



# Lecture 9:

## MPI Parallelization, part II

### (code structure and changes)

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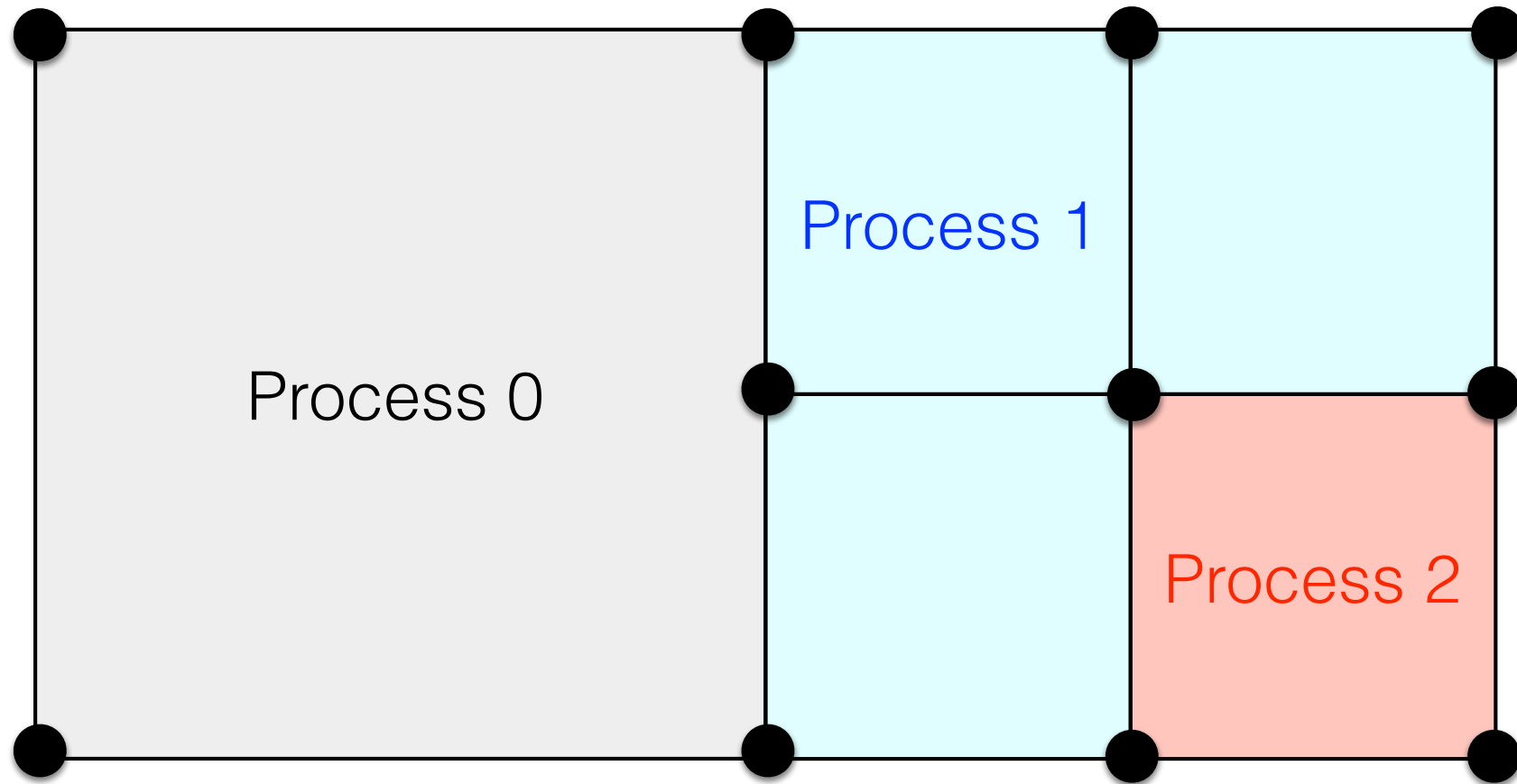
# Aims for this module

- First introduction into parallel computing with deal.II
- Parallel distribution of degrees-of-freedom
  - Ownership concepts
- Setup data structures for parallel processing
- Assembly in parallel
- Synchronization of distributed data
- Visualization of distributed solutions

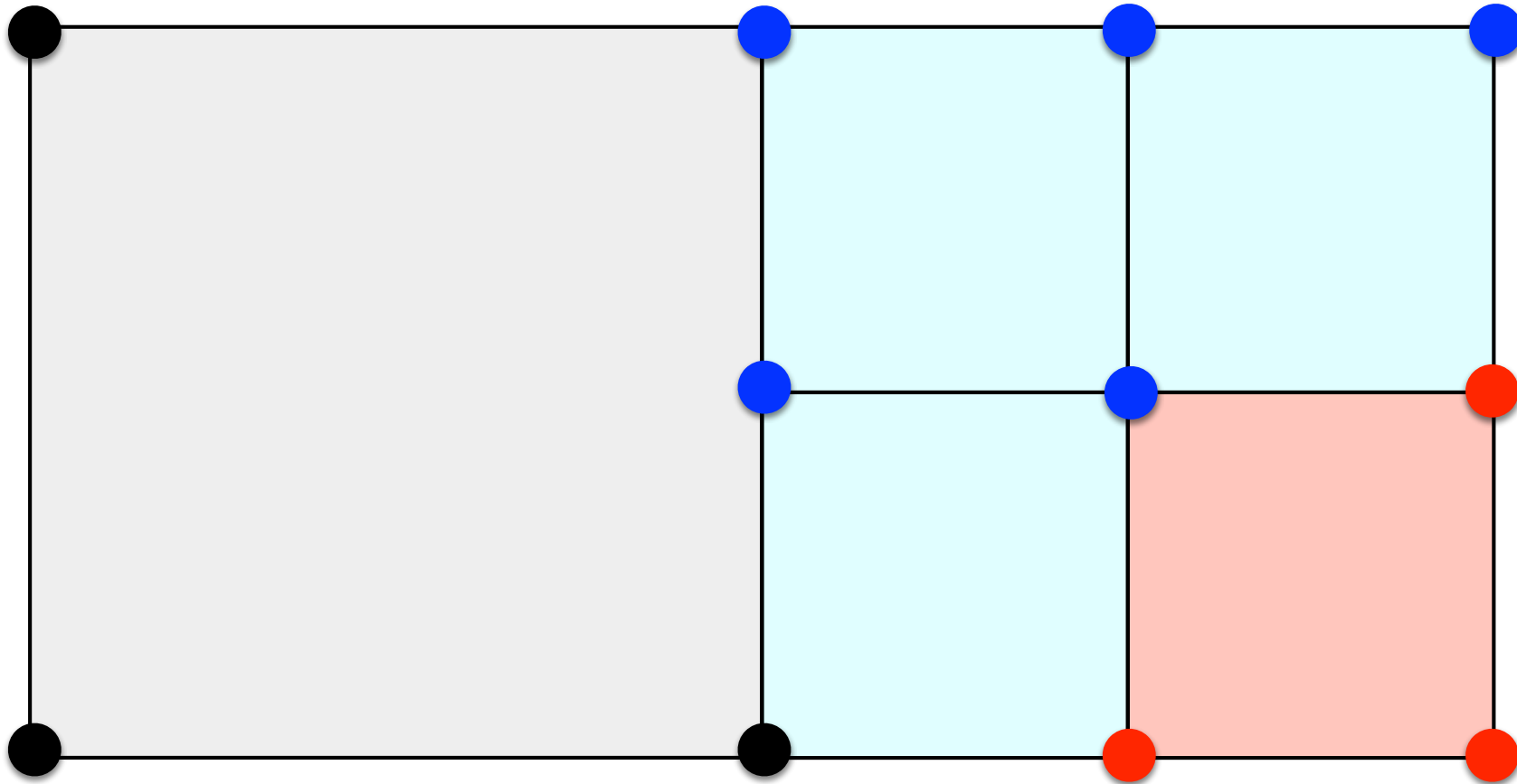
# Reference material

- Tutorials
  - [https://dealii.org/current/doxygen/deal.II/step\\_17.html](https://dealii.org/current/doxygen/deal.II/step_17.html)
  - [https://dealii.org/current/doxygen/deal.II/step\\_18.html](https://dealii.org/current/doxygen/deal.II/step_18.html)
  - <http://www.math.colostate.edu/~bangerth/videos.676.39.html>
  - <http://www.math.colostate.edu/~bangerth/videos.676.41.html>
  - <http://www.math.colostate.edu/~bangerth/videos.676.41.25.html>
  - <http://www.math.colostate.edu/~bangerth/videos.676.41.5.html>
  - <http://www.math.colostate.edu/~bangerth/videos.676.41.75.html>
- Documentation:
  - [https://www.dealii.org/developer/doxygen/deal.II/group\\_\\_distributed.html](https://www.dealii.org/developer/doxygen/deal.II/group__distributed.html)

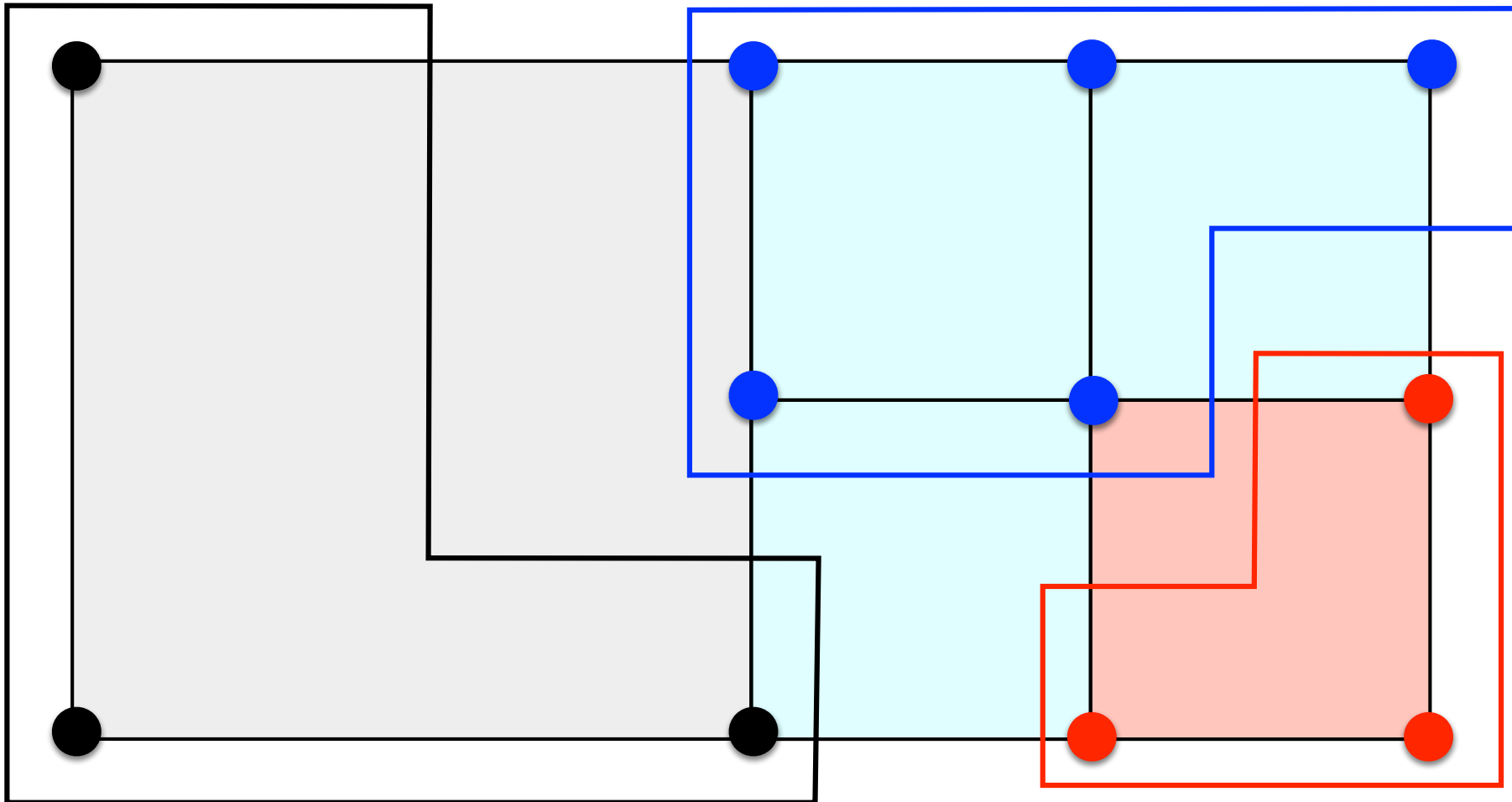
# Distribution of degrees-of-freedom: Colouring of cell ownership via graph partitioner



# Distribution of degrees-of-freedom: Colouring of DoFs based on cell colouring

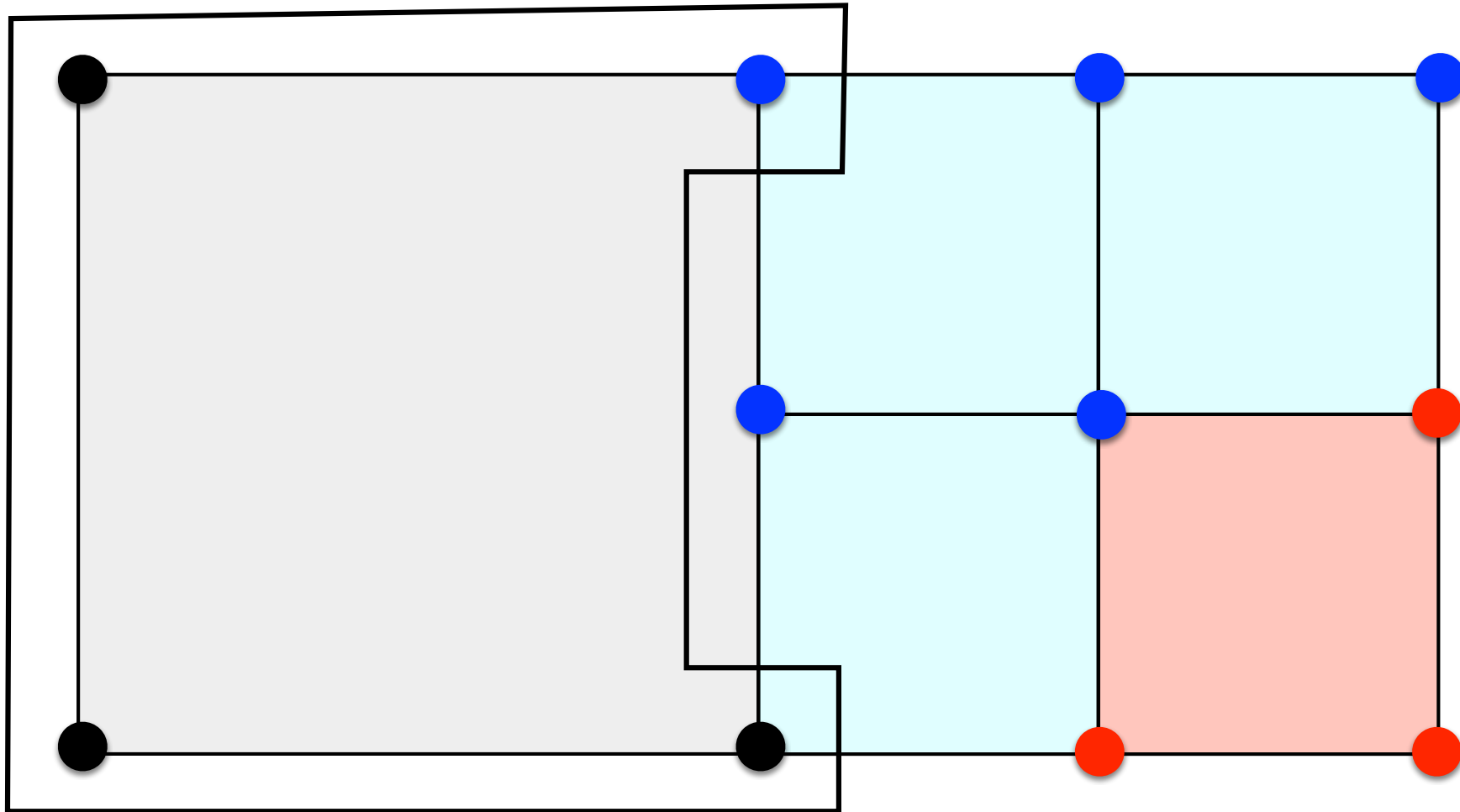


# Ownership of distributed DoFs: Locally owned degrees-of-freedom



# Distribution of DoFs:

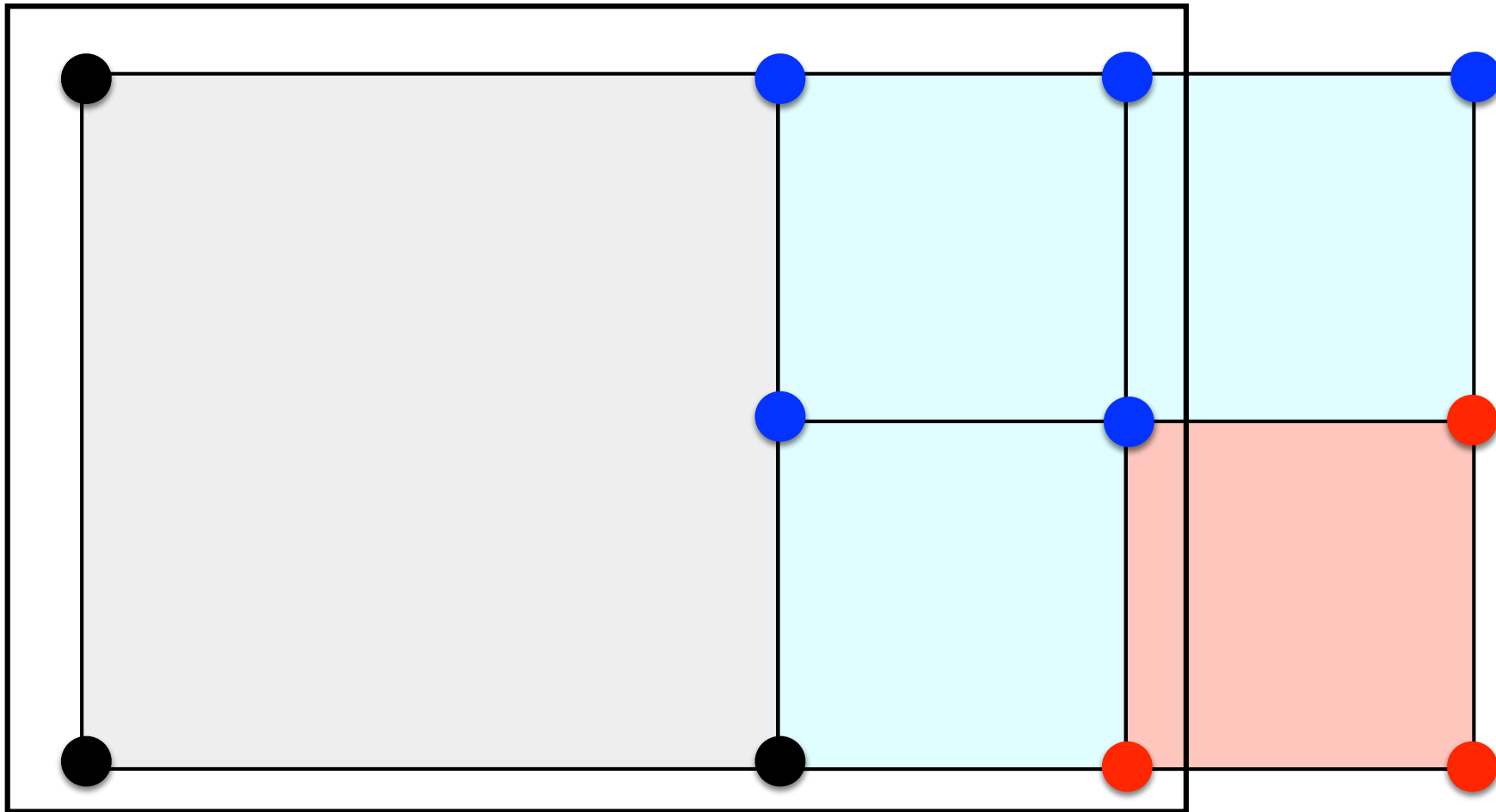
## Locally relevant degrees-of-freedom (process 0)



- Required for assembly, data output

## Distribution of DoFs:

# Locally relevant degrees-of-freedom (process 0)

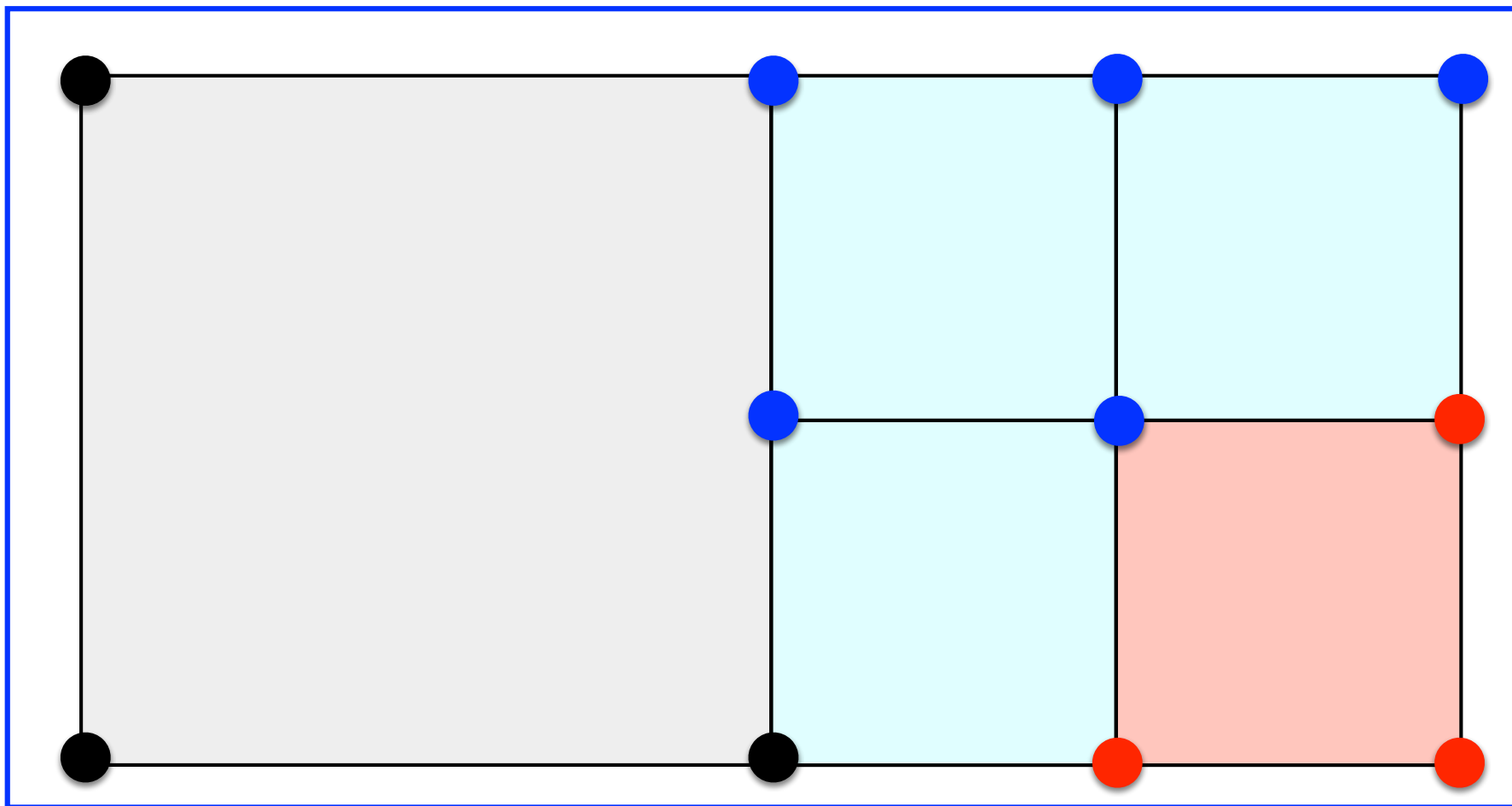


- Required for Kelly error estimator



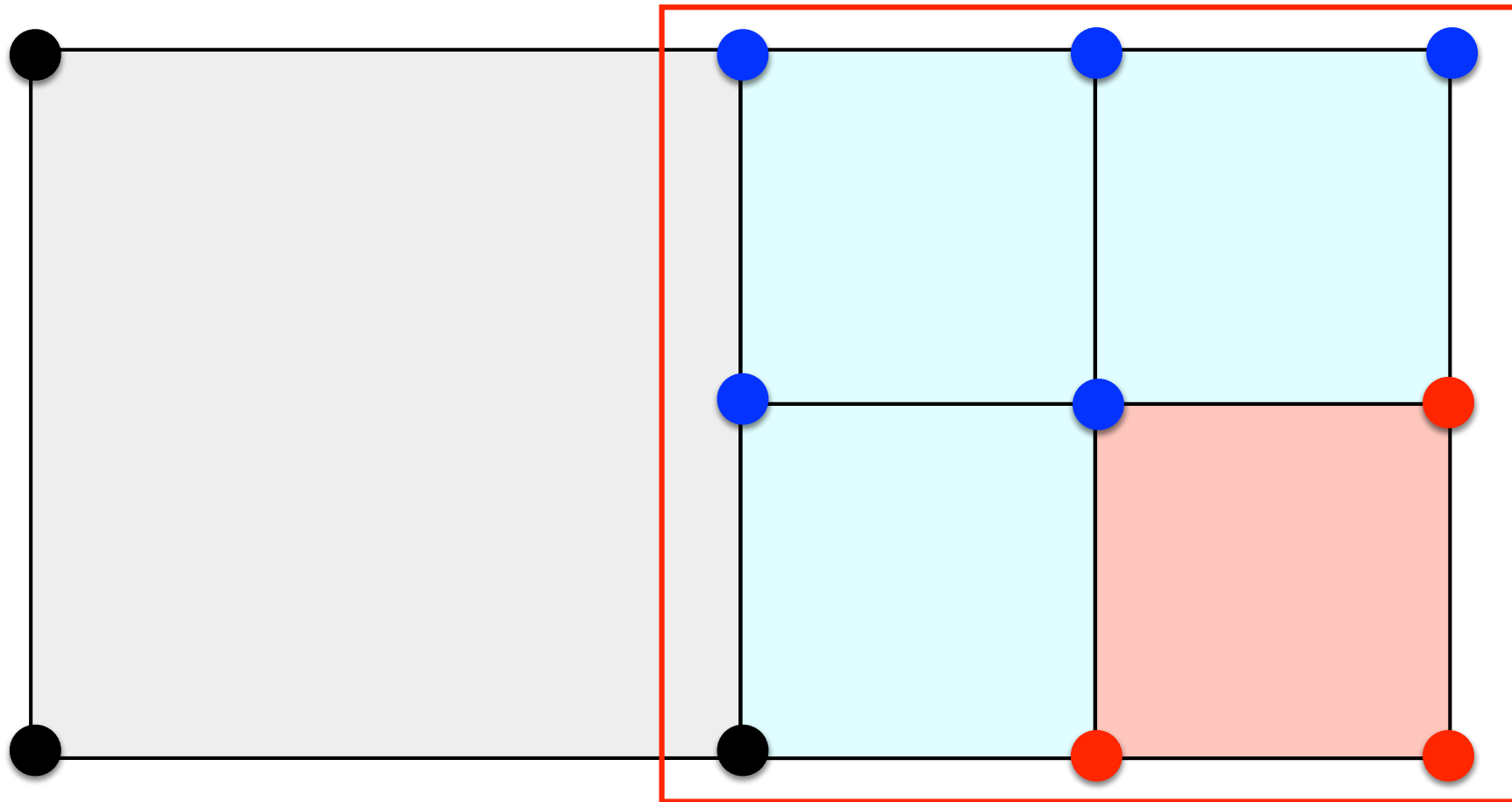
# Distribution of DoFs:

## Locally relevant degrees-of-freedom (process 1)



- Required for Kelly error estimator

# Distribution of DoFs: Locally relevant degrees-of-freedom (process 2)



- Required for Kelly error estimator

# Sketch...

- Each CPU has sets:
  - owned: we store vector and matrix entries of these rows
  - active: we need those for assembling, computing integrals, output, etc.
  - relevant: error estimation
- These set are subsets of  $\{0, \dots, n\_global\_dofs\}$
- Represented by objects of type `IndexSet`
- How to get? `DoFHandler::locally_owned_dofs()`,  
`DoFTools::extract_locally_relevant_dofs()`,  
`DoFHandler::locally_owned_dofs_per_processor()`, ...

# Sketch...

- 🐾 reading from owned rows only (for both vectors and matrices)
- 🐾 writing allowed everywhere (more about compress later)
- 🐾 what if you need to read others?
- 🐾 Never copy a whole vector to each machine!
- 🐾 instead: **ghosted vectors**

# Sketch...

- 🐾 read-only
- 🐾 create using  
`Vector(IndexSet owned, IndexSet ghost, MPI_COMM)`  
where ghost is relevant or active
- 🐾 copy values into it by using `operator=(Vector)`
- 🐾 then just read entries you need



# Compressing

## 🐾 Why?

- 🐾 After writing into foreign entries communication has to happen
- 🐾 All in one go for performance reasons

## 🐾 How?

- 🐾 `object.compress (VectorOperation::add);` if you added to entries
- 🐾 `object.compress (VectorOperation::insert);` if you set entries
- 🐾 This is a collective call

## 🐾 When?

- 🐾 After the assembly loop (with `::add`)
- 🐾 After you do `vec(j) = k;` or `vec(j) += k;` (and in between add/insert groups)
- 🐾 In no other case (all functions inside `deal.II` compress if necessary!)  
(this is new!)



# Changes: New headers

- MPI  
`#include <deal.II/base/mpi.h>`
- Parallel shared triangulation  
`#include <deal.II/distributed/shared_tria.h>`
- Filtered iterator  
`#include <deal.II/grid/filtered_iterator.h>`
- IndexSet  
`#include <deal.II/base/index_set.h>`
- Trilinos linear algebra  
`#include <deal.II/lac/trilinos_*>`
- Output filter  
`#include <deal.II/base/conditional_ostream.h>`

# Changes: Class definition

- MPI utility objects
  - MPI communicator
  - Number of processes, number of “this” process
  - Stream output assistant (filter)
- Triangulation type
- Sparse linear algebra objects
- IndexSets
  - Locally owned
  - Locally relevant



# Changes: System setup

- Must determine the set of locally owned and locally relevant DoFs
  - Locally owned = those assigned to a particular MPI process
  - Locally relevant = those assigned to other processors, but are required to perform some action on the current process
- Distribution of sparsity pattern
  - Tell the locally defined sparsity pattern which entries require data exchange / may be written into by other processes
- Can interrogate information about problem distribution as viewed from other processors

# Changes: Assembly

- Cell loop: Only cells owned by the MPI process
  - Can use filtered iterators
  - Can check within the cell loop  
`cell->locally_owned( ) ;`
- All assembled data is initially localised to a process
- Final synchronisation of data between MPI processes
  - Accumulation of values on DoFs written into by more than one process  
`[matrix/  
vector].compress(VectorOperations::add) ;`
    - Only once (outside of cell loop)

# Changes: Linear solver

- Solver templated on Vector type
- Preconditioner type

# Changes: Mesh refinement

- Kelly error estimator needs to know solution values on cells that are not owned by this current MPI process
- Need to provide view of solution with values for all locally owned and locally relevant DoFs

# Changes: Postprocessing

- Write out portion of solution from each processor
  - deal.II writes these outputs on a per-cell basis
  - Need to provide view of solution with values for all locally owned and locally relevant DoFs

# Changes: Main function

- Setup MPI environment

```
Utilities::MPI::MPI_InitFinalize  
mpi_initialization(argc, argv, 1);
```