

Here is our pastel color box theorem.

Theorem A. There exists a nice pastel-colored blue box.

Proof. This is a nice pastel-colored blue box. ■

Let's try an actual theorem which has some use in number theory.

Proposition B. Consider a weakly modular function $f : \mathcal{H} \rightarrow \mathbb{C}$ of weight k with respect to Γ . Then f is periodic of some period h and there exists a function $g : \mathcal{D}^* \rightarrow \mathbb{C}$ such that $f(z) = g(q_h)$ where $q_h(z) = e^{2\pi iz/h}$.

Proof. It should be clear that any congruence subgroup $\Gamma \subseteq \mathrm{SL}_2(\mathbb{Z})$ contains a translation matrix of the form

$$\gamma = \begin{pmatrix} 1 & h \\ 0 & 1 \end{pmatrix}.$$

Computing the factor of automorphy under γ , we have $j(\gamma, z) = 1$ for any $z \in \mathcal{H}$ which, by weakly modularity of f , implies that

$$f(z) = f[\gamma]_k = f(\gamma(z)) = f(z + h).$$

In other words, f is periodic of period h . It is obvious that the function q_h is a holomorphic function $\mathcal{H} \rightarrow \mathcal{D}^*$ and is periodic of period h as well. So we can consider the function $g : \mathcal{D}^* \rightarrow \mathbb{C}$ defined by

$$g(q_h) = f\left(\frac{h \log q_h}{2\pi i}\right),$$

which satisfies $f(z) = g(q_h)$. Since f is periodic of period h , we can choose any branch of $\log q_h$ in \mathcal{H} and so g is well-defined. This completes the proof. ■