Page1 there are 12 pages in the uploaded document

School of Mathematics and Statistics Honour Code:

I have not given aid or sought and/or received aid during this examination

Signed: Salome Hayes-Shuptar

Name: SALOME HAYES-SHUPTAR

Student No: 19312956

ACM 20030

Q1a) False you can only fit a unique 4th order polynomial through 5 data points

Q1b)

i	1	2	3	4	5
x_i	-2	-1	1	2	3
y_i	-0.9	7.2	0.8	-3.1	0.1
FD1	8.1	-3.2	-3.9	3.2	
FD2	-3.77	-2.37	3.55		
FD3	0.85	1.48			

$$FD1 = \frac{7.2 - -0.9}{-1 - -2} = 8.1$$

$$FD2 = \frac{-3.2 - 8.1}{1 - -2} - 3.77$$

$$FD1 = \frac{0.8 - 7.2}{1 - -1} = -3.2$$

$$FD2 = \frac{-3.9 - 3.2}{2 - -1} = -2.367$$

$$FD1 = \frac{-3.1 - 0.8}{2 - 1} = -3.9$$

$$FD2 = \frac{3.2 - -3.9}{3 - 1} = 3.55$$

$$FD1 = \frac{0.1 - -3.1}{3 - 2} = 3.2$$

$$FD3 = \frac{-2.37 - -3.77}{2 - -2} = 0.35$$

$$FD3 = \frac{3.55 - -2.37}{3 - -1} = 1.48$$

est. 0.5, choose $x = 1$, $i = 3$, closest to interpolating value

$$f_n(0.5) = 0.8 + (0.5-1)(-3.9) + (0.5-1)(0.5-2)(3.55)$$

$$\boxed{f_2(0.5) = 5.4125}$$

Q1c) $f_3(0.5) = 5.4125 + (0.5-1)(0.5-2)(0.5-3)(FD3)$

but $FD3 = 0$

$$\boxed{f_3(0.5) = 5.4125}$$

Page 3

Q2a) When the function does not meet the criteria of the intermmediate value theorem

- not continuous
- there does not exist an $f(a) < 0$ and $f(b) > 0$

Q2b) Newton Raphson Method:

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

1D maps is when the next iteration of an algorithm is defined by the previous iteration.

A period-2 orbit is when the iterations' outputs oscillate between two fixed values.

In otherwords :

$$F(F(x_*)) = x_*$$

Thus the condition to get caught is when the output of a previous iteration, when taken as the new input, maps back onto itself:

$$x_i = x_{i+1} = x_*$$

$$x_* = x_* - \frac{f(x_*)}{f'(x_*)}$$

$$f(x_*) = 0$$

Scanned with CamScanner

Q 2c)

$$f(x) = \frac{1-2x}{x} \qquad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f'(x) = \frac{x(-2)-(1-2x)}{x^2}$$

$$f'(x) = \frac{-2x-1+2x}{x^2}$$

$$f'(x) = \frac{-1}{x^2}$$

$x_0 = 0.1$

$$x_1 = 0.1 - \frac{\left(\frac{1-2(0.1)}{(0.1)}\right)}{\frac{-1}{(0.1)^2}} = 0.18$$

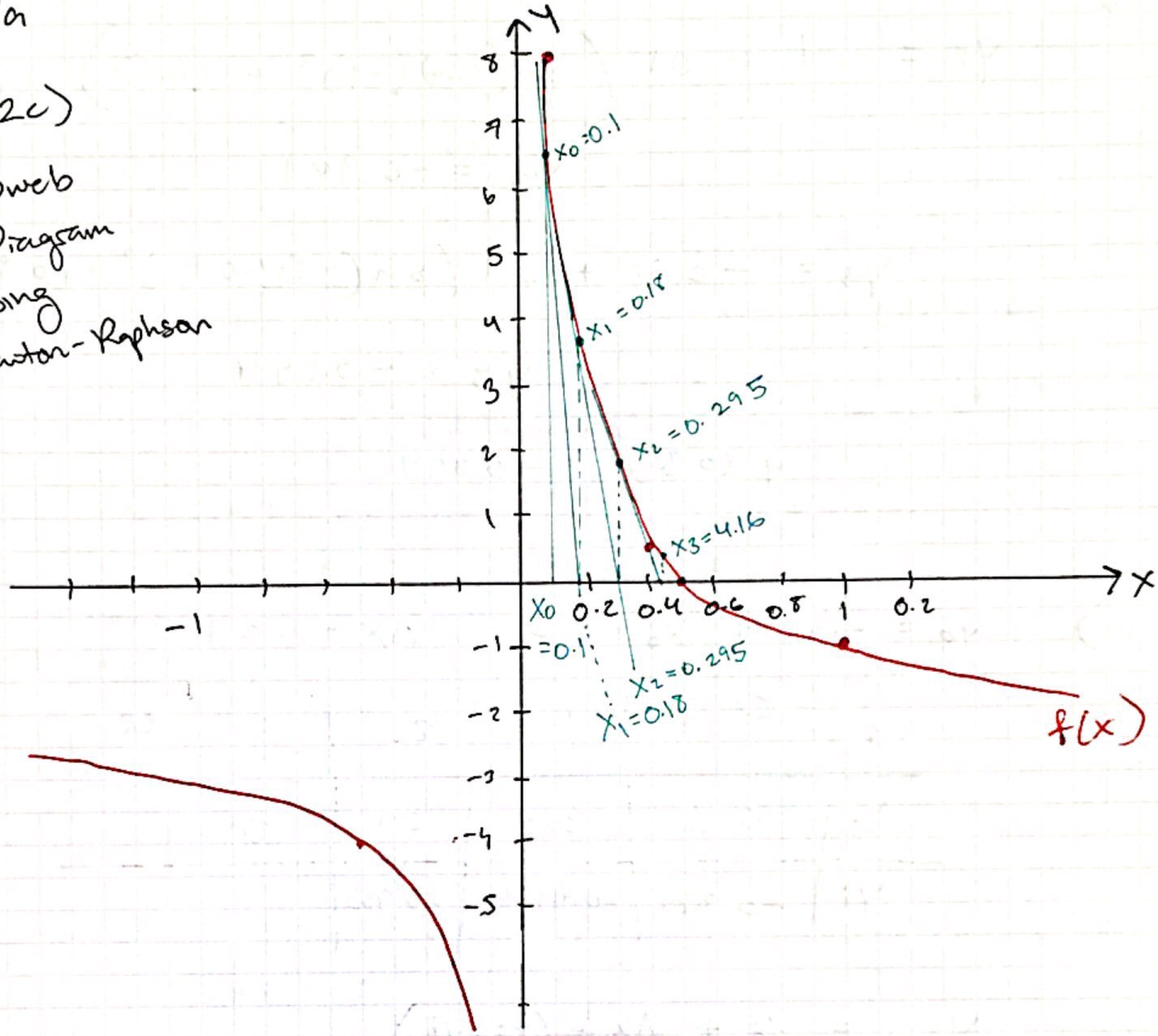
$$x_2 = 0.18 - \frac{\left(\frac{1-2(0.18)}{0.18}\right)}{\frac{-1}{(0.18)^2}} = 0.2952$$

$$x_3 = 0.2952 - \frac{\left(\frac{1-2(0.2952)}{0.2952}\right)}{\frac{-1}{(0.2952)^2}} = 0.416$$

Page 4a

Q2c)

Cobweb Diagram Using Newton-Raphson



at each new iteration the tangent line is found, and then the x-intercept of the tangent becomes the new input

$$x_{i+1} = x_i - f(x_i) \left(\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} \right)$$

$$f(x_0 = 3.1) = \sin(3.1) + \sqrt{3.1} - 2 = -0.1977$$
$$f(x_1 = 3) = \sin(3) + \sqrt{3} - 2 = -0.1268$$

① $x_2 = 3 - -0.1268 \left(\frac{3 - 3.1}{-0.1268 - -0.1977} \right)$

$$x_2 = 2.8212$$

$$f(x_2) = \sin(2.821) + \sqrt{2.8212} - 2 = -5.418 \times 10^{-3}$$

② $x_3 = 2.8212 + 5.418 \times 10^{-3} \left(\frac{2.8212 - 3}{-5.418 \times 10^{-3} - -0.1268} \right)$

$$\boxed{x_3 = 2.8132}$$

Q3 a) Initial Value problems specify all the data at a single value of the independent variable.

BVP specify data at multiple values of the independent variable

Q3b

$$y'(x) = f(x_i, y_i) = \sin(x)y^2 + x \quad y(0) = -1$$
$$x_0 = 0 \quad y_0 = -1$$
$$\Delta x = 0.1$$

i)

i	0	1	2	3
x_i	0	0.1	0.2	0.3
y_i	-1	-0.98	-0.951	-0.894

$$y_{i+1} = y_i + \Delta x \, f(x_i, y_i)$$

$$y_1 = -1 + 0.1 \left(\sin(0.1)(-1)^2 + (0.1) \right)$$
$$y_1 = -0.98$$

$$y_2 = -0.98 + 0.1\left(\sin(0.2)(-0.98)^2 + (0.2)\right)$$

$$y_2 = -0.951$$

$$y_3 = -0.951 + 0.1\left(\sin(0.3)(-0.951)^2 + (0.3)\right)$$

$$y_3 = -0.894$$

$$\boxed{y(0.3) = -0.894}$$

ii) $y_0 = -0.894$ $\quad x_0 = 0.3$ $\quad \Delta x = -0.1$

i	0	1	2	3
x_i	0.3	0.2	0.1	0
y_i	-0.894	-0.9476	-0.985	

$$y_{i+1} = y_i + \Delta x\, f(x_i, y_i)$$

$$= -0.894 - 0.1\left(\sin(0.2)(-0.894)^2) + 0.2\right)$$

$$y_1 = -0.9299$$

$$y_2 = -0.9299 - 0.1\left(\sin(0.1)(-0.9299)^2 + (0.1)\right)$$

$$y_2 = -0.9485$$

$$y_3 = -0.9485 - 0.1\left(\sin(0)(-0.9485)^2 + 0\right)$$

$$y_3 = -0.9485$$

actual $y(0) = -1$ and after forwards and backwards integration arrived at -0.9485

rel error: $1 - \frac{-0.9485}{-1} = 5\%$ error

Page 7

Q3c)

$$x_{i+1} = x_i + \Delta x$$
$$k_1 = f(x_i, y_i)$$
$$k_2 = f\left(x_i + \frac{\Delta x}{2}, y_i + \frac{\Delta x}{2} k_1\right)$$
$$k_3 = f\left(x_i + \Delta x, y_i + \Delta x(2k_2 - k_1)\right)$$
$$y_{i+1} = y_i + \frac{\Delta x}{6}(k_1 + 4k_2 + k_3)$$

$y(0.4)$ $\Delta x = 0.1$ $y_0 = 1$ $x_0 = 0$

$f(x_i, y_i) = x - y^2$

$i = 0$

$k_1 = -1$

$\boxed{k_2} \Rightarrow x_i + \frac{\Delta x}{2} = 0 + \frac{0.1}{2} = 0.05$

$y_i + \frac{\Delta x}{2} k_1 = 1 + 0.5(-1) = 0.95$

$\Rightarrow 0.05 - (0.95)^2 = \boxed{-0.8525}$

$\boxed{k_3} \Rightarrow x_i + \Delta x = 0.1$

$y_i + \Delta x(2k_2 - k_1) = 1 + 0.1(2(-0.8525) + 1)$

$= \boxed{0.9295}$

$y_1 = 1 + \frac{0.1}{6}\left(-1 + 4(-0.8525) + 0.9295\right)$

$y_1 = 0.942$ $x_1 = 0.1$

$\boxed{k_1} = \boxed{-0.7874}$

$\boxed{k_2} \Rightarrow x_1 + \frac{\Delta x}{2} = 0.1 + 0.05 = 0.15$

$y_1 + \frac{\Delta x}{2} k_1 = 0.942 + \frac{0.5}{2}(-0.7874) = \boxed{0.74515}$

$\boxed{k_3} \Rightarrow x_i + \Delta x = 0.2$

messed up the maths here - no time to fix it!

$y_1 + 0.1(2k_2 - k_1)$

$= 0.942 + 0.1(2(0.74515) + 0.7874)$

$\Rightarrow \boxed{-1.2689}$ $= 1.170$

$$y_2 = 0.942 + \frac{0.1}{6}\left(-0.7874 + 4(0.74515) - 1.2689\right)$$

$$y_2 = 0.998 \quad x = 0.2$$

$i = 2$

$$\boxed{k_1 = -0.796}$$

$k_2 \Rightarrow 0.2 + 0.05 = 0.25$

$$y_2 + 0.05(-0.796) = 0.9582$$

$$\Rightarrow \boxed{-0.7181}$$

$k_3 \Rightarrow x_2 + 0.1 = 0.3$

$$y_2 + 0.1(2k_2 - k_1)$$

$$= 0.998 + 0.1(2(-0.7181) + 0.796)$$

$$= 0.934$$

$$\Rightarrow 0.2 - (0.934)^2 = \boxed{-0.772}$$

$$y_3 = 0.998 + \frac{0.1}{6}\left(-0.796 + 4(-0.7181) + (-0.772)\right)$$

$$y_3 = 0.924 \quad x = 0.3$$

$i = 3$

$$k_1 = 0.3 - (0.924)^2 = \boxed{-0.554}$$

$k_2 \Rightarrow 0.3 + 0.05 = 0.35$

$$y_3 + \frac{\Delta x}{2} k_1 = 0.924 + 0.05(-0.554)$$

$$= 0.896$$

$$\Rightarrow 0.35 - (0.896)^2 = \boxed{-0.453}$$

$k_3 \Rightarrow x = 0.4$

$$y = 0.924 + 0.1(2(-0.453) + 0.554)$$

$$= 0.889$$

$$f(\quad) = 0.4 - (0.889)^2 \Rightarrow \boxed{-0.39}$$

$$Y_4 = 0.924 + \frac{0.1}{6}\left(-0.584 + 4(-0.453) + (-0.39)\right)$$

$$\boxed{Y_4 = 0.878 \quad x = 0.4}$$

Q4a) for 3 points of data, a unique 2nd order polynomial can be interpolated using the Lagrange Polynomials.

This polynomial can be integrated.

$\hookrightarrow f_n(x) = \sum_{i=1}^{n+1} f(x_i)L_i(x)$
and $L_i = \prod_{j=1, i \neq 1}^{n+1} \frac{x - x_i}{x_i - x_j}$

Q4b) $\Delta x = \frac{0.5 + 0.1}{6} = 0.1$

$$I_{total} = \frac{\Delta x}{3}\left[y_0 + 4\sum_{j=odd} y_j + 2\sum_{j=even} y_j + y_N\right]$$

i	0	1	2	3	4	5	6
x_i	-0.1	0	0.1	0.2	0.3	0.4	0.5
y_i	-0.199	0	0.199	0.389	0.565	0.717	0.841

$y_i = \sin(2x_i)\,dx$

$$I_{total} = \frac{0.1}{3}\left[-0.199 + 4(0 + 0.389 + 0.717) + 2(0.199 + 0.565) + 0.841\right]$$

$$\boxed{I_{total} = 0.2198}$$

$$\left[\frac{1}{2}\cos(2x)\right]_{-0.1}^{0.5} = 0.21988$$

rel error $= 1 - \frac{0.2198}{0.21988}$

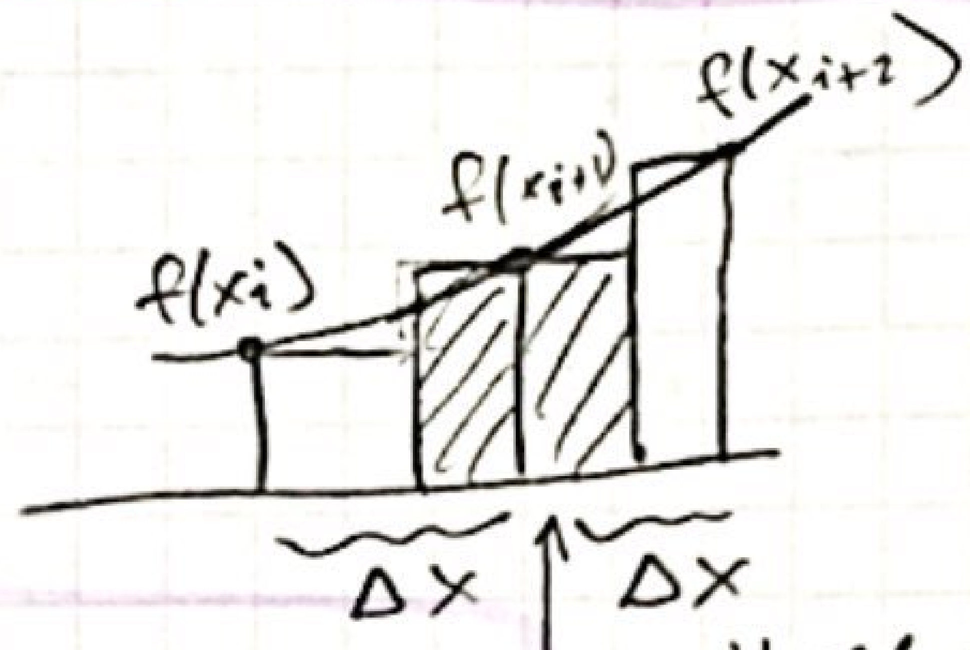
$= 0.036\%$

P10

Q4ci) $I_i = \left(\frac{\Delta x}{2}\right) f(x_i) + \left(\frac{\Delta x}{2}\right) f(x_i + \Delta x)$

$$\boxed{I_i = \left(\frac{\Delta x}{2}\right)\left[f(x_i) + f(x_{i+1})\right]}$$

Q4cii) $\Delta x = \frac{b-a}{N}$



these two are the same size, but they're not doubled counted

$$\boxed{I_{tot} = \sum_{i=0}^{N-1}\left(\frac{\Delta x}{2}\right)\left[f(x_i) + f(x_{i+1})\right]}$$

$$I_{tot} = \frac{1}{2}\sum_{i=0}^{N-1}(\Delta x)\left[f(x_i) + f(x_{i+1})\right]$$

Q4ciii) $\Delta x = \frac{0.4 - 0}{4} = 0.1$

$y_i = \sqrt{x_i}$

i	0	1	2	3	4
x_i	0	0.1	0.2	0.3	0.4
y_i	0	0.316	0.447	0.548	0.632

$I_{tot} = 0.05(0 + 0.316) + 0.05(0.316 + 0.447) + 0.05(0.447 + 0.548)$

$+ 0.05(0.548 + 0.632)$

$$\boxed{I_{tot} = 0.1627}$$

Scanned with CamScanner

P11

Q5a) ill conditioned means that for a small change in b (δb), there is a large change in the result x (Δx).

Q5b) $\|A\|_\infty$ = max $(4.53, 6.796)$

max (of sums of abs. value of rows)

$\|A\|_\infty = 6.796$

Q5c)

R1:	3	12	1	0	2
R2:	6	25	0	1	3
	3	12	1	0	2
R2 − 2R1	0	1	−2	1	−1
$\frac{R1}{3}$	1	4	1/3	0	2/3
	0	1	−2	1	−1
R1 − 4R2	1	0	25/3	−4	14/3
	0	1	−2	1	−1
			inverse		$\begin{bmatrix} x \\ y \end{bmatrix}$

inverse : $\begin{bmatrix} \frac{25}{3} & -4 \\ -2 & 1 \end{bmatrix}$

solution : $\begin{bmatrix} 14/3 \\ -1 \end{bmatrix}$