

$$f(t) = \ln \left( \frac{\sqrt{t^2+1} - t}{\sqrt{t^2-1} + t} \right)$$

$\frac{1}{2} - 1$

Sol:

$$f(t) = \ln \sqrt{t^2+1} - t - \ln \sqrt{t^2-1} + t$$

Differentiate w.r.t  $t$

$$\begin{aligned} y' &= \left\{ \frac{1}{\sqrt{t^2+1} - t} \cdot \frac{d}{dt} (\sqrt{t^2+1} - t) \right\} - \left\{ \frac{1}{\sqrt{t^2-1} + t} \cdot \frac{d}{dt} (\sqrt{t^2-1} + t) \right\} \\ &= \frac{1}{\sqrt{t^2+1} - t} \left\{ \frac{1}{2} (t^2+1)^{-\frac{1}{2}} \cdot 2t - 1 \right\} - \left\{ \frac{1}{\sqrt{t^2-1} + t} \left\{ \frac{1}{2} (t^2-1)^{-\frac{1}{2}} \cdot 2t + 1 \right\} \right\} \\ &= \frac{1}{\sqrt{t^2+1} - t} \left\{ \frac{2t}{2\sqrt{t^2+1}} - \frac{1}{2} \right\} - \left\{ \frac{1}{\sqrt{t^2-1} + t} \left\{ \frac{2t}{2\sqrt{t^2-1}} + 1 \right\} \right\} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\sqrt{t^2+1} - t} \left\{ \frac{2t - 2\sqrt{t^2+1}}{2\sqrt{t^2+1}} \right\} - \frac{1}{\sqrt{t^2-1} + t} \left\{ \frac{2t + 2\sqrt{t^2-1}}{2\sqrt{t^2-1}} \right\} \\ &= \left\{ \frac{2t - 2\sqrt{t^2+1}}{(\sqrt{t^2+1} - t)(2\sqrt{t^2+1})} \right\} - \left\{ \frac{2t + 2\sqrt{t^2-1}}{(\sqrt{t^2-1} + t)(2\sqrt{t^2-1})} \right\} \end{aligned}$$

$$\begin{aligned} &= \frac{2t - 2\sqrt{t^2+1}}{(\sqrt{t^2+1} - t)(2\sqrt{t^2+1})} - \frac{2t + 2\sqrt{t^2-1}}{(\sqrt{t^2-1} + t)(2\sqrt{t^2-1})} \\ &= \frac{2}{2} \left\{ \frac{t - \sqrt{t^2+1}}{(\sqrt{t^2+1} - t)(\sqrt{t^2+1})} \right\} - \frac{2}{2} \left\{ \frac{t + \sqrt{t^2-1}}{(\sqrt{t^2-1} + t)(\sqrt{t^2-1})} \right\} \\ &= \frac{t - \sqrt{t^2+1}}{(\sqrt{t^2+1} - t)(\sqrt{t^2+1})} - \frac{t + \sqrt{t^2-1}}{(\sqrt{t^2-1} + t)(\sqrt{t^2-1})} \\ &= \frac{\sqrt{t^2+1} - t}{\sqrt{t^2+1} - t(\sqrt{t^2+1})} - \frac{\sqrt{t^2-1} + t}{\sqrt{t^2-1} + t(\sqrt{t^2-1})} \\ &= \frac{1}{\sqrt{t^2+1}} - \frac{1}{\sqrt{t^2-1}} = \frac{\sqrt{t^2-1} - \sqrt{t^2+1}}{(\sqrt{t^2+1})(\sqrt{t^2-1})} + \dots \end{aligned}$$