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# Design of The University of Akron's 2015 FSAE Electric Vehicle Braking System

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**Design of the 2015**  
**University of Akron's**  
**FSAE Electric Vehicle**  
**Braking System**

**By: Nicholas D. Galbincea**



## Abstract

The following report encompasses the design of the 2015 University of Akron's FSAE formula electric braking system. The system is one based on hydraulic braking and designed for a one man performance racing vehicle. The objective of the system is to convert the kinetic energy of the vehicle into thermal energy, allowing the vehicle to decelerate optimally and safely. The design includes three major categories: calculation and evaluation of the hydraulic system in order to select calipers and master cylinders, the design of the pedal box, and the design of the rotors.

The results and findings of the proceeding report rendered a symmetrical front and rear braking system. The Wilwood Single Dynalite two piston floating caliper and the 77-Series Tilton master cylinders with a 1 inch bore were selected for the system. An optimized pedal box was designed consisting of two pedals, (gas and brake), eight mounting tabs for the two pedals and master cylinders, and two electrical sensor supports (one for the throttle sensor and the other for the emergency stop switch). The pedal box was optimized for minimal mass and satisfied the design criteria (later defined in the text). Lastly, custom rotors were designed based on the optimal material choice, analyzing thermal effects and considering cost.

## Acknowledgments

I would like to thank Dr. Siamak Farhad for being my primary advisor for this design project and Dr. Richard Gross for being my secondary advisor for this project and making this experience challenging and enjoyable.

A special thanks to Mike, Bill and Dale in the machine shop for all of their help and advice in fabricating the pedal box.

I would like to dedicate this project to my family, my father David, mother Annette and sister Hannah.

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## Introduction

The braking system was designed as a hydraulic system for a one man performance racing vehicle. Generic parameters of the vehicle that were used in the system design were a total vehicle weight, including the driver, of 625 *lbs* and a maximum velocity of 60 *mp*<sup>2</sup>. Further details of parameters will be defined in the report corresponding to the braking system.

The three main categories/ components designed and analyzed for this system are the following: calculation and evaluation of the hydraulic system in order to select calipers and master cylinders, the design of the pedal box and the design of the rotors. For the first component, the overall functionality and free variables of the braking system will be analyzed for optimum caliper, master cylinder and rotor size selection. The second main component of the design encompasses the pedal box design. This design includes the gas pedal, brake pedal and master cylinder orientation as well as the throttle sensor and emergency stop placement and orientation. The third and final main component involves the design of the rotors. The main focus on the design of the rotors was material selection as well as the geometry of the rotors.

The constraints and limitations based on the competition rules of these components will be defined later in each of their sections. The objective of these major component designs was to minimize the weight for the lightest design possible while at the same time designing to all of the mechanical and thermal conditions that the system would be subjected to. In the pedal box and rotor design sections the optimization processes will be discussed as well as the finite element analysis that support the final design. In the final sections of the report the manufacturing of the pedal box and rotors will be discussed.

## Braking System Theory, Calculation & Design

### Design Objective and Overview

The main objective of the braking system is to convert the kinetic energy of the vehicle into thermal energy, thus allowing the vehicle to decelerate. The braking system was designed as a hydraulic system with two master cylinders, one for the braking of the front two tires and one for braking of the rear two tires. Attached to each master cylinder are two floating calipers, one located at each of the tires for a total of four calipers for the system, as well as four rotors or brake disks.

The flow of the braking system is as follows: the driver exerts a force on the brake pedal, the brake pedal channels that force to the master cylinders, thus displacing the braking fluid in the master cylinders. The displaced fluid then exerts a pressure on each of the calipers allowing

the caliper pistons to exert a clamping force on the rotors. Therefore, the input of the system is the driver's applied foot force and the output is the clamping force of the calipers exerted on the rotors.

Based on the competition rules and conditions the designed braking system must be able to lock all four tires of the vehicle completely during an emergency stop braking scenario. What this translates to physically is that the moment generated from the caliper force placed on the rotor must be equal to or greater than the moment the tire exerts on the surface of the road.

Therefore for this braking hydraulic system design the main objective is to design the system so that the driver has to place a substantial, but not excessive force on the brake pedal in order to completely lock the tires of the vehicle. The system was designed for this foot force to approximately be 150 lbs. This value was decided upon by referencing other design teams and Dr. Richard Gross.

## Theory for Design

During braking, under the condition the vehicle is moving in the direction of its front tires, the greatest deceleration rate will translate as a weight transfer from the rear tires to the front tires. Therefore, it is necessary to calculate this weight transfer based on the average coefficient of friction between the tires and the road  $\mu_B$ , the weight of the vehicle  $W$ , the height of the center of mass above the road  $H$ , and the wheelbase  $L$ . This weight transfer will be denoted as  $F_T$  and calculated [1]:

$$F_T = \frac{\mu_B WH}{L} \quad (1)$$

From this weight transfer calculation the normal force exerted on the front two tires ( $F_{NF}$ ) and the rear two tires ( $F_{NR}$ ) are calculated [1]:

$$F_{NF} = \frac{WL_R}{L} + F_T \quad (2)$$

$$F_{NR} = \frac{WL_F}{L} - F_T \quad (3)$$

Where  $L_F$  &  $L_R$  are the lengths of the front and rear axles to the center of mass of the vehicle [1].

Neglecting the weight on the tires, wheels and rotors and taking a summation of moments about the center of rotation of one of the tires, it is observed for the front and rear systems [1]:

$$\Sigma M_O = 0$$

$$\text{Front: } F_{FCP}R_{FCP} - \mu_B F_{NF}R_{FT} = 0 \quad (4)$$

$$\text{Rear: } F_{RCP}R_{RCP} - \mu_B F_{NR}R_{RT} = 0 \quad (5)$$

Where,  $F_{FCP}$  &  $F_{RCP}$  are the sum of the friction forces generated from the front two calipers and rear two calipers,  $R_{FCP}$  &  $R_{RCP}$  are the mean radii of these friction forces, and  $R_{FT}$  &  $R_{RT}$  are the radii of the front and rear tires [1].

The total friction force of the calipers for the front and rear can be written in terms of the normal force or “clamping force” that the calipers exert on the rotor multiplied by the brake pad coefficient of friction [1]:

$$F_{FCP} = \mu_{CP}F_{NFCP} \quad (6)$$

$$F_{RCP} = \mu_{CP}F_{NRCP} \quad (7)$$

The normal forces of each set of calipers, both front and rear, can be written in terms of the pressure in each brake line and the area of each caliper piston [1]:

$$F_{NFCP} = (\# \text{ of front caliper pistons})P_F A_{FCP} \quad (8)$$

$$F_{NRCP} = (\# \text{ of rear caliper pistons})P_R A_{RCP} \quad (9)$$

Thus, the pressure in both the front and rear braking lines can be written [1]:

$$P_F = \frac{F_{NFCP}}{\# \text{ of front caliper pistons} * A_{FCP}} \quad (10)$$

$$P_R = \frac{F_{NRCP}}{\# \text{ of rear caliper pistons} * A_{RCP}} \quad (11)$$

The forces on both master cylinders can then be expressed from the pressure in the front and rear lines multiplied by the bore area of each master cylinder [1]:

$$F_{FMC} = P_F A_{FMC} \quad (12)$$

$$F_{RMC} = P_R A_{RMC} \quad (13)$$

Taking a moment about the braking pedal, the foot force applied to the pedal can be written in terms of the forces from the master cylinders [1]:

$$aF_{Foot} = b(F_{FMC} + F_{RMC}) \quad (14)$$

Where  $a$  is the distance from the foot force to the pivot point and  $b$  is the distance from where the master cylinder mounts to the brake pedal to the pivot point [1].



## Results from Calculations & Selection of Components

The above theory and equations were implemented in the following calculations and the results & selection of the calipers, brake pads, rotor size and master cylinders are as follows:

Based on the calculations from the teams lead suspension engineer the following values of  $\mu_B$ ,  $H$  &  $L$  were used:

$$\mu_B = 1.8, H = 11in, L = 61in, L_F = 29in \text{ \& } L_R = 32in$$

From these values the force transferred was determined:

$$F_T = 203 \text{ lbs}$$

The normal forces applied to the front and rear tires during breaking:

$$F_{NF} = 531 \text{ lbs}$$

$$F_{NR} = 94 \text{ lbs}$$

From the suspension engineer the radii of the tires for both the front and rear were given as 9.5 *in*. Given the hollow rim design, where the rotor sits inside of the rim with the caliper mounted, the rotor's diameter was restricted. The inner diameter of the rim was approximated to 12 *in* and the caliper clearance was approximated to be 1.5 *in*, (given a final selection has not been made yet). After modeling the rotor, caliper and hub in SolidWorks the largest rotor diameter that could fit the assembly was 8.25 *in*. This was diameter used, as the largest rotor diameter would produce the greatest torque from the caliper, for a lower line pressure, during braking. Thus the friction forces for the front and rear calipers could be solved for:

$$F_{FCP} = \frac{\mu_B F_{NF} R_{FT}}{R_{FCP}} = \frac{(1.8)(531 \text{ lbs})(9.5in)}{3.5in} = 2594.3 \text{ lbs}$$

$$F_{RCP} = \frac{\mu_B F_{NR} R_{RT}}{R_{RCP}} = \frac{(1.8)(94 \text{ lbs})(9.5in)}{3.5in} = 459.3 \text{ lbs}$$

At this stage in the design, it was necessary to select a caliper and brake pad. The number of pistons in the caliper and the coefficient of friction of the brake pads were the free variables for these selections. After researching on the internet possible caliper choices and drafting a caliper selection matrix (see below) based on number of pistons, weight, size and cost, the Wilwood *Billet Dynalite Single* caliper was selected [2] & [3].

Company/ Manufacturer	Make/ Model	Body Material	Piston Material	Weight (lbs)	Cost	Disk OD (in)	Disk thickness (in)	Piston Diam (in)	Piston Diam 2(in)	Piston Area (in^2)	Piston Area 2 (in^2)	# of Pistons
AP Racing	Aluminium - Lug Mount - CP2576	Aluminum	Aluminum	2.4		10.5118	0.38189	1.625984		2.08		2
AP Racing	Aluminium - Lug Mount - CP2577	Aluminum	Aluminum	2.4		10.5118	0.38189	1.751969		2.41		2
AP Racing	Aluminium - Lug Mount - CP3176	Aluminum	Aluminum	2.4		10.5118	0.38189	1.5		1.77		2
AP Racing	Aluminium - Lug Mount - CP3177	Aluminum	Aluminum	2.4		10.5118	0.38189	1.41732		1.58		2
AP Racing	Aluminium - Lug Mount - CP3178	Aluminum	Aluminum	2.4		10.5118	0.38189	1.251969		1.23		2
AP Racing	Aluminium - Radial Mount 130 ctrs - Solid Disc - CP6120	Cast Aluminum Alloy	Aluminum	3.3		11.1024	0.5	1.751969		2.41		2
AP Racing	Aluminium - Radial Mount 130 ctrs - Solid Disc - CP6121	Cast Aluminum Alloy	Aluminum	3.3		11.1024	0.5	1.5		1.77		2
AP Racing	Aluminium - Radial Mount 130 ctrs - Ventilated Disc - CP6126	cast Aluminium alloy	Aluminum	3.3		11.0236	0.7007874	1.751969		2.41		2
AP Racing	Aluminium Lug Mount - CP3696-6E0	Aluminium alloy	Aluminum	1.8		10.5118	0.279528	1.625		2.07		2
AP Racing	Aluminium Radial Mount - CP3676	Aluminium alloy	Aluminum	2.4		10.5118	0.38189	1.625984		2.08		2
AP Racing	Aluminium Radial Mount - CP3677	Aluminium alloy	Aluminum	2.4		10.5118	0.38189	1.751969		2.41		
AP Racing	Aluminium Radial Mount - CP4586	Aluminium alloy	Aluminum	2.4		10.5118	0.38189	1.41732		1.58		
AP Racing	Aluminium Radial Mount - CP4596	Aluminium alloy	Aluminum	2.4		10.5118	0.38189	1.251969		1.23		
AP Racing	Forged Aluminium Billet - Radial Mount - Ø36mm Bores	aluminium alloy	Aluminum	2.4		11.811	0.629921	1.41732		1.58		
AP Racing	4 Piston - Cast Body - 30mm Wide Disc - CP5090	cast aluminium alloy	Aluminum	5.1		11.0236	1.1811	1.5	1.75	1.77	2.4052819	4
Wilwood	Billet Dynalite Billet Dynalite Single	Aluminum	Stainless	1.8	115	13	0.38			2.4		2
Wilwood	Dynalite Single IIIA	Aluminum	Stainless	2	89	13	0.25			2.4		2
Wilwood	Dynapro Single	Aluminum	Stainless	2.3	107	13	0.19			2.4		2
Wilwood	Narrow Dynapro Lug Mount	Aluminum	Stainless	4.1	192	12.7	0.25			1.98		4
Wilwood	Forged Narrow Dynalite	Aluminum	Stainless	4.1	140	12.7	0.25			1.98		4
Wilwood	Narrow Dynapro Lug Mount	Aluminum	Stainless	4.1	192	12.72	0.25			1.98		4
Wilwood	Dynapro Lug Mount	Aluminum	Stainless	3.8	160	13	0.25			3		4
Wilwood	Forged Dynalite	Aluminum	Stainless	3.4	135	13	0.25			3		4
Wilwood	Narrow Dynapro Lug Mount	Aluminum	Stainless	4.1	163	12.72	0.25			3		4
Wilwood	Billet Dynalite - Side Inlet	Aluminum	Stainless	3.7	168	13	0.25			4.8		4
Wilwood	Dynapro Lug Mount	Aluminum	Stainless	3.8	160	13	0.25			4.8		4
Wilwood	Forged Dynalite	Aluminum	Stainless	3.4	135	13	0.25			4.8		4
Wilwood	Narrow Dynapro Lug Mount	Aluminum	Stainless	4.1	178	12.72	0.25			4.8		4

Figure 1: Comparative Caliper Matrix [2] & [3]

After selecting a Wilwood caliper, a Wilwood brake pad was chosen that would fit the style and size of the caliper. *The Polymatrix Compound A* Wilwood brake pad was chosen for its overall high coefficient of friction. Based off of the company's performance chart (see below) between temperatures of 100° & 700° the average coefficient of friction was approximated as 0.6215 [2].

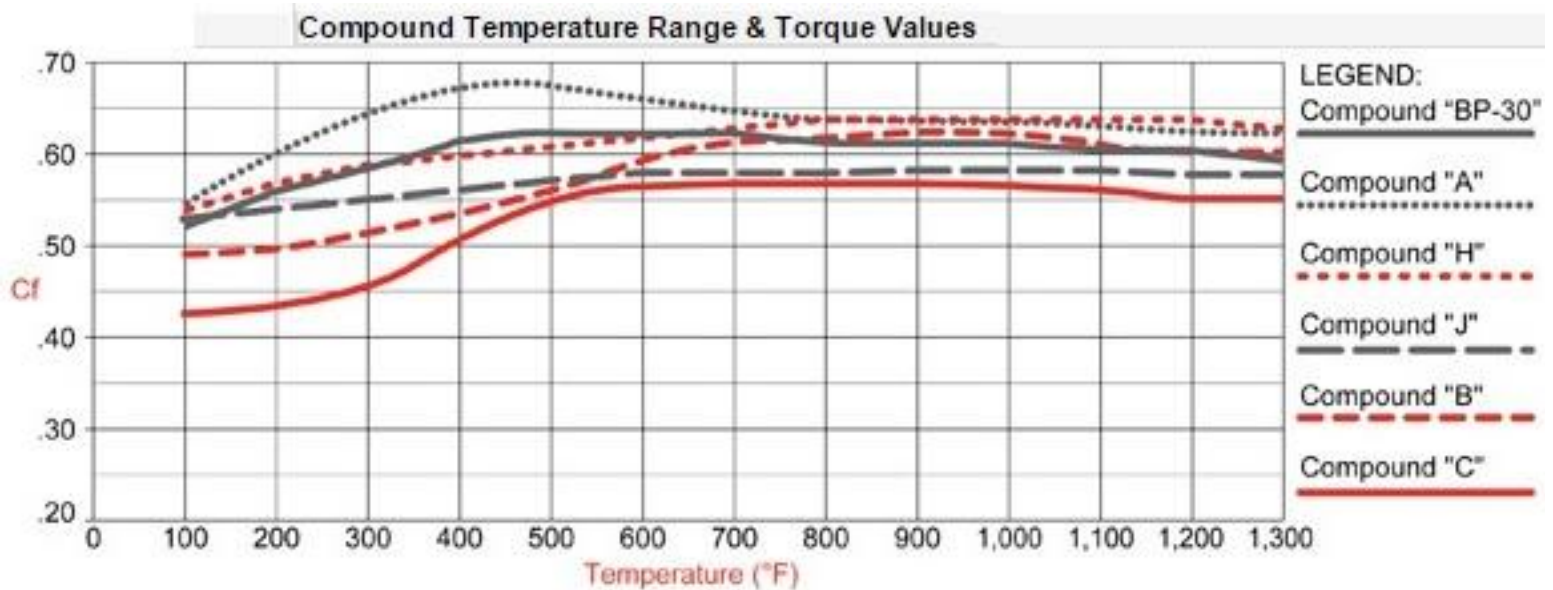


Figure 2: Wilwood Brake Pad Performance Graph [4]

The normal forces of the calipers then could then be solved:

$$F_{NFCP} = \frac{F_{FCP}}{\mu_{CP}} = 4218.4 \text{ lbs}$$

$$F_{NRCP} = \frac{F_{RCP}}{\mu_{CP}} = 746.8 \text{ lbs}$$

The pressure in both the front and rear braking lines could be solved:

$$P_F = \frac{F_{NFCP}}{\# \text{ of front caliper pistons} * A_{FCP}} = \frac{4218.4 \text{ lbs}}{4 \times (2.4 \text{ in})} = 439 \text{ psi}$$

$$P_R = \frac{F_{NRCP}}{\# \text{ of rear caliper pistons} * A_{RCP}} = \frac{746.8 \text{ lbs}}{4 \times (2.4 \text{ in})} = 78 \text{ psi}$$

At this step in the design it was necessary to select a set of master cylinders. The bore area of the master cylinder is the free variable in this selection. The bore area of the master cylinder, in addition to the pressure in each of the lines, dictate the total applied foot force needed to lock the tires during braking. After researching on the internet and drafting a master cylinder selection matrix, (see below), the Tilton 77-Series master cylinders with a 1" diameter bore were selected for both the front and rear systems [4], [5], [6] & [7]. This larger bore size was chosen because the selection rendered an applied foot force of just under 150 *lbs*, which was the target applied force. Have to minimal of an applied foot force would cause the braking system to be very sensitive, thus making braking and overall operation not optimal for the driver.

Master Cylinder Component Matrix	Part #	Price- each	Stroke in.	Bore in.	Length (end of rod to center of bearing)	MC Area
Tilton						
	77-Series	395-340	1.1	1	5.66	0.785
	77-Series	395-340	1.1	0.9375	5.66	0.690
	77-Series	395-340	1.1	0.8750	5.66	0.601
	77-Series	395-340	1.1	0.8125	5.66	0.518
	77-Series	395-340	1.1	0.7500	5.66	0.442
	77-Series	395-340	1.1	0.7000	5.66	0.385
	77-Series	395-340	1.1	0.6250	5.66	0.307
Brembo						
	BRE-XA3G144	827.45	.906	.669		0.000
	BRE-XA3G145	827.45	.907	.75		0.352
	BRE-XA2L2A8	827.45	.908	.886		0.442
	BRE-XA2L2A9	827.45	.909	.789		0.617
AP Racing						
	CP7855-88PRTE	291.54?	1.1	.5	6.13	0.000
	CP7855-89PRTE	291.54?	1.1	.59	6.13	0.196
	CP7855-90PRTE	291.54?	1.1	.625	6.13	0.273
	CP7855-905PRTE	291.54?	1.1	.66	6.13	0.307
	CP7855-91PRTE	291.54?	1.1	.7	6.13	0.342
	CP7855-92PRTE	291.54?	1.1	.75	6.13	0.385
	CP7855-93PRTE	291.54?	1.1	0.8125	6.13	0.442
	CP7855-94PRTE	291.54?	1.1	0.8750	6.13	0.518
	CP7855-95PRTE	291.54?	1.1	0.9375	6.13	0.601
	CP7855-96PRTE	291.54?	1.1	1.0000	6.13	0.690
Alcon USA						
	MAR52xxHM161MBB	865.19		1	6.34	0.785
	Charted DWG	865.19		.9375	6.34	0.690
	Charted DWG	865.19		.875	6.34	0.601
	Charted DWG	669.97		.812	6.34	0.518
	Charted DWG	669.97		.85	6.34	0.567
	Charted DWG	669.97		.7	6.34	0.385
	Charted DWG	669.97		.625	6.34	0.307

Figure 3: Master Cylinder Selection Matrix [5], [6], [7] & [8]

In order to calculate the applied foot force, the last free variable that needed to be selected was the mechanical advantage of the pedal, from Equ.14,  $b/a$ . This factor is determined upon how the master cylinders are mounted to the pedal. This leads to the pedal box geometry design. The main constraints of this design were prescribed from the team's lead frame engineer, who requested that the surface area space of the pedal box be reduced as much as possible. Due to the allotted width and for the comfort of the driver, the frame engineer requested that the width of the pedal box be no more than 10 *in*. Further details of the pedal box design will be discussed later in the text. Based on the frame engineer's requests it was decided that the most optimal way to mount the master cylinders would be at an angle. A kinematic diagram was drafted of the set up (see below) where the lengths  $a$  and  $b$  are denoted as well as  $c$ , the distance between the pivot point of the pedal and the master cylinders,  $d_f$ , the distance the pedal or foot of driver travels and  $l$  and  $l_0$ , the original master cylinder stroke and the displaced master cylinder stroke.

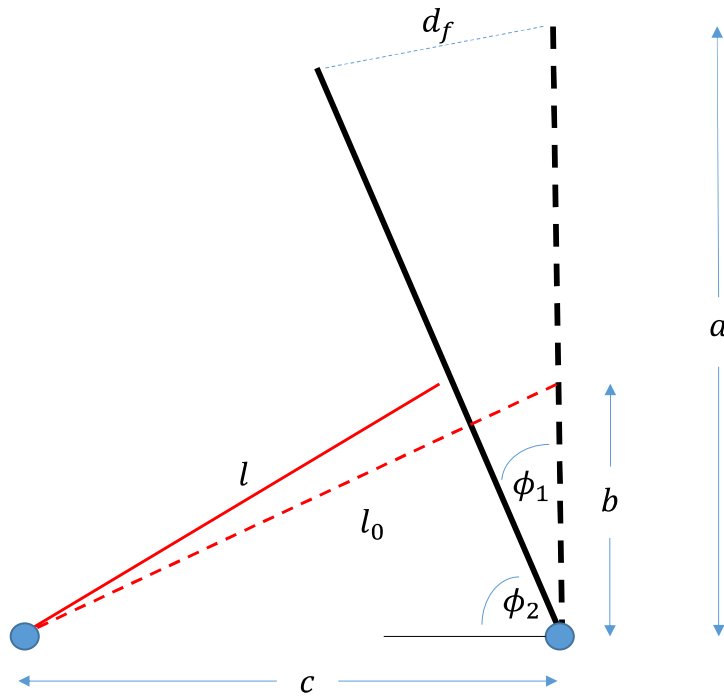


Figure 4: Kinematic Diagram of Brake Pedal and Master Cylinders

Applying the Law of Cosines and Pythagorean Theorem to the above diagram the following equations can be derived:

$$d_f = a^2 + a^2 - 2a^2 \cos \phi_1 \quad (15)$$

$$l_0^2 = c^2 + b^2 \quad (16)$$

$$l^2 = c^2 + b^2 - 2cb \cos(\phi_2) \quad (17)$$

As recommended by Dr. Richard Gross, the value of the displacement of each caliper piston should approximately be 0.03125 *in*. Therefore, the only remaining free variables in the pedal box geometry are lengths *a, b and c*.

After running a parametric sweep on these variables to find the optimal combination of the three, the following lengths were selected:

$$a = 8.5 \text{ in}$$

$$b = 3.54 \text{ in}$$

$$c = 6.72 \text{ in}$$

Substituting these values into Equ.15, 16 & 17 rendered:

$$d_f = 0.3 \text{ in}$$

$$l_0 = 7.77 \text{ in}$$

$$l = 7.39 \text{ in}$$

$$\phi_1 = 0.11 \text{ radians or } 6.3 \text{ degrees}$$

$$\phi_2 = 1.46 \text{ radians or } 83.65 \text{ degrees}$$

Based upon the selected pedal box geometry and master cylinder setup the mechanical advantage and applied foot force can be solved for:

$$\frac{b}{a} = 0.3412$$

$$F_{Foot} = 139 \text{ lbs}$$

Based on the above results and data, the torque generated from each front and rear caliper are:

$$T_F = 4538 \text{ in} - \text{lb}$$

$$T_R = 806 \text{ in} - \text{lb}$$

## Pedal Box Design

### Design Criteria

The constraints of the pedal box design are as follows:

- The pedal box needed to be designed around the geometry constraints set in the results from the braking system calculations. That is the master cylinder mount height and the distance between the master cylinder pivot and the brake pedal are fixed.
- The team's frame engineer also asked that the total width be no more than 10 *in*.
- The length of both the gas and the brake pedal need to be approximately the length of a human foot (~ 9.5 *in*).
- The competition rules designate that the brake pedal must be able to withstand a force of 2000 *Newtons*.
  - o Due to this constraint the material selection of the brake pedal and the overall pedal box must be strong, stiff and have high toughness.

The objective of the pedal box design was to satisfy all of the constraints mentioned above and optimize for minimal weight.

A recommendation from the team members was that the pedal box be easily adjustable in the lateral direction, that is be comfortable for a taller driver as well as a smaller driver. Thus, the first stage in the design was this adjustment. The first draft was to have a tension line with retractable shafts in the base plate of the pedal box. That is the driver would push a button or grip a handle causing tension in the line that the shafts, which connected the pedal box to the floor of the vehicle, would retract and the pedal box could be slide into a new set of holes making it closer or further away from the driver. It was decided that this design would be too time consuming and a more robust design was desired. The second draft was similar to the first, however, omitted the tension lines and quick release pins would be used in place and the pedal box could be slide and adjusted just as before. However, this design was dismissed as an even more rigid design would be favorable. The third and chosen design, was simply adjustable pedal heads. Where the pedal would have extended mounts to accommodate additional holes and quick release pins would be used for easy adjustment.

The rest of the design encompassed mounting both pedals, mounting the master cylinders and mounting the throttle sensor and emergency stop switch to the pedal box.

The pedal box was designed with a single base plate; that is, all of the mounts would be fixed to this single plate. The brake pedal and master cylinder mounts were designed to be symmetrical for ease of manufacturing and were short, approximately 1.5 *in* in height. The brake pedal would be supported by this mount as well as the support of the master cylinders; accordingly the moment generated from the applied foot force would be supported. The gas pedal however, would only be supported by the mounts and therefore were designed to be taller, approximately 4.5 *in* in height. The throttle sensor mount would be subjected to very little force, but the height of it needed to be able to accommodate



for the sensor's ability to rotate and move in a full stroke without being restricted by the rotation of the gas pedal. The throttle sensor mount would also provide a support for a tension spring, which would be used to keep the gas pedal at a neutral state and provide some resistance during rotation. The emergency stop mount needed to be positioned where, under the circumstance that there was a leak in the brake line and there was no longer substantial pressure in the line, the pedal would strike the switch at approximately 90% displacement of the master cylinders.

## Material Selection

The material selection process of the pedal box was based around the initial constraints of the design mentioned prior, but more specifically, the component of the pedal box that would be subjected to the most stress, the brake pedal. Based on the type of loading the pedal box would be subjected to it was modeled as a beam in bending and was decided that the desired material needed to have the following mechanical properties:

- High Strength and be stiff
- Good toughness
- Machinability

From these constraints it was decided that metals, composites and possibly some polymers would make optimal choices. From these constraints, two objective functions were derived by substituting the equations for bending stress of a beam and the stiffness of a beam into the equation for the mass of the brake pedal. From these objective equations two material indices,  $M_1$  &  $M_2$  could be derived; with  $M_1$  corresponding to the Elastic Modulus and  $M_2$  corresponding to Yield Strength:

$$M_1 = \frac{\rho}{\sigma_y^{1/2}} \quad (18)$$

$$M_2 = \frac{\rho}{E^{1/2}} \quad (19)$$

Using the material selection software, *Granta CES EduPack 2014* [9], a graphical approach to material selection was chosen. The first material index was plotted on the y-axis and the second material index as plotted on the x-axis. Optimal material selection resulted in the lowest possible values for both these indices, thus the area of interest on the graph is in the lower-left corner of the graph.

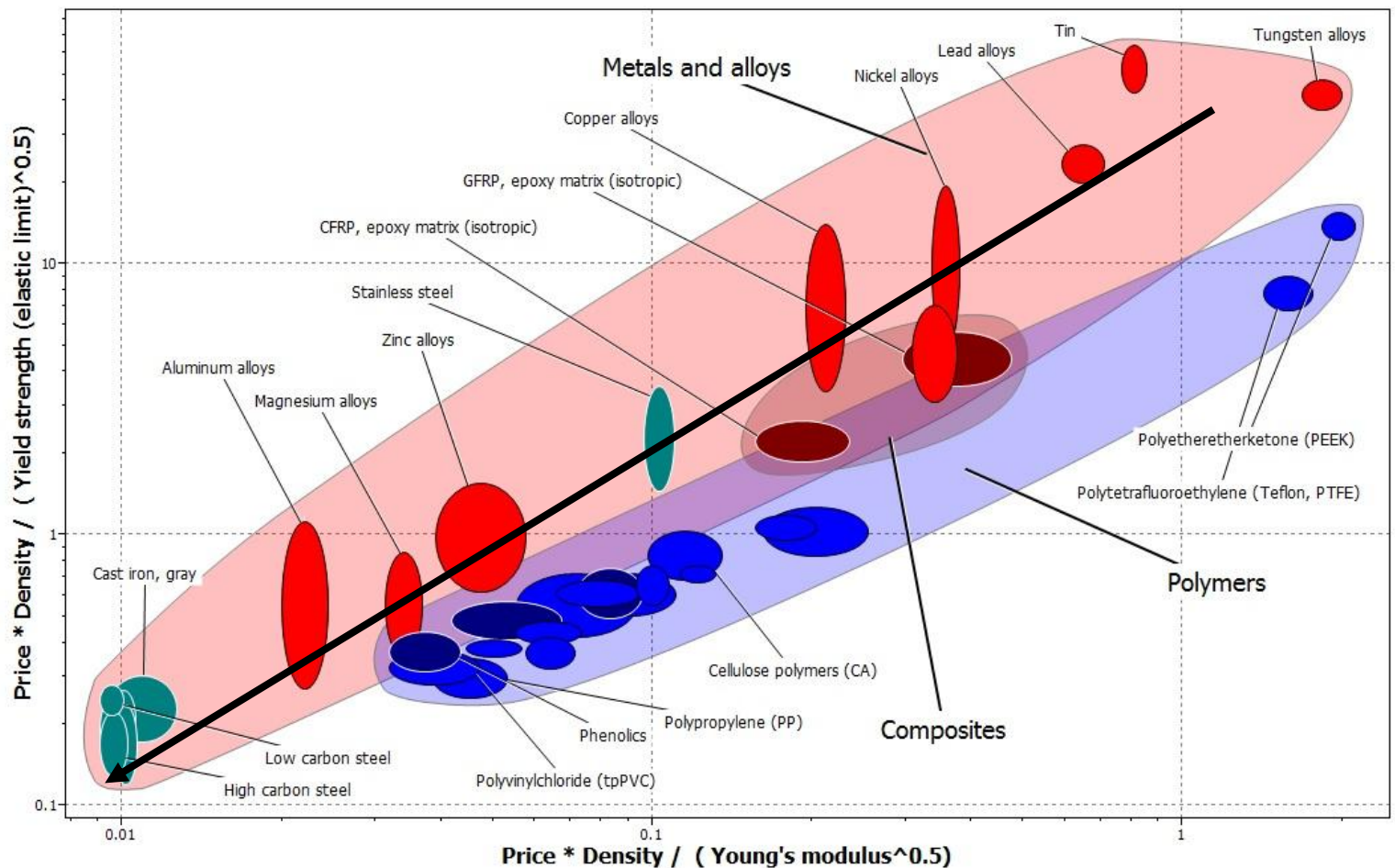


Figure 5: Pedal Box Material Selection Chart [9]

Analysis of this chart rendered the optimal choice was, based on the material properties that were most dependent on functionality and taking cost into consideration, carbon steel, cast iron or an aluminum alloy. Since out of the three, the aluminum alloys are the lightest, this material group was selected for the design. 6061 T6 Aluminum was the final selection as the material choice of the pedal box and the material properties for 6061 T6 Aluminum will be used from this point on in design of the components of the pedal box.

## Initial Design

The initial design of the pedal box was drawn using the three-dimensional graphics software SolidWorks [10], seen in Figure 5 below, weighed approximately 7.25 *lbs*.

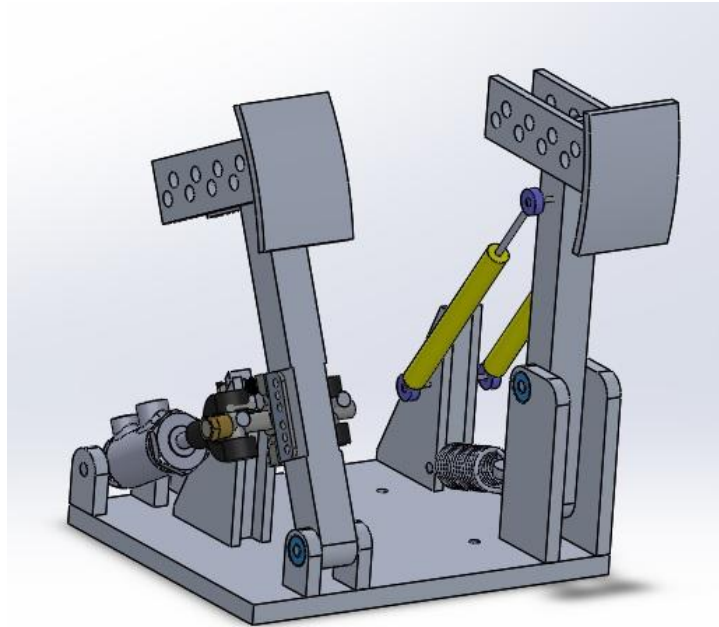


Figure 6: First Draft of Pedal Box [10]

## Optimization & Weight Reduction

Optimization of the pedal box was conducted using the finite element analysis simulations in SolidWorks [10]. Each of the components was subjected to a specific force, based on application, and a minimum factor of safety of 1.2 was used as a design criteria. The following is a breakdown of each component and the force that it was subjected to during simulation:

Brake Pedal: 2000 *N*

Gas Pedal: 1000 *N*

Pedal Heads: 2000 *N*

Base Plate: 2000 *N* (Distributed)

Brake Pedal Mount: 350 *N*

Master Cylinder Mounts: 350 *N*

Gas Pedal Mount: 500 *N*

Throttle Sensor Mount: 500 *N*

Emergency Stop Switch Mount & Shield: 500 *N*

After running FEA on the individual components, sections of each component, where the least amount of stress occurred, were removed. This process resulted in many holes and skeletal cuts in most of the components. FEA was conducted again on these optimized components to verify each still passed the force criteria. This process was repeated until each component was reduced to its minimal mass and still passed the FEA simulations. The original components are shown below as well as the optimized final components:

#### Brake Pedal

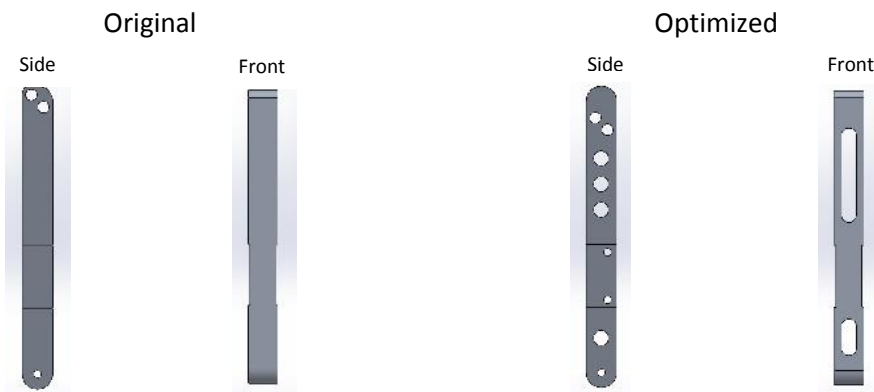


Figure 7: Initial & Optimized Brake Pedal [10]

#### Gas Pedal

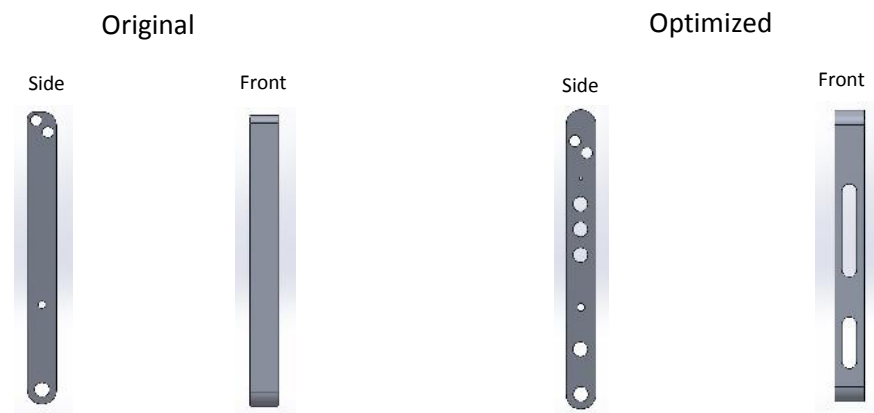


Figure 8: Initial & Optimized Gas Pedal [10]

### Pedal Heads

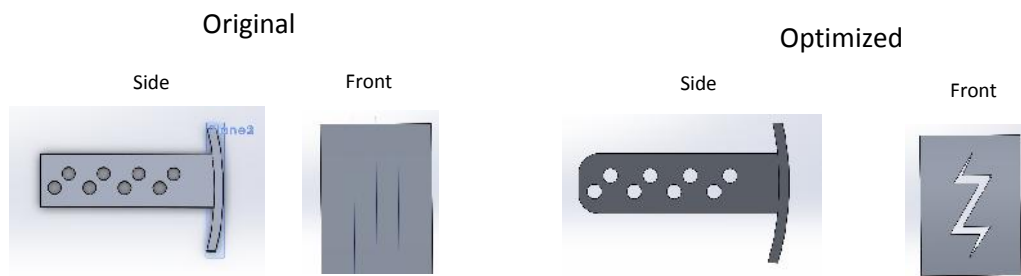


Figure 9: Initial & Optimized Pedal Head [10]

### Base Plate



Figure 10: Initial & Optimized Base Plate [10]

### Brake Pedal Mount

Original & Optimized

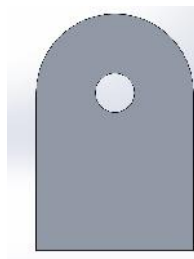


Figure 11: Initial & Optimized Brake Pedal Mount [10]

### Master Cylinder Mount

Original & Optimized

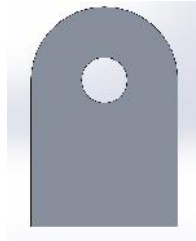
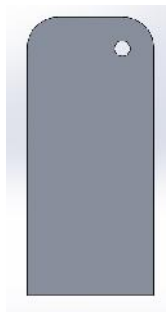


Figure 12: Initial & Optimized MC Mount [10]

### Gas Pedal Mount

Original



Optimized

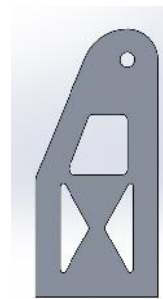
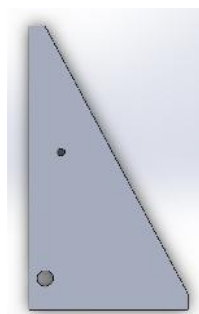


Figure 13: Initial & Optimized Gas Pedal Mount [10]

### Throttle Sensor Mount

Original



Optimized

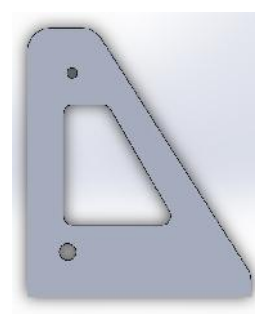


Figure 14: Initial & Optimized Throttle Sensor Mount [10]

### Emergency Stop Mount & Shield

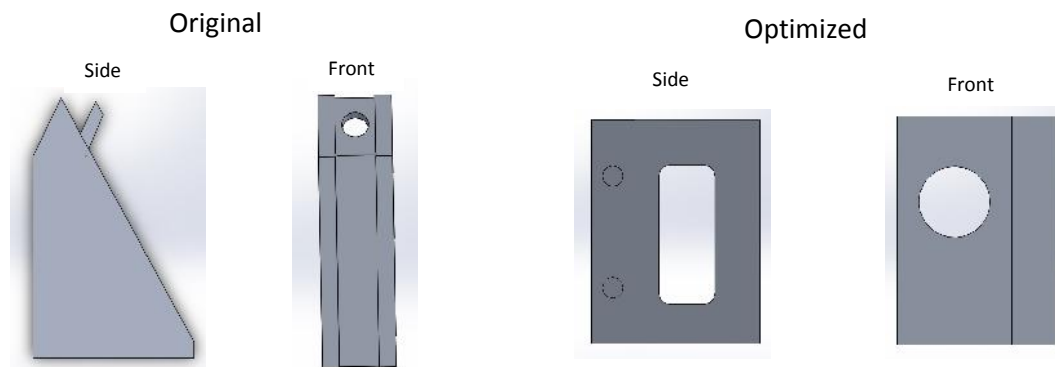


Figure 15: Initial & Optimized Emergency Stop Mount & Shield [10]

The only major design change between the first and the final draft was the emergency stop mount and shield. Originally the mount was directly behind the brake pedal. This design was changed after realizing the potential of destroying the switch if the pedal exceeded its designed rotation. In the final design the emergency stop switch was changed from a push button to a two stage switch and was mounted perpendicular to the brake pedal. With this design if the brake pedal exceeded its rotation the emergency stop switch would be flipped and not crushed from brake failure.

## Finite Element Analysis

The following figures show the FEA stress results on all of the optimized pedal box components [10]:

### Brake Pedal

Minimum Factor of Safety: 1.4

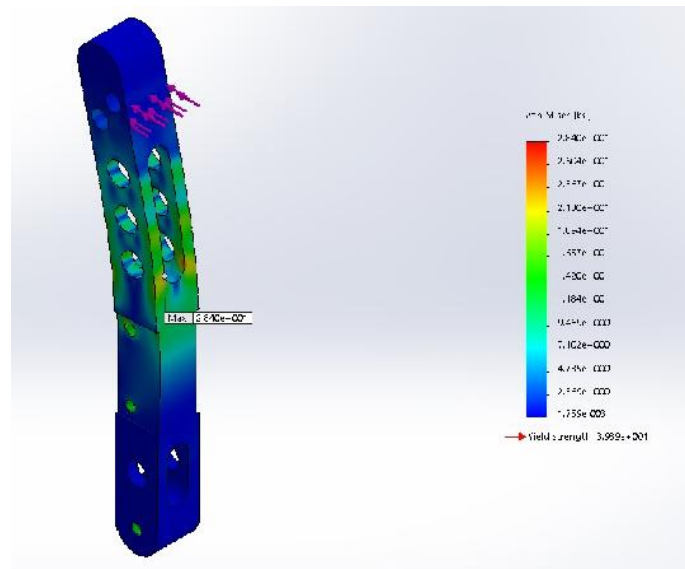


Figure 16: FEA Stress Analysis of Brake Pedal [10]

### Gas Pedal

Minimal Factor of Safety: 1.6

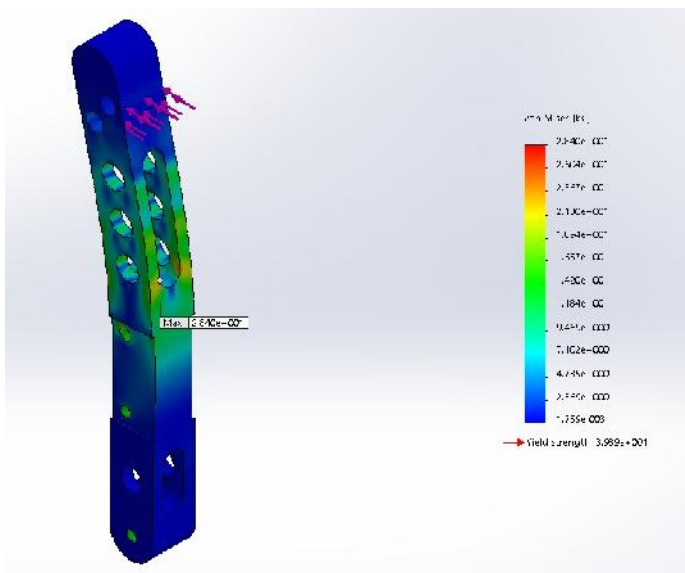


Figure 17: FEA Stress Analysis of Gas Pedal [10]



### Pedal Heads

Minimum Factor of Safety: 1.8

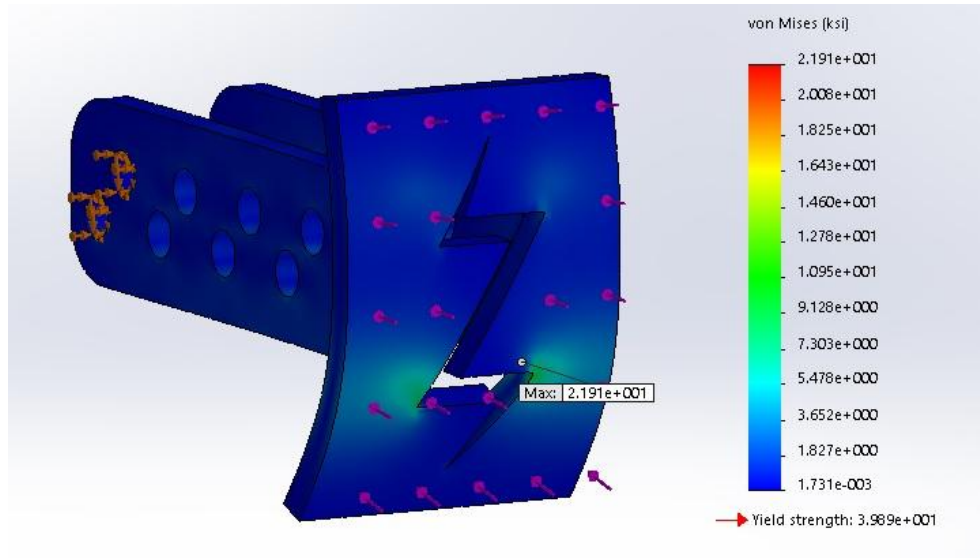


Figure 18: FEA Stress Analysis of Pedal Head [10]

### Base Plate

Minimum Factor of Safety: 8

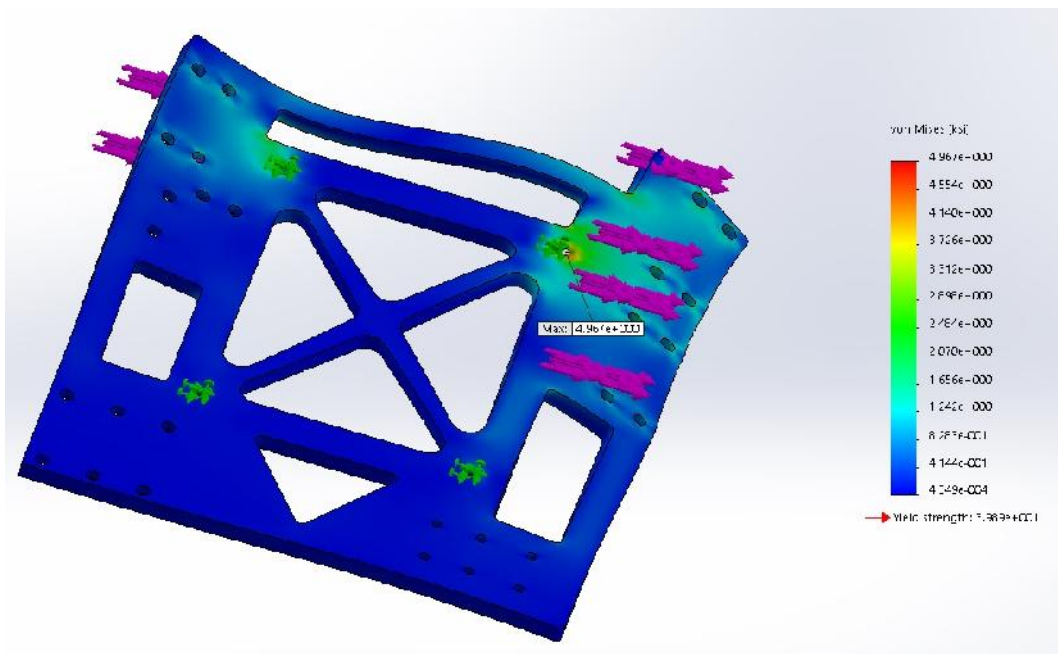


Figure 19: FEA Stress Analysis of Base Plate [10]

### Brake Pedal Mount

Minimum Factor of Safety: 21

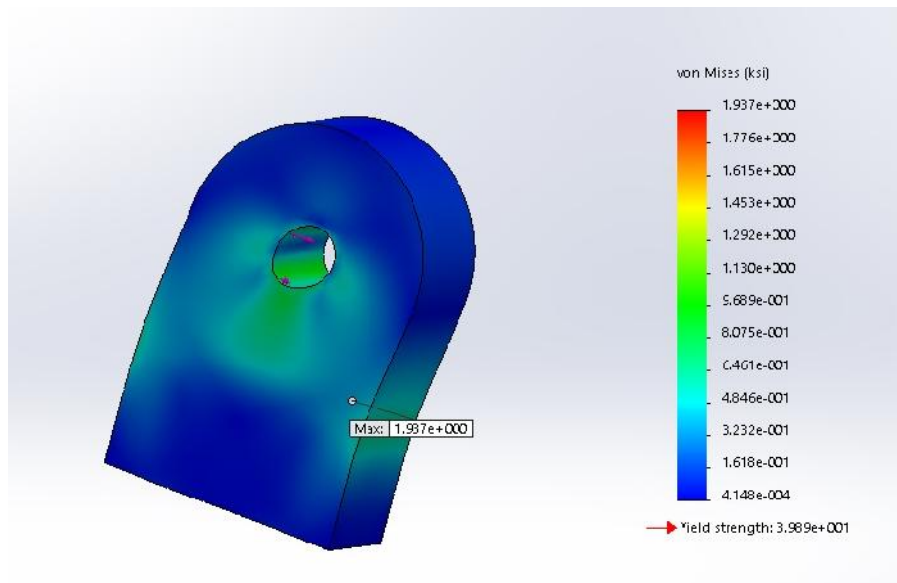


Figure 20: FEA Stress Analysis of Brake Pedal Mount [10]

### Master Cylinder Mount

Minimum Factor of Safety: 19

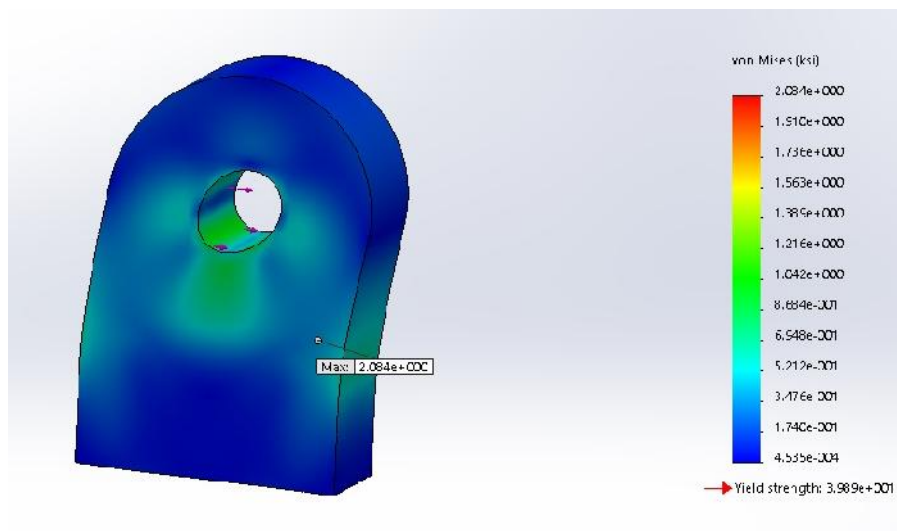


Figure 21: FEA Stress Analysis of MC Mount [10]

### Gas Pedal Mount

Minimum Factor of Safety: 8.2

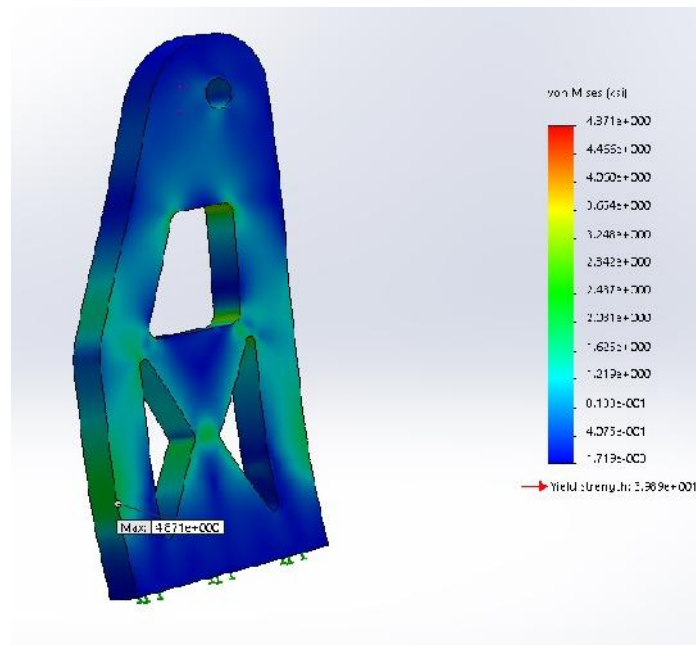


Figure 22: FEA Stress Analysis of Gas Pedal Mount [10]

### Throttle Sensor Mount

Minimum Factor of Safety: 8.6

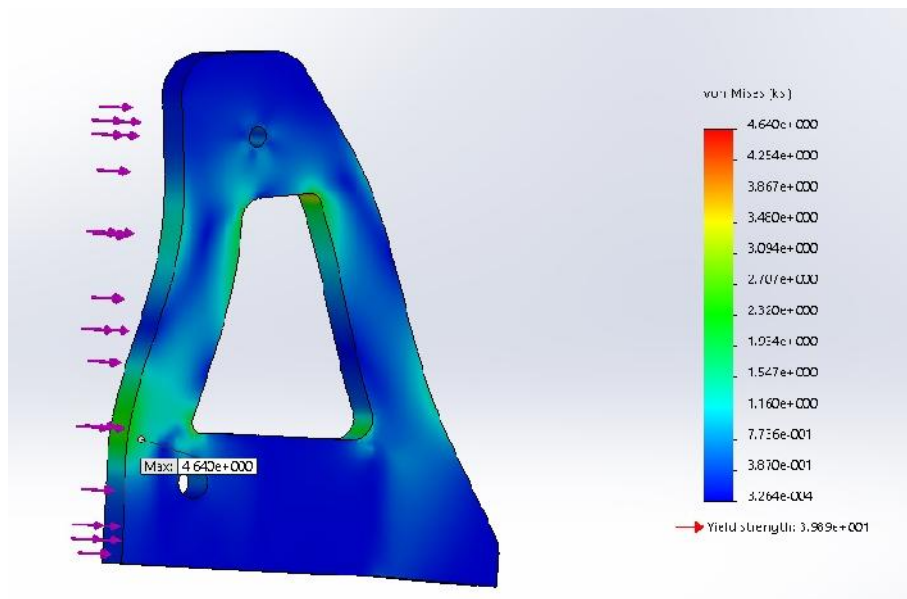


Figure 23: FEA Stress Analysis of Throttle Sensor Mount [10]

## Emergency Stop Mount & Shield

### Mount

Minimum Factor of Safety: 11

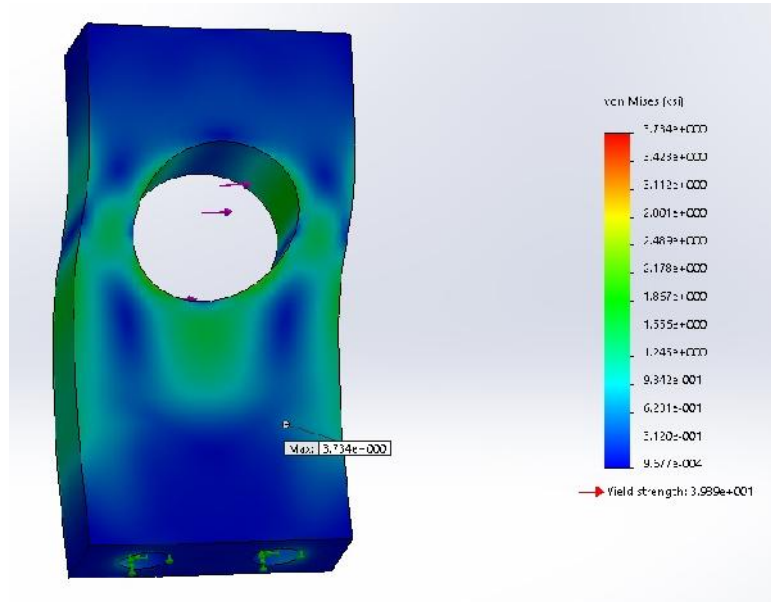


Figure 24: FEA Stress Analysis of Emergency Stop Mount [10]

### Shield

Minimum Factor of Safety: 9

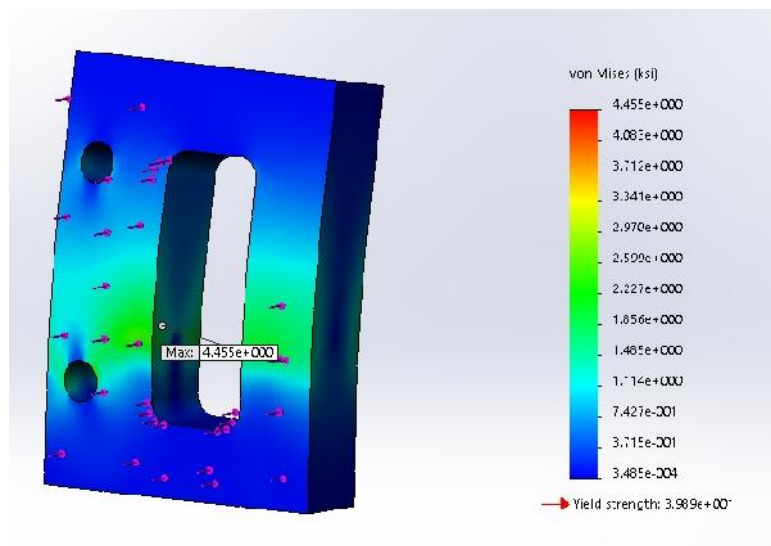


Figure 25: FEA Stress Analysis of Emergency Stop Shield [10]

## FEA Verification

In order to verify that FEA software was rendering appropriate results, the maximum stress on the most critical component, the brake pedal, was calculated by hand. The following are the results:

The area of max stress of the brake pedal was near the half inch hole just above where the master cylinders connect to the pedal, denoted by the red dashed line, see Figure 25.

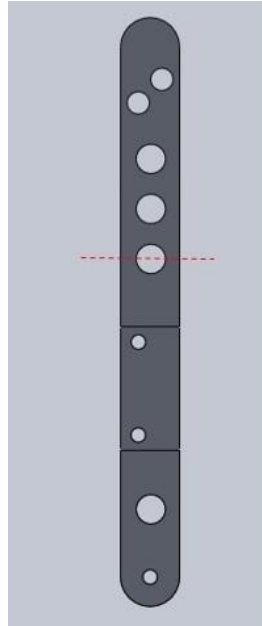


Figure 26: Max Area Section of Brake Pedal

The Brake Pedal was modeled as a beam in bending due to the applied foot force. The bending stress can be calculated from the following equation:

$$\sigma_B = \frac{K \times M \times y}{I} \quad (20)$$

Where  $K$ , is the stress concentration factor,  $M$  is the moment induced by the applied foot force,  $y$ , is the distance from the neutral axis of the cross-section to the point at which the stress is being calculated and  $I$  is the second moment of Inertia of the cross-section.

The second moment of inertia was calculated using the following cross section, see Figure 26, with  $b = 1 \text{ in}$  &  $t = 0.25 \text{ in}$ .

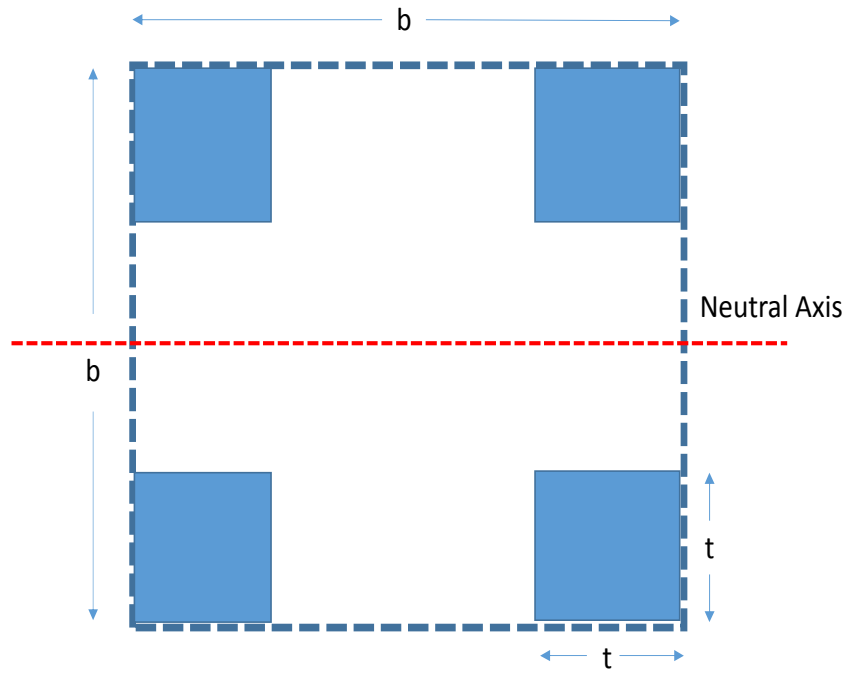


Figure 27: Max Stress Cross Section of Brake Pedal

From these dimensions the second moment of inertia was calculated:

$$I = 4 \left( \frac{1}{12} t^4 + t^2 \left( \frac{1}{2} (b - t) \right)^2 \right) \quad (21)$$

$$I = 3.646 \times 10^{-2} \text{ in}^4$$

The moment from the applied foot force, the distance from the neutral axis and stress concentration factor were calculated:

$$M = 1.395 \text{ kip} - \text{in}$$

$$y = 0.5 \text{ in}$$

$$k = 1.5$$

The hand calculated max bending stress resulted in verification of the results from the FEA simulations:

$$\sigma_{B_{max}} = 28.7 \text{ ksi}$$

## Final Design

The final optimized design of the pedal box weighed approximately 4.25 *lbs*, excluding the master cylinders and throttle sensors and can be seen below in Figure 27 & 28.

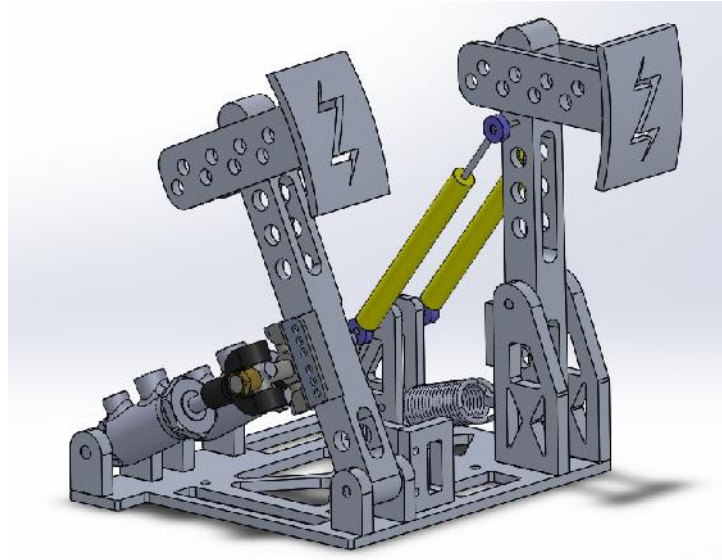


Figure 28: Final Design of Pedal Box [10]

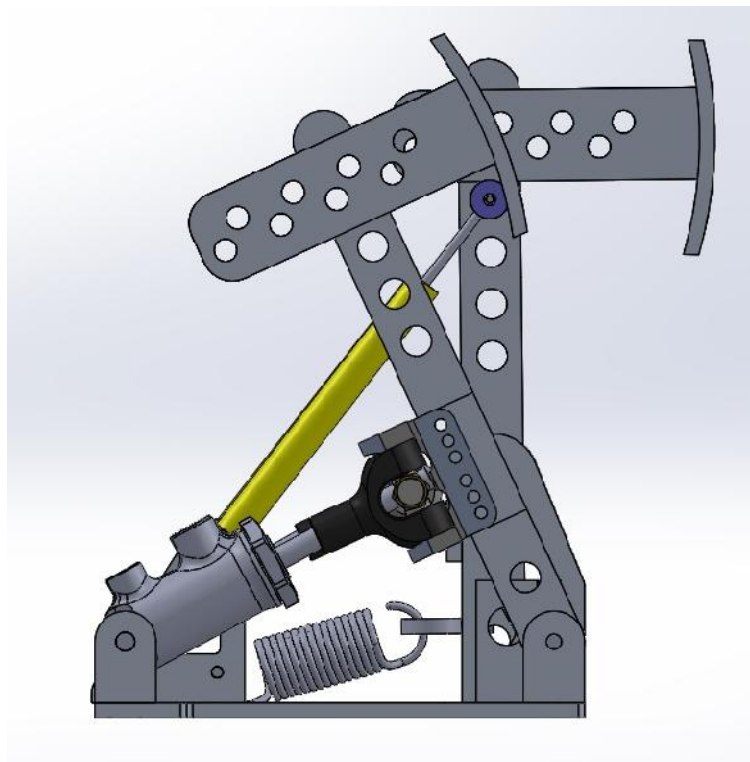


Figure 29: Final Design of Pedal Box (Side View) [10]

## Rotor Design

### Design Criteria

The rotors of the vehicle are the medium in which the kinetic energy from the motion of the vehicle is converted into thermal energy and then dissipated. From the braking system calculations earlier in the text the geometrical constraints of the rotors are:

- 8.25 *in* in diameter
- Thickness between 0.38 *in* & 0.25 *in*, (dependent on caliper selection)

Based on the application the other constraints that will be imposed on design are:

- Mechanical endurance at high temperatures
- Ability to dissipated or with stand high temperatures

### Material Selection

The following rotor calculations were done in metric units for ease of conversion and calculation.

The most crucial selection in the design of the rotors based on the above constraints was the material choice. Based on the application, a metal, ceramic or composite material was assumed to be optimal, however based on the team's budget a metal was the only realistic choice. In order to make a selection, the conservation of energy was employed only for the linear motion of the vehicle at its maximum velocity to compare the temperature change of the rotors between materials. In this calculation it was assumed that each rotor would share evenly in the overall heat transfer. This is not the actual case, however for the purpose of comparison this method was utilized.

$$\text{Kinetic Engery} = \text{Thermal Energy}$$

$$\frac{1}{2}m_{vehicle}v_{vehicle}^2 = (\# \text{ of rotors})m_{rotor}C_{p_{rotor}}\Delta T \quad (22)$$

Where  $m$  denotes mass,  $v$  denotes linear velocity,  $C_p$  is heat capacity and  $\Delta T$  denotes temperature change. The following know values were substituted into the equation:



$$m_{vehicle} = 283.5 \text{ kg}$$

$$v_{vehicle} = 26.8 \frac{m}{s}$$

$$\# \text{ of rotors} = 4$$

Thus the remaining variables,  $m_{rotor}$  &  $C_{p_{rotor}}$ , were strictly dependent on material choice. Evaluating the conservation of energy equation established and solving for  $\Delta T$ , the equation becomes:

$$\Delta T = \frac{25452.63 \text{ J}}{m_{rotor} C_{p_{rotor}}} \quad (23)$$

Using this model, several different metals were input and the results were graphed [11]. Materials with the lowest temperature change were favored as these materials would increase in temperature the least during braking.

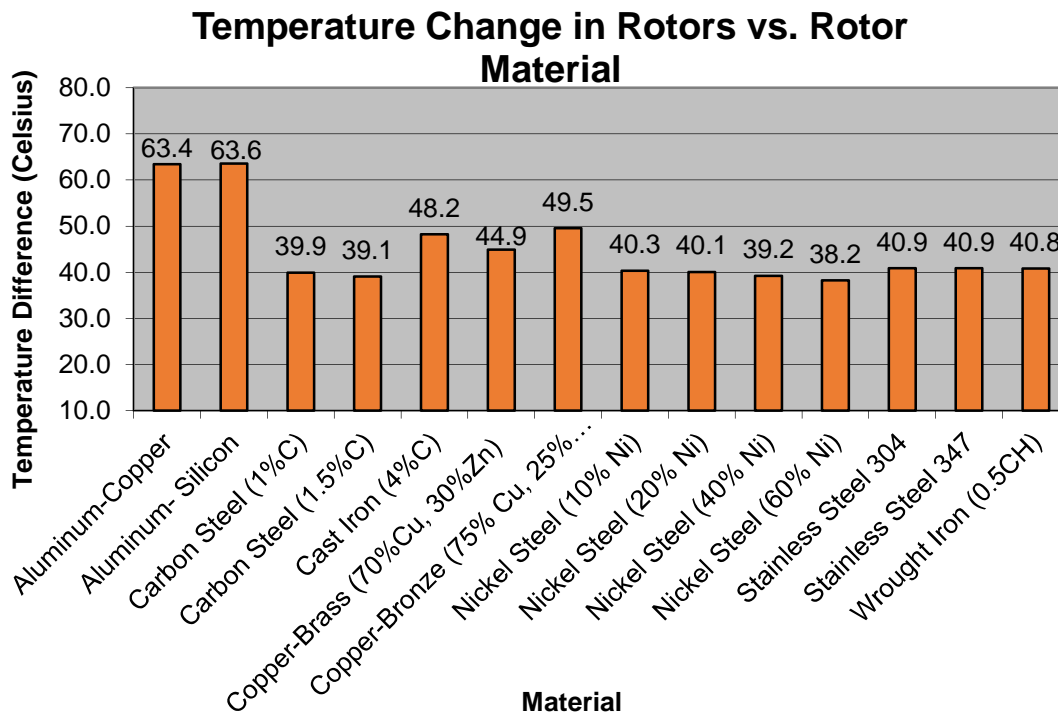


Figure 30: Rotor Temperature Change vs. Rotor Material

Based on this graph, the optimal choice are:

- High Carbon Steel
- Nickel Steel
- Stainless Steel

The cheapest of these was selected for the rotor material, thus the high carbon steel was chosen. However, the final material choice was A36 Carbon Steel, a low carbon steel. This was due to a donation, by a local steel vender; once again the team's budget ultimately factored into the final design.

### COMSOL Simulations: Rotor Sensitivity Analysis

In addition to the conservation of energy equation, the multi-physics software, COMSOL [12], was used to run heat transfer simulations modeling a brake pad and a rotor during emergency braking. The example used was a tutorial example from the software and therefore the rotor geometry and vehicle simulation were not exact to the electric vehicle's. However, the COMSOL simulations were explicitly used to run parametric sweeps on the different material variables of the rotor. The tutorial simulation was for a 1800 Kg vehicle traveling at 25 m/s which suddenly decelerates at  $10 \text{ m/s}^2$  for two seconds and then coasts at 5 m/s for eight seconds. The results would determine which variables had the greater impact on the rotor or the rotor's sensitivity to these variables. From these results a better understanding of the heat transfer could be observed and therefore translated into an optimal rotor design.

The variables of the rotor that the parametric sweeps were conducted for were the following:

- Thermal Conductivity
- Heat Capacity
- Density\*Heat Capacity

The maximum rotor temperature of each scenario was used among the parametric sweeps for comparative and rotor sensitivity analysis.

The results from the rotor thermal density parametric sweep rendered a decreasing power function, see below; as the thermal conductivity of the rotor increased the maximum temperature of the rotor decreased. However, it can be observed from the graph that between the thermal conductivity values of  $50 \frac{\text{J}}{\text{m K}}$  &  $200 \frac{\text{J}}{\text{m K}}$  the rate at which the maximum temperature decreases is reduced severally and the graph appears to plateau. The overall temperature range of this parametric sweep was an approximate maximum of  $400^\circ\text{C}$  and a minimum of  $125^\circ\text{C}$ .

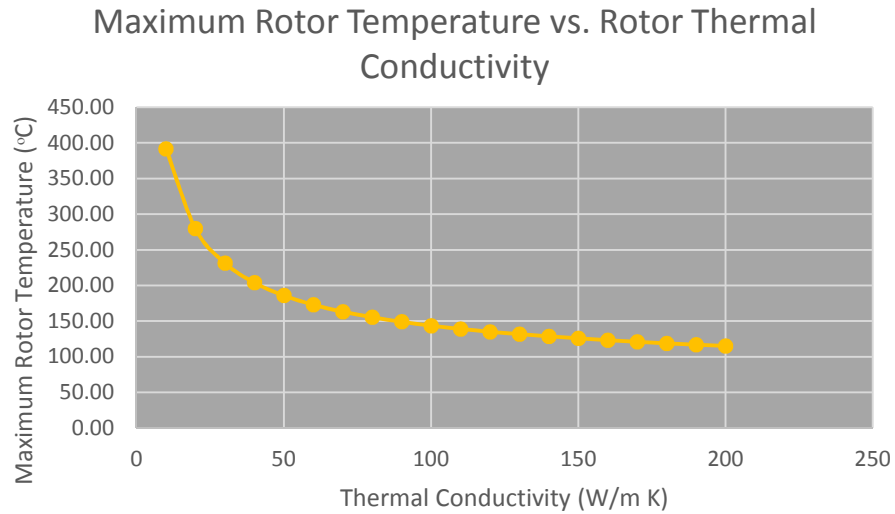


Figure 31: COMSOL Parametric Sweep of Rotor Thermal Conductivity [12]

The results from the rotor heat capacity parametric sweep rendered another decreasing power curve, however this one behaving more linearly; as the heat capacity of the rotor increased the maximum rotor temperature decreased. The overall temperature range of this parametric sweep was an approximate maximum of 183 °C and a minimum of 120°C .

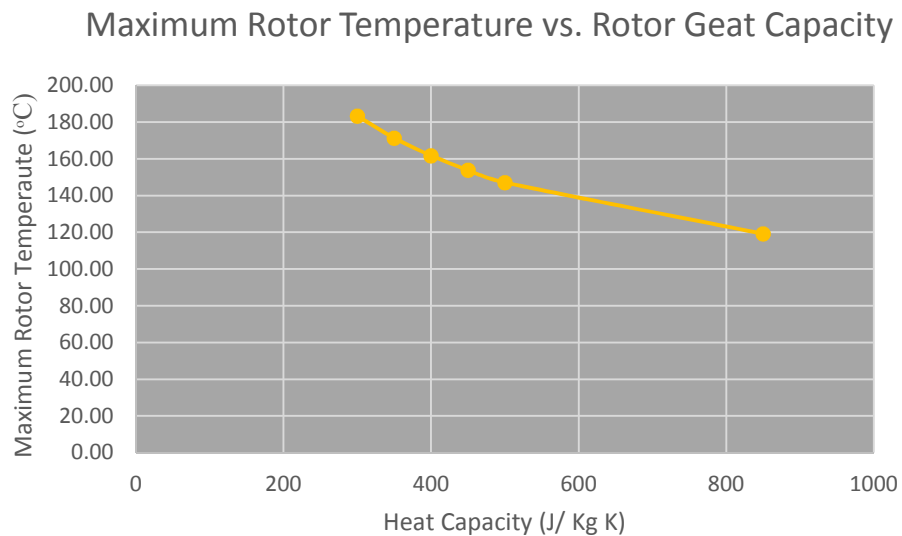


Figure 32: COMSOL Parametric Sweep of Rotor Heat Capacity [12]

The results from the rotor density\*heat capacity parametric sweep rendered an approximate decreasing line, see below; as the density\*capacity increased the maximum rotor temperature decreased. The overall temperature range of this parametric sweep was an approximate maximum of 183 °C and a minimum of 150 °C .

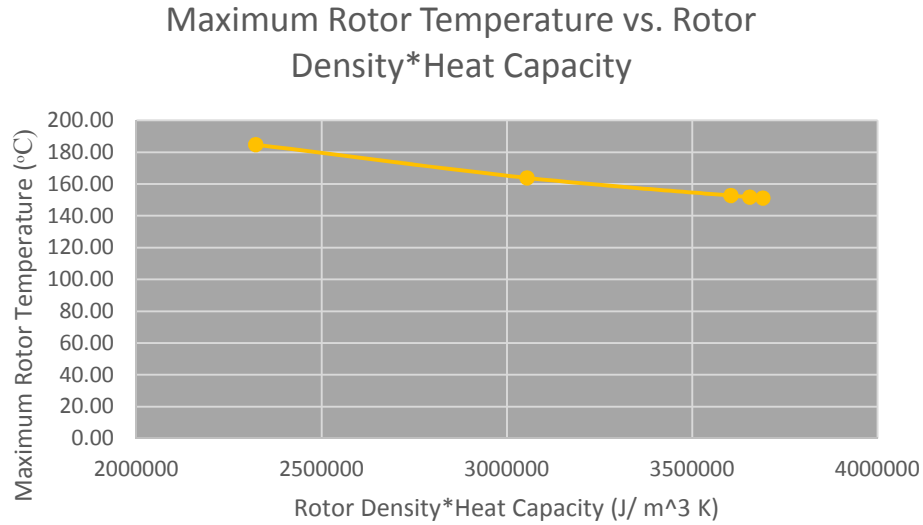


Figure 33: COMSOL Parametric Sweep of Rotor Density\*Heat Capacity [12]

After analyzing and comparing these parametric sweeps, it was concluded that the most sensitive parameter was the rotor's thermal conductivity, followed by the rotor's heat capacity and then the rotor's density\*heat capacity, because the temperature ranges decrease in the same order. Analyzing where the material selection of A36 carbon steel, from the previous section, falls in these graphs, it is concluded that it is in fact an optimal choice based on the following values from the graphs:

$$k_{A36} = 50 \frac{W}{m K}, Rotor Temperature_{max} = 173^{\circ}C$$

$$C_{pA36} = 480 \frac{J}{kg K}, Rotor Temperature_{max} = 140^{\circ}C$$

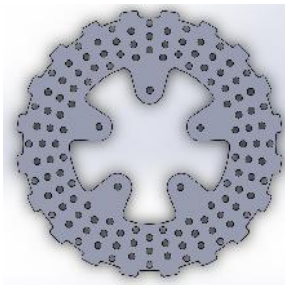
$$\rho * C_{pA36} = 3744000 \frac{J}{m^3 K}, Rotor Temperature_{max} = 148^{\circ}C$$

## Rotor Geometry

In most rotor designs there are cuts or holes in the surface where the rotor comes in contact with the brake pad. The main intention of these cuts is to reduce brake fade during braking. As the temperature of the brake pad begins reaching its design limit a faint layer of gas from the deterioration of the brake pad forms between the pad and the rotor. The effect of this gas layer is a severe decrease in the brake pad coefficient of friction, which translates as an ineffective brake to the driver. This effect gives the driver a spongy feel to the brake pedal; as the applied foot force of the driver is used to applying barley brakes, thus hindering the deceleration of the vehicle.

Having cuts or holes in the rotor creates a vent or fan like effect, which flushes out this gas layer that forms. Two side effects of having these cuts in the rotor are reduced mass and an increase in the cooling. Thus, it was decided that the rotor would have a cut geometry. The initial method of attachment of the rotor to the hub was a bolted design, as seen by the following initial rotor designs A through G:

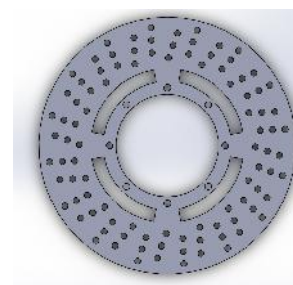
Design A



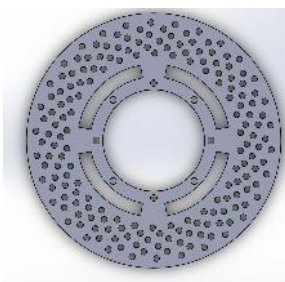
Design B



Design C



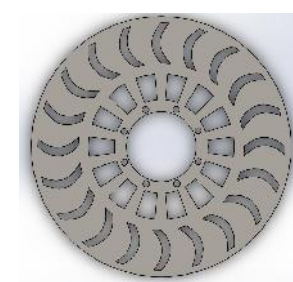
Design D



Design E



Design F



Design G

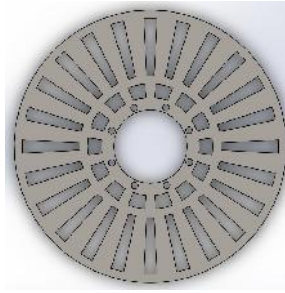


Figure 34: Initial Rotor Designs [10]

In selecting a rotor geometry there were two concepts that were taken into account:

- 1) The cut pattern needed to wear the brake pad relatively uniformly, as a deformed pad would cause the contact surface between the pad and the rotor to decrease, thus inhibiting the effectiveness of the braking system.
- 2) Decreasing the mass of the rotor needed to be optimal; that is, there is a certain amount of material that can be removed where the rotor has enough mass to endure the heat transfer and there are enough cuts for venting.

After consulting with the team's hub engineer, the rotor attachment design was changed from a "bolt on" design to a "button pin" design. This design allowed there to be less rotor material for mounting, as well as less hub material for mounting. In addition, the stress endured from the torque of the braking system would be spread amongst the rotor, hub and button pin as opposed to the "bolt on design", where the bolt would be the only component enduring stress. The final rotor design is shown below. In order to accommodate the button pins an inner lip needed to cut into the inner radius of the rotor just past the button pin holes.



Figure 35: Final Rotor Design [10]

## COMSOL Simulation: Actual Trial

Once the final rotor geometry and material were selected, a COMSOL simulation was run mimicking an emergency stop of the electric vehicle. The above rotor geometry from Figure 34 was input along with the necessary material properties for A36 carbon steel. The input parameters of the simulation were changed for the electric vehicle: the mass was changed to 283.5 Kg and the brake pad coefficient of friction was changed to 0.615. The rest of the parameters from the parametric sweep were kept, that is the vehicle started at 25 m/s, decelerated at  $10 \text{ m/s}^2$  for 2 seconds and then coasted at 5 m/s for eight seconds with the initial temperature of the entire system at  $27^\circ\text{C}$ .

As noted from prior section, a weight transfer from the rear to the front of the vehicle occurs during braking. Because of this weight transfer the front rotors will be subjected to more weight and therefore more heat transfer. Thus, one of the front rotor will be targeted during this simulation.

The following is a graphical representation of the maximum temperature versus time of the rotor and a thermal rendering of the rotor at its peak temperature throughout the simulation.

The maximum temperature during the entire simulation was found to be  $62.7^\circ\text{C}$

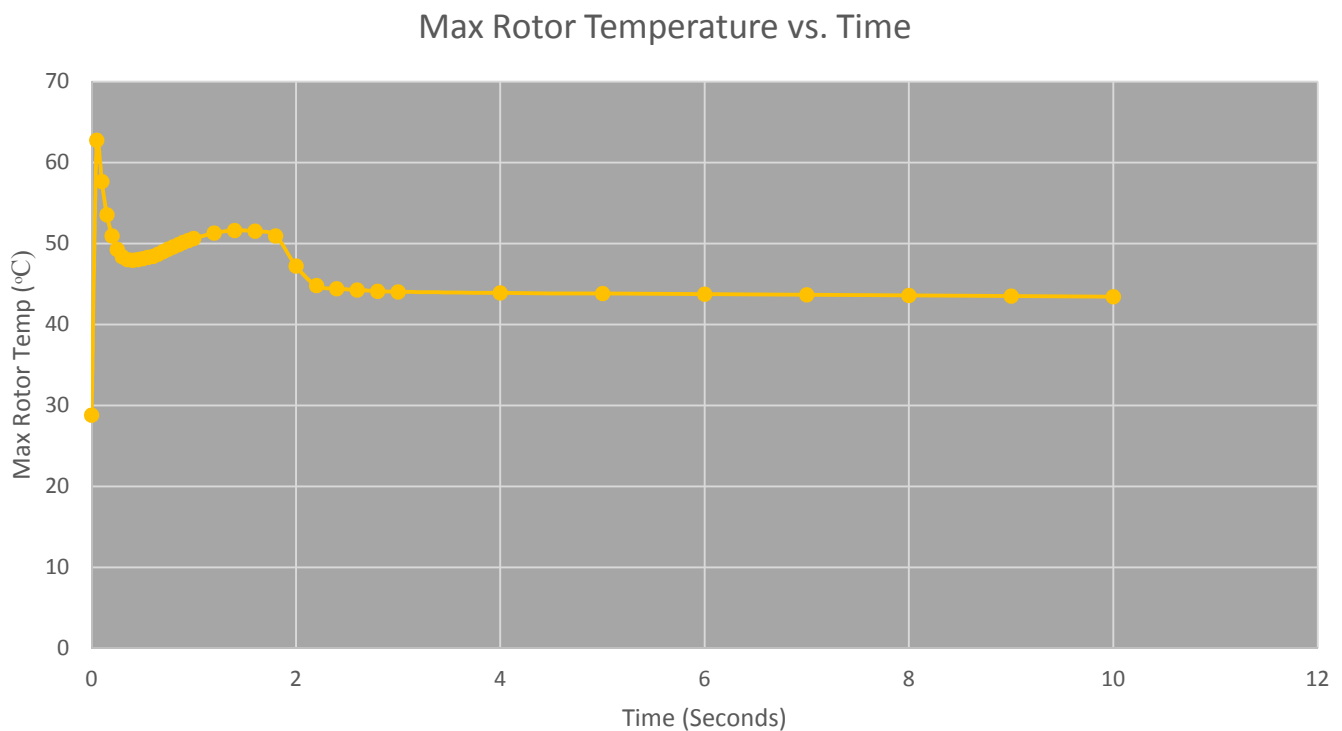


Figure 36: Maximum Rotor Temperature from Actual COMSOL Simulation [12]

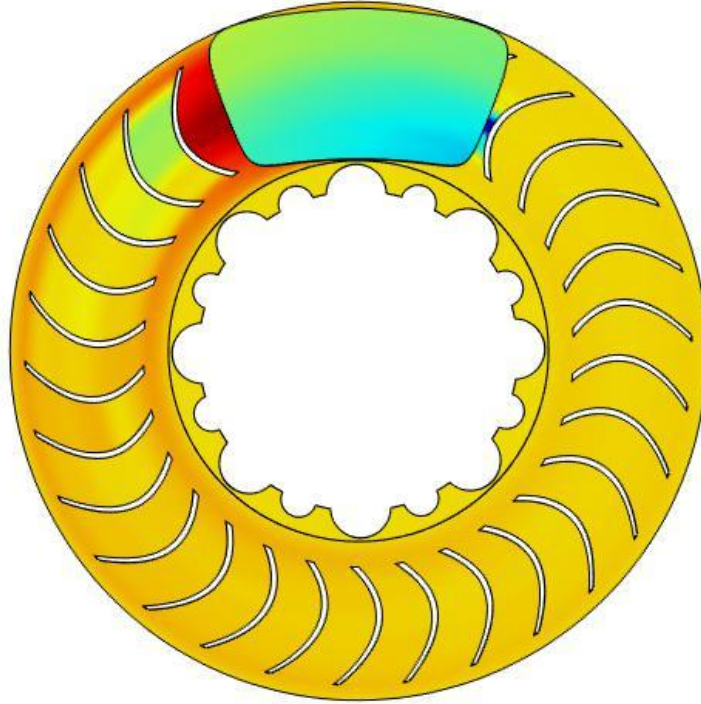


Figure 37: Thermal Rendering of rotor at maximum temperature (62.7 °C) [12]

#### COMSOL Actual Trial Verification

The COMSOL simulation was verified using the conservation of energy. In this calculation the linear kinetic of the vehicle was accounted for along with the rotational kinetic energy of the rotors and tires.

$$\text{Kinetic Engery} = \text{Thermal Energy}$$

$$\frac{1}{2}I_{\text{rotor}}\omega_{\text{rotor}}^2 + \frac{1}{2}I_{\text{wheel}}\omega_{\text{wheel}}^2 + \frac{1}{2}m_{\text{vehicle}}v_{\text{vehicle}}^2 = m_{\text{rotor}}C_{p_{\text{rotor}}}\Delta T$$

(24)

Where  $I$  denotes moment of inertia of the rotating parts,  $\omega$  denotes the angular velocity of the rotating parts,  $m$  is the mass,  $v$  is linear velocity,  $C_p$  is heat capacity and  $\Delta T$  denotes temperature change. The following know values were substituted into the equation:



$$I_{rotor} = 0.03632 \text{ Kg m}^2$$

$$\omega_{rotor} = 190.89 \frac{1}{s}$$

$$I_{wheel} = 0.25 \text{ Kg m}^2$$

$$\omega_{wheel} = 80 \frac{1}{s}$$

$$m_{vehicle} = 283.5 \text{ Kg}$$

$$v_{vehicle} = 20 \frac{m}{s}$$

Thus the remaining variables,  $m_{rotor}$  &  $C_{p_{rotor}}$ , were strictly dependent on material choice. Evaluating the conservation of energy equation established and solving for  $\Delta T$ , the equation becomes:

$$\Delta T = \frac{14404.05 \text{ J}}{m_{rotor} C_{p_{rotor}}} \quad (25)$$

Inputting the mass of the rotors and the heat capacity of A36 carbon steel, the temperature difference is calculated to be:

$$\Delta T = 25.82 \text{ }^{\circ}\text{C}$$

Adding this to the initial temperature of the system (27 °C) renders a maximum rotor temperature of:

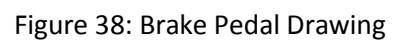
$$T_{rotor \text{ max}} = 52.82 \text{ }^{\circ}\text{C}$$

The COMSOL simulation rendered a maximum rotor temperature of 62.7 °C. However, this temperature is observed early in the simulation. From the data, the temperature increased immensely initially and then decreased rapidly, increased at a more gradual and reasonable rate, then reached a temperature of 51.5 °C at time 1.6 seconds. It was concluded that the initial results of the COMSOL simulation are inaccurate and after 0.25 seconds the results are reasonable. Therefore, the maximum rotor temperature was taken as approximately 52 °C and the simulation was verified.

## Manufacturing

Manufacturing of the rotors and pedal box components was performed by a water jetting process. In this process, a water and sand mixture is forced through a nozzle at approximately 60,000 *psi*, cutting the raw material into the desired components. This process was chosen because the opportunity of warping was minimal, which was crucial for the rotor manufacturing. The rest of the manufacturing was done in the university's machine shop by the braking system designer, which included: final touches on weight reduction cuts for pedal components, fabricating of pins for pedal and master cylinder, spacers and cutting the inner diameter lip out of the rotors. The components of the pedal box were TIG-welded to base plate. The use of a drill press, vertical mill, lathe and arc welder were used to accomplish these tasks.

## Drawings



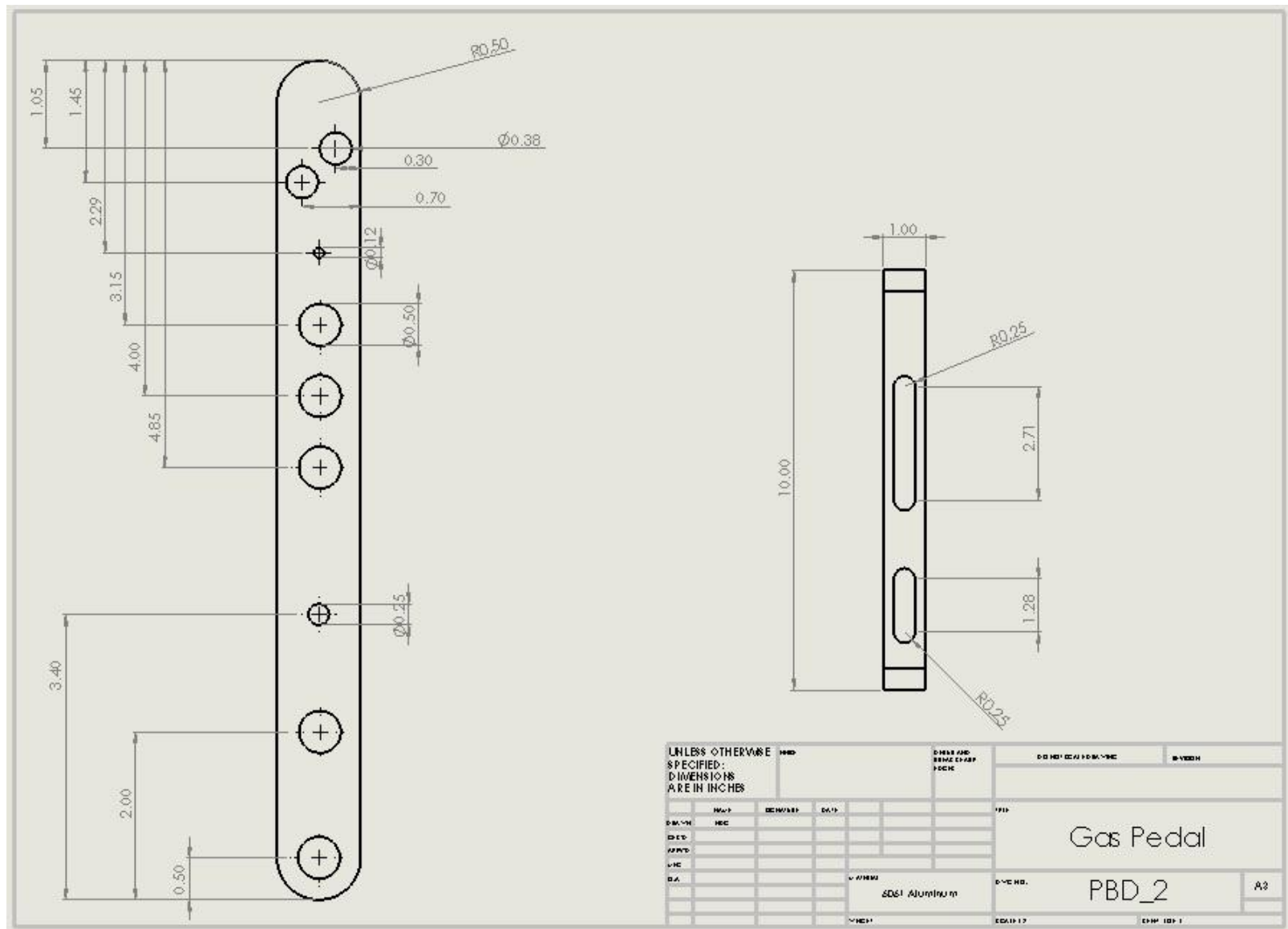


Figure 39: Gas Pedal Drawing



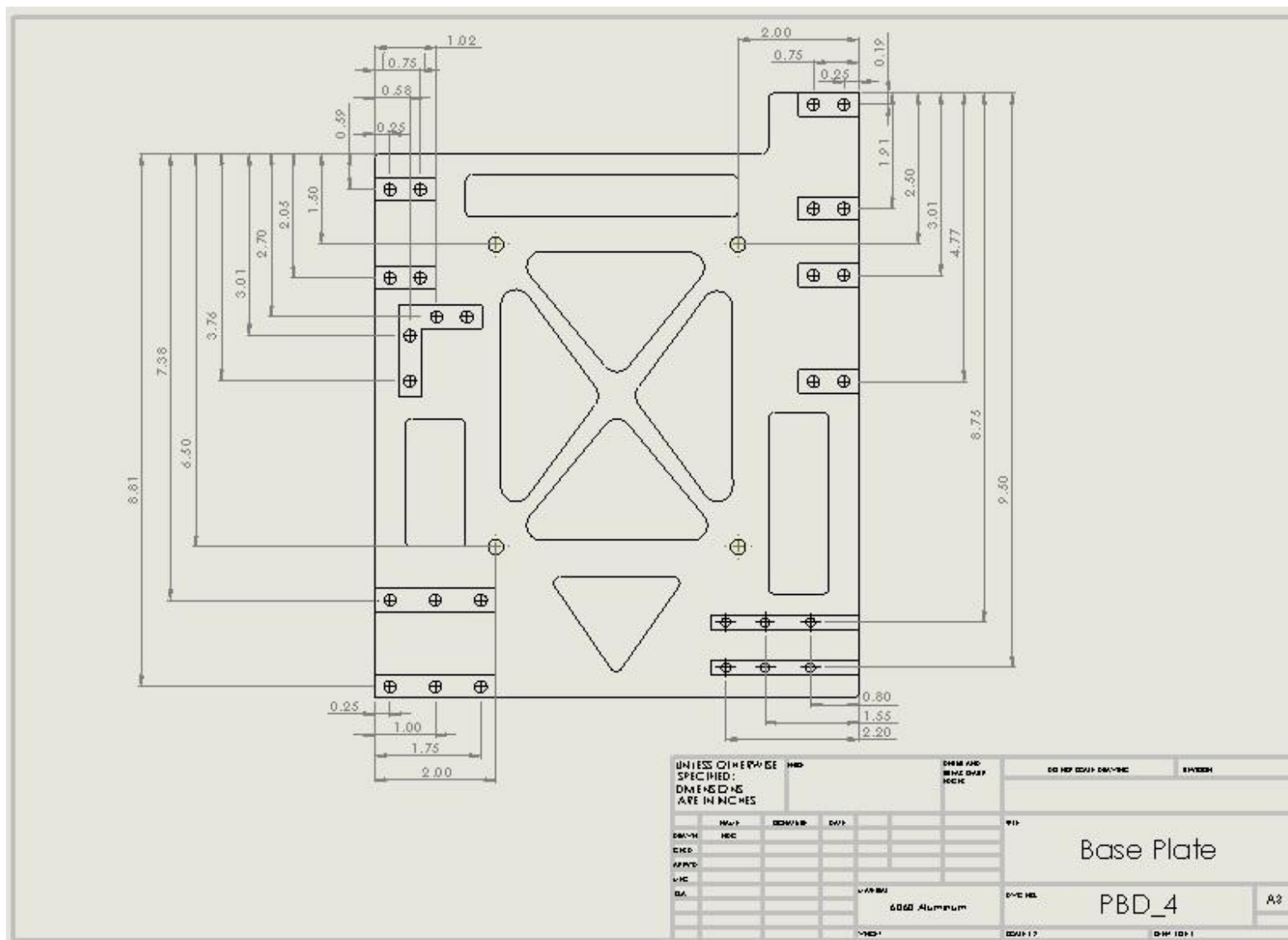


Figure 41: Base Plate Drawing

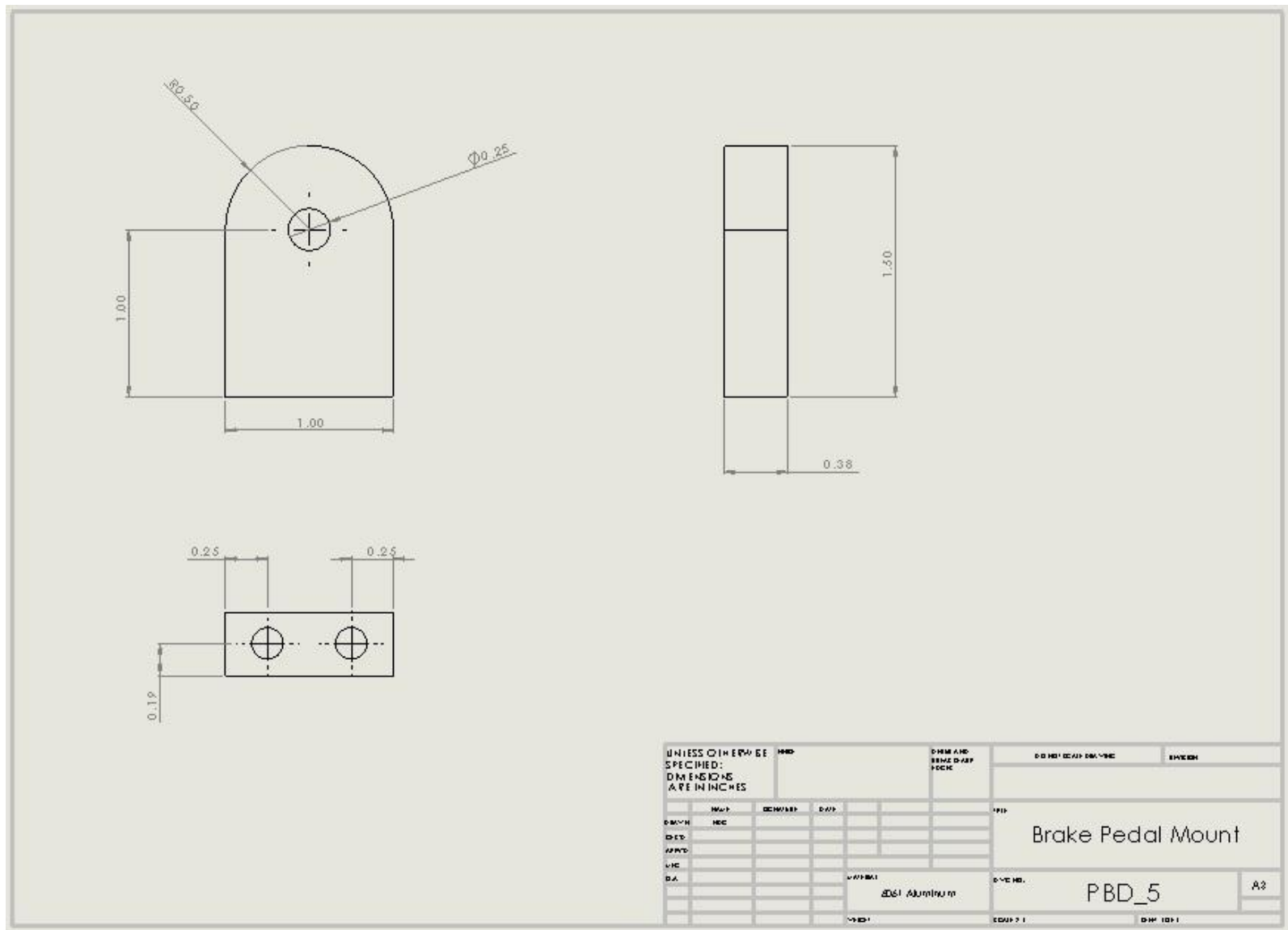


Figure 42: Brake Pedal Mount





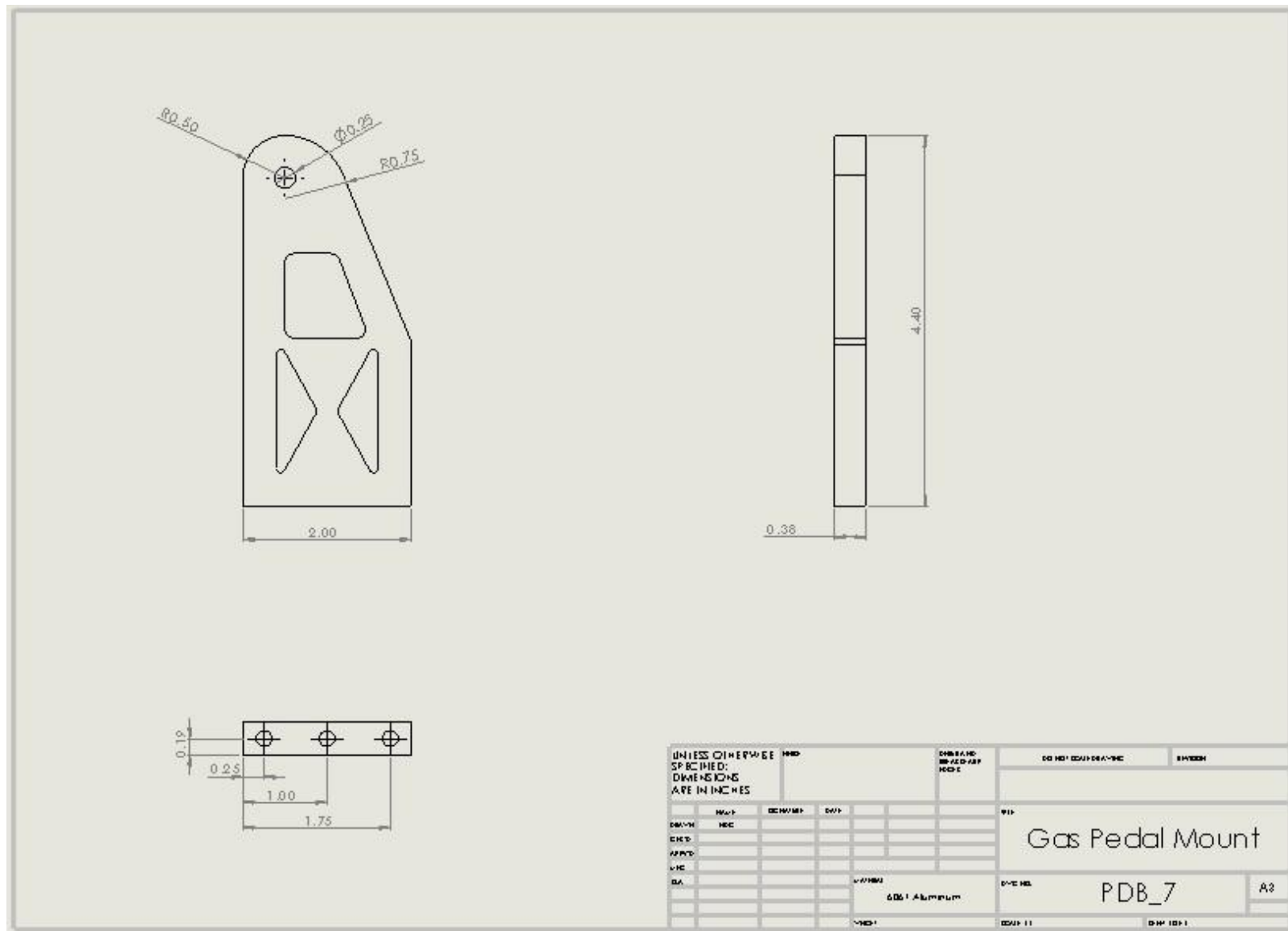


Figure 44: Gas Pedal Mount

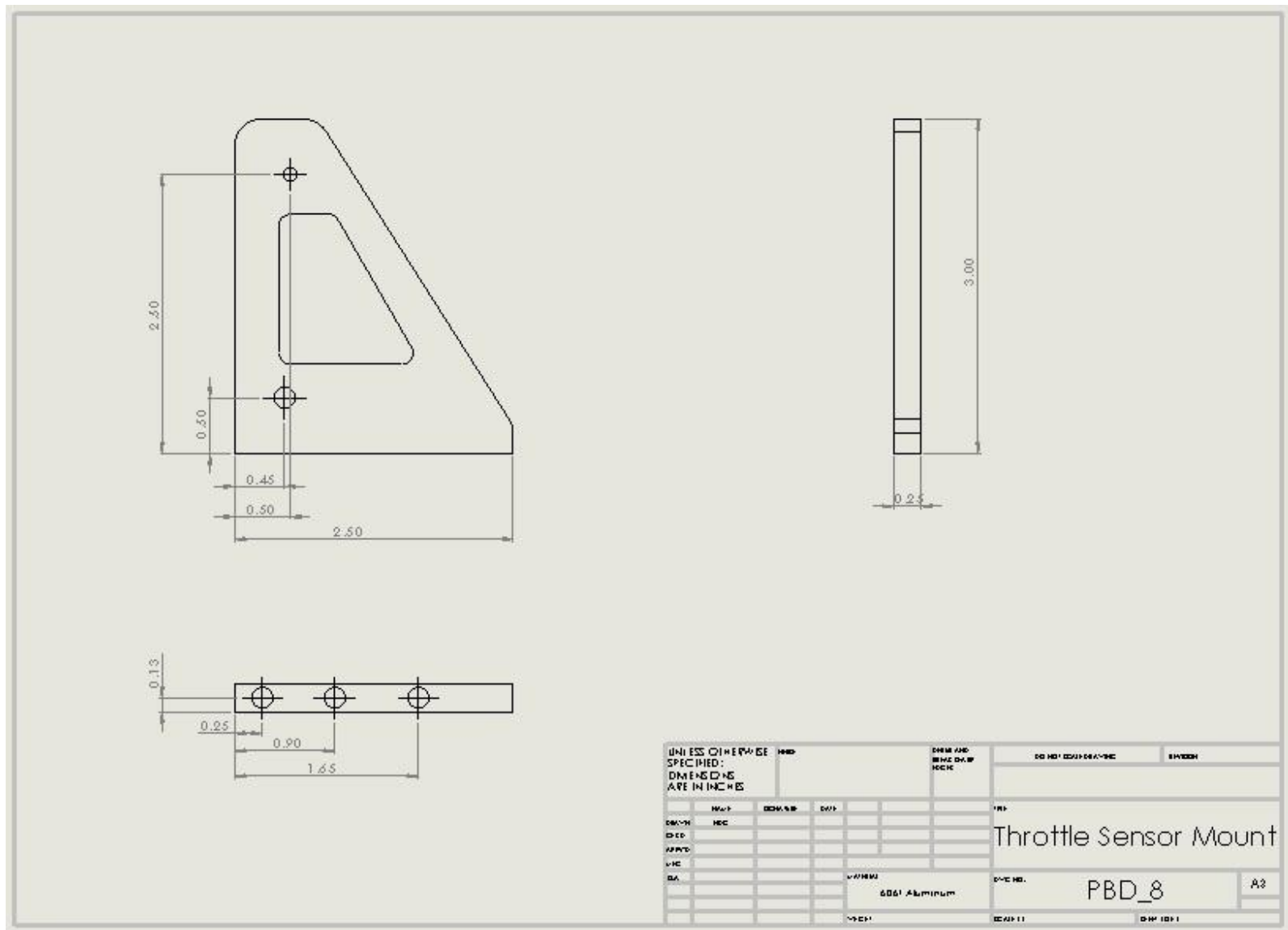


Figure 45: Throttle Sensor Mount







## Conclusion

In conclusion, the University of Akron's FSAE Electric Vehicle Braking System was designed for a weight of 625 *lbs*, including driver. The front and rear braking systems were symmetrical and consisted of two 1 *in* bore, Tilton 77-series master cylinders, four Wilwood *Billet Dynalite Single* calipers and four 8.25 *in* diameter spline vented rotors. The pedal box was manufactured from 6061 T6 Aluminum and consisted of two pedals and mountings for the pedals along with mountings for the master cylinders and throttle sensor. The final design of the entire system weighed approximately 22.81 *lbs*, reducing the weight by approximately 24.4% from the initial design.

SolidWorks was used to draw all of the components of the pedal box and also to run FEA on each component. After subjecting each component to the stress criteria mentioned earlier in the text the following factors of safety were found:

Brake Pedal: 1.4

Gas Pedal: 1.6

Pedal Head: 1.8

Base Plate: 8

Brake Pedal Mount: 21

Master Cylinder Mount: 19

Gas Pedal Mount: 8.2

Throttle Sensor Mount: 8.6

Emergency Stop Mount: 11

Emergency Stop Shield: 9

The component of the pedal box that would be subjected to the maximum stress, the brake pedal, was verified through hand calculations for this finding. The brake pedal arm was modeled as a beam in bending and the maximum stress was found to be:

$$\sigma_{max} = 28.7 \text{ ksi}$$

COMSOL was utilized to perform heat transfer simulations on the rotors, specifically targeting one of the front rotors. The results from the COMSOL simulations rendered a maximum rotor temperature during an emergency stop scenario:

$$\text{Rotor Max Temp} = 62.7^{\circ}\text{C}$$

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However, after verification of this finding using the conservation of energy, it was concluded that the maximum rotor temperature on of the front rotors would experience during an emergency stop would be approximately 52.8 °C.



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