475	-0.0035	0.0042	0.0033

## **BLCS Study Cohort**

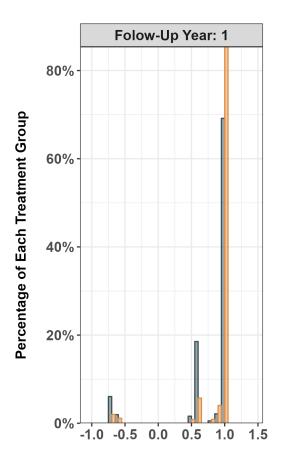


### Dist. of Confounders Differs Across First-Line Trt. Groups

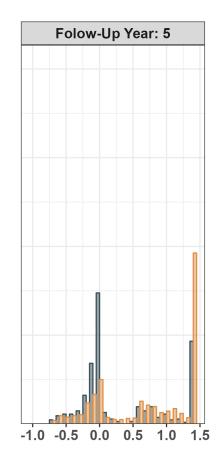
Characteristic*	Overall, N = 4,700 <sup>1</sup>	Other, N = 706 <sup>1</sup>	Surgery, N = 3,994 <sup>1</sup>	p-value <sup>2</sup>
Stage at Diagnosis				< 0.001
Stage 1	3,007 (64%)	223 (32%)	2,784 (70%)	
Stage 2	753 (16%)	100 (14%)	653 (16%)	
Stage 3a	940 (20%)	383 (54%)	557 (14%)	
Age at Diagnosis (yrs.)	68 (61, 74)	69 (62, 77)	67 (61, 74)	< 0.001
Female	2,603 (55%)	367 (52%)	2,236 (56%)	0.077
Body Mass Index	26.6 (23.3, 31.1)	26.1 (22.8, 30.8)	26.7 (23.4, 31.1)	0.013
Smoking Status				0.003
Never Smoker	592 (13%)	65 (9%)	527 (13%)	
Former Smoker	2,821 (60%)	422 (60%)	2,399 (60%)	
Current Smoker	1,171 (25%)	205 (29%)	966 (24%)	
Smoker, Status Unknown	116 (2%)	14 (2%)	102 (3%)	
Pack-Years of Smoking	40 (19, 53)	40 (24, 61)	40 (18, 51)	< 0.001
EGFR Mutation	158 (3%)	22 (3%)	136 (3%)	< 0.001
Not Tested	3,805 (81%)	610 (86%)	3,195 (80%)	
KRAS Mutation	265 (6%)	17 (3%)	248 (6%)	< 0.001
Not Tested	3,805 (81%)	610 (86%)	3,195 (80%)	

<sup>&</sup>lt;sup>1</sup> n (%); Median (IQR); <sup>2</sup> Pearson's Chi-squared test; Wilcoxon rank sum test; Fisher's exact test \* Subsequent models further adjusted for race, ethnicity, and education level

### **Distribution of BLCS Pseudo-Values**



Distribution of pseudo-recurrence probabilities shifts from one to zero over time, moreso for other first-line treatments



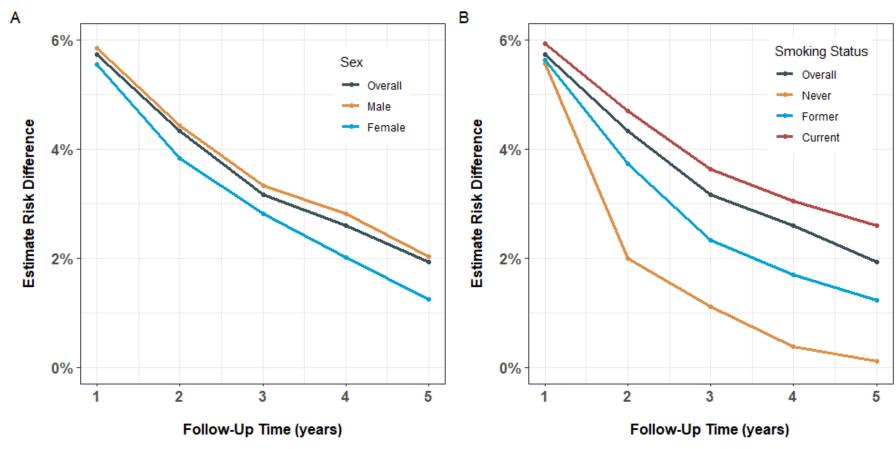
Estimated Pseudo-Values

First-Line Treatment: Other Surgery

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### Risk Difference Attenuates over Time

#### Differently by Patient Subgroups



### **BLCS Results**

- Overall difference in risk of recurrence between first-line therapies attenuates over time: 5.7% at 1 year vs. 1.9% at 5 years
- Stratified by sex risk difference is slightly higher among male patients, attenuates similarly
- Larger differences were observed when stratifying by smoking status
  - Treatment differences slightly higher among current smokers (5.9% at 1-year vs. 2.5% at 5-years)
  - Greater attenuation among former (range: 5.6% to 1.2%) and never smokers (range: 5.6% to 0.1%)

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### **Conclusions**

- Method to estimate the causal effect of treatment on NSCLC recurrence, respecting semi-competing risks
- Demonstrated the **performance** of this approach on simulated and real-world data
- Emphasized the importance of accounting for dependent censoring
- **Observed** differences in the efficacy of surgical resection compared to other first-line therapies, which attenuated over time in BLCS

### **Planned Future Work**

- Develop an approach for quantifying uncertainty in our estimated ATE and drawing inference
- Extend this method to account for other sources of confounding such as immortal time bias
- Improve the **computational efficiency** and usability of software before making it **available** via an R package
- Consider additional target values such as restricted mean survival times

## **Questions?**

#### Thank you to my collaborators:

- My advisor: Prof. Yi Li
- My BLCS Collaborators: Dr. David Christiani,
   Dr. Xinan Wang, Ms. Jui Kothari

#### **Preprint:**



#### Slides & Code:



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### **Challenges We Address**

- 1. Semi-competing risks introduce dependent censoring, where death precludes disease recurrence
- 2. Current approaches necessitate **complicated** loss functions or **strong assumptions** (e.g., PH)
- 3. Parametric or semi-parametric methods are limited in their ability to model **complex** risk functions
- 4. Little work integrating causal inference and machine learning in settings of dependent censoring

## **Key Assumptions**

1. Consistency:  $T_{i1} = T_{i1}^{Z_i}$  almost surely

An individual's potential outcome under their assigned treatment group is the outcome that will be observed

**2. Positivity**:  $Z_i \in \{0, 1\} \ \forall X_i$ 

Every individual has a non-zero probability of being assigned to either treatment group

3. No Interference:  $T_{i1}^z$  is unaffected by the value of z for another subject, j

The potential outcomes of one individual are not affected by the treatment assignment of other individuals

4. Exchangeability:  $T_{i1}^1$ ,  $T_{i1}^0 \perp Z_i \mid X_i$ 

There is no unmeasured confounding

**5.** Non-Informative Censoring:  $T_{i1} \perp C_i \mid Z_i, X_i$ 

Subject's censoring time is independent of their failure time given their covariates

## **Proposed Three-Stage Approach**

Step 1. Estimate survival function for time-to-recurrence

Step 2. Calculate pseudo-survival probabilities at fixed times

- Consistent for survival probability
- Circumvents need for complex loss function
- Does not require assumptions like proportional hazards

Step 3. Train deep neural network to estimate causal target (ATE)



### 1. Survival Function for Time-to-Recurrence

• Assume time-to-recurrence  $(T_1)$  and death  $(T_2)$  are cont. non-negative r.v.'s with **survival functions**:

$$S_1(t_1) = \Pr(T_1 > t_1); \quad t_1 \ge 0$$
  
 $S_2(t_2) = \Pr(T_2 > t_2); \quad t_2 \ge 0.$ 

- Distribution of  $T_1$  is **non-parametrically identifiable** ONLY when recurrence ALWAYS precedes death
- The **joint survival function** of the event times is:

$$S(t_1, t_2) = Pr(T_1 > t_1, T_2 > t_2)$$

1  $S_1(t_1)$   $\theta$  2 Jackknife 3 DNN ATE

## Estimating $S_1(t_1)$ and $\theta$

• Consider a Clayton copula model for  $T_1$ ,  $T_2$  with dependence parameter  $\theta \ge 0$ :

$$S(t_1, t_2) = [S_1(t_1)^{-\theta} + S_2(t_2)^{-\theta} - 1]^{-1/\theta},$$
(4)

• Given (4) and  $S_2(t_2)$ ,  $S_1(t_1)$  is **monotonic** and **estimable**:

$$S_1(t) = [S_*(t)^{-\theta} - S_2(t)^{-\theta} + 1]^{-\frac{1}{\theta}},$$
 (5)

where  $S_*(t)$  is the recurrence-free survival function

- $S_*(t)$  and  $S_2(t)$  are estimable via KM, since both are always observable
- $\theta$  estimated via extension to concordance-based estimator of Fine et al. (2001)

1  $\rightarrow$   $S_1(t_1)$   $\rightarrow$   $\theta$  2 Jackknife 3 DNN ATE

## Estimating $\theta$ w/o Covariates

Fine et al. (2001) proposed a **concordance**-based estimator:

$$\hat{\theta} = \frac{\sum_{i < j} W(Y_{ij1}, Y_{ij2}) D_{ij} \Delta_{ij}}{\sum_{i < j} W(Y_{ij1}, Y_{ij2}) D_{ij} (1 - \Delta_{ij})} - 1$$

where (i, j) are independent observation pairs, and:

- $T_{ij1} = \min(T_{i1}, T_{j1}); T_{ij2} = \min(T_{i2}, T_{j2}); C_{ij} = \min(C_i, C_j)$
- $Y_{ij1} = \min(T_{ij1}, T_{ij2}, C_{ij}); Y_{ij2} = \min(T_{ij2}, C_{ij})$
- $D_{ij} = I(T_{ij1} < T_{ij2} < C_{ij}); \Delta_{ij} = I[(T_{i1} T_{j1})(T_{i2} T_{j2}) > 0]$
- $W_{a,b}^{-1}(x,y) = \frac{1}{n} \sum_{i} \{ I(Y_{i1} \ge \min(a,x), Y_{i2} \ge \min(b,y) \}$

*Note*:  $\Delta_{ij}$  is estimable only when  $D_{ij} = 1$  and a, b may be selected to dampen W for large x, y

### Estimating $\theta$ w/ Covariates

With covariates Z, X, the copula model (4) is extended to

$$S(t_1, t_2 \mid Z, X) = [S_1(t_1 \mid Z, X)^{-\theta} + S_2(t_2 \mid Z, X)^{-\theta} - 1]^{-1/\theta}$$
(6)

- $\theta$  quantifies correlation of  $T_1$  and  $T_2$  conditional on Z, X
- Conditioning on Z, X, model (6) implies

$$S_1(t \mid Z, X) = [S_*(t \mid Z, X)^{-\theta} - S_2(t \mid Z, X)^{-\theta} + 1]^{-\frac{1}{\theta}},$$

 $1 \longrightarrow S_1(t_1) \longrightarrow \theta$  2 Jackknife 3 DNN ATE

## Estimating $\theta$ – 'Leave-One-In'

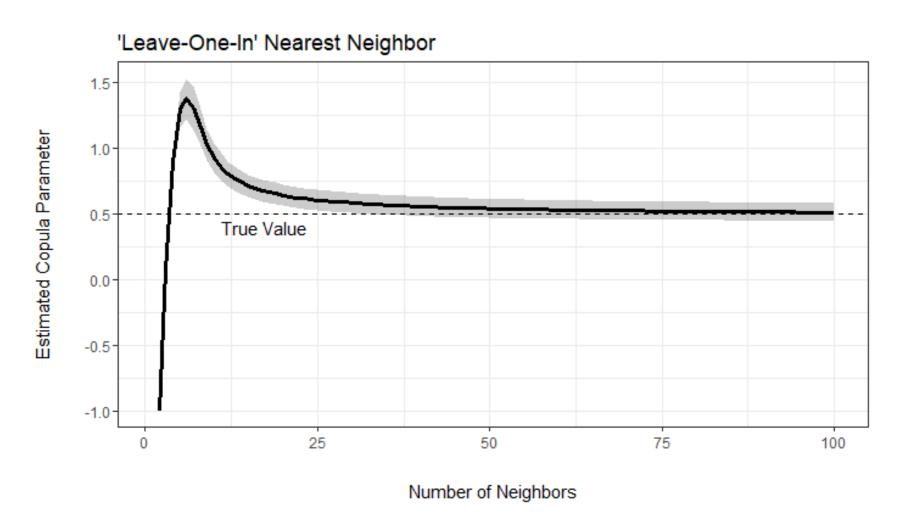
Propose a **conditional**  $\hat{\theta}$  using nearest k neighbors to subject i:

- 1. Let  $\tilde{X}_i = \{Z_i, X_i\}$  denote the (p+1)-vector of covariates for the *i*th patient
- **2.** Calculate distance between  $\tilde{X}_i$  and  $\tilde{X}_{i'}$  for  $1 \le i \ne i' \le n$ :  $||\tilde{X}_i \tilde{X}_{i'}||_2 = \{\sum_{j=1}^{p+1} (\tilde{x}_{ij} \tilde{x}_{i'j})^2\}^{1/2}$
- 3. For each i, identify k nearest neighbors,  $\mathcal{N}(i, k)$
- 4. Estimate  $\hat{\theta}^{(i)}$  based on subjects from  $\mathcal{N}(i,k)$  via  $\hat{\theta}^{(i)} = \frac{\sum_{j,l \in \mathcal{N}(i,k);j < l} W\left(Y_{jl1},Y_{jl2}\right)D_{jl}\Delta_{jl}}{\sum_{j,l \in \mathcal{N}(i,k);j < l} W\left(Y_{jl1},Y_{jl2}\right)D_{jl}\left(1-\Delta_{jl}\right)} 1$
- **5.** Estimate  $\hat{\theta}$  via  $\hat{\theta} = n^{-1} \sum_{i=1}^{n} \hat{\theta}^{(i)}$



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# Example $\hat{\theta}$ Calculation



### 2. Pseudo-Survival Probabilities

Interested in the risk difference at a given time point, want a model that reflects this:

- Common approaches (e.g., Cox model) impose structure across all time points (e.g., proportional hazards)
- However, treatment efficacy of a may change over time
- Pseudo-values are an intuitive alternative



### **Jackknife Pseudo-Values**

- Probability of **no recurrence** by time  $t_j$ , j = 1, ..., J, is  $S_1(t_j) = \Pr(T_1 > t_j)$
- Pseudo-survival probability for ith individual at time  $t_i$ :

$$\hat{S}_{i1}(t_j) = n \times \hat{S}_1(t_j) - (n-1) \times \hat{S}_1^{-i}(t_j)$$

where  $\hat{S}_1(t_i)$  and  $\hat{S}_1^{-i}(t_i)$  are estimates of  $S_1(t_i)$  using all n subjects and excluding the ith subject

- Represents contribution of *i*th individual in estimating  $\mathbb{E}[S_1(t_i)]$  in the sample of *n* subjects
- $\hat{S}_i(t)$  then used as **response**, similar to logistic model fit to  $I(T_{i1} > t_j)$  if the data were **fully observed**



## **Example Calculation**

Example pseudo-values for two individuals.

Observation		Simulated Outcomes				Treatment	Estimated
ID	t	<i>Y<sub>i1</sub></i>	D <sub>i1</sub>	<b>Y</b> <sub>i2</sub>	D <sub>i2</sub>	$Z_i$	Pseudo-Values
1	0.2	0.3991	1	0.4054	1	0	1.0302
1	0.4	0.3991	1	0.4054	1	0	-0.3260
1	0.6	0.3991	1	0.4054	1	0	0.1765
1	8.0	0.3991	1	0.4054	1	0	0.0968
1	1.0	0.3991	1	0.4054	1	0	0.0496
2	0.2	1.0401	0	1.0401	0	1	1.0302
2	0.4	1.0401	0	1.0401	0	1	1.1761
2	0.6	1.0401	0	1.0401	0	1	1.3082
2	8.0	1.0401	0	1.0401	0	1	1.4430
2	1.0	1.0401	0	1.0401	0	1	1.5688

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## **Some Useful Properties**

- 1. Survival probabilities are more natural to interpret than hazards ratios
- **2.**  $S_{i1}(t_i)$  is approximately independent of  $S_{i'1}(t_i)$  for  $i \neq i'$  as  $n \to \infty$
- 3.  $\lim_{n\to\infty} E[S_{i1}(t_j) \mid Z_i, X_i] = S_1(t_j \mid Z, X)$  (important for this method)
- With (2) and (3), these pseudo-values can be used as a response variables in a deep learning framework
- Imputed outcome is more efficient for deep learning



## 3. Deep Learning

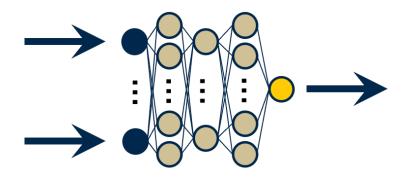
**Deep learning** mirrors how the brain functions:

- Neurons connected in a network of L layers,
   with k<sub>l</sub> neurons in the lth layer
- Predictions based on *L*-fold **composite**:

$$F_L(\cdot) = f_L \circ f_{L-1} \circ \cdots \circ f_1(\cdot)$$
 where  $(g \circ f)(\cdot) = g(f(\cdot))$ 

$$f_l(x) = \sigma_l(\mathbf{W}_l x + \mathbf{b}_l) \in \mathbb{R}^{k_{l+1}}$$

where  $\sigma_l$  is an activation function,  $\mathbf{W}_l$  are weights, and  $\mathbf{b}_L$  are biases

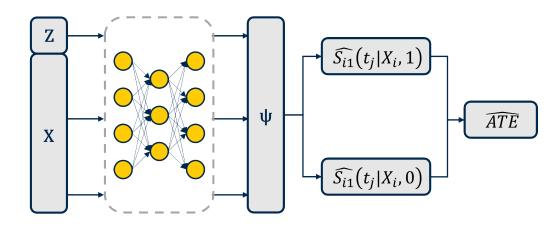


- > Allows for *non-parametric* estimation of risk functions
- > Circumvents the *curse of dimensionality* [4, 10]



### **Causal S-Learner**

- 1. Fit model for  $S_{i1}(t \mid X_i, Z_i)$  non-parametrically with a feed-forward, fully-connected S-Learner
- **2. Predict** potential outcomes  $\hat{S}_{i1}(t \mid X_i, Z_i = z); z \in \{0, 1\}$
- 3. Calculate  $\hat{ATE} = n^{-1} \sum_{i=1}^{n} \{ \hat{S}_{i1}(t \mid X_i, 1) \hat{S}_{i1}(t \mid X_i, 0) \}$





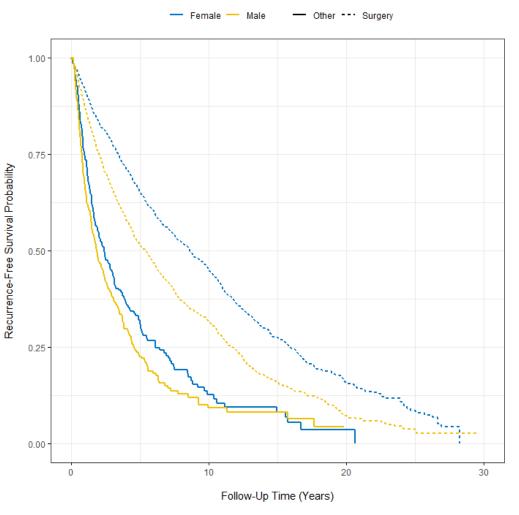
**Fred Hutch Cancer Center** 

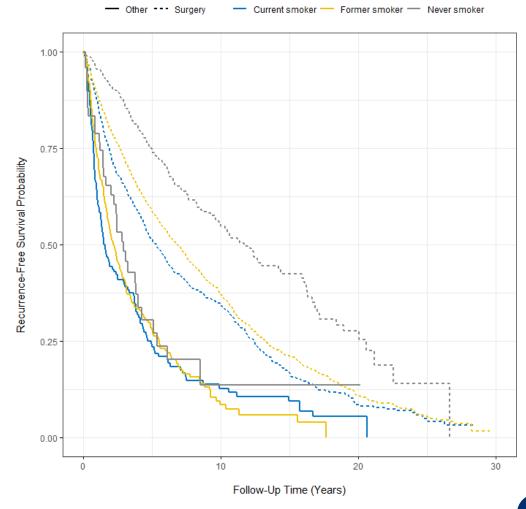
### **Advantages of Causal S-Learner**

- DNNs encode informative features into lower representative spaces
- These **embeddings** make downstream supervised learning tasks easier
- S-Learning for causal estimation outputs  $\Psi$ , which produces a **representation** of the covariates **decorrelated** from the treatment
- For our **pseudo-value** approach, network output optimized under the **binary cross-entropy loss** function
  - Faster learning rate/convergence than MSE due to steeper gradient when prediction is far from truth
  - More natural interpretation than common loss functions in survival analysis

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### **BLCS KM-Estimated Recurrence-Free Survival**





### **BLCS** Estimated $\theta$

- Estimated  $\theta$  using 'leave-one-in' extension to the concordance-based estimator
- Among all patients, estimated  $\hat{\theta} = 5.60$ , corresponding to a Kendall's  $\tau$  value of 0.737
  - This suggests a high degree of correlation between recurrence and death
- Stratified estimates of  $\theta$  differed across subgroups
  - E.g., by sex, estimated  $\theta$  higher among females (5.93;  $\tau = 0.748$ ) than males (4.85;  $\tau = 0.708$ )

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