$$f(t) = \ln\left(\frac{\sqrt{t^2+1}-t}{\sqrt{t^2-1}+t}\right)$$

$\frac{1}{2}-1$

Sol:-

$$f(t) = \ln \sqrt{t^2+1} - t - \ln \sqrt{t^2-1} + t$$

Diff w.r.t

$$y' = \left\{\frac{1}{\sqrt{t^2+1}-t}\,\frac{d}{dt}\left((t^2 \ldots\right) \ldots\right\} \left\{\frac{1}{\sqrt{t^2-1}+t}\,\frac{d}{dt}\left(t^2-1\right)^{\frac{1}{2}}+t\right\}$$

$$= \frac{1}{\sqrt{t^2+1}-t}\left\{\frac{1}{2}(t^2+1)^{-\frac{1}{2}}\cdot 2t - 1\right\} - \left\{\frac{1}{\sqrt{t^2-1}+t}\left\{\frac{1}{2}(t^2-1)^{-\frac{1}{2}}2t+1\right\}\right\}$$

$$= \frac{1}{\sqrt{t^2+1}-t}\left\{\frac{2t}{2\sqrt{t^2+1}}-1\right\} - \left\{\frac{1}{\sqrt{t^2-1}+t}\left\{\frac{2t}{2\sqrt{t^2-1}}+1\right\}\right\}$$

$$= \frac{1}{\sqrt{t^2+1}-t}\left\{\frac{2t-2\sqrt{t^2+1}}{2\sqrt{t^2+1}}\right\} - \frac{1}{\sqrt{t^2-1}+t}\left\{\frac{2t+2\sqrt{t^2-1}}{2\sqrt{t^2-1}}\right\}$$

$$= \left\{\frac{2t-2\sqrt{t^2+1}}{(\sqrt{t^2+1}-t)(2\sqrt{t^2+1})}\right\} - \left\{\frac{2t+2\sqrt{t^2-1}}{(\sqrt{t^2-1}+t)(2\sqrt{t^2-1})}\right\}$$

$$= \frac{2t-2\sqrt{t^2+1}}{(\sqrt{t^2+1}-t)(2\sqrt{t^2+1})} - \frac{2t-2\sqrt{t^2-1}}{(\sqrt{t^2-1}+t)(2\sqrt{t^2-1})}$$

$$= \frac{2}{2}\left\{\frac{t-\sqrt{t^2+1}}{(\sqrt{t^2+1}-t)(\sqrt{t^2+1})}\right\} - \frac{2}{2}\left\{\frac{t-\sqrt{t^2-1}}{(\sqrt{t^2-1}+t)(\sqrt{t^2-1})}\right\}$$

$$= \frac{t-\sqrt{t^2+1}}{(\sqrt{t^2+1}-t)(\sqrt{t^2+1})} - \frac{t-\sqrt{t^2-1}}{(\sqrt{t^2-1}+t)(\sqrt{t^2-1})}$$

$$= \frac{\sqrt{t^2+1}-t}{(\sqrt{t^2+1}-t)(\sqrt{t^2+1})} - \frac{\sqrt{t^2-1}+t}{(\sqrt{t^2-1}+t)(\sqrt{t^2-1})}$$

$$= \frac{1}{\sqrt{t^2+1}} - \frac{1}{\sqrt{t^2-1}} \rightarrow = \frac{\sqrt{t^2-1}-\sqrt{t^2+1}}{(\sqrt{t^2+1})(\sqrt{t^2-1})} +$$