Trade Credit, Risk Sharing, and Inventory Financing Portfolios

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As an integrated part of a supply contract, trade credit has intrinsic connections with supply chain coordination and inventory management. Using a model that explicitly captures the interaction of firms' operations decisions, financial constraints, and multiple financing channels (bank loans and trade credit), this paper attempts to better understand the risk-sharing role of trade credit – that is, how trade credit enhances supply chain efficiency by allowing the retailer to partially share the demand risk with the supplier. Within this role, in equilibrium, trade credit is an indispensable external source for inventory financing, even when the supplier is at a disadvantageous position in managing default relative to a bank. Specifically, the equilibrium trade credit contract is net terms when the retailer's financial status is relatively strong. Accordingly, trade credit is the only external source that the retailer uses to finance inventory. By contrast, if the retailer's cash level is low, the supplier offers two-part terms, inducing the retailer to finance inventory with a portfolio of trade credit and bank loans. Further, a deeper early-payment discount is offered when the supplier is relatively less efficient in recovering defaulted trade credit, or the retailer has stronger market power. Trade credit allows the supplier to take advantage of the retailer's financial weakness, yet it may also benefit both parties when the retailer's cash is reasonably high. Finally, using a sample of firm-level data on retailers, we empirically observe the inventory financing pattern that is consistent with what our model predicts.

Key words: trade credit; supply chain management; supply contract; inventory management; newsvendor; operations–finance interface; financial constraint; capital structure; cost of financial distress

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1. Introduction

Extended by sellers to buyers to allow the latter to purchase goods or service from the former without making immediate payment, trade credit is an important source of external financing. As of June 2016, accounts payable, the amount of trade credit owed by buyers to suppliers, are 3.3 times as large as bank loans on the aggregated balance sheet of non-financial US businesses (Federal Reserve Board 2016). Focusing on large public retailers in North America, we find that accounts payable alone amount to approximately one third of their total liabilities.[1] Directly linked

[1] Section 6 explains the data set and selection criteria in detail.

to inventory investment, the amount of trade credit received by retailers is significant relative to firms' inventory holdings. As shown in Table 1, across all US public retailers, the median days payable outstanding (DPO), i.e., the number of days that it takes retailers to pay their suppliers, is 37.3, amounting to half of the days sales of inventory (DSI). Furthermore, variations in DPO across different subcategories in the retail sector are closely related to variations in DSI, with a correlation of 0.83 between the median DPO and median DSI across different subcategories. Such a high correlation suggests that trade credit is closely related to firms' inventory investment.

Table 1 Day payable outstanding (DPO) and days sales of inventory (DSI) of retailers by sub-category

Subcategory in Retail (NAICS)	Number of obs.	DPO 25%	DPO 50%	DPO 75%	DSI 25%	DSI 50%	DSI 75%
Motor vehicle and parts dealers (441)	258	5.4	9.7	61.4	58.8	74.2	168.7
Furniture and home furnishings stores (442)	109	36.8	45.0	66.5	73.1	100.7	136.5
Electronics and appliance stores (443)	151	32.8	50.3	86.3	58.6	79.1	132.9
Building material and garden equipment and supplies dealers (444)	96	20.6	32.8	40.6	64.9	83.5	115.5
Food and beverage stores (445)	274	22.9	29.4	46.1	28.5	33.7	42.9
Health and personal care stores (446)	263	27.5	38.6	56.9	12.1	49.2	97.4
Gasoline stations (447)	57	7.2	13.0	21.8	8.7	12.3	54.9
Clothing and clothing accessories stores (448)	749	27.8	38.0	48.3	64.0	81.0	117.1
Sporting goods, hobby, book, and music stores (451)	227	42.3	56.3	71.7	112.6	137.4	165.4
General merchandise stores (452)	326	27.7	34.5	42.7	72.2	95.4	113.7
Miscellaneous stores (453)	168	22.7	32.4	46.1	59.0	88.7	115.3
Non-store retailers (454)	426	31.9	49.0	74.7	23.5	52.4	92.4
All Retailers	3104	26.8	37.3	56.3	53.7	75.1	113.1

Notes. Data based on 3,104 firm-year observations of US retailers between 2000 and 2013. See Section 6 for detailed data selection criteria. DPO = accounts payable/(COGS/365), DSI = inventory/(COGS/365). Correlation between median DPO and DSI across different sub-catogeries is 0.83.

Empirical evidence also indicates significant variation in trade credit terms both across and within industries and sometimes even within a company (Ng et al. 1999, Klapper et al. 2012). The most common trade credit terms are *net terms*, which are essentially interest-free loans extended by suppliers to buyers. For example, under the term "net 30," buyers need to pay suppliers within 30 days of invoice issuance. Another type of commonly seen terms is *two-part terms*. For example, under the often-used two-part terms "2/10 net 30," trade credit must also be paid off within 30 days; however, if it is paid off within 10 days, a 2% discount applies. Such variations are also indirectly supported by Table 1, which highlights the variability of trade credit usage even within a retail subcategory.

Despite the widespread use of trade credit, offering this financing option exposes suppliers to buyers' demand risk. For example, after several seasons of disappointing sales, Circuit City, the second-largest consumer electronics retailer in the US at the time, filed for bankruptcy in November

2008. As disclosed in its bankruptcy filing, 48 out of its 50 largest unsecured creditors were trade creditors and the three largest trade creditors (Hewlett-Packard, Samsung, and Sony) had total claims worth $284 million (Circuit City Stores, Inc. 2008). In addition to default risk, a more common risk associated with trade credit is payment timing. Stretching payment beyond the agreed trade credit terms is extremely common. For instance, in 2012, large UK companies paid up more than 20 days beyond the agreed terms on average (Hurley 2012). Such delays are often associated with buyers' disappointing sales (Strom 2015, Armstrong 2016). As such, trade credit payment is (partly) contingent on buyers' demand risk, in terms of both the received amount and the time. While seemingly undesirable for suppliers, this contingency shares some similarities with other mechanisms used in supply chain coordination in terms of its role in sharing retailers' demand risk, which often leads to enhanced supply chain efficiency (Cachon 2003).

Motivated by the above anecdotal and empirical evidence, this paper aims to deepen our understanding of the (demand) risk-sharing role of trade credit. Specifically, we examine the following three questions under this role of trade credit. First, how do operational and financial factors drive trade credit terms? Second and relatedly, facing multiple financing sources, including trade credit, how does a buyer finance inventory? Finally, what are the operational implications of trade credit?

To answer the above questions, the paper extends the classic selling-to-the-newsvendor model to explicitly capture the retailer's external financing options (bank loans and trade credit) and costs of financial distress. The model focuses on the class of supply contracts under which suppliers offer two prices: a unit cash price, which is paid by the buyer upon delivery, and a unit credit price, which is paid after demand is realized. This contract captures the two types of trade credit terms most commonly used in practice: it corresponds to net terms if the cash price equals the credit price and to two-part terms if not. In response to this contract, the retailer decides the order quantity, as well as how to finance the inventory through (a portfolio of) different sources. This model reveals that under trade credit, the supplier's payoff depends on the buyer's realized demand through the (partial) contingency of the buyer's trade credit payment to the supplier. Such a contingency effectively lowers the buyer's marginal cost, allowing the buyer to stock at a higher inventory level, and, hence, boosts sales for the supplier.

Within the risk-sharing role of trade credit, we find that, when a buyer's internal capital (cash) is not too low, the supplier offers net terms as the operational benefits of trade credit dominate. However, as the buyer's cash reserves become extremely limited, the supplier offers two-part terms to balance the operational benefit and the financing cost associated with trade credit default. Furthermore, we find that the supplier offers larger early-payment discounts when the bank is relatively more efficient than the supplier at collecting default claims or when the buyer has more market power.

The optimal trade credit contract determines the composition of a retailer's *inventory financing portfolio*, i.e., the amount of trade credit and bank loans that the retailer uses to finance inventory. In equilibrium, the operational benefit associated with trade credit dictates that it is always an indispensable part of the retailer's inventory financing portfolio, even when the supplier is less efficient in managing default than a bank. In other words, acting as a risk-sharing mechanism, trade credit is the preferred source for retailers financing their inventory, while bank loans are used only as a supplement. Specifically, when a retailer's cash level is high, i.e., his financing need is low, the retailer receives net terms, and, hence, uses cash and trade credit alone to finance inventory. By contrast, with high financing need, the retailer, facing two-part terms, finances inventory using a portfolio of cash, bank loans, and trade credit. This inventory financing pattern is supported by a preliminary empirical test that we conducted using COMPUSTAT data. Finally, our model also reveals that, in equilibrium, the buyer receives more trade credit when the chain profit margin is high, and possibly when the buyer has more market power.

The adoption of trade credit connects the equilibrium operational decisions and supply chain performance to the retailer's financial situation and financial frictions in the chain. Compared to the classic price-only contract (Lariviere and Porteus 2001), trade credit always leads to higher order quantities, hence enhancing channel efficiency and social welfare. In addition, the supplier may also enjoy a higher nominal profit margin under trade credit, especially when the retailer is financially weak. Combining its impact on quantity and margin, trade credit allows suppliers to better take advantage of buyers' financial weaknesses. Buyers, on the other hand, may also benefit from trade credit when they are not extremely financially constrained. In addition, while the supplier always benefits from being more efficient in managing trade credit default, retailers may prefer a supplier who is less efficient in doing so.

The contribution of the paper is twofold. First, by developing a novel model that incorporates endogenous supply contracts, multiple financing sources, and financial frictions, this paper arrives at a range of results on trade credit terms and usage that are consistent with empirical and anecdotal evidence. This suggests that demand risk sharing, a concept at the core of supply chain management, could be an important reason behind the widespread use of trade credit in practice. Second, the paper highlights how suppliers should adjust trade credit terms (together with the wholesale price) based on both financial and operational characteristics. Accordingly, buyers' inventory and financing choices should jointly depend on financial and supply chain factors.

The rest of the paper is organized as follows. Section 2 discusses prior research in related areas. Section 3 lays out the model of a financially constrained supply chain and a stylized trade credit contract. Section 4 examines the retailer' response under a given trade credit contract. Section 5 studies the optimal trade credit contract, the structure of the inventory financing portfolio, and the operational implications of trade credit. We provide preliminary empirical evidence on the composition of the inventory financing portfolio in Section 6. Section 7 concludes the paper. All proofs and technical lemmas are in the Appendix, which also includes a list of notations.

2. Related Literature

Focusing on the role of trade credit in channel coordination and inventory management, our work is closely related to three streams of literature: the operations–finance interface, inventory management in the presence of trade credit, and research on trade credit in finance.

The rapidly growing literature on the operations–finance interface examines the interplay between firms' operational decisions and financial frictions. Papers such as Babich and Sobel (2004), Buzacott and Zhang (2004), Xu and Birge (2004), Gaur and Seshadri (2005), Caldentey and Haugh (2006), and Ding et al. (2007), Boyabatlı and Toktay (2011), Alan and Gaur (2011), Dong and Tomlin (2012), Li et al. (2013), and Dong et al. (2015) focus on joint operational and financial decision-making in individual companies. Several more closely related papers in this stream have examined how financial constraints influence supply chain performance: Dada and Hu (2008) study a cash-constrained retailer's optimal ordering quantity when facing a profit-maximizing bank; Lai et al. (2009) discuss whether a cash-constrained supplier should operate in pre-order or consignment mode; Caldentey and Chen (2010) propose a contract where the supplier offers partial credit to the budget-constrained retailer; Kouvelis and Zhao (2011) study the optimal price-only contract when selling to a cash-constrained newsvendor when bankruptcy is costly.

Several recent works in the operations-finance interface literature examine the role of trade credit in operational settings. Babich and Tang (2012) and Rui and Lai (2015) show how trade credit can be used to mitigate a supplier's moral hazard. Cai et al. (2014) and Chod (2015) identify that, by tying financing with physical transactions, trade credit controls the buyer's opportunistic behavior. Peura et al. (2016) find that adopting trade credit may improve the profitability of firms who engage in price competition. Other related works include Yang and Birge (2011), Tunca and Zhu (2014), Devalkar and Krishnan (2014), and Dong et al. (2016).

Within this literature, our work is most closely related to Kouvelis and Zhao (2012), who compare supplier financing (trade credit) with bank financing in the absence of costs of financial distress.[2] Under certain technical conditions, they find that when the supplier can only choose one of the two channels, supplier financing is the superior option. Our paper differs from Kouvelis and Zhao (2012) in two crucial modeling assumptions. First, instead of having the supply chain partners *choose* between trade credit and a bank loan, we adopt a portfolio framework under which the retailer can

[2] This paper is a revision of the first author's job market paper and part of this author's dissertation (Yang and Birge 2009, Yang 2010), which were written without prior awareness of Kouvelis and Zhao (2012).

simultaneously borrow from a bank and use trade credit. Second, we explicitly capture the costs of financial distress in the event of bank loan or trade credit default. These two key differences intertwine, allowing the model to generate insights that complement Kouvelis and Zhao (2012) in several respects. First, under milder technical conditions, we generalize Kouvelis and Zhao (2012) by finding that when optimally endogenized, trade credit is always an indispensable financing source for the retailer and enhances supply chain efficiency, and that when the cost of financial distress associated with trade credit is negligible or the retailer's cash level is sufficiently high, the supplier should still offer net terms. Second, we enrich Kouvelis and Zhao (2012) by characterizing conditions when the supplier should offer two-part trade credit terms to induce the retailer to use a bank loan in addition to trade credit. Specifically, suppliers offer a larger early-payment discount when the retailer's cash level is low, the deadweight loss associated with trade credit default is high, or when the retailer has more market power (§5.1 and §5.2). Third, we characterize the impact of financial frictions (e.g., the supplier's efficiency in managing default) on operational decisions and supplier chain performance (§5.3). Finally, we offer some empirical support to the analytical findings by empirically examining the composition of retailers' inventory financing portfolios (§6).

Our paper is related to the literature on inventory policy in the presence of trade credit, which can be traced back at least as far as Haley and Higgins (1973), who study how companies make order quantity and payment time decisions simultaneously under an economic order quantity (EOQ) model. More recently, Moses and Seshadri (2000) identified that credit extension is critical for a periodically reviewed two-tier supply chain to reach the first-best safety inventory level. Gupta and Wang (2009) extend the above result by introducing stochastic demand. Luo and Shang (2013) characterize a firm's inventory policy under payment default. Our paper complements this literature by characterizing how firms finance their inventory under endogenous trade credit contracts and multiple external financing sources.

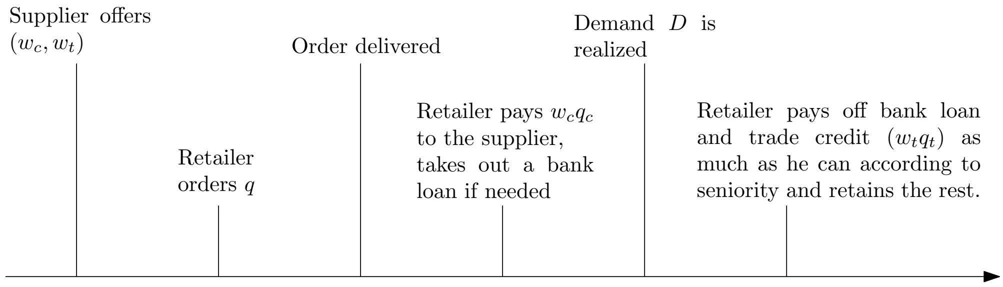
Our paper is also closely related to the trade credit literature in finance. On the theory side, we refer to Giannetti et al. (2011), Klapper et al. (2012) and Chod et al. (2016) for extensive reviews on the literature of trade credit theories. Among these theories, Wilner (2000) and Cuñat (2007) point out that suppliers are more willing to offer help when their customers are in trouble (e.g. due to liquidity shocks) as they seek to continue to do business with those customers in the future. Our model, however, argues that another advantage for suppliers originates from the risk-sharing mechanism created by the possibility of trade credit default, which boosts sales for suppliers. On the empirical side, our paper enriches the literature by linking a retailer's financial situation to the composition of the inventory financing portfolio through endogenous trade credit contracts. In addition, our analytical results provide plausible explanations for many previous empirical observations. For example, we find that when trade credit terms are endogenously determined based on the retailer's financial situation, the early-payment discount may exhibit great variation. Such variation is consistent with the long-standing debate over whether trade credit is an expensive substitute for institutional funding (Petersen and Rajan 1997, Ng et al. 1999) or a low-cost financing source (Giannetti et al. 2011, Klapper et al. 2012). Specifically, Klapper et al. (2012) find that early-payment discounts are often offered to buyers who are small and non-investment grade, which is in line with Proposition 3. Furthermore, our prediction of the composition of retailers' inventory financing portfolios is consistent with Nilsen (2002), who document that trade credit is less cyclical than bank debt. Our results (Section 5.2) also support those of Giannetti et al. (2011), who find that companies with more market power normally receive a large early discount. Finally, the risk-sharing role of trade credit that we propose is also consistent with the empirical finding in Lee and Zhou (2015) that aggressiveness in trade credit provision is positively correlated with the supplier's operating margin.

3. Model

Our model is built on the classic selling-to-the-newsvendor model (Lariviere and Porteus 2001). The supply chain consists of a supplier (S, "she") and a retailer (R, "he"). A single type of good is produced by the supplier at unit cost c and sold by the retailer to end consumers at unit price p, which is normalized to 1. As in the classic selling-to-the-newsvendor model, the retailer determines order quantity q before the stochastic demand $\tilde{D}$ is realized. The stochastic demand has support $[0, +\infty)$, cumulative distribution function (CDF) $F(\cdot)$, and probability density function (PDF) $f(\cdot)$. Other notations for the probability distribution include: complimentary CDF (CCDF) $\bar{F}(\cdot) = 1 - F(\cdot)$, failure rate $h(\cdot) = \frac{f(\cdot)}{\bar{F}(\cdot)}$, and generalized failure rate $g(\cdot) = (\cdot)h(\cdot)$. Let D devote the realized demand. Similarly to the literature, we assume that the demand distribution has an increasing failure rate (*IFR*). All unmet demand is lost, and all leftover inventory is salvaged at price $s = 0$. After demand D is realized, all revenues are realized.

One notable deviation of our model from the classic selling-to-the-newsvendor model is that the retailer is assumed to be financially constrained. Specifically, the retailer only has cash $K > 0$, and has to rely on external financing options, such as bank loan, trade credit, or both, to finance his inventory investment. In addition, he can only use future revenue to repay such external liabilities. This assumption captures the various financing difficulties that retailers face (Gaur et al. 2014, Heider and Ljungqvist 2015, Milliot 2010). Figure 1 illustrates the sequence of events detailed as follows. The supply contract between the supplier and the retailer is detailed in Section 3.1; external financing channels are discussed in Section 3.2.

Figure 1 **Sequence of events**



3.1. A supply contract with trade credit

Related to the retailer's financial constraint, the supplier offers a supply contract with trade credit that includes two parameters: a unit cash price w_c if the retailer chooses to pay cash upon delivery and a unit credit price w_t, which is due at the end of the sales horizon.[3] To avoid trivial cases, w_c and w_t should satisfy $c \le w_c \le w_t \le 1$. This abstraction of a trade credit contract allows us to model two forms of trade credit commonly used in practice: The case with $w_c = w_t$ corresponds to *net terms (one-part terms)*, and the case with $w_c < w_t$ corresponds to *two-part terms* with *early-payment discount* $d_t = 1 - \frac{w_c}{w_t}$.

In response to this contract, the retailer determines order quantity q and, in addition, the amount to pay upon delivery to take advantage of the early-payment discount. Let q_c be the amount of inventory the retailer purchases at the cash price, while $q_t = q - q_c$ denotes the retailer's purchase at the credit price. When the retailer's internal financial resources are not sufficient to cover $w_c q_c$, the difference is financed by a bank loan. $w_t q_t$ represents the amount of trade credit extended by the supplier to the retailer, i.e., $w_t q_t$ appears in the supplier's balance sheet as accounts receivable and the retailer's balance sheet as accounts payable. Note that $w_t q_t$ is only the nominal amount owed by the retailer to the supplier; the actual amount that the supplier receives depends on the retailer's cash level and the realized demand. Finally, note that the traditional price-only contract can be seen as a special case in our model by setting q_t to zero.

3.2. Financing sources and financial frictions

When facing the above trade credit contract, the retailer has two means of external financing – bank loans and trade credit – and, hence, needs to determine the composition of his inventory financing portfolio. Specifically, suppose that the retailer's order quantities under cash and credit prices are q_c and q_t, respectively; the retailer takes out a bank loan in the amount $B = (w_c q_c - K)^+$ and also owes the supplier $w_t q_t$. When the retailer does not have sufficient revenue to repay both claims fully, following common practice (Schwartz 1997, Longhofer and Santos 2000), we assume

[3] Despite its widespread use, trade credit is not the only form of supplier financing. For example, automobile manufacturers often finance their dealers through different programs. Our model can also cover such situations.

that the retailer's bank loan is senior to trade credit; i.e., when the retailer first repays the bank loan and then only repays the trade credit once the bank loan has been paid in full. This seniority arrangement is also reflected in the fact that trade credit often has a lower recovery rate relative to bank loans (Moody's 2007, Ivashina and Iverson 2014).[4] Finally, the retailer retains the earnings only after both bank loan and trade credit are fully repaid.

Under this seniority arrangement, intuitively, the bank loan defaults only if the retailer's demand realization is sufficiently low. Thus, we denote θ_b as the *bank loan default threshold*, i.e., the bank loan is fully repaid if and only if the realized demand $D \geq \theta_b$. Note that θ_b also equals to the total amount the retailer should pay back to the bank, i.e., $\theta_b = (1 + r_b)B$, where r_b represents the interest rate of the bank loan.

Similarly, let θ_t be the *trade credit default threshold*. That is, the retailer cannot fully repay trade credit when $D < \theta_t$. As the total amount of trade credit owed by the retailer to the supplier is $w_t(q - q_c)$, which is paid *after* the bank loan is repaid in full, the trade credit default threshold θ_t satisfies:

$$\theta_t = \theta_b + w_t(q - q_c). \tag{1}$$

In addition, we note that defaulting on financial claims is not without cost (Townsend 1979, Almeida and Philippon 2007, Korteweg 2007). Such costs of financial distress normally include legal, monitoring, and administrating costs. Similar to Leland (1994), Xu and Birge (2004) and others, we assume that upon default, creditors (the bank or supplier) incur (deadweight) costs proportional to the value they seize. Let the cost proportion associated with recovering unpaid bank loan and trade credit to be α_b and α_t respectively, where α_b, $\alpha_t \in [0, 1]$. A smaller α_b (α_t) reflects that the bank (the supplier) is more efficient at recovering from default. At an extreme, $\alpha_i = 0$ suggests that the corresponding creditor does not incur any deadweight loss when recovering the claim.

Table 2 **Payoffs to different parties at the end of the sales horizon**

	$D \in [0, \theta_b)$	$D \in [\theta_b, \theta_t)$	$D \in [\theta_t, q]$	$D > q$
Bank	$(1 - \alpha_b)D$	θ_b	θ_b	θ_b
Supplier	0	$(1 - \alpha_t)(D - \theta_b)$	$(\theta_t - \theta_b)$	$(\theta_t - \theta_b)$
Retailer	0	0	$D - \theta_t$	$q - \theta_t$

[4] Yang and Birge (2011) discuss the various laws that govern the seniority of trade credit and bank loans in greater detail. By comparing different seniority arrangements, i.e., when trade credit is more or less senior than or equally senior to bank loans, they find that supply chain efficiency is highest when trade credit is junior to bank loans, as assumed in this paper.

Combining the seniority arrangement and costs of financial distress, each party's payoff is shown in Table 2. For $D \in [0,\, \theta_b)$, the retailer defaults on both the bank loan and trade credit, the retailer's bank receives $(1-\alpha_b)D$, and the supplier receive nothing. For $D \in [\theta_b,\, \theta_t)$, the bank loan is repaid in full but the retailer defaults on the trade credit and the supplier's retained payoff is $(1-\alpha_t)(D-\theta_b)$. Finally, when $D \in [\theta_t,\, +\infty)$, the retailer fully repays both claims and retains the rest, $q-\theta_t$.

With the exception of the costs of defaults, we assume that the financial market is perfectly competitive. All parties are risk-neutral and have symmetric information about the retailer's financial situation and demand distribution. Our model also assumes away any moral hazard, which has been extensively studied in the literature. The supplier and retailer aim to maximize their expected discounted profits, which are denoted as Π_s and Π_r respectively and are detailed later. The risk-free rate r_f is normalized to 0.

4. The Retailer's Response to the Trade Credit Contract

Faced with a trade credit contract (w_c, w_t), the retailer decides not only the total order quantity q but also how to allocate q between the units financed via cash and bank loans (q_c), which allow the retailer to enjoy the early payment discount, and those financed by trade credit $(q_t = q - q_c)$. To characterize the retailer's optimal response, we first establish how the retailer's inventory financing portfolio and profit are affected by (q_c, q_t). Depending on whether the retailer exhausts his own cash, i.e. $w_c q_c \geq K$, or not, we consider the following two cases.

First, when $w_c q_c < K$, as the retailer has not exhausted his cash, he will not use a bank loan. Further, as the cash price w_c is (weakly) lower than the trade credit price w_t, i.e., $w_c \leq w_t$, the retailer should not use trade credit before exhausting cash either, i.e. $q_t = 0$.[5] As such, the retailer's profit is $\Pi_r = \int_0^q \bar{F}(x)dx - w_c q$, which is identical to the classic newsvendor problem.

Second, when $w_c q_c \geq K$, the retailer may finance his inventory with a bank loan and/or trade credit. For a bank loan, as the bank loan market is competitive, the loan should be priced such that the bank's expected payoff, $\int_0^{\theta_b}(1-\alpha_b)x dF(x) + \theta_b \bar{F}(\theta_b)$, equals the borrowed amount (the principal of the loan) $B = w_c q_c - K$. Rearranging the terms, the bank loan default threshold θ_b follows:

$$w_c q_c - K - \int_0^{\theta_b} \hat{F}_b(x)dx = 0, \tag{2}$$

where $\hat{F}_b(x) := \bar{F}(x)[1-\alpha_b g(x)]$. Intuitively, $\hat{F}_b(x)$ is the expected proportion the retailer repays the bank for the x-th dollar of the bank loan. It is easy to see that $\hat{F}_b(x)$ decreases in both x and

[5] Intuitively, if $q_c < \frac{K}{w_c}$ and $q_t > 0$, as $w_t \geq w_c$, the retailer can always shifts an arbitrarily small quantity from the units under the credit price to those under the cash price to (weakly) improve his profit. See Lemma C.1 and the proof for technical details.

α_b, capturing the fact that the bank should expect higher repayment risk when the loan amount is larger or the bank is less efficient at collecting payment from the retailer when the bank loan defaults. Accordingly, the bank adjusts the interest rate r_b as follows:

$$r_b = \frac{\theta_b - B}{B} = \frac{\theta_b}{\int_0^{\theta_b} \hat{F}_b(x)dx} - 1. \tag{3}$$

Two observations are notable. First, there is one-to-one correspondence between the principal borrowed (B) and the amount to repay (θ_b), as well as interest rate r_b. For technical convenience, we focus on θ_b as the retailer's decision variable instead of B. Second, as the bank loan is senior to trade credit, for any given amount borrowed (B), the pricing of the loan is independent of the trade credit contract and whether the retailer uses trade credit or not.

In addition to the bank loan, the retailer can also use trade credit to finance (part of) his inventory. Under q_c and q, the corresponding trade credit default threshold θ_t is as defined in (1). As the retailer exhausts cash reserves K and retains revenue only when $D \geq \theta_t$, his profit is

$$\Pi_r = \int_{\theta_t}^{q} \bar{F}(x)dx - K. \tag{4}$$

Hence, the retailer chooses q_c and q, or equivalently, q_c and q_t, to maximize (4) subject to (1) and (2). Combining the above two scenarios ($w_c q_c < K$ and $w_c q_c \geq K$), the retailer's optimal response is summarized in Proposition 1.[6]

PROPOSITION 1. *Let* $\kappa^{nb}(w_c) = w_c \bar{F}^{-1}(w_c)$, $\kappa_r^b(w_c, w_t) = w_c \bar{F}^{-1}\left(\frac{w_c}{1-\alpha_b g(\hat{F}_b^{-1}(w_c/w_t))}\right) - \int_0^{\hat{F}_b^{-1}(w_c/w_t)} \hat{F}_b(x)dx$. *Under trade credit contract* (w_c, w_t), *the retailer's optimal inventory and financing decisions are:*

1. *for* $K \in [\kappa^{nb}(w_c), +\infty)$, *the retailer does not use external financing. The optimal order quantity and corresponding default thresholds are:* $q^w = q_c^w = \bar{F}^{-1}(w_c)$, $\theta_b^w = \theta_t^w = 0$;
2. *for* $K \in [\kappa^b(w_c, w_t), \kappa^{nb}(w_c))$, *the retailer uses a bank loan only.* $q^w = q_c^w = \bar{F}^{-1}\left(\frac{w_c}{1-\alpha_b g(\theta_b^w)}\right)$, *and* $\theta_t^w = \theta_b^w$ *is uniquely determined by*

$$w_c \bar{F}^{-1}\left(\frac{w_c}{1-\alpha_b g(\theta_b^w)}\right) - \int_0^{\theta_b^w} \hat{F}_b(x)dx = K; \tag{5}$$

3. *for* $K \in [0, \kappa^b(w_c, w_t))$, *the retailer uses both a bank loan and trade credit.* $\theta_b^w = \hat{F}_b^{-1}(w_c/w_t)$; $q_c^w = \frac{K + \int_0^{\theta_b^w} \hat{F}_b(x)dx}{w_c}$; (q^w, θ_t^w) *jointly satisfy* $w_t \bar{F}(\theta_t^w) = \bar{F}(q^w)$ *and*

$$[\theta_t^w + C(\theta_b^w)]\bar{F}(\theta_t^w) = q^w \bar{F}(q^w), \tag{6}$$

where $C(\theta) = \frac{K + \int_0^{\theta} \hat{F}_b(x)dx}{\hat{F}_b(\theta)} - \theta$.

[6] With slight abuse of notation, we use superscript w to represent quantities related to the supplier's optimal response under *exogenous* trade credit contract (w_c, w_t). In Section 5, we use superscript $*$ to represent all quantities under the *endogenous*, i.e., *equilibrium*, trade credit contract.

Proposition 1 has three important implications. First, fixing the supply contract (w_c, w_t), as the retailer's cash position decreases, the retailer follows a simple pecking order when financing inventory. He first uses cash reserves to take advantage of the early-payment discount and to avoid the distress cost associated with the bank loan. After exhausting K, the retailer switches to a bank loan and still enjoys the early-payment discount until the *marginal* discount factor of the bank loan, which equals $1 - \hat{F}_b(\theta_b^w)$, exceeds the early-payment discount $d_t = 1 - \frac{w_c}{w_t}$. After that, he foregoes the early-payment discount and finances inventory through trade credit.

Second, as shown in Statement 3, when trade credit is used, at the optimal order quantity q^w, the retailer's marginal cost is $w_t\bar{F}(\theta_t^w)$, i.e., the nominal trade credit price w_t discounted at the repayment probability $\bar{F}(\theta_t^w)$, which is directly associated with the demand uncertainty. This expression reveals that as a larger order quantity inflates the risk of trade credit default, the retailer's marginal cost decreases as the order size increases. In this way, by aligning the retailer's marginal cost with the demand risk he faces, trade credit (partially) alleviates double marginalization and incentivizes the retailer to order more. Indeed, as shown in Corollary 1, when using trade credit, the retailer actually stocks a larger amount of inventory at a lower cash level, where the retailer's marginal cost is more aligned with the demand risk.

Corollary 1. *Under trade credit contract (w_c, w_t), the retailer's optimal order quantity q^w decreases in K for $K \in [0,\, \kappa^b(w_c, w_t)]$.*

Last but not least, Proposition 1 reveals the relationship between the implicit interest rate of the trade credit, $r_t := \frac{w_t - w_c}{w_c} = \frac{d_t}{1-d_t}$, and the interest rate of the bank loan (3), as formalized in Corollary 2.

Corollary 2. *When $d_t > 0$, under the retailer's optimal response, $r_b^w < r_t$.*

As shown, under the retailer's best response, when using both a bank loan and trade credit to finance inventory (Statement 3), the bank loan interest rate is strictly less than r_t. Intuitively, this is because the retailer switches from bank loans to trade credit when r_t is equal to the *marginal* interest rate of the bank loan, which is strictly greater than r_b^w, the *average* interest rate of the bank loan. This provides an alternative explanation for why firms use trade credit even though the corresponding implicit interest rate is much higher than the interest rate on a bank loan.

Before closing this section, we present a corollary that will facilitate the analysis when identifying the optimal trade credit contract in the next section.

Corollary 3. *Under the retailer's best response function as specified in Proposition 1, there exists a one-to-one mapping between $(\theta_b^w, \theta_t^w) \in \Theta := \{(\theta_b, \theta_t) \mid \theta_t \geq \theta_b \geq 0,\ [\theta_t + C(\theta_b)]\bar{F}(\theta_t) \leq g^{-1}(1)\bar{F}(g^{-1}(1))\}$ and (w_c, w_t) such that $w_c \leq w_t \leq 1$.*

Under this mapping, the supplier's order quantity $q^w = Q(\theta_b^w, \theta_t^w) := \min\{q : q\bar{F}(q) = [\theta_t^w + C(\theta_b^w)]\bar{F}(\theta_t^w)\}$, and $Q(\theta_b, \theta_t)$ increases in both θ_b and θ_t.

5. The Optimal Trade Credit Contract and Its Implications

With an understanding of how the retailer responds to a supply contract with trade credit (w_c, w_t), we move to explore the optimal contract that the supplier offers in anticipation of such response, and its operational and financial implications for the supply chain.

Under our model, the supplier's expected profit under (w_c, w_t) is:

$$\Pi_s = -cq^w + w_c q_c^w + (1-\alpha_t)\int_{\theta_b^w}^{\theta_t^w}(x-\theta_b^w)dF(x) + (\theta_t^w - \theta_b^w)\bar{F}(\theta_t^w), \tag{7}$$

where q_c^w, θ_b^w, θ_t^w, and q^w are the retailer's best response as depicted in Proposition 1. As shown, the supplier's expected profit consists of three components: cost of production (cq^w), cash sales $(w_c q_c^w)$, and, finally, effective trade credit sales as expressed using the retailer's default thresholds (θ_b^w and θ_t^w) with the cost associated with default. Clearly, when the retailer does not employ any external financing, i.e., $\theta_t^w = \theta_b^w = 0$ and $q_c^w = q^w$, the supplier's profit function is identical to the classic selling-to-the-newsvendor model.

When external financing is used, according to Proposition 1, $w_c q_c^w = K + \int_0^{\theta_b^w} \hat{F}_b(x)dx$. Therefore, we can express Π_s solely as a function of the two thresholds θ_b^w and θ_t^w:

$$\Pi_s = K + \int_0^{\theta_t^w}\bar{F}(x)dx - cQ(\theta_b^w, \theta_t^w) - \left[\alpha_b \int_0^{\theta_b^w} x dF(x) + \alpha_t \int_{\theta_b^w}^{\theta_t^w}(x-\theta_b^w)dF(x)\right], \tag{8}$$

As shown, the supplier's profit function includes three components: revenue $K + \int_0^{\theta_t^w}\bar{F}(x)dx$, cost of production ("operating cost") $cQ(\theta_b^w, \theta_t^w)$, and costs related to defaults ("financing cost"). Also, note that as long as the supplier is exposed to trade credit default risk $(\theta_t^w > 0)$, she retains all revenue from the retailer's sales when $D \le \theta_t^w$ and, at the same time, incurs costs associated with production and both trade credit and bank loan defaults, the latter of which is passed to her through the pricing of the bank loan. As such, the supplier's revenue – as well as part of her costs – is contingent on the realization of demand. This contingency forms the most important difference between trade credit and a traditional price-only contract, under which neither the supplier's revenue nor cost is influenced by demand realization.

Given (8) and Corollary 3, we treat (θ_b, θ_t) as the supplier's decision variables in the rest of the section for technical convenience. Taking a closer look at how the two default thresholds θ_t^w and θ_b^w influence the supplier's profit function, we have:

$$\frac{\partial \Pi_s}{\partial \theta_t^w} = \bar{F}(\theta_t^w) - c\frac{\partial Q}{\partial \theta_t^w} - \alpha_t(\theta_t^w - \theta_b^w)f(\theta_t^w). \tag{9}$$

As shown, θ_t^w affects Π_s through three channels. As θ_t^w increases, which corresponds to the supplier becoming more aggressive in offering trade credit, the supplier can collect higher revenue. However, according to Corollary 3, the operating cost increases in θ_t^w. Finally, also note that the financing cost also increases in θ_t^w. Therefore, the optimal θ_t^w should trade off these three impacts. As for θ_b^w, we have:

$$\frac{\partial \Pi_s}{\partial \theta_b^w} = -c\,\frac{\partial Q}{\partial \theta_b^w} + \left\{[\alpha_t - \alpha_b g(\theta_b^w)]\bar{F}(\theta_b^w) - \alpha_t \bar{F}(\theta_t^w)\right\}. \tag{10}$$

This suggests that increasing θ_b^w, which is equivalent to increasing the early-payment discount in trade credit, increases the production cost but has no positive impact on revenue. Therefore, the net operational impact is always negative for a larger θ_b^w. In other words, from a purely operational perspective, offering early payment discount cannot be beneficial. However, on the financial side, note that a larger θ_b^w may lower the costs related to default. To see this, note that at $\theta_b^w = 0$, the marginal impact of θ_b^w on the total costs associated with default, i.e., the second term in (10), is $-\alpha_t F(\theta_t^w)$. Clearly, when trade credit default is costly ($\alpha_t > 0$), increasing θ_b^w slightly above zero *reduces* the total financing related costs. Therefore, the optimal θ_b^w should balance the negative operational impact with the potential financial benefit. Formalizing this intuition, the following result gives the necessary condition for a trade credit contract to be optimal.

LEMMA 1. *Under the optimal trade credit contract, the cash and credit prices (w_c^*, w_t^*) follow:*

$$w_c^* = \frac{\hat{F}_b(\theta_b^*)\bar{F}(q^*)}{\bar{F}(\theta_t^*)} \quad \textit{and} \quad w_t^* = \frac{\bar{F}(q^*)}{\bar{F}(\theta_t^*)}; \tag{11}$$

where θ_b^*, θ_t^*, and $q^* = Q(\theta_b^*, \theta_t^*)$ satisfy:

1. $\theta_b^* = 0$ *and*

$$\frac{c}{\bar{F}(q^*)[1-g(q^*)]} = \frac{1-\alpha_t g(\theta_t^*)}{1-(\theta_t^*+K)h(\theta_t^*)} \geq \frac{\alpha_t[1-\bar{F}(\theta_t^*)]}{f(0)K\bar{F}(\theta_t^*)}, \quad \textit{or} \tag{12}$$

2. $\theta_b^* > 0$ *and*

$$\frac{c}{\bar{F}(q^*)[1-g(q^*)]} = \frac{1-\alpha_t(\theta_t^*-\theta_b^*)h(\theta_t^*)}{1-[\theta_t^*+C(\theta_b^*)]h(\theta_t^*)} = \frac{[\alpha_t - \alpha_b g(\theta_b^*)]\bar{F}(\theta_b^*) - \alpha_t \bar{F}(\theta_t^*)}{C'(\theta_b^*)\bar{F}(\theta_t^*)}. \tag{13}$$

Lemma 1 characterizes the necessary condition for the optimal solution. Note that the magnitude of θ_b^* has an intuitive explanation: when $\theta_b^* = 0$, the corresponding trade credit contract is *net terms*, while $\theta_b^* > 0$ suggests that the optimal trade credit contract is *two-part terms*.

Before moving to the details of the trade credit contract and its implication for how the retailer finances inventory, we first highlight the risk-sharing role of trade credit through the following proposition.

PROPOSITION 2. *Under the optimal trade credit contract, the retailer orders strictly more than under the optimal price-only contract with only a bank loan.*

It is well-known that a price-only contract does not fully coordinate the supply chain (Lariviere and Porteus 2001). How does trade credit influence supply chain performance? As formalized in Proposition 2, trade credit can incentivize the retailer to order more from the supplier, hence enhancing supply chain performance, even when trade credit can be associated with significant deadweight loss upon default. The mechanism that allows trade credit to induce higher order quantities lies in its risk-sharing role: as the repayment of trade credit, $\min(\theta_t, D) - \theta_b$, partly depends on the realized demand D, the supplier essentially bears part of the (downside) demand risk, which the retailer must bear alone under a price-only contract. As such, the retailer can afford to order more from the supplier, similar to the response to other supply chain coordinating mechanisms (Cachon 2003).

In the remainder of this section, we first examine the optimal trade credit terms, the retailer's corresponding inventory financing portfolio, and how these decisions are influenced by financial (§5.1) and operational characteristics (§5.2), and then examine the operational implications of trade credit under its risk-sharing role (§5.3).

5.1. Optimal trade credit terms and the inventory financing portfolio

Previously, we have shown that when trade credit terms are exogenous to the retailer's financial status, the retailer first uses bank loan and then trade credit due to the seniority arrangement. However, as the following proposition shows, when trade credit terms become endogenous to the retailer's financial situation, the risk-sharing role of trade credit induces the supplier to offer terms such that trade credit is always an external source the retailer uses to finance inventory.

Proposition 3. *Under the optimal trade credit contract, when the retailer uses external sources to finance inventory, trade credit is always used. In addition,*

1. *when $\alpha_t = 0$ or when K is sufficiently large, the optimal trade credit contract is net terms ($d_t^* = 0$). In response, the retailer does not use bank loans to finance inventory;*
2. *when $\alpha_t > 0$ and K is sufficiently small, the supplier offers two-part terms ($d_t^* > 0$). In response, the retailer finances inventory using both bank loans and trade credit.*

As revealed in Proposition 3, under an endogenous supply contract, trade credit should always be adopted by the retailer, regardless of the cost proportions associated with trade credit or bank loan defaults. In other words, the supplier offers financing even when she is less efficient at collecting defaulted claims than the bank ($\alpha_b < \alpha_t$). The reason lies in the risk-sharing role of trade credit: by extending trade credit, the supplier lowers the retailer's effective marginal cost $w_t \bar{F}(\theta_t)$, as the probability of default increases, and hence induces the retailer to order more. This results in a higher operational profit for the supplier – an effect absent in the bank loan as the bank and the retailer have no such operational connection. As such, the supplier can afford to offer subsidized financing to the retailer. Relating this to (10), we note that in our model, the only benefit of offering an early-payment discount, which induces the retailer to use a bank loan, is to reduce the cost associated with trade credit default. Clearly, such a cost is only present when trade credit is used. Therefore, trade credit is the preferred channel for financing inventory, and bank loans are only used to complement trade credit.

Proposition 3 highlights two factors that influence the optimal terms of trade credit: the retailer's financial situation, as captured by his cash level K, and the supplier's (relative) efficiency at recovering defaulted trade credit. As shown, when the deadweight loss associated with trade credit default is not too high, the need for financial diversification does not exist, thereby eliminating the need to use bank loans. Similarly, when the retailer's financing need is low, he is unlikely to default. As such, the supplier's gains from operations by offering net terms outweigh the reduction in distress cost by offering two-part terms.

By contrast, when collecting defaulted trade credit is costly, the supplier has an incentive to lower the distress cost associated with trade credit. This incentive increases when the retailer's cash level is low and the probability of trade credit default is high. As a result, when the retailer's cash level is sufficiently low, the supplier limits the retailer's trade credit usage by offering a discount on trade credit, luring the retailer into diversifying his inventory financing portfolio by using bank loans. Combined, it reconciles the risk-sharing role of trade credit with the variety of trade credit terms used in practice.

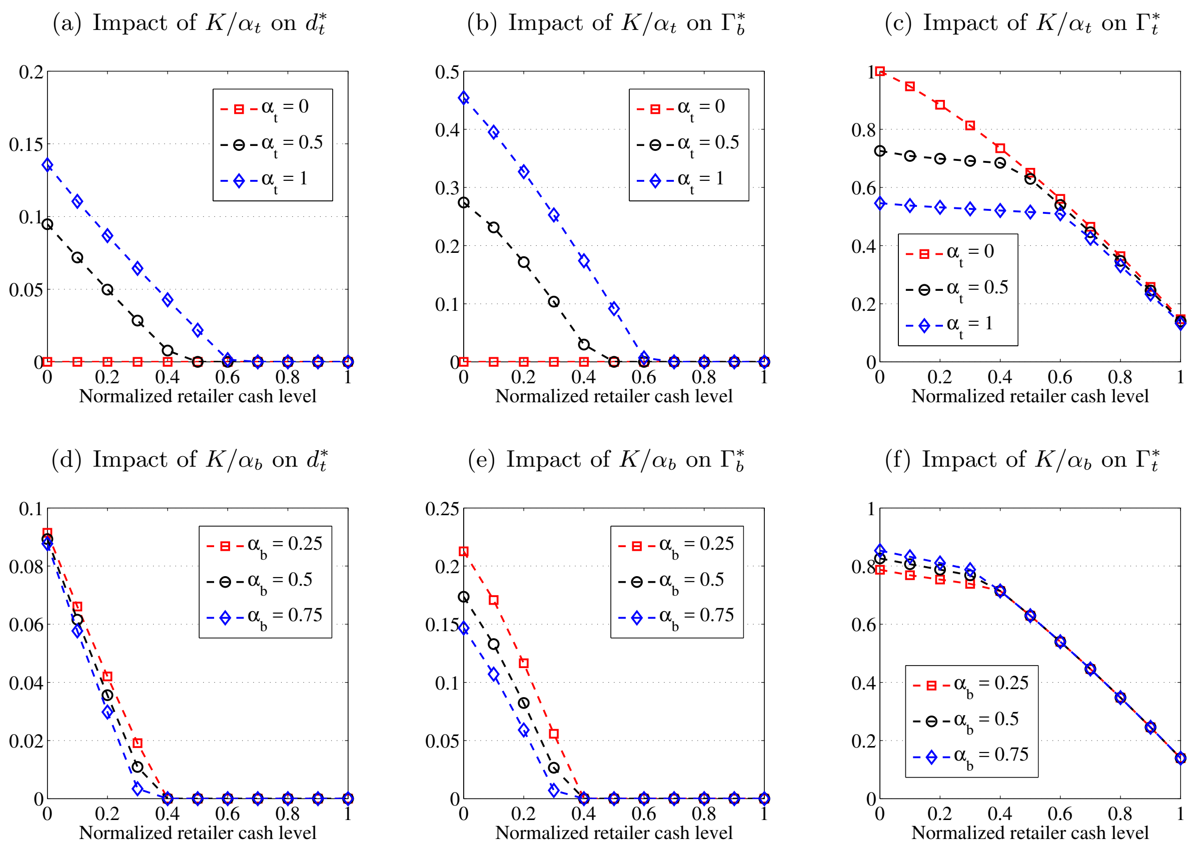
Under the equilibrium trade credit terms, the pecking order of financing sources the retailer adopts with respect to his financing need forms a sharp contrast with that shown in Proposition 1. Specifically, when the trade credit term is exogenous to the retailer's financial situation (Proposition 1), the retailer with low financing need (high K) may only use bank loans to finance inventory. This is because if the retailer's financing need is low, as the bank loan is senior to trade credit, the bank is willing to offer an attractive rate for the bank loan. Thus, the retailer always has an incentive to use bank loans to take advantage of an early-payment discount, if there is any. Trade credit is only used when the retailer's financing need is high, and the resulting marginal bank loan rate exceeds the early-payment discount in trade credit. However, when the trade credit contract is endogenous to the retailer's financing need (K), the risk-sharing role of trade credit incentivizes the supplier to offer net terms to a retailer with low financing need, and an early-payment discount is only offered when the retailer's financing need is high. Consequently, the retailer first adopts trade credit and then bank loans as his financing need increases. This trend is further refined in Proposition 4 as follows.

PROPOSITION 4. *When the condition in Lemma 1 is sufficient for* (θ_b^*, θ_t^*) *to be globally optimal:*

1. *for sufficiently small c, the early payment discount (d_t^*) and the face value of the bank loan used by the retailer (θ_b^*) (weakly) decrease in K and α_b, and increase in α_t;*

2. *when the optimal trade credit term is net terms ($d_t^* = 0$), the amount of trade credit used in equilibrium ($w_t^* q_t^*$) decreases in K and α_t.*

Figure 2 The impact of financial characteristics on early-payment discount and inventory finance portfolio under the optimal trade credit contract



Notes. The x-axis represents the retailer's cash level K normalized by k^{po}, the retailer's procurement cost under the optimal price-only contract ($k^{po} = w^{po} q^{po}$, where w^{po} and q^{po} are the wholesale price and quantity under the optimal price-only contract without financial constraint. q^{po} satisfies $\bar{F}(q^{po})[1 - g(q^{po})] = c$, and $w^{po} = \bar{F}(q^{po})$.) Bank loan usage $\Gamma_b^* = \frac{w_c^* q_c^* - K}{w_c^* q_c^* + w_t^* q_t^*}$; trade credit usage $\Gamma_t^* = \frac{w_t^* q_t^*}{w_c^* q_c^* + w_t^* q_t^*}$. Parameters used: $\tilde{D} \sim \text{Unif}[0,1]$; $c = 0.5$; $\alpha_b = 0$ in Figure 2(a)–2(c); $\alpha_t = 0.5$ in Figure 2(d) –2(f).

Figure 2 illustrates Propositions 3 – 4.[7] As shown in Figures 2(a) and 2(d), when the retailer has relatively high cash levels, the optimal trade credit contract is net terms, i.e., $d_t^* = 0$. However, as the retailer's financing need increases, the supplier increases d_t^*, inducing the buyer to rely more

[7] Although we require the assumption that c should be sufficiently small to analytically prove Statement 1 in Proposition 4 and some other following results, extensive numerical results suggest that these results continue to hold for all range of $c \in (0, 1)$.

on the bank loan, limiting the supplier's exposure to trade credit default. Further, the supplier also offers a larger discount when she is relatively more efficient in managing trade credit default (lower α_t or higher α_b).

An important implication of the optimal trade credit contract is how the retailer finances inventory using different financing channels. Figures 2(b) and 2(c) illustrate the proportions of trade credit ($\Gamma_t^* = \frac{w_t^* q_t^*}{w_c^* q_c^* + w_c^* q_c^*}$) and bank loans ($\Gamma_b^* = \frac{w_c^* q_c^* - K}{w_c^* q_c^* + w_c^* q_c^*}$) in the retailer's inventory financing portfolio. Consistent with Proposition 3, under the optimal trade credit contract, trade credit is an indispensable component of the retailer's inventory financing portfolio, although the fraction of trade credit decreases in the supplier's inefficiency at collecting trade credit (α_t). By contrast, bank loans are used to finance inventory when the retailer's financing need is high, where the proportion increases in α_t. Further, both the amounts of the bank loan and accounts payable that the retailer carries increase as the cash level K declines, consistent with the previous rationale that bank loan complements trade credit within the risk-sharing role. Similarly, as shown in Figure 2(d)–2(f), both d_t^* and the fraction of bank loan (Γ_b^*) are higher when the bank is more efficient at collecting defaulted claims (lower α_b), while the fraction of trade credit increases in α_b.

5.2. The impact of operational characteristics on trade credit terms

In the previous sections, we have shown how the retailer's financial status (K) and the distress costs (α_b and α_t) influence trade credit terms. In addition to these financial characteristics, the risk-sharing role of trade credit also relates trade credit terms and usage to operational characteristics, in particular, the supplier's production cost c, and the retailer's market power, as detailed in the following results.

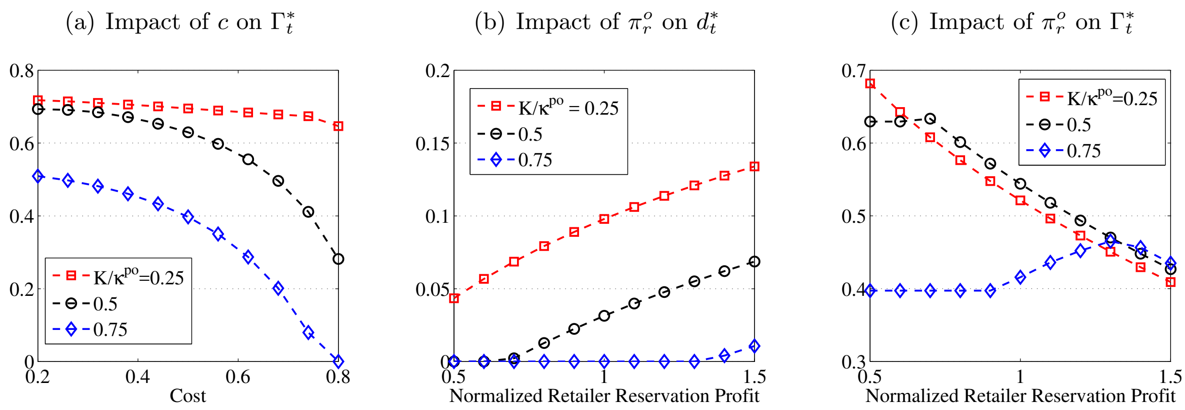
COROLLARY 4. *When the condition in Lemma 1 is sufficient for (θ_b^*, θ_t^*) to be globally optimal, if the optimal trade credit term is net terms ($d_t^* = 0$), the amount of trade credit used in equilibrium ($w_t^* q_t^*$) decreases in c.*

As illustrated in Figure 3(a), Corollary 4 continues to hold under two-part terms. The basic logic is that when the supply profit margin $(p - c)$ is higher, the supplier has a stronger incentive to push products downstream, and, hence, offers more trade credit.

The other operational characteristic that we examine is the retailer's market power. In measuring that, we extend the basic model by incorporating a retailer participation constraint. Specifically, let $\pi_r^o \in \left[0,\, \int_0^{\bar{F}^{-1}(c)} \bar{F}(x)dx - c\bar{F}^{-1}(c)\right]$ represent the retailer's *reservation profit*, i.e., the minimum profit the retailer will accept.[8] Therefore, the supplier's problem is to maximize (8) subject to $\int_{\theta_t}^{Q(\theta_b,\theta_t)} \bar{F}(x)dx - K \geq \pi_r^o$.

[8] Note that $\int_0^{\bar{F}^{-1}(c)} \bar{F}(x)dx - c\bar{F}^{-1}(c)$ is the optimal supply chain profit, which equals to a financially unconstrained retailer's profit when the supplier offers a wholesale price equal to her production cost c.

Figure 3 **The impact of operational characteristics (c and π_r^o) on trade credit terms and usage**



Notes. κ^{po} is the (financially unconstrained) retailer's procurement cost under the optimal price-only contract. In Figure 3(b)–3(c), the x-axis is π_r^o normalized by $\pi_r^{po} = \int_0^{q^{po}} \bar{F}(x)dx - w^{po}q^{po}$, the retailer's profit under the optimal price-only contract without financial constraint. Parameters used: $\tilde{D} \sim \text{Unif}[0,1]$, $c = 0.5$, $\alpha_t = 0.5$, $\alpha_b = 0$.

Proposition 5. *When the condition in Lemma 1 is sufficient for (θ_b^*, θ_t^*) to be globally optimal, for sufficiently small c, the optimal early-payment discount d_t^* (weakly) increases in π_r^o.*

Proposition 5 is depicted in Figure 3(b); as the retailer's market power increases, the supplier offers a larger early-payment discount, which is consistent with the finding of Giannetti et al. (2011). The reason for this is as follows: when the retailer's market power increases, the supplier has to sacrifice her profit by lowering profit margin and, hence, has less incentive to share the demand risk with the retailer by offering trade credit. Further, Figure 3(c) reveals that the fraction of trade credit in the retailer's inventory financing portfolio may increase or decrease as the buyer's market power increases. This offers a plausible explanation for recent empirical findings that some large retailers with strong market share receive more trade credit (Murfin and Njoroge 2015).

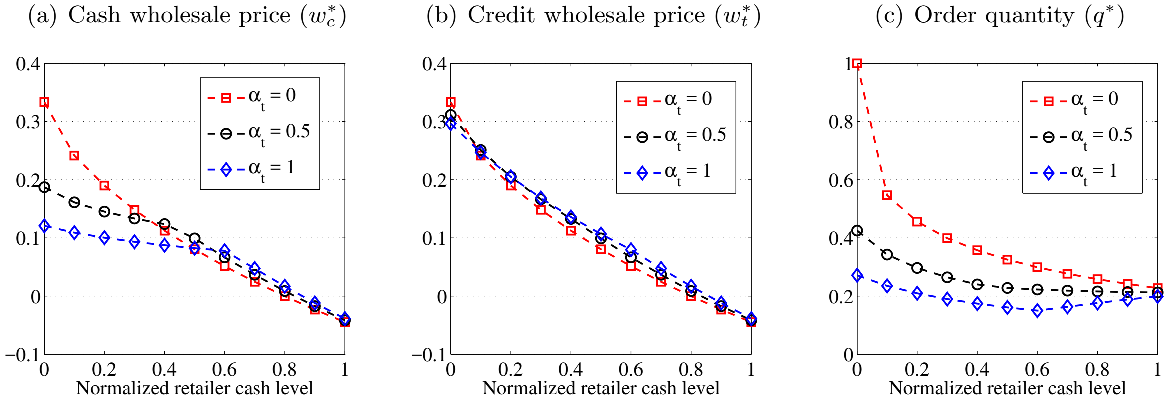
5.3. The operational implications of trade credit as a risk-sharing mechanism

With an understanding of how trade credit terms and the retailer's inventory financing portfolio are influenced by financial and operational characteristics, in this section, we examine how the financial characteristics influence supply chain decisions and performance.

Corollary 5. *When the condition in Lemma 1 is sufficient for (θ_b^*, θ_t^*) to be globally optimal, if the optimal trade credit term is net terms ($d_t^* = 0$), as α_t increases, the equilibrium order quantity (q^*) decreases, and the wholesale price ($w_c^* = w_t^*$) increases.*

The risk-sharing role of trade credit has notable implications for operational decisions, as summarized in Figure 4. As shown in Figure 4(a) and 4(b), both the cash and credit price increase in the retailer's financing needs (lower K). This is due to two related reasons. First, the retailer's

Figure 4 Operational decisions under the optimal trade credit contract



Notes. The y-axis represents the relative differences from the benchmark under the optimal price-only contract without financial constraint (q^{po} and w^{po}). Parameters used: $\tilde{D} \sim \text{Unif}[0,1]$, $c = 0.5$, $\alpha_b = 0$.

limited liability effect increases in his financing need, boosting the demand faced by the supplier. This allows the supplier to charge a higher cash price. Second, as K decreases, the retailer's default probability, $F(\theta_t)$, increases. Consequently, the supplier needs to increase the credit price to compensate for that. The impact of the supplier's financial efficiency (α_t) on wholesale prices is also notable. First, when K is high, the supplier offers net terms. In this case, as suggested by Corollary 5, the credit price, which also equals the cash price, increases in α_t. This is because the supplier can only control distress costs by limiting the total amount of trade credit $w_t^* q_t^*$. As a result, the supplier increases w_t^* to suppress the quantity ordered by the retailer. However, for lower K, the optimal term is two-part, which gives the supplier an additional lever, d_t^*, to limit distress cost. Consequently, as α_t increases, the supplier lowers the cash price to induce the retailer to use a bank loan.

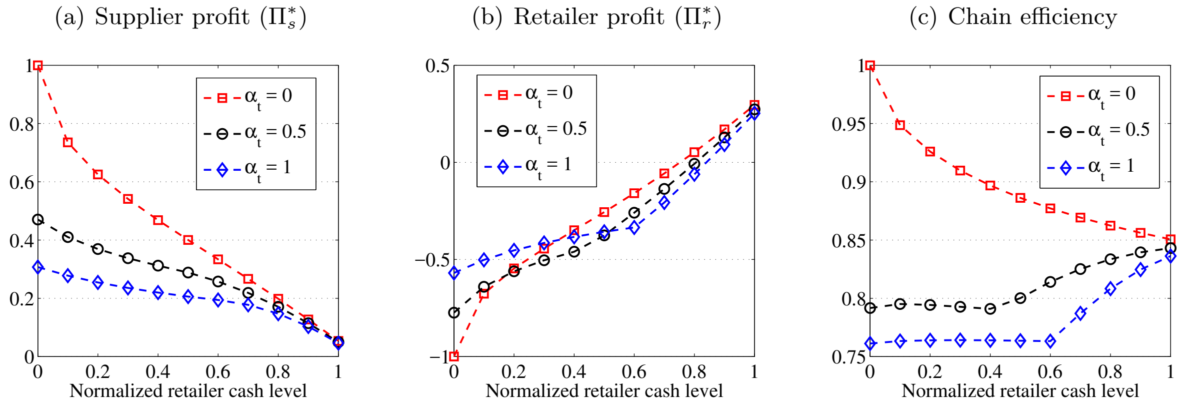
The retailer's order quantity q^* is also closely related to the chain's financial characteristics. First, we observe that q^* increases as the supplier becomes more efficient at recovering defaulted trade credit (smaller α_t). Intuitively, this is because, when offering trade credit is less costly to the supplier, she is more willing to offer generous terms, hence enhancing the risk-sharing role of trade credit. However, q^* may increase or decrease in the retailer's cash level due to two competing forces. First, as the retailer's cash level declines, in general, the repayment probability of trade credit also declines, driving down the retailer's effective marginal cost under trade credit, $w_t^* \bar{F}(\theta_t^*)$. Therefore, the retailer has the incentive to order a larger quantity. In this sense, the risk-sharing feature of trade credit is more pronounced when the retailer's cash level is low. Second, offering trade credit can be costly for the supplier, especially when α_t is large. As a result, the supplier may tighten trade credit terms to reduce exposure to distress costs, leading to a lower order quantity. Combining these two forces, the retailer's quantity generally decreases in K when α_t is sufficiently small, suggesting that the risk-sharing incentive dominates. However, for large α_t, the supplier's incentive to reduce distress costs may be a stronger force, leading to higher order quantities as K increases.

PROPOSITION 6. *Under the optimal trade credit contract, the supplier's profit* Π_s^* *decreases in both* α_b *and* α_t.

When the condition in Lemma 1 is sufficient for (θ_b^*, θ_t^*) *to be globally optimal, if the optimal trade credit term is net terms* $(d_t^* = 0)$,

1. Π_s^* *decreases in* K*; and*
2. *the retailer's profit* Π_r^* *decreases in* α_t.

Figure 5 Supply chain performance under the optimal trade credit contract



Notes. The y-axis in 5(a) and 5(b) represents the relative differences from the benchmark under the optimal price-only contract without financial constraint. In Figure 5(c), chain performance is measured by the sum of the chain profit $(\pi_r^* + \pi_s^*)$ divided by the integrated chain profit. Parameters used: $\tilde{D} \sim \text{Unif}[0,1]$, $c = 0.5$, $\alpha_b = 0$.

As shown in Proposition 6, and further illustrated in Figure 5, in terms of profitability, it is clear that offering trade credit always benefits the supplier, since she can extract more value from the retailer by sharing demand risk, while the benefit decreases as financial frictions (α_b or α_t) are more notable. Furthermore, as shown in Figure 5(a), which extends Proposition 6 to two-part terms, the supplier can take advantage of a weaker retailer through trade credit, as the risk-sharing role of trade credit is more pronounced when the retailer's cash level is low, although the benefit is partially offset by the higher distress cost.

On the other hand, trade credit hurts the retailer under low cash levels, as shown in Figure 5(b). This is consistent with the previous observation about the supplier's profit: when the retailer's cash level is low, the supplier is better able to extract surplus from the retailer using trade credit, leaving less for the retailer. That said, the retailer may also be better off under trade credit when his cash level is not too low, suggesting that moderate risk-sharing can create a win–win situation for both parties in the supply chain. Furthermore, note that unlike the supplier, the retailer may not necessarily benefit when the supplier becomes more efficient at collecting defaulted trade credit (lower α_t) as smaller α_t not only lowers deadweight loss but also enables the supplier to extract profit from the retailer more efficiently.

Combining the influence of trade credit on the two parties' profits, we observe from Figure 5(c) that for smaller α_t, a lower K better enables risk-sharing, leading to higher chain efficiency. However, for larger α_t, a higher K is more beneficial to the supply chain as it reduces the total distress cost, consistent with the trend we observe in order quantity q^*.

6. Empirical Evidence for the Inventory Financing Portfolio

Among the financial and operational implications of trade credit under its risk-sharing role, one is that, under an endogenous trade credit contract, the composition of the retailer's inventory financing portfolio depends on the retailer's financing need: for retailers with relatively larger internal resources and, hence, lower financing need, the only type of external source that the retailer uses to finance inventory is trade credit. However, retailers with higher financing need employ a diversified inventory financing portfolio comprised of both trade credit and short-term debt. In this section, we provide preliminary empirical support for this analytical finding. Before discussing our empirical test in detail, we note that the results presented later by no means imply any causal relationship between inventory, accounts payable and/or short-term debt. Indeed, the results in Section 5 state that both inventory and the financing sources used are endogenously determined by the trade credit contract faced by the retailer, which is in turn influenced by the retailer's financing need. However, as the trade credit contracts are not available for direct testing, we indirectly test our theory by quantifying the composition of the inventory financing portfolio for retailers with different financing needs.

6.1. Data description

Our empirical test is based on quarterly financial statement data of North American retailers from COMPUSTAT (North American Industry Classification System, NAICS Code: 441–454) gathered between 2000 and 2013.[9] Different from firms in other sectors, who often simultaneously extend

[9] The data we use is at firm level. Due to data limitation, it is not uncommon for papers in operations management and trade credit to match firm-level data to models based on a single product or bi-lateral supply chain (Gaur and Seshadri 2005, Giannetti et al. 2011, Bray and Mendelson 2012). In addition, as a robustness check, we conducted the same test with data dating back as early as the 1980s. However, we found that some empirical results (Hypothesis 1 in the following) became unstable. This could be because the processing of trade credit was less synchronized with inventory due to lack of information systems in the early years. An alternative reason is that some firm-years belonging to the high (low) group could be mis-classified to the low (high) group, and such misclassification is amplified by including more data due to serial correlation. Given the focus of the paper, we leave a full-scaled empirical study for future research.

trade credit to their customers and receive trade credit from their suppliers, as the end of the entire supply chain, retailers sell directly to consumers and, hence, offer little trade credit. As shown in Table 3, their median accounts receivable amount to just 4.6% of the firms' total assets, which is less than one third of the total median accounts payable of 13.9%. This property allows us to better isolate the relationship between trade credit received (accounts payable) and inventory. Furthermore, to control for seasonality and fiscal-year-end effects, we confine our sample to firms with fiscal year ends on December 31 or January 31, which covers approximately 75% of the retailers in our sample. As we show later, the alignment of the fiscal and calendar years in the retail sector allows us to use the high demand faced by some retailers during the holiday season to construct groups of retailers with different financing needs. Considering these criteria, our data contains 3,104 firm-year observations. Descriptive statistics are summarized in Table 3.

Table 3 Descriptive statistics

		Balance sheet items as a fraction of total assets				
	Total assets ($ Million)	Inventory (INV)	Cash & Eq (CASH)	Accts receivable (AR)	Accts payable (AP)	Debt in current liabilities (DLC)
Mean	2878.3	31.1%	13.3%	8.7%	16.9%	8.1%
25%	153.6	16.4%	2.4%	1.8%	8.5%	0.0%
50%	462.2	27.7%	7.7%	4.6%	13.9%	0.9%
75%	1867.1	43.7%	19.1%	10.6%	21.0%	6.4%

Notes. LIFO adjustment is applied to inventory, assets, and COGS.

6.2. Using an operational measure to proxy for financing need

In our empirical test, to proxy for financing need, we design an operational metric, namely the fourth-quarter cost of goods sold (COGS) as a fraction of the annual COGS, which we refer to as *Q4 COGS Fraction*. Dividing our sample in half according to Q4 COGS Fraction, we argue that the *high* (*low*) group proxies for firms with high (low) financing need. The intuition for this is as follows. If a retailer, such as Macy's or Toys "R" Us, expects sales during Q4 (holiday season) to be significantly higher than in other quarters, he would need more financial resources to build up inventory at the end of Q3. As the demand spike is cyclic, the retailer is more likely to finance this inventory investment with short-term financing sources. As such, the retailer's *short-term* financing need increases. By contrast, for some other retailers, such as Dollar General, sales are relatively smooth over the year, and, hence, they are more likely to operate under moderate (low) financing need throughout the year.

Using this operational proxy allows us to reconcile our model and data available in three aspects. First, there is no direct metric in the dataset that corresponds to the retailer's financing need in our model, which is captured by K, the cash position *before* the inventory investment, which is unobservable in the empirical data. In contrast, the amount of cash in financial statements is *after* procuring inventory.

Second and relatedly, to focus on the implications of various external financing sources on firms' short-term operational decisions, our model assumes that a retailer's long-term capital structure, as captured by cash level K, is exogenous. Yet, in practice, a firm's long-term capital structure is also endogenously determined by its operational characteristics (Alan and Gaur 2011). However, even when firms operate under their own optimal long-term capital structure, the difference in demand patterns between the two groups of firms means that they should still face different financing needs temporarily, i.e., firms in the high group are likely to have higher financing need at the end of Q3 than those in the low group.

Finally, across different companies, firms' financing needs (lack of internal capital) may be negatively correlated with their market power, which, according to Proposition 5, influences trade credit terms and usage in the opposite direction to financing need. However, Q4 COGS Fraction captures (cyclic) changes in financing need within a company, and such changes are unlikely to influence market power. Therefore, using Q4 COGS Fraction as a proxy for firms' financing needs allows us to partly control the market power effect.

Table 4 Descriptive statistics of observations sorted by Q4 COGS Fraction.

	Q4 COGS Fraction	
	Low	High
Medium Assets (\$ Million)	388.6	460.9
Medium Q4 COGS Fraction	25.9%	32.9%
Medium amount as a fraction of Q3 assets		
Q3 AP	15.2%	17.3%
Q3 AR	6.4%	3.2%
Q3 INV	29.0%	40.3%
Q3 CASH	3.9%	3.1%
Q3 DLC	2.0%	1.8%
ΔAP	-0.7%	-2.7%
ΔAR	0.0%	0.0%
ΔINV	-0.6%	-6.7%
ΔCASH	0.2%	3.4%
ΔDLC	0.0%	0.0%

Notes. ΔX = Q4 X - Q3 X, where X = AP, AR, INV, CASH, or DLC.

This proxy is partially supported by the summary statistics presented in Table 4. As shown, the median Q4 COGS Fraction in the low group is around 25%; that is, sales in Q4 are roughly the average of sales across all quarters. In the high group, however, the median Q4 COGS Fraction is close to one third of annual sales. In addition, both median accounts payable and inventory at the end of Q3 are larger in the high group than in the low group, which suggests that firms in the high group increase their inventory to meet Q4 demand and are more likely to have high financing need. Comparing the changes from the end of Q3 to the end of Q4, the high group reduces inventory and payable more significantly than the low group; cash levels in the high group also increase much more than those in the low group.

6.3. Regression model and hypotheses

Combining the above classification with results in Propositions 3–4, we should expect that trade credit is the main source of external financing for firms in the low group, while firms in the high group use not only more trade credit but also a significant amount of short-term debt to finance inventory. To quantify the composition of the retailer's inventory financing portfolio, we use the correlation between *changes* in accounts payable (short-term debt) and *changes* in inventory from Q3 to Q4. Intuitively, if the retailer uses trade credit extensively to finance inventory at the end of Q3, the level of accounts payable should also come back to the normal level by the end of Q4 when inventory returns to the normal level, as the duration of trade credit is normally less than 90 days. Therefore, the changes in accounts payable between Q3 and Q4 are likely to correspond to the changes in inventory during the same period. This method is similar to that in Shyam-Sunder and Myers (1999) and Frank and Goyal (2003), both of which test the pecking order theory by regressing changes in debt and equity on an aggregate measure of financial deficit.

As such, the regression regarding changes in accounts payable is as follows:

$$\Delta\mathrm{AP}^i = \alpha^i_{ap} + \beta^i_{ap}\Delta\mathrm{INV}^i + \text{control variables} + \epsilon, \tag{14}$$

where $i \in \{H, L\}$ represents the high and low groups classified above, and control variables include changes in accounts receivable (ΔAR), changes in cash (ΔCASH), year, industry (three digit NAICS code), and/or firm fixed effects.

According to Section 5, under optimal endogenous trade credit, retailers with both high and low financing need should have trade credit as part of their inventory financing portfolio, while retailers with high financing need to use more trade credit compared to those with low financing need. Relating this to the above regression model, our hypothesis can be formally written as follows.

Hypothesis 1. *$\beta^H_{ap} > \beta^L_{ap} > 0$.*

Similarly, the regression used to quantify the correlation between changes in current debt and changes in inventory is specified as follows:

$$\Delta\mathrm{DLC}^i = \alpha^i_{dlc} + \beta^i_{dlc}\Delta\mathrm{INV}^i + \text{control variables} + \epsilon, \quad i \in \{H, L\}. \tag{15}$$

Following Propositions 3–4, the optimal endogenous trade credit should be structured so that only retailers with high financing need should have short-term debt in their inventory financing portfolio, while retailers with low financing need should be offered attractive trade credit terms so that short-term debt is not related to inventory, leading to the following hypothesis.

HYPOTHESIS 2. $\beta_{dlc}^{H} > \beta_{dlc}^{L} = 0$.

Table 5 **Regressions results for changes in accounts payable and debt in current liabilities from $Q3$ to $Q4$**

Panel A: $\Delta\text{AP} = \alpha_{ap} + \beta_{ap}\Delta\text{INV} + \text{control variables}$

	L	H	L	H	L	H	L	H
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ΔINV	0.384***	0.648***	0.377***	0.656***	0.276***	0.580***	0.261***	0.591***
	(0.043)	(0.022)	(0.044)	(0.022)	(0.057)	(0.028)	(0.058)	(0.028)
ΔAR			0.200***	0.291***			0.300***	0.275***
			(0.048)	(0.041)			(0.060)	(0.040)
ΔCASH			0.086***	0.151***			0.094***	0.036*
			(0.032)	(0.018)			(0.036)	(0.020)
Year fixed effects	included	included	included	included	included	included	included	included
NAICS fixed effects	included	included	included	included				
Firm fixed effects					included	included	included	included
R^2	0.091	0.475	0.106	0.518	0.381	0.807	0.397	0.816
Number of obs.	1,371	1,371	1,336	1,319	1,371	1,371	1,336	1,319

Panel B: $\Delta\text{DLC} = \alpha_{dlc} + \beta_{dlc}\Delta\text{INV} + \text{control variables}$

	L	H	L	H	L	H	L	H
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ΔINV	0.042	0.290***	-0.028	0.277***	-0.009	0.232***	-0.071	0.253***
	(0.213)	(0.037)	(0.223)	(0.039)	(0.271)	(0.054)	(0.279)	(0.056)
ΔAR			0.169	0.217***			0.809***	0.170***
			(0.242)	(0.071)			(0.286)	(0.078)
ΔCASH			-0.258	0.092***			-0.115	0.201***
			(0.164)	(0.031)			(0.177)	(0.039)
Year fixed effects	included	included	included	included	included	included	included	included
NAICS fixed effects	included	included	included	included				
Firm fixed effects					included	included	included	included
R^2	0.023	0.091	0.026	0.102	0.393	0.515	0.398	0.532
Number of obs.	1,364	1,338	1,328	1,284	1,364	1,338	1,328	1,284

Notes. ***, **, * indicate significance at the 1%, 5%, and 10% level, respectively. Combining the two groups (L and H) into one regression with a dummy variable on Q4 COGS Fraction lead to qualitatively similar results. However, due to the difference in the variances of the two groups, we choose to report them as separate regressions.

6.4. Regression results

The results for the above regressions are summarized in Table 5. Panel A shows that, after controlling for various factors, the correlations between changes in accounts payable and changes in inventory are highly significant. Furthermore, the correlation between inventory change and trade credit change is significantly lower in the low group than in the high group, consistent with our results in Section 5. Loosely speaking, β_{ap} can be interpreted as the proportion of marginal inventory financed through trade credit. As such, retailers with low financing need in Q4 normally use trade credit to finance around 30% to 40% of their marginal inventory, depending on model specifications. In contrast, the proportion increases to up to 65% for retailers with high financing need.[10] This result suggests that trade credit is an indispensable component of the inventory financing portfolio for retailers with various financing needs and becomes more important when the retailer's financing need is high.

Similarly, Panel B summarizes the correlations between changes in current debt and changes in inventory under various model specifications. Comparing Panel B with Panel A, the most notable observation is that the correlation between debt and inventory is not significant for a retailer in the low group, which is consistent with our analytical result. This suggests that when retailers have low financing need, they can simply use internal resources and trade credit to finance inventory. However, for retailers with high financing need, our regression results show that retailers need to use debt to finance inventory. Similar to the previous explanation, for retailers with high financing need, around 25% to 30% of marginal inventory is financed by short-term debt. Combining this with the correlation between accounts payable and inventory, we conclude that retailers with high financing need use a diversified portfolio of trade credit and short-term debt to finance their inventory.

7. Conclusion

As an integrated part of a supply chain contract, trade credit can act as a risk-sharing mechanism between suppliers and buyers. Such risk-sharing is embedded in the contingency of trade credit payment and the realization of stochastic demand. As such, buyers are induced to order higher quantities than under the classic price-only contract. This risk-sharing role of trade credit has important implications for trade credit terms and, consequently, for how buyers finance their inventory. When determining trade credit terms, suppliers balance the operational benefit through risk-sharing and the distress cost associated with trade credit default. Our model reveals that trade credit is an indispensable component of a buyer's inventory financing portfolio, while bank loans are only used to limit a supplier's risk exposure to trade credit default.

Adopting trade credit as a risk-sharing mechanism also has significant operational implications. By offering trade credit, suppliers are able to take advantage of buyers' weaker financial situation and extract higher profits. On the other hand, when a buyer's financial situation is relatively strong, trade credit can enhance both the supplier's and the buyer's profits, leading to a win–win situation.

[10] The difference between β_{ap}^H and β_{ap}^L for all model specifications reported is statistically significant at 1% level.

In addition, we find that while the supplier always is always better off as she becomes more efficient in collecting defaulted trade credit, the retailer may prefer a supplier who is less efficient in doing so.

As an initial attempt at linking trade credit with supply chain coordination and inventory financing, this work can be extended in several directions. Analytically, this model can be extended to settings such as multiple retailers and/or suppliers. On the empirical side, while this paper has offered cursory evidence on the composition of firms' inventory financing portfolio, the study is limited by data availability and the scope of the paper. Should trade credit terms be observed, the risk-sharing role of trade credit can be validated directly. In addition, other results on the operational and financial implications presented in this paper may also serve as testable hypotheses for future empirical studies.

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Appendix A: List of Notation

In the paper, we use superscript w to represent all quantities associated with the retailer's optimal response under a fixed trade credit contract, and use superscript $*$ to represent all quantities under the *endogenous*, i.e., *equilibrium* trade credit contract.

Table 6 Notation

$\tilde{D}$	stochastic demand; $\tilde{D} \in [0, +\infty)$, CDF $F()$, PDF $f()$, Complementary PDF $\bar{F}()$; failure rate $h() = f()/\bar{F}()$; generalized failure rate $g() = (\cdot)h()$.
D	demand realization
p	retail price, $p = 1$
c	unit production cost
w_c	unit cash wholesale price $c \leq w_c$
w_t	unit trade credit price, $w_c \leq w_t \leq p = 1$
d_t	trade credit early discount: $d_t = 1 - \frac{w_c}{w_t}$
q	retailer's order quantity, $q \geq 0$
q_c	retailer's order quantity paid in cash price, $q_c \in [0, q]$
q_t	retailer's order quantity paid in trade credit; $q_t = q - q_c$
K	retailer's cash, $K \geq 0$
α_b	the proportion of cost the bank incurs when bank loan defaults, $\alpha_b \in [0, 1]$
α_t	the proportion of cost the supplier incurs when trade credit defaults, $\alpha_t \in [0, 1]$
θ_b	the bank loan default threshold. The bank loan defaults if and only if $D < \theta_b$
θ_t	the trade credit default threshold. Trade credit defaults if and only if $D < \theta_t$
$\hat{F}_b()$	$\hat{F}_b(x) = \bar{F}(x)[1 - \alpha_b g(x)]$, $\hat{f}_b(x) = -\frac{\partial \hat{F}_b(x)}{\partial x}$, $\hat{h}_b(x) = \frac{\hat{f}_b(x)}{\hat{F}_b(x)}$
Γ_b	bank loan usage, $\Gamma_b = \frac{w_c q_c - K}{w_c q_c + w_t q_t}$
Γ_t	trade credit usage, $\Gamma_t = \frac{w_t q_t}{w_c q_c + w_t q_t}$
(q^{po}, w^{po})	the equilibrium quantity and wholesale price under the optimal price-only contract without financial constraints (Lariviere and Porteus 2001), where q^{po} satisfies: $\bar{F}(q^{po})[1 - g(q^{po})] = c$ and $w^{po} = \bar{F}(q^{po})$.
κ^{po}	$\kappa^{po} = w^{po} q^{po}$
π_r^o	the retailer's reservation profit

Appendix B: Proofs.

Proof of Proposition 1. To prove this proposition, we first identify the possible candidates for the retailer's optimal response $(q_c^w,\, q_t^w)$ when facing the contract $(w_c,\, w_t)$. Consider the following two scenarios depending on whether the order quantity (q_c, q_t) satisfies $w_c q_c < K$ or not (i.e., $w_c q_c \geq K$).

1. When $w_c q_c < K$, as shown in Lemma C.1, the firm should not use trade credit, i.e., $q_t = 0$. The retailer's profit function hence degenerates to the classic newsvendor one, and the optimal cash quantity q^c is $\bar{F}(w_c)$. Thus, among all (q_c, q_t) that satisfy $w_c q_c < K$, the only order quantity that can be globally optimal is: $q_c = q_c^{w,1} := \bar{F}(w_c)$ and $q_t = q_t^{w,1} := 0$.

Further, note that this solution satisfies $w_c q_c < K$, which holds if and only if $K > \kappa^{nb}(w_c)$, where $\kappa^{nb}(w_c)$ is defined in Proposition 1.

2. When $w_c q_c \geq K$, we further consider two cases depending on whether the retailer uses trade credit.

(a) When $q_t = 0$ (no trade credit used), the retailer's objective is to find q_c and θ_b to maximize (4), which becomes $\Pi_r = \int_{\theta_b}^{q_c} \bar{F}(x)dx - K$ subject to (2). For technical convenience, we treat θ_b as the decision variable and q_c as a function of θ_b through (2). Accordingly, we have $\frac{\partial q_c}{\partial \theta_b} = \frac{\hat{F}_b(\theta_b)}{w_c}$. Taking the derivative of Π_r with respect to θ_b, we have:

$$\frac{d\Pi_r}{d\theta_b} = \frac{\partial \Pi_r}{\partial \theta_b} + \frac{\partial \Pi_r}{\partial q_c}\frac{\partial q_c}{\partial \theta_b} = -\bar{F}(\theta_b) + \bar{F}(q_c)\left(\frac{\hat{F}_b(\theta_b)}{w_c}\right). \tag{16}$$

Setting $\frac{d\Pi_r}{d\theta_b} = 0$ leads to:

$$q_c = \bar{F}^{-1}\left(\frac{w_c}{1 - \alpha_b g(\theta_b)}\right). \tag{17}$$

Note that under (17), q_c decreases in θ_b, while under (2), q_c increases in θ_b. Therefore, there is at most one (θ_b, q_c) that satisfy both (2) and (17), and the corresponding θ_b satisfies $\theta_b = \theta_b^{w,2}$, where $\theta_b^{w,2}$ is defined by (5) with θ_b^w replaced by $\theta_b^{w,2}$. Thus, among all (q_c, q_t) that satisfy $w_c q_c \geq K$ and $q_t = 0$, the only order quantity that can be globally optimal is: $q_c = q_c^{w,2} := \bar{F}^{-1}\left(\frac{w_c}{1-\alpha_b g(\theta_b^{w,2})}\right)$, where $\theta_b^{w,2}$ is defined by (5), and $q_t = q_t^{w,2} := 0$.

Further, note that such $(\theta_b^{w,2}, q_c^{w,2})$ is feasible if and only if $K < \kappa^{nb}(w_c)$.

(b) When $q_t > 0$ (trade credit is used), the only possible candidate that can be globally optimal is defined $(q_c^{w,3}, q_t^{w,3}, \theta_b^{w,3}, \theta_t^{w,3})$ as characterized by Lemma C.3.

Combining these scenarios, the retailer's optimal response (q_c^w, q_t^w) has to equal one of he above three solutions, i.e., $(q_c^{w,1}, q_t^{w,1})$, $(q_c^{w,2}, q_t^{w,2})$, and $(q_c^{w,3}, q_t^{w,3})$. To find the optimal one among the three for different K, we consider the following three cases depending on the magnitude of K.

1. For $K \in (\kappa^{nb}(w_c), +\infty)$, clearly, the optimal solution with $q_t = 0$ is $(q_c^{w,1}, q_t^{w,1})$, as $(q_c^{w,2}, q_t^{w,2})$ is infeasible based on the above analysis. This corresponding to Statement 1 in Proposition 1.

Next, we show that $(q_c^{w,1}, q_t^{w,1})$ dominates any solution with $q_t^w > 0$ by contradiction. Assume $q_t^w > 0$. According to Lemma C.3, the optimal solution $(q_c^{w,3}, q_t^{w,3}, \theta_b^{w,3}, \theta_t^{w,3})$ satisfies $\theta_b^{w,3} = \hat{F}_b^{-1}\left(\frac{w_c}{w_t}\right)$, and $q_c^{w,3} = \frac{K + \int_0^{\theta_b^{w,3}} \hat{F}_b(x)dx}{w_c} > \frac{K}{w_c} > \bar{F}^{-1}(w_c)$, where the second inequality follows from $K > \kappa^{nb}(w_c)$. Under $\theta_b^{w,3}$ and $q_c^{w,3}$, we write Π_r as a function of only q_t:

$$\Pi_r(q_t) = \int_{w_t q_t + \theta_b^{w,3}}^{q_t + q_c^{w,3}} \bar{F}(x)dx. \tag{18}$$

Following the assumption that $q_t^w > 0$, according to Lemma C.3, q_t^w must satisfy

$$w_t \bar{F}(\theta_b^{w,3} + w_t q_t) = \bar{F}(q_c^{w,3} + q_t). \tag{19}$$

However, note that according to the definition of $q_c^{w,3}$ and $\theta_b^{w,3}$ as in (1) and (69),

$$\frac{d\Pi_r}{dq_t}|_{\{q_t=0\}} = \bar{F}(0+q_c^{w,3}) - w_t\bar{F}(w_t\cdot 0+\theta_b^{w,3}) = \bar{F}(q_c^{w,3}) - w_c < 0, \tag{20}$$

where the last inequality holds because $K > \kappa^{nb}(w_c)$.

On the other hand, for q_t^w to be the optimal solution, we must have $\frac{d\Pi_r}{dq_t}|_{\{q_t=q_t^w\}} = 0$ and $\frac{d^2\Pi_r}{dq_t^2}|_{\{q_t=q_t^w\}} < 0$. By the continuity of $\frac{d\Pi_r}{dq_t}$, $\exists\ q_t' \in (0, q_t^w)$ such that $\frac{d\Pi_r}{dq_t}|_{\{q_t=q_t'\}} = 0$ and $\frac{d^2\Pi_r}{dq_t^2}|_{\{q_t=q_t'\}} > 0$. However, this contradicts with the proof of Lemma C.3 that shows for any q_t with $\frac{d\Pi_r}{dq_t} = 0$, we must have $\frac{d^2\Pi_r}{dq_t^2} < 0$. Therefore, $q_t^w > 0$ cannot be an optimal solution. Instead, we should have $q_t^w = 0$, and $q_c^w = q_c^{w,1}$ and $\theta_b^w = \theta_b^{w,1}$, corresponding to the first set of condition in Proposition 1.

2. For $K \in [\kappa^b(w_c, w_t), \kappa^{nb}(w_c))$, similar to the previous case, we can show that the optimal solution is $q_t^w = 0$, and θ_b^w and q_c^w follows from the second set of conditions in Proposition 1.

3. For $K \in [0, \kappa^b(w_c, w_t))$, clearly, if the optimal solution (q_c^w, q_t^w) satisfies $q_t^w = 0$, we must have $\frac{w_c}{\hat{F}_b(\theta_b^w)} > w_t$. It is easy to show that the solution $(\theta_b'', q_c'', q_t'')$ with $\theta_b'' = \hat{F}_b^{-1}\left(\frac{w_c}{w_t}\right)$, $q_c'' = \frac{K+\int_0^{\theta_b''}\hat{F}_b(x)dx}{w_c}$, and $q_t'' = q_c^* - q_c''$ leads to a higher profit for the retailer. Therefore, the optimal solution must satisfy $q_t^w > 0$, and the optimality condition follows Lemma C.3.

Finally, we notice that $q^w = q_c^w + q_t^w$, (70) can be re-written as $w_t = \frac{\bar{F}(q^w)}{\bar{F}(\theta_t^w)}$. Therefore, by rearranging the optimality conditions (69) and (70) and combining them with (2) and (1), we have:

$$q_c^w(\theta_t^w - \theta_b^w)\bar{F}(\theta_b^w) - q_t^w\left(K + \int_0^{\theta_b^w}\hat{F}_b(x)dx\right) = 0; \tag{21}$$

$$(\theta_t^w - \theta_b^w)\bar{F}(\theta_t^w) - q_t^w\bar{F}(q^w) = 0. \tag{22}$$

which can be further simplified to (6), as desired. □

Proof of Corollary 1. According to Statement 3 in Proposition 1, for $K < \kappa^b(w_c, w_t)$, q^w and θ_t^w follow jointly $w_t\bar{F}(\theta_t^w) = \bar{F}(q^w)$ and (6).

Using $w_t\bar{F}(\theta_t^w) = \bar{F}(q^w)$, we can define θ_t^w as an implicit function of q^w, and accordingly, we have

$$\frac{\partial\theta_t^w}{\partial q^w} = \frac{f(q^w)}{w_t f(\theta_t^w)} = \frac{\bar{F}(\theta_t^w)f(q^w)}{f(\theta_t^w)\bar{F}(q^w)} = \frac{h(q^w)}{h(\theta_t^w)} > 1. \tag{23}$$

where the last inequality following from IFR.

Next, define $\Phi(q^w, K) = q^w\bar{F}(q^w) - [\theta_t^w + C(\theta_b^w)]\bar{F}(\theta_t^w)$, where θ_b^w does not depend on either q_w or K, and θ_t^w depends only on q^w through the equation $w_t\bar{F}(\theta_t^w) = \bar{F}(q^w)$.

Applying the Implicit Function Theorem to $\Phi(q^w, K)$, we have: $\frac{\partial q^w}{\partial K} = -\frac{\Phi_{q^w}}{\Phi_K}$, where

$$\Phi_K = \frac{\partial\Phi(q^w, K)}{\partial K} = -\frac{\bar{F}(\theta_t^w)}{\bar{F}(\theta_b^w)} < 0, \tag{24}$$

$$\Phi_{q^w} = \frac{\partial\Phi(q^w, K)}{\partial K} = \bar{F}(q^w)\left[1 - g(q^w)\right] - \bar{F}(\theta_t^w)\left[1 - (\theta_t^w + C(\theta_b^w))h(\theta_t^w)\right]\left(\frac{\partial\theta_t^w}{\partial q^w}\right). \tag{25}$$

According to (6), $[\theta_t^w + C(\theta_b^w)]h(\theta_t^w) = \frac{q^w\bar{F}(q^w)}{\bar{F}(\theta_t^w)}h(\theta_t^w) < g(q^w)$, therefore,

$$\Phi_{q^w} = \bar{F}(q^w)\left[1 - g(q^w)\right] - \bar{F}(\theta_t^w)\left[1 - (\theta_t^w + C(\theta_b^w))h(\theta_t^w)\right]\left(\frac{h(q^w)}{h(\theta_t^w)}\right) < \left[\bar{F}(q^w) - \bar{F}(\theta_t^w)\right]\left[1 - g(q^w)\right] < 0, \tag{26}$$

and, hence, $\frac{\partial q^w}{\partial K} = -\frac{\Phi_{q^w}}{\Phi_K} < 0$, i.e., the retailer's order quantity decreases in K. □

Proof of Corollary 2. By definition, $r_t = \frac{w_t - w_c}{w_t}$ and $r_b = \frac{\theta_b}{\int_0^{\theta_b} \hat{F}_b(x)dx} - 1$, $r_t > r_b^w$ is equivalent to $\int_0^{\theta_b^w} \hat{F}_b(x)dx > \theta_b^w \left(\frac{w_c}{w_t}\right)$. To show that, consider the following two cases.

1. When trade credit is not used, which corresponds to Statement 2 of Proposition 1, i.e., $K > \kappa^b(w_c, w_t)$. In this region, we can verify that $\hat{F}_b(\theta_b^w) > \frac{w_c}{w_t}$. As $\hat{F}_b(x)$ decreases in x,

$$\int_0^{\theta_b^w} \hat{F}_b(x)dx > \int_0^{\theta_b^w} \hat{F}_b(\theta_b^w)dx > \theta_b^w \left(\frac{w_c}{w_t}\right) \tag{27}$$

2. When trade credit is used, according to Statement 3 in Proposition 1, we have $\hat{F}_b(\theta_b^w) = \frac{w_c}{w_t}$. Similar to the previous case, we have: $\int_0^{\theta_b^w} \hat{F}_b(x)dx > \int_0^{\theta_b^w} \hat{F}_b(\theta_b^w)dx = \theta_b^w \left(\frac{w_c}{w_t}\right)$, as desired. □

Proof of Corollary 3. We first show the one-to-one mapping between (θ_b^w, θ_t^w) and (w_c, w_t). It is clear from Proposition 1 that for fixed (w_c, w_t) in the range of interests, there exists a unique (θ_b^w, θ_t^w).

Conversely, given (θ_b^w, θ_t^w), note that $1 - \frac{w_c}{w_t} = \hat{F}_b(\theta_b^w)$, therefore, w_c is uniquely determined by θ_b and w_t. Furthermore, because $w_t \bar{F}(\theta_t^w) = \bar{F}(q^w)$, w_t is uniquely determined by θ_t^w and q^w, while q^w in turn is uniquely determined by (θ_b^w, θ_t^w) according to (6).

Further, we note that under IFR, $q\bar{F}(q)$ is concave, first increasing in q and then decreasing. With fixed θ_b and θ_t, Π_s in (8) decreases in q. Therefore, if more than one q satisfies the constraint $q\bar{F}(q) = [\theta_t + C(\theta_b)]\bar{F}(\theta_t)$ for the given (θ_b, θ_t), the supplier should pick the smaller one, which guarantees that $g(q) \leq 1$. Thus, we focus on the smallest q^w that satisfies (6), as defined in $Q(\theta_b, \theta_t)$.

Next, we the monotonicity of $q^w = Q(\theta_b^w, \theta_t^w)$ with respect to θ_b^w and θ_t^w. To do so, we define $G_x(\theta_b, \theta_t, q) = [\theta_t + C(\theta_b)]\bar{F}(\theta_t) - q\bar{F}(q) = 0$ as the implicit function that determines q given θ_b and θ_t, we have:

$$\frac{\partial G_x}{\partial q} = -\bar{F}(q)(1 - g(q)) \leq 0; \tag{28}$$

$$\frac{\partial G_x}{\partial \theta_b} = C'(\theta_b)\bar{F}(\theta_t); \tag{29}$$

$$\frac{\partial G_x}{\partial \theta_t} = \bar{F}(\theta_t)(1 - (C(\theta_b) + \theta_t)h(\theta_t)) = \bar{F}(\theta_t)\left[1 - \frac{q\bar{F}(q)}{\bar{F}(\theta_t)}h(\theta_t)\right]. \tag{30}$$

According to the Implicit Function Theorem,

$$\frac{\partial q}{\partial \theta_b} = \frac{C'(\theta_b)\bar{F}(\theta_t)}{\bar{F}(q)[1 - g(q)]}, \tag{31}$$

$$\frac{\partial q}{\partial \theta_t} = \frac{\bar{F}(\theta_t)(1 - (\theta_t + C(\theta_b))h(\theta_t))}{\bar{F}(q)[1 - g(q)]} = \frac{\bar{F}(\theta_t)\left(1 - q\frac{\bar{F}(q)}{\bar{F}(\theta_t)}h(\theta_t)\right)}{\bar{F}(q)[1 - g(q)]}. \tag{32}$$

It is easy to check that $\frac{\partial q}{\partial \theta_b} \geq 0$ as $C'(\theta_b) > 0$ according to Lemma C.4, and $\frac{\partial q}{\partial \theta_t} > \frac{\bar{F}(\theta_t)[1-g(q)]}{\bar{F}(q)[1-g(q)]}$. □

Proof of Lemma 1. We treat Π_s as a function of only θ_b and θ_t by letting $q = Q(\theta_b, \theta_t)$. The derivatives of Π_s with respect to θ_b and θ_t follows:

$$\frac{\partial \Pi_s}{\partial \theta_t} = \bar{F}(\theta_t)\left[1 - \alpha_t(\theta_t - \theta_b)h(\theta_t)\right] - c\frac{\partial q}{\partial \theta_t}; \tag{33}$$

$$\frac{\partial \Pi_s}{\partial \theta_b} = \left\{[\alpha_t - \alpha_b g(\theta_b)]\bar{F}(\theta_b) - \alpha_t\bar{F}(\theta_t)\right\} - c\frac{\partial q}{\partial \theta_b}. \tag{34}$$

where $\frac{\partial q}{\partial \theta_b}$ and $\frac{\partial q}{\partial \theta_t}$ follow (31) and (32).

For this optimization problem, the necessary condition for global optimality is that the solution satisfies the first-order condition or the solution is at the boundary of Θ, i.e., $\theta_b^*=0$, $\theta_t^*=\theta_b^*$, or $[\theta_t^*+C(\theta_b^*)]\bar{F}(\theta_t^*)=g^{-1}(1)\bar{F}(g^{-1}(1))$. Clearly, for an interior solution (θ_b^*,θ_t^*) to be optimal, one must have $\frac{\partial \Pi_s}{\partial \theta_t}|_{(\theta_b^*,\theta_t^*)}=\frac{\partial \Pi_s}{\partial \theta_b}|_{(\theta_b^*,\theta_t^*)}=0$, i.e.,

$$\bar{F}(\theta_t)\left[1-\alpha_t(\theta_t-\theta_b)h(\theta_t)\right]-c\left(\frac{\bar{F}(\theta_t)(1-(\theta_t+C(\theta_b))h(\theta_t))}{\bar{F}(q)[1-g(q)]}\right)=0; \tag{35}$$

$$\left\{[\alpha_t-\alpha_b g(\theta_b)]\bar{F}(\theta_b)-\alpha_t\bar{F}(\theta_t)\right\}-c\left(\frac{C'(\theta_b)\bar{F}(\theta_t)}{\bar{F}(q)[1-g(q)]}\right)=0. \tag{36}$$

Combining the above two equations leads to (13).

Next, we consider the necessary condition for the boundary solution to be the global optima. Consider the three options.

1. If (θ_b^*,θ_t^*) with $\theta_b^*=0$ is optimal, one must have $\frac{\partial \Pi_s}{\partial \theta_b}|_{(0,\theta_t^*)}<0$ and $\frac{\partial \Pi_s}{\partial \theta_t}|_{(0,\theta_t^*)}=0$, equivalent to (12).
2. For the boundary $\theta_t^*=\theta_b^*$, it is easy to check that $\frac{\partial \Pi_s}{\partial \theta_b}|_{(\theta_b^*,\theta_t^*)}<0$, i.e., the supplier is always better off by setting θ_b to be slightly smaller than θ_b^* while keeping θ_t^*. Therefore, (θ_b^*,θ_t^*) cannot be optimal.
3. For (θ_b^*,θ_t^*) such that $[\theta_t^*+C(\theta_b^*)]\bar{F}(\theta_t^*)=g^{-1}(1)\bar{F}\left(g^{-1}(1)\right)$, note that $\frac{\partial q}{\partial \theta_t}=+\infty$, and, hence, $\frac{\partial \Pi_s}{\partial \theta_t}<0$, which again cannot be optimal.

Combining the interior solution and boundary one, (θ_b^*,θ_t^*) can only be optimal if they are an interior optima, or $\theta_b^*=0$ and $\frac{\partial \Pi_s}{\partial \theta_t^*}=0$.

Finally, the expression of w_t^* follows directly from (70), and that of w_c^* follows from $w_c^*=w_t^*\bar{F}_b(\theta_b^*)$. $\square$

Proof of Proposition 2. To show that that the optimal quantity under trade credit is greater than that without, we first show that without trade credit, the optimal order quantity $q^*\le q^{po}$ where q^{po} satisfies $\bar{F}(q^{po})[1-g(q^{po})]=c$. To see that, consider the following two cases.

1. When the retailer does not use the bank loan, i.e., $q\le \frac{K}{w_c}$, according to the first statement of Proposition 1, the retailer's best response is satisfies: $q=\bar{F}^{-1}(w_c)$. We further consider two scenarios.

   (a) When the retailer's cash constraint is not binding, i.e., $q<\frac{K}{w_c}$, clearly, the supplier's problem degenerates to the classic selling-to-the-newsvendor model, with the optimal quantity $q^*=q^{po}$. Clearly, this solution is feasible if and only if $K\ge\kappa^{po}$, where $\kappa^{po}=q^{po}\bar{F}(q^{po})$.

   (b) When the retailer's cash constraint is binding, i.e., $q=\frac{K}{w_c}$, combining the cash constraint and the retailer's best response $q^*=\bar{F}^{-1}(w_c)$, the optimal quantity q^* is determined by $q^*\bar{F}^{-1}(q^*)=K$.

Comparing the two cases, the supplier's profit Π_s is higher in the second case if and only if $K<\kappa^{po}$, under which $q^*<q^{po}$.

2. When the retailer uses the bank loan, i.e., $q^*>\frac{K}{w_c}$, according to the second statement of Proposition 1, the retailer's best response quantity q_c and the corresponding bank loan default threshold θ_b satisfy: $q_c=\bar{F}^{-1}\left(\frac{w_c}{1-\alpha_b g(\theta_b)}\right)$, where θ_b satisfies (5), and the supplier's objective is to choose w_c to maximize $(w_c-c)q_c$. For technical convenience, we treat $(q_c,\,\theta_b)$ as the decision variables, therefore, the supplier's problem becomes:

$$\max_{q,\theta_b}\quad K+\int_0^{\theta_b}\hat{F}_b(x)dx-cq \tag{37}$$

$$\text{s.t.}\quad q\bar{F}(q)=[\theta_b+C(\theta_b)]\bar{F}(\theta_b). \tag{38}$$

Treating q as a function of θ_b (through the constraint) and taking the first-order derivative with respect to θ_b, we have:

$$\frac{d\Pi_s}{d\theta_b} = \hat{F}_b(\theta_b) - c\frac{\partial q}{\partial \theta_b} = \hat{F}_b(\theta_b) - c\frac{\bar{F}(\theta_b)\left\{1 - [\theta_b + C(\theta_b)]\left[h(\theta_b) - \hat{h}_b(\theta_b^*)\right]\right\}}{[1-\alpha_b g(\theta_b)]\bar{F}(q)[1-g(q)]}. \tag{39}$$

where the second equality follows from the Implicit Function Theorem and $C'(\theta) = [\theta + C(\theta)]\hat{h}_b(\theta_b)$ from the proof of Lemma C.4. The first-order condition follows:

$$\bar{F}(q^*)[1-g(q^*)] = c\frac{\bar{F}(\theta_b^*)\left\{1 - [\theta_b^* + C(\theta_b^*)]\left[h(\theta_b^*) - \hat{h}_b(\theta_b^*)\right]\right\}}{1-\alpha_b g(\theta_b^*)}. \tag{40}$$

According to the proof in Lemma C.4, $h(\theta_b) \leq \hat{h}_b(\theta_b^*)$, therefore, the optimal quantity q^* satisfies:

$$\bar{F}(q^*)[1-g(q^*)] > c. \tag{41}$$

It is obvious that under IFR and for $q < g^{-1}(1)$, $\bar{F}(q)[1-g(q)]$ decreases in q. Therefore, $q^* < q^{po}$.

Summarizing the above two scenarios, without trade credit, the optimal quantity $q^* \leq q^{po}$.

Next, we consider the order quantity under the optimal trade credit contract. According to Lemma 1, under the optimal quantity q^*, $(\theta_b^*, \theta_t^*, q^*)$ has to satisfy $\frac{\partial \Pi_s}{\partial \theta_t^*} = 0$, therefore, we have

$$\bar{F}(q^*)[1-g(q^*)] = \frac{1 - q^*\frac{\bar{F}(q^*)}{\bar{F}(\theta_t^*)}h(\theta_t^*)}{1-\alpha_t(\theta_t^* - \theta_b^*)h(\theta_t^*)}c < c. \tag{42}$$

Therefore, $q^* > q^{po}$, as desired. $\square$

Proof of Proposition 3. To show that the retailer uses trade credit when adopting external financing, it is sufficient to prove that if the optimal solution satisfies $\theta_b^* > 0$, then we must also have $\theta_t^* > \theta_b^*$. We show this by contradiction. Assuming $\theta_t^* = \theta_b^* > 0$ is optimal, then we must have $\frac{\partial \Pi_s}{\partial \theta_b}|_{\theta_b = \theta_b^*} \geq 0$, otherwise, $\exists\ \epsilon > 0$ where $(\theta_b^* - \epsilon, \theta_t^*)$ is a feasible solution and generates a strictly greater Π_s. However, note that at $\theta_b = \theta_t$, $\frac{\partial \Pi_s}{\partial \theta_b} = -c\frac{\partial q}{\partial \theta_b} < 0$, which contradicts with the assumption. Therefore, trade credit is always used.

Next, we show the optimal terms of trade credit. For the first part, we first show the stated result holds for $\alpha_b = 0$, i.e., there is no deadweight cost associated with bank loan default. Then we show the result continues to hold for the general case, i.e., $\alpha_b > 0$.

At $\alpha_b = 0$, according to Lemma C.8, $\frac{\partial^2 \Pi_s}{\partial \theta_b^2} < 0$, and therefore, a sufficient condition for the net terms to be optimal is that $\frac{\partial \Pi_s}{\partial \theta_b} \leq 0$ for all $\theta_t \in [0, \theta_t^m]$ and $\theta_b = 0$, where θ_t^m satisfies: $(\theta_t^m + K)\bar{F}(\theta_t^m) = g^{-1}(1)\bar{F}(g^{-1}(1))$. Note that $\frac{\partial \Pi_s}{\partial \theta_b} \leq 0$ is equivalent to $\alpha_t[1 - \bar{F}(\theta_t)] - c\frac{Kf(0)\bar{F}(\theta_t)}{\bar{F}(q)[1-g(q)]} \leq 0$, which clearly holds when $\alpha_t = 0$.

When $\alpha_t > 0$, $\frac{\partial \Pi_s}{\partial \theta_b}|_{\theta_b = 0} \leq 0$ is equivalent to

$$\frac{\bar{F}(q)[1-g(q)]}{c} \leq \left(\frac{Kf(0)}{\alpha}\right)\frac{\bar{F}(\theta_t)}{1-\bar{F}(\theta_t)}. \tag{43}$$

For any θ_t, as $q\bar{F}(q) = [\theta_t + C(\theta_b)]\bar{F}(\theta_t)$, as K increases, $C(\theta_b)$ increases, and, hence, q increases. As a result, the left hand side of the above equation decreases. In the meantime, the right hand side increases. Therefore, if the inequality holds for a certain cash level K, it holds for all cash levels that are greater K.

Next, we show that there are indeed some K such that $\theta_b^* = 0$ while $\theta_t^* > 0$. Consider $K = g^{-1}(1)\bar{F}(g^{-1}(1)) - \epsilon$ for sufficiently small $\epsilon > 0$. Clearly, $\lim_{\epsilon \to 0} \theta_t^m = 0$, and the right hand side of (43) goes to infinity when

$f(0) > 0$, and the left hand size goes to 1, and, hence, (43) holds, i.e., $\theta_b^* = 0$. Furthermore, we can show that $\frac{\partial \Pi_s}{\partial \theta_t}|_{\theta_t=0} > 0$ and $\frac{\partial \Pi_s}{\partial \theta_t}|_{\theta_t=\theta_t^m} < 0$. Therefore, by continuity and concavity of Π_s with respect to θ_t (according to Lemma C.9), $\exists\ \theta_t^* \in (0, \theta_t^m)$ such that $\frac{\partial \Pi_s}{\partial \theta_t}|_{\theta_t=\theta_t^*} = 0$. Therefore, we can conclude that for $K > g^{-1}(1)\bar{F}(g^{-1}(1)) - \epsilon$, the optimal supply contract satisfies $\theta_b^* = 0$ and $\theta_t^* > 0$.

For general α_b, let $(\theta_b^{*,0}, \theta_t^{*,0})$ be the optimal solution for $\alpha_b = 0$. Consider the region where $\theta_b^{*,0} = 0$. Let $\Pi_s^{*,1} = \max_{\theta_b,\theta_t} \Pi_s(\theta_b, \theta_t; \alpha_b^1)$ for $\alpha_b^1 > 0$. By this definition, $\Pi_s^{*,1}$ is the supplier's profit under the optimal contract under α_b^1. According to Proposition 6 and the proof (which does not depend on this result), we have that $\Pi_s^{*,1} \le \Pi_s(\theta_b^{*,0}, \theta_t^{*,0}; 0)$.

On the other hand, note that when $\theta_b = 0$, the supplier's profit is independent of α_b, i.e., $\Pi(\theta_b^{*,0}, \theta_t^{*,0}; \alpha_b^1) = \Pi(\theta_b^{*,0}, \theta_t^{*,0}; 0)$. Combining the above two equations, and by the definition that $\Pi_s^{*,1}$ is the supplier's profit under the optimal contract under α_b^1, we should have $\Pi_s^{*,1} = \Pi(\theta_b^{*,0}, \theta_t^{*,0}; \alpha_b^1)$. i.e., $(\theta_b^{*,0}, \theta_t^{*,0})$ is also the optimal solution for $\alpha_b^1 > 0$. Therefore, the stated result in the proposition also holds for $\alpha_b > 0$.

Finally, to show that two-part terms are optimal, it is equivalent to show that $\theta_b^* > 0$. Clearly, a sufficient condition for $\theta_b^* > 0$ is $\frac{\partial \Pi_s}{\partial \theta_b} > 0$ at $(0, \theta_t^0)$ where $\frac{\partial \Pi_s}{\partial \theta_t}|_{\theta_t=\theta_t^0} = 0$. Note that $\frac{\partial \Pi_s}{\partial \theta_t}|_{\theta_t=0} > 0$, and, hence, $\theta_t^0 > 0$. At such $(0, \theta_t^0)$, we re-write $\frac{\partial \Pi_s}{\partial \theta_t}|_{\theta_t=\theta_t^0}$ as follows.

$$\frac{c}{\bar{F}(q^0)[1 - g(q^0)]} = \frac{1 - \alpha_t g(\theta_t^0)}{1 - (\theta_t^0 + K)h(\theta_t^0)}, \tag{44}$$

where $q^0 = Q(0, \theta_t^0)$.

On the other hand, $\frac{\partial \Pi_s}{\partial \theta_b}|_{(0,\theta_t^0)} > 0$ if and only if

$$\frac{c}{\bar{F}(q^0)[1 - g(q^0)]} < \frac{\alpha_t[1 - \bar{F}(\theta_t^0)]}{K f(0)\bar{F}(\theta_t^0)}. \tag{45}$$

Clearly, at $\alpha_t > 0$ and $K = 0$, because $\theta_t^0 > 0$, the above condition always holds. According to the continuity of the above derivatives, the conditions also hold for sufficiently small K, as desired. □

Proof of Proposition 4. For the first part of the proposition, we note that $d_t^* = 1 - \hat{F}_b(\theta_b^*)$ is monotonically increasing in θ_b^*. Therefore, we focus on the monotonicity of θ_b^* on K, α_t and α_b. We discuss the three cases separately as follows.

1. For the monotonicity of θ_b^* on K, to show that the θ_b^* that satisfy the first-order conditions (weakly) decreases in K, i.e., $\frac{d\theta_b^*}{dK} \le 0$, we consider two scenarios.

First, when $\theta_b^* > 0$, (θ_b^*, θ_t^*) are determined by the following first-order conditions $\frac{\partial \Pi_s}{\partial \theta_b} = 0$ and $\frac{\partial \Pi_s}{\partial \theta_t} = 0$. By the implicit function theorem, for that (θ_b^*, θ_t^*), we have:

$$\frac{\partial^2 \Pi_s}{\partial \theta_b^2} d\theta_b + \frac{\partial^2 \Pi_s}{\partial \theta_b \partial \theta_t} d\theta_t + \frac{\partial^2 \Pi_s}{\partial \theta_b \partial K} dK = 0; \tag{46}$$

$$\frac{\partial^2 \Pi_s}{\partial \theta_b \partial_t} d\theta_b + \frac{\partial^2 \Pi_s}{\partial \theta_t^2} d\theta_t + \frac{\partial^2 \Pi_s}{\partial \theta_t \partial K} dK = 0. \tag{47}$$

Therefore,

$$\frac{d\theta_b}{dK} = -\frac{\frac{\partial^2 \Pi_s}{\partial \theta_b \partial K}\frac{\partial^2 \Pi_s}{\partial \theta_t^2} - \frac{\partial^2 \Pi_s}{\partial \theta_t \partial K}\frac{\partial^2 \Pi_s}{\partial \theta_b \partial \theta_t}}{\frac{\partial^2 \Pi_s}{\partial \theta_b^2}\frac{\partial^2 \Pi_s}{\partial \theta_t^2} - \left(\frac{\partial^2 \Pi_s}{\partial \theta_b \partial \theta_t}\right)^2} \tag{48}$$

At the interior global maxima, the Hessian is negative definite, and, hence, the denominator in the above equation is positive. Therefore, $\frac{d\theta_b}{dK} \leq 0$ is equivalent to:

$$\frac{\partial^2 \Pi_s}{\partial \theta_b \partial K} \frac{\partial^2 \Pi_s}{\partial \theta_t^2} - \frac{\partial^2 \Pi_s}{\partial \theta_t \partial K} \frac{\partial^2 \Pi_s}{\partial \theta_b \partial \theta_t} \geq 0. \tag{49}$$

Note that for (θ_b^*, θ_t^*) to be the interior global maxima, we must have $\frac{\partial^2 \Pi_s}{\partial \theta_t^2} < 0$. For the other three the second-order derivatives, we have:

$$\frac{\partial^2 \Pi_s}{\partial K \partial \theta_b^*} = -c \frac{\partial^2 q}{\partial K \partial \theta_b^*} = -c \frac{\partial q}{\partial K} \left(\hat{h}_b(\theta_b) + \frac{\bar{F}(q) \{[2 - g(q)] h(q) + q h'(q)\}}{\bar{F}(q)[1 - g(q)]} \frac{\partial q}{\partial \theta_b} \right), \tag{50}$$

where $\hat{h}_b(\theta_b) = -\frac{\hat{F}_d'(\theta_b)}{\hat{F}_b(\theta_b)} > 0$ according to the proof of Lemma C.4, and $\frac{\partial q}{\partial K} > 0$ according to Lemma C.7. In addition, we have $h'(q) > 0$ according to IFR, and $\frac{\partial q}{\partial \theta_b} > 0$ according to Lemma C.5. Combined, we have $\frac{\partial^2 \Pi_s}{\partial K \partial \theta_b^*} < 0$.

For $\frac{\partial^2 \Pi_s}{\partial K \partial \theta_t^*}$, we have:

$$\frac{\partial^2 \Pi_s}{\partial K \partial \theta_t^*} = -c \frac{\partial^2 q}{\partial \theta_t \partial K} = c \frac{\partial q}{\partial K} \left[h(\theta_t) - \frac{\bar{F}(q) \{[2 - g(q)] h(q) + q h'(q)\}}{\bar{F}(q)[1 - g(q)]} \frac{\partial q}{\partial \theta_t} \right]. \tag{51}$$

To determine the sign of $\frac{\partial^2 \Pi_s}{\partial K \partial \theta_t^*}$, note that according to the proof in Lemma C.5, $\frac{\partial q}{\partial \theta_t} = \frac{\bar{F}(\theta_t)[1 - [\theta_t + C(\theta_b)] h(\theta_t)]}{\bar{F}(g)[1 - g(q)]}$, and, hence,

$$h(\theta_t) + \frac{\bar{F}(q) \{[2 - g(q)] h(q) + q h'(q)\}}{\bar{F}(q)[1 - g(q)]} \frac{\partial q}{\partial \theta_t} < 0 \tag{52}$$

$$\Leftarrow \quad \bar{F}(\theta_t)[1 - [\theta_t + C(\theta_b)] h(\theta_t)] \bar{F}(q^*) \{h(q^*)[2 - g(q^*)] + q h'(q^*)\} > \bar{F}^2(q^*)[1 - g(q^*)]^2 h(\theta_t^*) \tag{53}$$

$$\Leftarrow \quad \bar{F}(\theta_t^*) \left[1 - \frac{q \bar{F}(q^*)}{\bar{F}(\theta_t^*)} h(\theta_t^*) \right] h(q^*)[2 - g(q^*)] > \bar{F}(q^*)[1 - g(q^*)]^2 h(\theta_t^*) \tag{54}$$

$$\Leftarrow \quad \bar{F}(\theta_t^*) [1 - g(q^*)] [2 - g(q^*)] > \bar{F}(q^*)[1 - g(q^*)]^2. \tag{55}$$

Therefore, we have $\frac{\partial^2 \Pi_s}{\partial K \partial \theta_t^*} < 0$. Finally, for $\frac{\partial^2 \Pi_s}{\partial \theta_b^* \partial \theta_t^*}$, note that

$$\frac{\partial^2 \Pi_s}{\partial \theta_b^* \partial \theta_t^*} = \alpha_t f(\theta_t) - c \frac{\partial q^2}{\partial \theta_b^* \partial \theta_t^*}. \tag{56}$$

Note that $\frac{\partial q^2}{\partial \theta_b^* \partial \theta_t^*}$, which is determined by (6), is independent of c. Therefore, when $\alpha_t > 0$, $\frac{\partial^2 \Pi_s}{\partial \theta_b^* \partial \theta_t^*}$ is positive for sufficiently small c. Combining the above terms, we have $\frac{\partial^2 \Pi_s}{\partial \theta_b \partial K} \frac{\partial^2 \Pi_s}{\partial \theta_t^2} - \frac{\partial^2 \Pi_s}{\partial \theta_t \partial K} \frac{\partial^2 \Pi_s}{\partial \theta_b \partial \theta_t} \geq 0$, i.e., $\frac{d\theta_b^*}{dK} \leq 0$, as desired.

Second, when $\theta_b^* = 0$, the optimal solution is determined by (θ_b^*, θ_t^*) that satisfy $\frac{\partial \Pi_s}{\partial \theta_t}_{|(\theta_b = 0, \theta_t = \theta_t^*} = 0$. To show that $\theta_b^* = 0$ for $K' > K$, it is sufficient to show that $\frac{d}{dK} \left(\frac{\partial \Pi_s}{\partial \theta_b} \right) < 0$, or equivalently,

$$\frac{\partial^2 \Pi_s}{\partial \theta_t \partial \theta_b} \frac{d\theta_t^*}{dK} + \frac{\partial^2 \Pi_s}{\partial \theta_t \partial K} < 0. \tag{57}$$

As $\frac{d\theta_t^*}{dK} = -\frac{\frac{\partial^2 \Pi_s}{\partial \theta_t \partial K}}{\frac{\partial^2 \Pi_s}{\partial \theta_t^2}}$ and $\frac{\partial^2 \Pi_s}{\partial \theta_t^2} < 0$ at the global optima, the above equation is equivalent to:

$$\frac{\partial^2 \Pi_s}{\partial \theta_b \partial K} \frac{\partial^2 \Pi_s}{\partial \theta_t^2} - \frac{\partial^2 \Pi_s}{\partial \theta_t \partial \theta_b} \frac{\partial^2 \Pi_s}{\partial \theta_b \partial K} < 0. \tag{58}$$

which holds for the same reason as in the case with $\theta_b^* > 0$. Combining the above two cases ($\theta_b^* > 0$ and $\theta_b^* = 0$), we have that for sufficiently small c, θ_b^* (weakly) decreases in K.

2. For the monotonicity of θ_b^* on α_t, similar to the above case, θ_b^* (weakly) increases in α_t is equivalent to:

$$\frac{\partial^2\Pi_s}{\partial\theta_b\partial\alpha_t}\frac{\partial^2\Pi_s}{\partial\theta_t^2}-\frac{\partial^2\Pi_s}{\partial\theta_t\partial\alpha_t}\frac{\partial^2\Pi_s}{\partial\theta_b\partial\theta_t}\geq 0. \tag{59}$$

Note that $\frac{\partial^2\Pi_s}{\partial\theta_t^*\partial\alpha_t}=-(\theta_t^*-\theta_b^*)h(\theta_t^*)<0$, and $\frac{\partial^2\Pi_s}{\partial\theta_b^*\partial\alpha_t}=\bar{F}(\theta_b^*)-\bar{F}(\theta_t^*)>0$. Combining these two results with $\frac{\partial^2\Pi_s}{\partial\theta_b\partial\theta_t}>0$ (for sufficiently small c) and $\frac{\partial^2\Pi_s}{\partial\theta_t^2}<0$ (according to optimality), we have $\frac{\partial^2\Pi_s}{\partial\theta_b\partial\alpha_t}\frac{\partial^2\Pi_s}{\partial\theta_t^2}-\frac{\partial^2\Pi_s}{\partial\theta_t\partial\alpha_t}\frac{\partial^2\Pi_s}{\partial\theta_b\partial\theta_t}\geq 0$ as desired.

3. For the monotonicity on α_b, similar to the previous cases, we only need to show that

$$\frac{\partial^2\Pi_s}{\partial\theta_b\partial\alpha_b}\frac{\partial^2\Pi_s}{\partial\theta_t^2}-\frac{\partial^2\Pi_s}{\partial\theta_t\partial\alpha_b}\frac{\partial^2\Pi_s}{\partial\theta_b\partial\theta_t}\leq 0. \tag{60}$$

Considering each term separately, similar to the above cases, we have $\frac{\partial^2\Pi_s}{\partial\theta_b\partial\theta_t}>0$ (for sufficiently small c) and $\frac{\partial^2\Pi_s}{\partial\theta_t^2}<0$ (according to optimality), and

$$\frac{\partial^2\Pi_s}{\partial\alpha_b\partial\theta_b^*}=-c\frac{\partial^2 q}{\partial\alpha_b\partial\theta_b^*}=-c\frac{\bar{F}(\theta_t)}{\bar{F}(q)[1-g(q)]}\left[\frac{\partial C'(\theta_b)}{\partial\alpha_b}+\frac{\bar{F}(q)\left\{[2-g(q)]h(q)+qh'(q)\right\}}{\bar{F}(q)[1-g(q)]}\frac{\partial q}{\partial\alpha_b}\right]. \tag{61}$$

According to Lemma C.7, $\frac{\partial q}{\partial\alpha_b}>0$. Finally, according to the proof in Lemma C.4, $C'(\theta_b)=[\theta_b+C(\theta_b)]\hat{h}_b(\theta_b)$. Therefore, we have:

$$\frac{\partial C'(\theta_b)}{\partial\alpha_b}=\frac{\partial C(\theta_b)}{\partial\alpha_b}\hat{h}_b(\theta_b)+[\theta_b+C(\theta_b)]\frac{\partial\hat{h}_b(\theta_b)}{\partial\alpha_b} \tag{62}$$

$$=\frac{\partial C(\theta_b)}{\partial\alpha_b}\hat{h}_b(\theta_b)+\frac{\theta_b+C(\theta_b)}{[\hat{F}_b(\theta_b)]^2}\left(\frac{\partial\hat{f}_b(\theta_b)}{\partial\alpha_b}\hat{F}_b(\theta_b)-\hat{f}_b(\theta_b)\frac{\partial\hat{F}_b(\theta_b)}{\partial\alpha_b}\right). \tag{63}$$

where $\hat{f}_b(\theta_b)=-\frac{\partial\hat{F}_b(\theta_b)}{\partial\theta_b}=\bar{F}(\theta_b)\left\{h(\theta_b)-\alpha_b[g(\theta_b)-g'(\theta_b)]\right\}$. Therefore, we have:

$$\frac{\partial\hat{f}_b(\theta_b)}{\partial\alpha_b}=\bar{F}(\theta_b)[g'(\theta_b)-g(\theta_b)h(\theta_b)] \tag{64}$$

$$\frac{\partial\hat{F}_b(\theta_b)}{\partial\alpha_b}=-\bar{F}(\theta_b)g(\theta_b) \tag{65}$$

and, hence,

$$\frac{\partial C'(\theta_b)}{\partial\alpha_b}=\frac{\partial C(\theta_b)}{\partial\alpha_b}\hat{h}_b(\theta_b)+\frac{[\theta_b+C(\theta_b)]\bar{F}(\theta_b)^2}{[\hat{F}_b(\theta_b)]^2}g'(\theta_b), \tag{66}$$

which is greater than zero as $\frac{\partial C(\theta_b)}{\partial\alpha_b}>0$ (according to Lemma C.6) and $g'(\theta_b)>0$ (according to IFR). Finally, for $\frac{\partial^2\Pi_s}{\partial\alpha_b\partial\theta_t^*}$, we have:

$$\frac{\partial^2\Pi_s}{\partial\alpha_b\partial\theta_t^*}=-c\frac{\partial^2 q}{\partial\alpha_b\partial\theta_t^*}=-c\left[h(\theta_t)-\frac{\bar{F}(q)\left\{[2-g(q)]h(q)+qh'(q)\right\}}{\bar{F}(q)[1-g(q)]}\frac{\partial q}{\partial\theta_t}\right]\frac{\partial q}{\partial\alpha_b}. \tag{67}$$

According to the above proof of the monotonicity of θ_b^* with respect to K, we have $h(\theta_t)-\frac{\bar{F}(q)\left\{[2-g(q)]h(q)+qh'(q)\right\}}{\bar{F}(q)[1-g(q)]}\frac{\partial q}{\partial\theta_t}<0$, and according to Lemma C.7, $\frac{\partial q}{\partial\alpha_b}\geq 0$. Therefore, $\frac{\partial^2\Pi_s}{\partial\alpha_b\partial\theta_t^*}\leq 0$. Combining the sign of the above terms, we have $\frac{\partial}{\partial\alpha_b}\left(\frac{d\Pi_s}{d\theta_b^*}\right)<0$, as desired.

For the second part, note that when $\theta_b^*=0$, the amount of account payables $w_tq_t=\theta_t^*$. Therefore, we only need to consider the monotonicity of θ_t^* with respect to K and α_t. Consider them separately.

1. For the monotonicity of θ_t^* on K, we note that θ_t^* which satisfy the first-order conditions decreases in K is equivalent to $\frac{\partial^2\Pi_s}{\partial\theta_t\partial K}<0$, which holds according to the previous proof.

2. For the monotonicity of θ_t^* on α_t, similarly, θ_t^* decreases in α_t if $\frac{\partial^2\Pi_s}{\partial\theta_t\partial\alpha_t}<0$, which we have also shown in the earlier part of the proof. □

Proof of Corollary 4. The proof is similar to the proof of the second part of Proposition 4. When $\theta_b^*=0$, for θ_t^* to be decreasing in c, we only need to to show that $\frac{\partial^2\Pi_s}{\partial\theta_t\partial c}<0$, which holds because $\frac{\partial^2\Pi_s}{\partial\theta_t\partial c}=-\frac{\partial q}{\partial\theta_t^*}$ and $\frac{\partial q}{\partial\theta_t^*}>0$. □

Proof of Proposition 5. To show that d_t^* (weakly) increases in π_r^o, we focus on the non-trivial case where the retailer's participation constraint is binding, i.e., $\int_{\theta_t^*}^{Q(\theta_b^*,\theta_t^*)}\bar{F}(x)dx=\pi_r^o$. Next, consider two outside options for the retailer, $\pi_r^{o,1}$ and $\pi_r^{o,2}$, where $\pi_r^{o,2}>\pi_r^{o,1}$. Let the corresponding optimal solution be $(\theta_b^{*,1},\theta_t^{*,1})$ and $(\theta_b^{*,2},\theta_t^{*,2})$. Clearly, $(\theta_b^{*,1},\theta_t^{*,1})$ is infeasible for $\pi_r^o=\pi_r^{o,2}$. In fact, according to Lemma C.10, any solution (θ_b,θ_t) with $\theta_b\le\theta_b^{*,1}$ and $\theta_t\le\theta_t^{*,1}$ is infeasible for $\pi_r^{o,2}>\pi_r^{o,1}$. Therefore, $(\theta_b^{*,2},\theta_t^{*,2})$ must satisfy $\theta_b^{*,2}>\theta_b^{*,1}$ or $\theta_t^{*,2}>\theta_t^{*,1}$. The first condition is the same as what we need to show. For the second condition, note that for $(\theta_b^{*,2},\theta_t^{*,2})$ to be optimal, we must have $\frac{\partial\Pi_s}{\partial\theta_t}=0$ at $(\theta_b^{*,2},\theta_t^{*,2})$. Therefore, if $\theta_t^{*,2}>\theta_t^{*,1}$, for sufficiently small c, according to the proof of Proposition 4, we have that $\frac{\partial^2\Pi_s}{\partial\theta_b\partial\theta_t}>0$. Thus, if we express θ_b as a function of θ_t as defined by $\frac{\partial\Pi_s}{\partial\theta_t}=0$, according to the Implicit Function Theorem, we have: $\frac{d\theta_b^*}{d\theta_t^*}=-\frac{\frac{\partial^2\Pi_s}{\partial\theta_b\partial\theta_t}}{\frac{\partial^2\Pi_s}{\partial\theta_t^2}}>0$. Therefore, if $\theta_t^{*,2}>\theta_t^{*,1}$, we must also have $\theta_b^{*,2}>\theta_b^{*,1}$, i.e., the early-payment discount for $\pi_r^o=\pi_r^{o,2}$ is greater than that in $\pi_r^o=\pi_r^{o,1}$, as desired. □

Proof of Corollary 5. The proof follows from the proof of Proposition 4. When $d_t^*=0$,

1. for q^*, we have $\frac{dq^*}{d\alpha_t}=\frac{\partial q}{\partial\theta_t}\frac{d\theta_t}{d\alpha_t}<0$ as $\frac{\partial q}{\partial\theta_t}>0$ (according to Lemma C.5), and $\frac{d\theta_t}{d\alpha_t}<0$ (according to the proof of Proposition 4).

2. for w_t^* and w_c^*, similarly, according to Lemma C.5, w_t^* decreases in θ_t^*, which in turn decreases in α_t according to the proof of Corollary 4. So w_t^* increases in α_t, as desired. □

Proof of Proposition 6. First, we prove the three monotonicity results on Π_s^* separately as follows.

1. For α_t, note that under given (θ_b,θ_t), $q=Q(\theta_b,\theta_t)$ is independent of α_t, therefore, the supplier's profit decreases in α_t under any given (θ_b,θ_t). Now consider two α_t, with $\alpha_t^1>\alpha_t^2$. Let $(\theta_b^{*,i},\theta_t^{*,i})$ be the optimal contract under α_t^i for $i=1,2$. Therefore, we have: $\Pi_s(\theta_b^{*,1},\theta_t^{*,1};\alpha_t^1)<\Pi_s(\theta_b^{*,1},\theta_t^{*,1};\alpha_t^2)\le\Pi_s(\theta_b^{*,2},\theta_t^{*,2};\alpha_t^2)$, where the second inequality holds as $(\theta_b^{*,2},\theta_t^{*,2})$ be the optimal contract under α_t^2. Therefore, the supplier's profit under the optimal trade credit contract decreases in α_t.

2. For α_b, the relationship is more complicated, as α_b influences the profit not only directly through the distress cost, i.e., $\alpha_b\int_0^{\theta_b}xdF(x)$, but also indirectly through $q=Q(\theta_b,\theta_t)$. Specifically, according to Lemma C.7, q increases in α_b. With these results, following the same steps for the above proof of α_t, we can show that the supplier's profit under the optimal trade credit contract (weakly) decreases in α_b.

3. For the monotonicity of Π_s^* on K, when $\theta_b^*=0$, θ_t^* is uniquely determined by $\frac{\partial\Pi_s}{\partial\theta_t^*}=0$. Seeing q^* as a function of θ_t^*, θ_b^* and K, we can apply the Envelope Theorem and have:

$$\frac{d\Pi_s}{dK}=\frac{\partial\Pi_s}{\partial K}=1-c\frac{\bar{F}(\theta_t^*)}{\bar{F}(q^*)[1-g(q^*)]}. \tag{68}$$

As $\frac{\partial\Pi_s}{\partial\theta_t^*}=0$, we have $c\frac{\bar{F}(\theta_t^*)}{\bar{F}(q^*)[1-g(q^*)]}=\frac{\bar{F}(\theta_t^*)}{1-(\theta_t^*+K)h(\theta_t^*)}$. Due to IFR, $h(\theta_t^*)\ge h(x)\ge f(x)$ for all $x\in[0,\theta_t^*)$. Therefore, $\bar{F}(\theta_t^*)=1-\int_0^{\theta_t^*}f(x)dx>1-(\theta_t^*+K)h(\theta_t^*)$, and, hence, $\frac{d\Pi_s}{dK}<0$.

Finally, for the monotonicity on Π_r^*, as $\Pi_r^* = \int_{\theta_t^*}^{q^*} \bar{F}(x)dx - K$, we have $\frac{d\Pi_r^*}{d\alpha_t} = \left[\bar{F}(\theta_t^*) - \bar{F}(q)\frac{\partial q^*}{\partial \theta_t^*}\right]\frac{d\theta_t^*}{d\alpha_t}$. According to the proof of Lemma C.5, $\frac{\partial q^*}{\partial \theta_t^*} = \frac{\bar{F}(\theta_t)[1-q\bar{F}(q)h(\theta_t)]}{\bar{F}(q)[1-g(q)]} > \frac{\bar{F}(\theta_t)}{\bar{F}(q)}$. Therefore, $\left[\bar{F}(\theta_t^*) - \theta F(q)\frac{\partial q^*}{\partial \theta_t^*}\right] < 0$, and, hence, $\frac{d\Pi_r^*}{d\alpha_t} > 0$, as desired. □

Appendix C: Technical Results and Proofs

Lemma C.1. Under $(w_c,\, w_t)$, for any $(q_c,\, q_t)$ with $q_c < \frac{K}{w_c}$ and $q_t > 0$, there exists a $(q_c',\, q_t')$ such that the retailer's profit under $(q_c',\, q_t')$ is no less than that under $(q_c,\, q_t)$.

Lemma C.2. Let $f(x)$ be twice differentiable over (a,b). If $f''(x) < 0$ $\forall x \in (a,b)$ with $f'(x) = 0$, $f(x)$ is pseudo-concave over (a,b).

Lemma C.3. Under given (w_c, w_t), $(q_c^{w,3}, q_t^{w,3}, \theta_b^{w,3}, \theta_t^{w,3})$ with $q_t^{w,3} > 0$ is the global optimal for $\Pi_r = \int_{\theta_t}^{q_c+q_t} \bar{F}(x)dx$ subject to (2) and (1) if and only if both of the following conditions hold:

$$\hat{F}_b(\theta_b^{w,3}) = \frac{w_c}{w_t}; \tag{69}$$

$$w_t \bar{F}(\theta_t^{w,3}) = \bar{F}(q_c^{w,3} + q_t^{w,3}). \tag{70}$$

Lemma C.4. $C(\theta)$ is increasing in θ. If $\alpha_b = 0$, then $C(\theta)$ is convex in θ.

Lemma C.5. Following Proposition 1 and Corollary 3, we can view d_t, w_c, w_t and q^w as functions of θ_b^w and θ_t^w, and we have:

1. d_t increases in θ_b.
2. w_c decreases in θ_b and θ_t
3. w_t decreases in θ_b and θ_t.
4. q is convex in θ_t. q is convex in θ_b if $\alpha_b = 0$.

Lemma C.6. $C(\theta)$ increases in α_b for $\theta > 0$ and is independent of α_b for $\theta = 0$.

Lemma C.7. $Q(\theta_b, \theta_t)$ increases in α_b and K.

Lemma C.8. For any θ_t, Π_s is concave on θ_b if $\alpha_b = 0$.

Lemma C.9. For any θ_b, Π_s is concave on θ_t.

Lemma C.10. For any $\pi_r^o \in \left[0,\, \int_0^{\bar{F}^{-1}(c)} \bar{F}(x)dx - c\bar{F}^{-1}(c)\right]$, let $\theta_b^m(\theta_t; \pi_r^o) = \min\left\{\theta_b \geq 0 : \int_{\theta_t}^{Q(\theta_b,\theta_t)} \bar{F}(x)dx \geq \pi_r^o\right\}$. $\theta_b^m(\theta_t; \pi_r^o)$ decreases in θ_t and increases in π_r^o.

C.1. Proofs of technical lemmas

Proof of Lemma C.1. We prove this result by construction. For any $(q_c,\, q_t)$ as stated in the Lemma, let $\epsilon = \min\left(\frac{K}{w_c} - q_c,\, q_t\right)$. Consider a new solution $(q_c',\, q_t')$ where $q_c' = q_c + \epsilon$, and $q_t' = q_t - \epsilon$. Clearly, the total inventory level $q_c' + q_t' = q_c + q_t$, therefore, the retailer's revenue $\int_0^q \bar{F}(x)dx$ is the same under the two solutions. However, as $w_c \leq w_t$, $(q_c',\, q_t')$ has a (weakly) lower cost. Therefore, the profit under $(q_c',\, q_t')$ is no lower than that under $(q_c,\, q_t)$, as desired. □

Proof of Lemma C.2. We prove the above statement by contradiction. Suppose $f(x)$ is not pseudo-concave, then $\exists\ x_1, x_2 \in (a,b)$ such that $f'(x_1)(x_2 - x_1) \le 0$, and $f(x_2) - f(x_1) > 0$. Without loss of generality, assume $x_1 < x_2$, and $f'(x_1) \le 0$. According to the mean-value theorem, $\exists\ x^m \in [x_1, x_2]$ such that $f(x_2) = f(x_1) + f'(x^m)(x_2 - x_1)$, which means that $f'(x^m) > 0$. On the other hand, as $f'(x)$ is continuous, $\exists$ $x^0 \in (x_1, x^m)$ such that $f'(x^0) = 0$ (if there are more than one x satisfies $f'(x) = 0$, let x^0 be the smallest); so, $\forall\ x \in (x^0 - \epsilon, x^0)$, we have $f'(x) < 0$. This in turn leads to $f''(x^0) \ge 0$, which contradicts the assumed condition. Therefore, $f(x)$ is pseudo-concave. □

Proof of Lemma C.3. To identify the optimal solution, we write out the Lagrangian relaxation of the math program defined by (4), (2), and (1) as follows:

$$L = \int_{\theta_t}^{q_c+q_t} \bar{F}(x)dx + \lambda_1 \left[w_c q_c - K - \int_0^{\theta_b} \hat{F}_b(x)dx \right] + \lambda_2 \left[w_t q_t - (\theta_t - \theta_b) \right]; \tag{71}$$

The derivatives for the Lagrangian are:

$$\frac{\partial L}{\partial q_c} = \bar{F}(q_c + q_t) + \lambda_1 w_c, \tag{72}$$

$$\frac{\partial L}{\partial q_t} = \bar{F}(q_c + q_t) + \lambda_2 w_t, \tag{73}$$

$$\frac{\partial L}{\partial \theta_b} = \lambda_1 \hat{F}_b(\theta_b) + \lambda_2, \tag{74}$$

$$\frac{\partial L}{\partial \theta_t} = -\lambda_2 - \bar{F}(\theta_t). \tag{75}$$

Setting all the derivatives to zero, we have $\hat{F}_b(\theta_b^{w,3}) = -\lambda_2/\lambda_1 = w_c/w_t$, and $q_c^{w,3}$ is uniquely determined by $\theta_b^{w,3}$ through (2).

Fixing $\theta_b^{w,3}$ and $q_c^{w,3}$, we can re-write the objective function (4) as a function only of q_t, i.e.,

$$\Pi_r = \int_{w_t q_t + \theta_b^{w,3}}^{q_t + q_c^{w,3}} \bar{F}(x)dx. \tag{76}$$

Taking the first-order derivative with respect to q_t, we have:

$$\frac{d\Pi_r}{dq_t} = \bar{F}(q_t + q_c^{w,3}) - w_t \bar{F}(w_t q_t + \theta_b^*). \tag{77}$$

Setting $\frac{d\Pi_r}{dq_t} = 0$, $q_t^{w,3}$ and the corresponding $\theta_t^{w,3}$ should satisfy: $\bar{F}(q_c^{w,3} + q_t^{w,3}) = w_t \bar{F}(\theta_t^{w,3})$, or equivalently, $w_t = \frac{\bar{F}(q_c^{w,3} + q_t^{w,3})}{\bar{F}(\theta_t^{w,3})}$. Taking the second-order derivative, we have:

$$\frac{d^2\Pi_r}{dq_t^2}|_{q_t = q_t^{w,3}} = -f(q_c^{w,3} + q_t^{w,3}) + w_t^2 f(\theta_t^{w,3}) = [\bar{F}(q_c^{w,3} + q_t^{w,3})]^2 \left\{ \left(\frac{1}{\bar{F}(\theta_t^{w,3})} \right)' - \left(\frac{1}{\bar{F}(q_c^{w,3} + q_t^{w,3})} \right)' \right\}. \tag{78}$$

It is easy to show that under IFR, $\left(\frac{1}{\bar{F}(x)}\right)' > 0$. As $\theta_t^{w,3} \le q_c^{w,3} + q_t^{w,3}$, we have that $\frac{d^2\Pi_r}{dq_t^2}|_{q_t = q_t^{w,3}} \le 0$. According to Lemma C.2, Π_r is pseudo-concave in q_t; thus, the first order condition is sufficient for global optimality. □

Proof of Lemma C.4. In preparation, taking derivative of $\hat{F}_b(\theta)$ with respect to θ, we have:

$$\hat{F}_b'(\theta) = -\bar{F}(\theta) \left\{ [1 + \alpha_b - \alpha_b g(\theta)] h(\theta) + \alpha_b \theta h'(\theta)] \right\} < 0. \tag{79}$$

Following this, by the definition of $C(\theta)$, we have:

1. $C'(\theta) = -\frac{[\theta + C(\theta)]\hat{F}_b'(\theta)}{\hat{F}_b(\theta)} > 0$.

2. For convexity, note that at $\alpha_b = 0$, $\hat{F}_b(x) = \bar{F}(x)$, and hence $C'(\theta) = [\theta + C(\theta)]h(\theta)$. Therefore, $C''(\theta) = h(\theta) + (\theta + C(\theta))[h'(\theta) + h^2(\theta)] \geq 0$. □

Proof of Lemma C.5. According to Corollary 3 and the definition of d_t, we can express d_t as a function of (θ_b^w, θ_t^w). Next, we show the monotonicity results.

1. For d_t, note that by definition, $d_t = 1 - \hat{F}_b(\theta_b^w)$, therefore, d_t increases in θ_b^w.

2. For w_t, re-write the related constraint as: $G_w(w_t, \theta_b^w, \theta_t^w) = w_t \bar{F}(\theta_t^w) - \bar{F}\left(\frac{\theta_t^w + C(\theta_b^w)}{w_t}\right) = 0$. Therefore,

$$\frac{\partial G_w}{\partial w_t} = -\frac{q^w f(q^w)}{w_t} + \bar{F}(\theta_t^w) = -\frac{q^w f(q^w) - \bar{F}(q^w)}{w_t} = \frac{\bar{F}(q^w)[1 - g(q^w)]}{w_t} \geq 0, \tag{80}$$

where the second equality holds as $w_t \bar{F}(\theta_t^w) = \bar{F}(q^w)$. Similarly,

$$\frac{\partial G_w}{\partial \theta_t^w} = -w_t f(\theta_t^w) + \frac{f(q)}{w_t} = \frac{f(q) - w_t^2 f(\theta_t^w)}{w_t} \geq 0, \tag{81}$$

where the last inequality follows from the proof of Lemma C.3.

Combining the above two results and applying the Implicit Function Theorem, we have: $\frac{\partial w_t}{\partial \theta_t^w} \leq 0$, that is, w_t decreases in θ_t^w.

Similarly, $\frac{\partial G_w}{\partial \theta_b^w} = f(q^w) C'(\theta_b^w) \geq 0$; hence, $\frac{\partial w_t}{\partial \theta_b^w} \leq 0$.

3. For w_c, as $w_c = w_t(1 - d_t)$, $\frac{\partial w_c}{\partial \theta_t^w} = \frac{\partial w_t}{\partial \theta_t^w}(1 - d_t) \leq 0$, and $\frac{\partial w_c}{\partial \theta_b^w} = (1 - d_t)\frac{\partial w_t}{\partial \theta_b^w} - w_t\left(\frac{\partial d_t}{\partial \theta_b^w}\right) \leq 0$.

Finally, we show the convexity of q^w with respect to θ_b^w and θ_t^w.

1. For the convexity of q^w with respect to θ_t^w, following (32), re-write $\frac{\partial q^w}{\partial \theta_t^w} = \frac{A(\theta_t^w)}{B(q^w)}$, where $A(\theta_t^w) = \bar{F}(\theta_t^w)[1 - (\theta_t^w + C(\theta_b^w))h(\theta_t^w)]$, and $B(q^w) = \bar{F}(q^w)[1 - g(q^w)]$. Therefore,

$$\frac{\partial^2 q^w}{\partial (\theta_t^w)^2} = \frac{A_t}{B} - \frac{A^2 B_q}{B^3}, \tag{82}$$

where

$$B_q = \frac{\partial B}{\partial q^w} = -\bar{F}(q^w)\{h(q^w)[1 - g(q^w)] + g'(q^w)\} < 0, \tag{83}$$

$$A_t = \frac{\partial A}{\partial \theta_t^w} = -\bar{F}(\theta_t^w)\left\{2h(\theta_t^w) + [\theta_t^w + C(\theta_b^w)][h'(\theta_t^w) - h^2(\theta_t^w)]\right\} < 0. \tag{84}$$

Therefore, $\frac{\partial^2 q^w}{\partial (\theta_t^w)^2} > 0$ is equivalent to $A_t - B_q\left(\frac{A}{B}\right)^2 > 0$. Note that $\frac{A}{B} = \frac{\partial q^w}{\partial \theta_t^w} \geq \frac{\bar{F}(\theta_t^w)}{\bar{F}(q^w)}$, and $B_q < 0$, and hence,

$$\begin{aligned}
\frac{\partial^2 q^w}{\partial (\theta_t^w)^2} > 0 \quad &\Leftarrow \quad A_t - B_q\left(\frac{\bar{F}(\theta_t^w)}{\bar{F}(q^w)}\right)^2 > 0 &(85)\\
&\Leftarrow \quad \bar{F}(\theta_t^w)\{[2 - g(q^w)]h(q^w) + qh'(q^w)\} - \bar{F}(q^w)\left\{2h(\theta_t^w) + [\theta_t^w + C(\theta_b^w)][h'(\theta_t^w) - h^2(\theta_t^w)]\right\} > 0 &(86)\\
&\Leftarrow \quad [2 - g(q^w)]h(q^w)\bar{F}^2(\theta_t^w) - q^w \bar{F}^2(q^w)h^2(\theta_t^w) > 0 &(87)\\
&\Leftarrow \quad 2[1 - g(q^w)]h(q^w)\bar{F}^2(\theta_t^w) > 0 &(88)
\end{aligned}$$

2. For the convexity of q^w with respect to θ_b^w, the first order derivative $\frac{\partial q^w}{\partial \theta_b^w} = \frac{C'(\theta_b^w)\bar{F}(\theta_t^w)}{\bar{F}(q^w)(1 - g(q^w))}$, and

$$\frac{\partial^2 q^w}{\partial (\theta_b^w)^2} = \frac{C''(\theta_b^w)\bar{F}(\theta_t^w)}{\bar{F}(q^w)[1 - g(q^w)]} + \frac{C'(\theta_b^w)\bar{F}(\theta_t^w)}{\{\bar{F}(q^w)[1 - g(q^w)]\}^2}\left(\frac{\partial q^w}{\partial \theta_b^w}\right)\bar{F}(q^w)[(1 - g(q^w))h(q^w) + g'(q^w)]. \tag{89}$$

According to Lemma C.4, at $\alpha_b = 0$, both $C''()$ and $C'()$ are positive, and hence $\frac{\partial^2 q^w}{\partial (\theta_b^w)^2} > 0$ at $\alpha_b = 0$. □

Proof of Lemma C.6. By definition, we have:

$$\frac{\partial C(\theta)}{\partial \alpha_b} = \frac{\left[K + \int_0^\theta \hat{F}_b(x)dx\right]\bar{F}(\theta)g(\theta) - \hat{F}_b(\theta)\int_0^\theta xf(x)dx}{\hat{F}_b^2(\theta)} = \frac{K\bar{F}(\theta)g(\theta) + \bar{F}(\theta)\int_0^\theta [g(\theta)-g(x)]dx}{\hat{F}_b^2(\theta)} \geq 0. \quad (90)$$

where the last inequality holds as $g(x)$ increases in x (according to IFR), and the equality only holds when $\theta = 0$. □

Proof of Lemma C.7. Let $G(q,K,\alpha_b) = [\theta_t + C(\theta_b)]\bar{F}(\theta_t) - q\bar{F}(q)$. Given IFR, $q\bar{F}(q)$ is concave on q. Thus, as $Q(\theta_b,\theta_t)$ is the smaller solution to $G(q,K,\alpha_b) = 0$, $\frac{\partial G}{\partial q} = -\bar{F}(q)[1-g(q)] < 0$. Further, $\frac{\partial G}{\partial K} = \frac{\bar{F}(\theta_t)}{\hat{F}_b(\theta_b)}$. Therefore, by the Implicit Function Theorem,

$$\frac{\partial Q}{\partial K} = -\frac{\frac{\partial G}{\partial K}}{\frac{\partial G}{\partial q}} = \frac{\bar{F}(\theta_t)}{\hat{F}_b(\theta_b)\bar{F}(q)[1-g(q)]} > 0. \quad (91)$$

For α_b, note that according to Lemma C.6, $\frac{\partial G}{\partial \alpha_b} = \bar{F}(\theta_t)\frac{\partial C(\theta)}{\partial \alpha_b} \geq 0$. Therefore, by the Implicit Function Theorem, $\frac{\partial Q}{\partial \alpha_b} = -\frac{\frac{\partial G}{\partial \alpha}}{\frac{\partial G}{\partial q}} \geq 0$, as desired. □

Proof of Lemma C.8. Consider the second order derivative of Π_s with respect to θ_b, at $\alpha_b = 0$, we have $\frac{\partial^2 \Pi_s}{\partial \theta_b^2} = -\alpha_t f(\theta_b) - c\frac{\partial^2 q}{\partial \theta_b^2}$. According to Lemma C.5, $\frac{\partial^2 q}{\partial \theta_b^2} > 0$ if $\alpha_b = 0$, and hence $\frac{\partial^2 \Pi_s}{\partial \theta_b^2} < 0$ at $\alpha_b = 0$. □

Proof of Lemma C.9. To show that Π_s is concave on θ_t, we have:

$$\frac{\partial^2 \Pi_s}{\partial \theta_t^2} = -f(\theta_t)[1 - \alpha_t(\theta_t - \theta_b)h(\theta_t)] - \alpha_t\bar{F}(\theta_t)[h(\theta_t) + (\theta_t - \theta_b)h'(\theta_t)] - c\frac{\partial^2 q}{\partial \theta_t^2} \quad (92)$$

$$= -\bar{F}(\theta_t)\left\{h(\theta_t) + \alpha_t[1 - (\theta_t - \theta_b)h(\theta_t)]h(\theta_t) + \alpha_t(\theta_t - \theta_b)h'(\theta_t)\right\} - c\frac{\partial^2 q}{\partial \theta_t^2}. \quad (93)$$

As $\theta_t < q$, hence $g(\theta_t) < 1$, which leads to $1 - (\theta_t - \theta_b)h(\theta_t) > 0$. So the first part of the second line is always negative. The second part is also negative as $\frac{\partial^2 q}{\partial \theta_t^2} > 0$ according to Lemma C.5. Therefore, $\frac{\partial^2 \Pi_s}{\partial \theta_t^2} < 0$, as desired. □

Proof of Lemma C.10. To show that θ_b^m decreases in θ_t, we focus on the non-trivial case, i.e., $\int_{\theta_t}^{Q(\theta_b^m,\theta_t)} \bar{F}(x)dx = \pi_r^o$. Taking derivative with respect to θ_t by treating θ_b^m and Q as functions of θ_t based on the retailer's participation constraint, we have:

$$\bar{F}(Q)\left(\frac{\partial Q}{\partial \theta_t} + \frac{\partial Q}{\partial \theta_b^m}\frac{\partial \theta_b^m}{\partial \theta_t}\right) = \bar{F}(\theta_t). \quad (94)$$

According to (31) and (32), $\frac{\partial Q}{\partial \theta_t} > \frac{\bar{F}(Q)}{\bar{F}(\theta_t)}$ and $\frac{\partial Q}{\partial \theta_b^m} > 0$. Therefore, $\frac{\partial \theta_b^m}{\partial \theta_t} < 0$.

To show that θ_b^m increases in π_r^o, recall that that according to Lemma C.5, $Q(\theta_b,\theta_t)$ increases in θ_b, therefore, fixing θ_t, as π_r^o increases, $Q(\theta_b,\theta_t)$ should increase, and, hence, θ_b^m increases. □

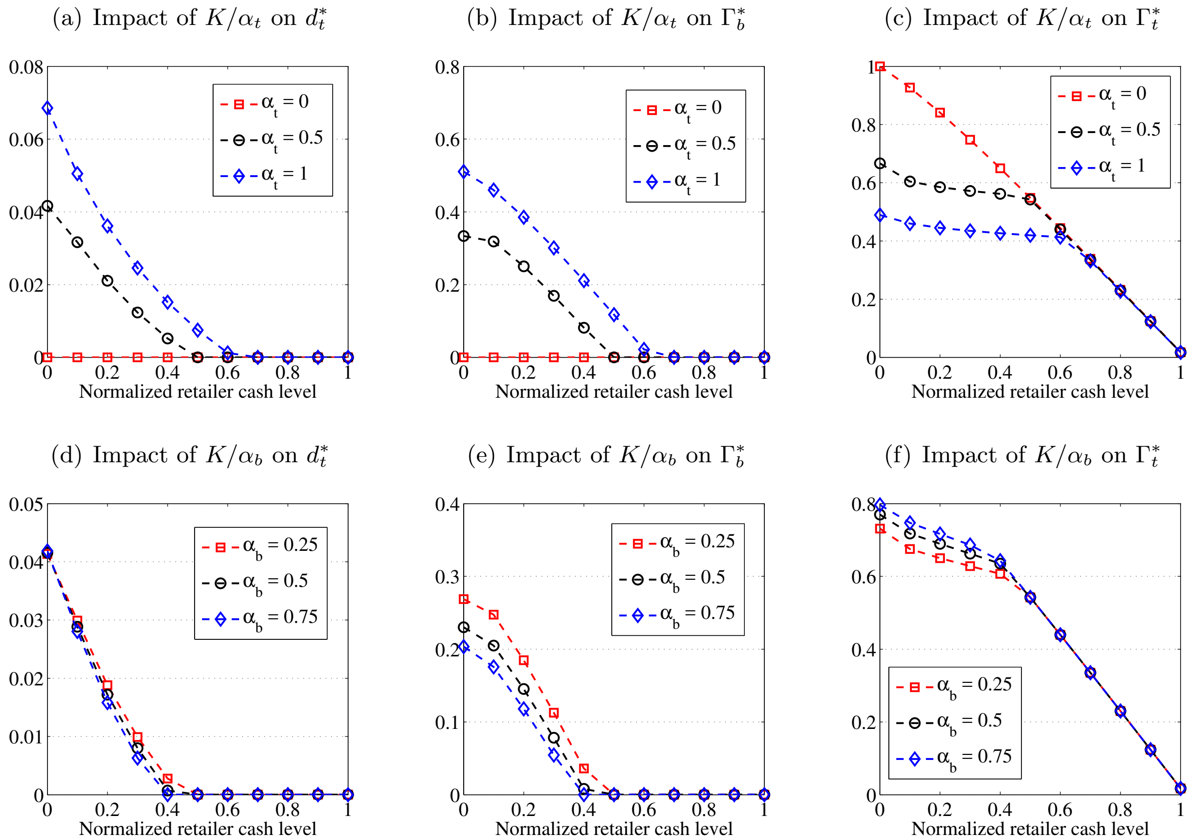
Appendix D: Additional Numerical Results

In this section, we present numerical results based on truncated normal and exponential distributions, which show that the results presented in the main body of the paper are robust under different distributions.

D.1. Numerical results based on truncated normal distribution

In this section, we present the numerical results where the demand follows a truncated normal distribution with mean $\mu = 50$, standard deviation $\sigma = 25$, and lower and upper limits at 0 and 100 respectively. The results are summarized in Figures 6 – 9, corresponding to Figures 2 – 5 in the main body of the paper. As shown, the patterns remain similar to the ones shown in the main body of the paper.

Figure 6 The impact of financial characteristics on early-payment discount and inventory finance portfolio under the optimal trade credit contract



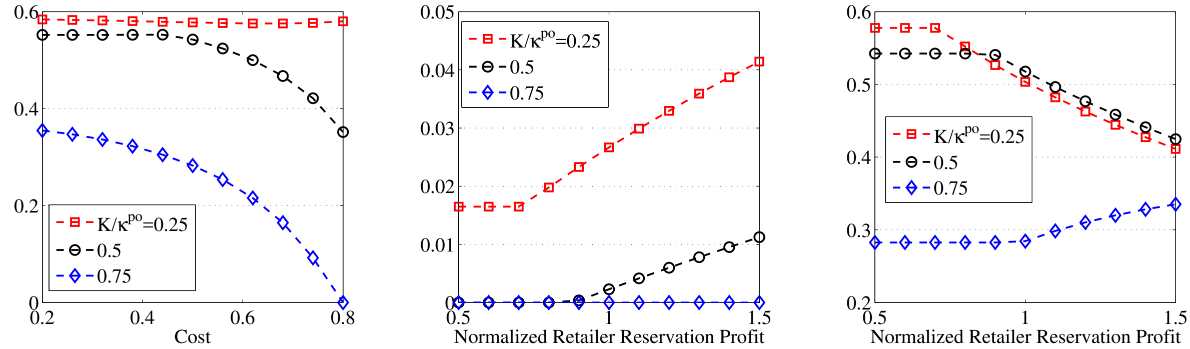
Notes. Bank loan usage $\Gamma_b^* = \frac{w_c^* q_c^* - K}{w_c^* q_c^* + w_t^* q_t^*}$; trade credit usage $\Gamma_t^* = \frac{w_t^* q_t^*}{w_c^* q_c^* + w_t^* q_t^*}$. Parameters used: $c = 0.5$. $\alpha_b = 0$ in Figure 6(a)–6(c) $\alpha_t = 0.5$ in Figure 6(d) –6(f).

D.2. Numerical results based on exponential distribution

In this section, the numerical results as presented in Figures 10 – 13 are based on the random demand that follows an exponential distribution with mean $\mu = 50$.

Figure 7 **The impact of operational characteristics (c and π_r^o) on trade credit terms and usage**

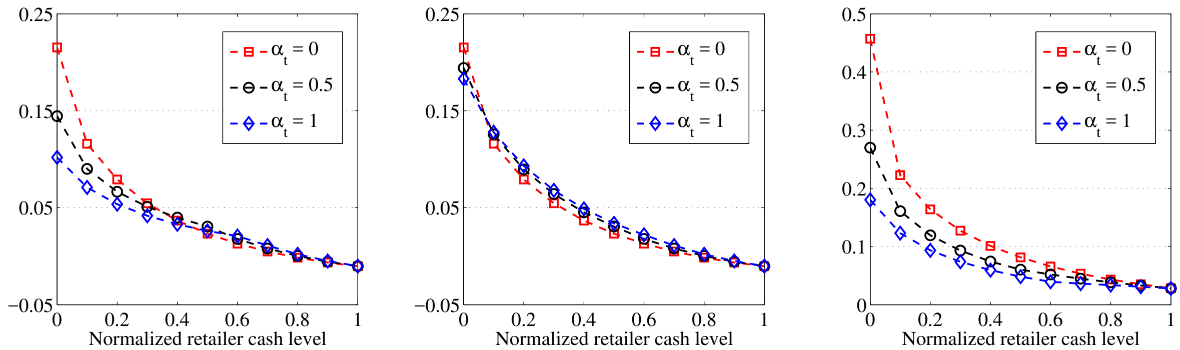
(a) Impact of c on Γ_t^* (b) Impact of π_r^o on d_t^* (c) The impact of π_r^o on Γ_t^*



Notes. $\kappa^{po} = w^{po}q^{po}$, where w^{po} and q^{po} are the wholesale price and quantity under the optimal price-only contract without financial constraint. In Figures 7(b) and 7(c), the x-axis is π_r^o normalized by $\pi_r^{po} = \int_0^{q^{po}} \bar{F}(x)dx - w^{po}q^{po}$, the retailer's profit under the optimal price-only contract without financial constraint. Parameters used: $c = 0.5$, $\alpha_t = 0.5$, $\alpha_b = 0$.

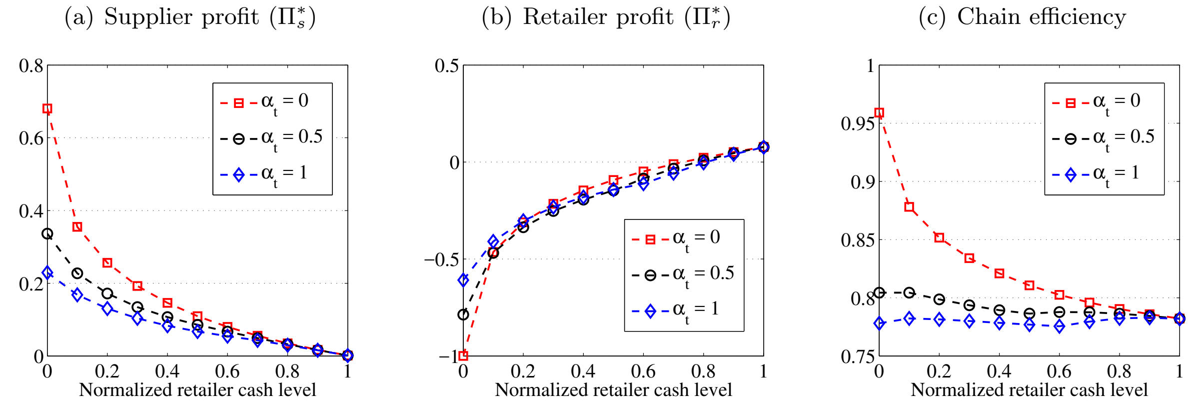
Figure 8 **Operational decisions under the optimal trade credit contract**

(a) Cash wholesale price (w_c^*) (b) Credit wholesale price (w_t^*) (c) Order quantity (q^*)



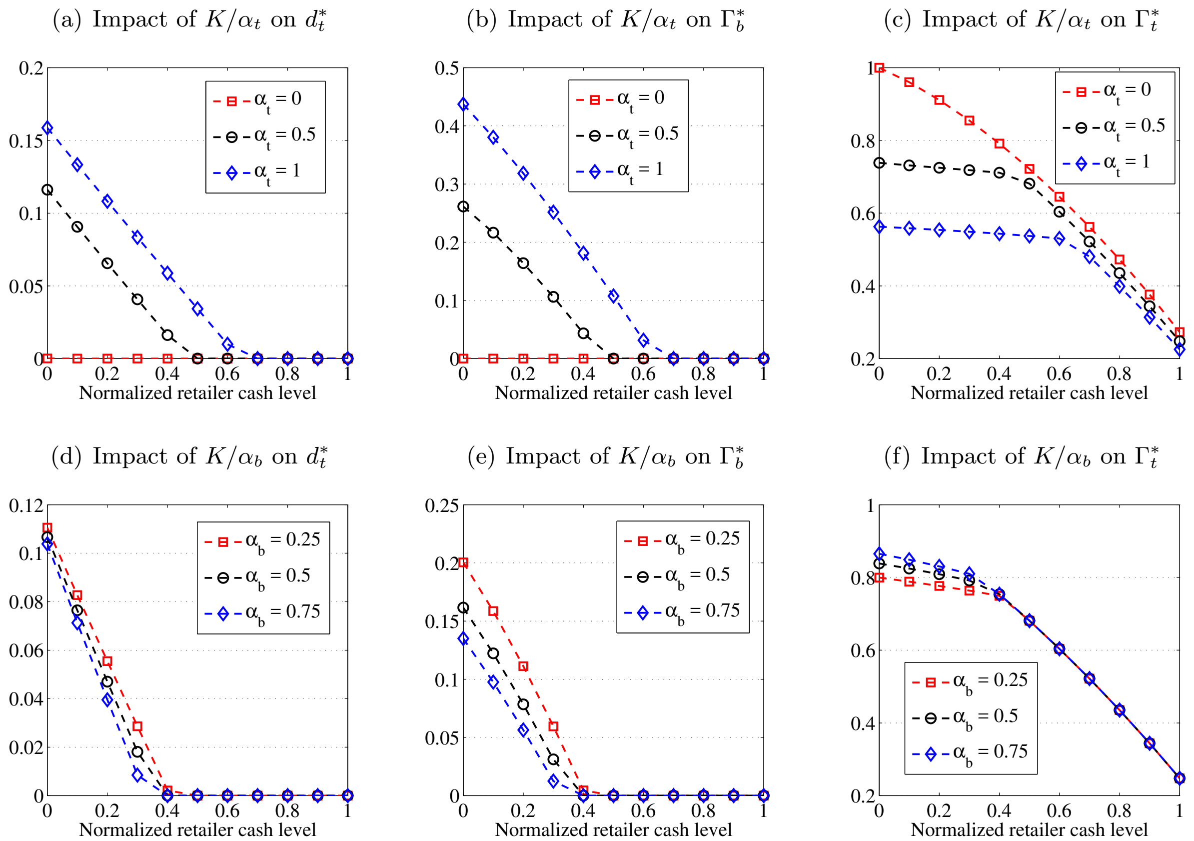
Notes. The y-axis represents the relative differences from the benchmark under the optimal price-only contract without financial constraint (q^{po} and w^{po}). Parameters used: $c = 0.5$, $\alpha_b = 0$.

Figure 9 **Supply chain performance under the optimal trade credit contract**



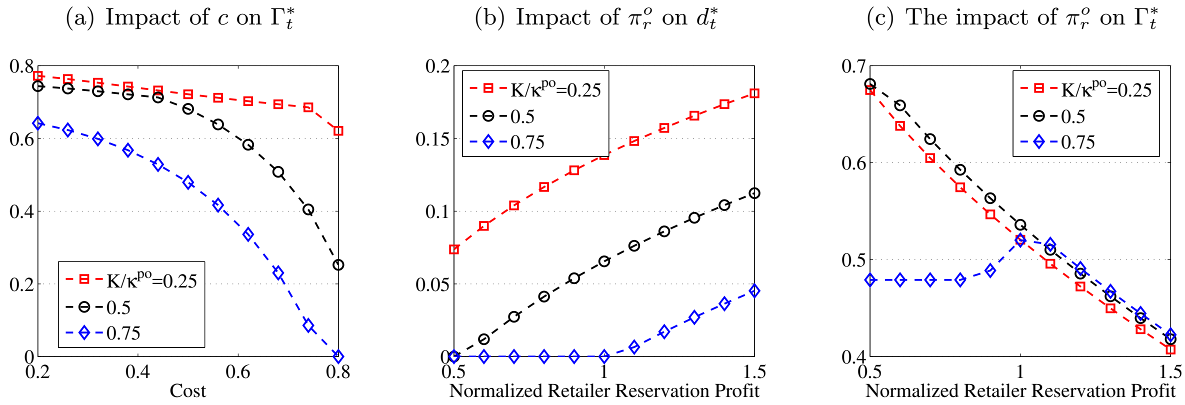
Notes. The y-axis in 9(a) and 9(b) represents the relative differences from the benchmark under the optimal price-only contract without financial constraint. In Figure 9(c), chain performance is measured by the sum of the chain profit $(\pi_r^* + \pi_s^*)$ divided by the integrated chain profit. Parameters used: $c = 0.5$, $\alpha_b = 0$.

Figure 10 **The impact of financial characteristics on early-payment discount and inventory finance portfolio under the optimal trade credit contract**



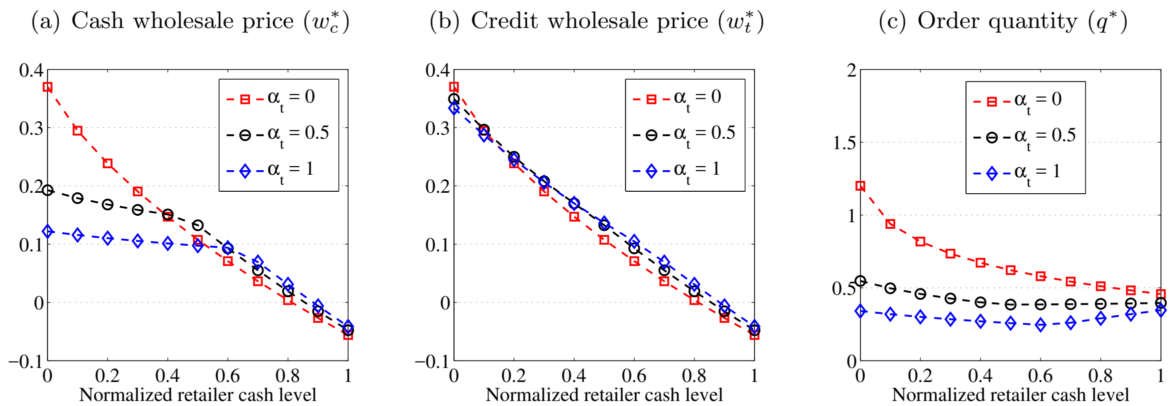
Notes. Bank loan usage $\Gamma_b^* = \frac{w_c^* q_c^* - K}{w_c^* q_c^* + w_t^* q_t^*}$; trade credit usage $\Gamma_t^* = \frac{w_t^* q_t^*}{w_c^* q_c^* + w_t^* q_t^*}$. Parameters used: $c = 0.5$. $\alpha_b = 0$ in Figure 10(a)–10(c) $\alpha_t = 0.5$ in Figure 10(d) –10(f).

Figure 11 The impact of operational characteristics (c and π_r^o) on trade credit terms and usage



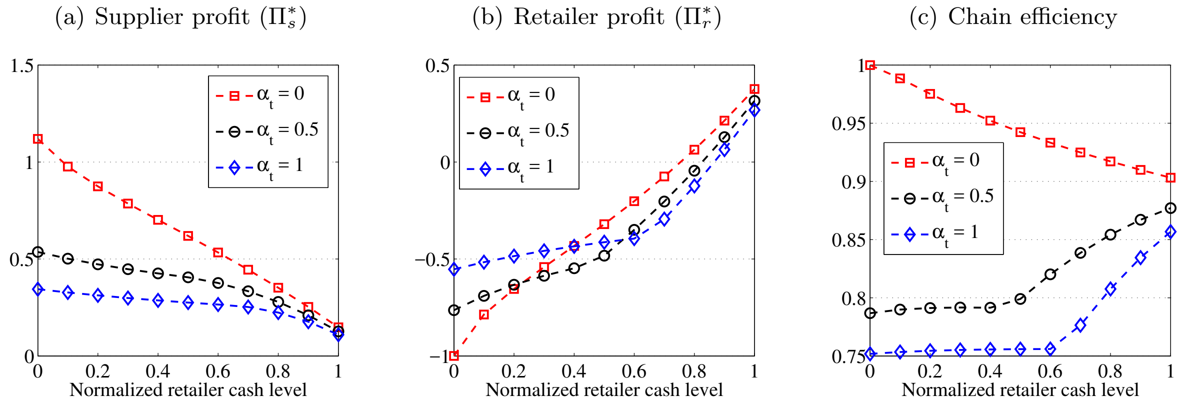
Notes. $\kappa^{po} = w^{po}q^{po}$, where w^{po} and q^{po} are the wholesale price and quantity under the optimal price-only contract without financial constraint. In Figures 7(b) and 7(c), the x-axis is π_r^o normalized by $\pi_r^{po} = \int_0^{q^{po}} \bar{F}(x)dx - w^{po}q^{po}$, the retailer's profit under the optimal price-only contract without financial constraint. Parameters used: $c = 0.5$, $\alpha_t = 0.5$, $\alpha_b = 0$.

Figure 12 Operational decisions under the optimal trade credit contract



Notes. The y-axis represents the relative differences from the benchmark under the optimal price-only contract without financial constraint (q^{po} and w^{po}). Parameters used: $c = 0.5$, $\alpha_b = 0$.

Figure 13 Supply chain performance under the optimal trade credit contract



Notes. The y-axis in 13(a) and 13(b) represents the relative differences from the benchmark under the optimal price-only contract without financial constraint. In Figure 13(c), chain performance is measured by the sum of the chain profit ($\pi_r^* + \pi_s^*$) divided by the integrated chain profit. Parameters used: $c = 0.5$, $\alpha_b = 0$.