CSCI/MATH 2113 Discrete Structures

6.3 - 6.4 Finite State Machines and Mealy Machines

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Contents

1 Finite State Machine

A *finite state machine* is a five-tuple

$$M = (S, \mathcal{I}, \mathcal{O}, v, w)$$

where

- S is the set of internal states for M,
- $\mathcal{I}$ is the input alphabet for M,
- $\mathcal{O}$ is the output alphabet for M,
- $v : S \times \mathcal{I} \to S$ is the next state function,
- $w : S \times \mathcal{I} \to \mathcal{O}$ is the output function.

1.1 Example

Let $M = (S, \mathcal{I}, \mathcal{O}, v, w)$ with $S = \{s_0, s_1, s_2\}$, $\mathcal{I} = \mathcal{O} = \{0, 1\}$ and v and w are defined as

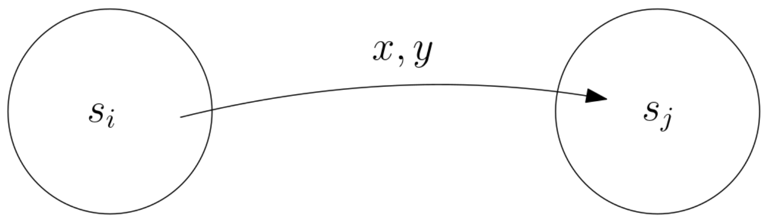
	v		w	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_2	s_1	0	0
s_2	s_0	s_1	0	1

On input 1010, starting in state s_0, the output is 0010:

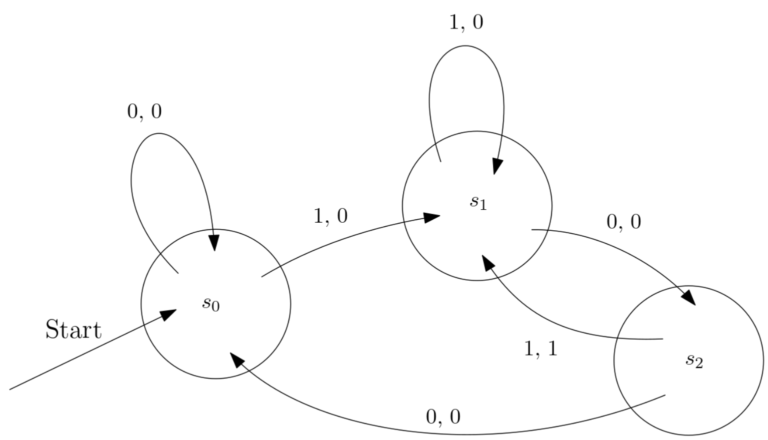
	t_0	t_1	t_2	t_3	t_4
State	s_0	$v(s_0, 1) = s_1$	$v(s_1, 0) = s_2$	$v(s_2, 1) = s_1$	$v(s_1, 0) = s_2$
Input	1	0	1	0	0
Output	$w(s_0, 1) = 0$	$w(s_1, 0) = 0$	$w(s_2, 1) = 1$	$w(s_1, 0) = 0$	-

2 State diagrams

Since we are primarily interested in the output, not in the sequence of transition states, the same machine can be represented by means of a *state diagram*. In such a diagram, each internal state s is represented by a circle with s inside of it. For states s_i, s_j, if $v(s_i, x) = s_j$ for $x \in \mathcal{I}$, and $w(s_i, x) = y$ for $y \in \mathcal{O}$, we represent this by drawing a *directed edge* from the circle for s_i to the circle for s_j and labeling the arc with the inptu x and output y.



The previous example becomes



We can think of a machine (such that $\mathcal{I} = \mathcal{O}$ as associating words over $\mathcal{I}$ to other words over $\mathcal{I}$. Hence, a machine represents a relation on $\mathcal{I}^*$.