AP Calculus Portfolio
Assignment #8

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Question I

a) $\int_0^6 R(t)\,dt = \int_0^6 2 + 5\sin\left(\frac{4\pi}{25}t\right) dt = 31.81593137\ldots$

$\approx \boxed{31.82 \text{ cubic yards}}$ of sand are removed from the beach during the 6-hour period

b) $Y(t) = Y(0) + \int_0^t S(x) - R(x)\,dx$

$\boxed{Y(t) = 2500 + \int_0^t S(x) - R(x)\,dx}$

Since at $t=0$, there's 2500 cubic yards of sand, and as time goes on, the tide removes some of it (which is modelled by $R(t)$) and the pumping station adds some of it (as modelled by $S(t)$).

c) $\frac{d}{dt}\left[Y(t) = 2500 + \int_0^t S(x) - R(x)\,dx\right]$

$Y'(t) = S(t) - R(t)\Big|_{t=4}$

$Y'(4) = S(4) - R(4)$

$Y'(4) = \boxed{-1.9088 \text{ cubic yards / hour}}$

$S(4) = \dfrac{15(4)}{1+3(4)} = 4.615384615\,38\ldots$

$R(4) = 2 + 5\sin\left(\frac{4\pi}{25}(4)\right) = 6.52413526233\ldots$

d) $Y'(t) = 0$

$0 = S(t) - R(t)$

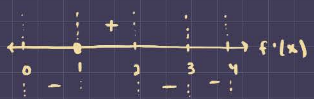
$R(t) = S(t)$

intersects at $(5.118, 4.694)$

t	Y(t)
0	2500
5.118	2492.3695
6	2493.2767

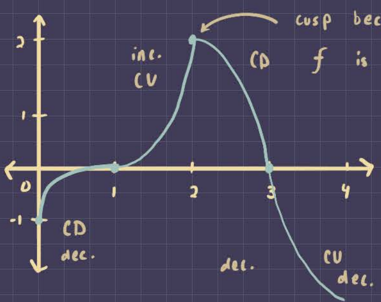
When $\boxed{t = 5.118}$, the amount of sand left are $\boxed{2492.3695 \text{ cubic yards}}$, which is the minimum value.

Question II

a) 

Relative maximum: ⊕ to ⊖

The graph of f has a relative maximum when $\boxed{x=2}$. This is because $f'(x)$ changes from positive to negative at $x=2$.

b) 

cusp because f(x) goes concave up then down. $x=2$ is not a discontinuity because f is a continuous function.

c) $g(x) = \int_1^x f(t)\,dt$

FTC

$g'(x) = f(x)$

g has a relative extremum at $x = 1, 3$. $\boxed{x=1}$ is a minimum because g' changes from negative to positive. Likewise, $\boxed{x=3}$ is a maximum because g' changes from positive to negative.

d) If $g'(x) = f(x)$ then:

$g''(x) = f'(x)$

g has an inflection pt. at $\boxed{x=2}$ because f' changes concavity at that point (i.e. goes from positive to negative OR concave up to concave down)

Question III

a) $T'(7) = \frac{T(8)-T(6)}{8-6} = \frac{55-62}{8-6} = \frac{-7}{2} = \boxed{-3.5\,^\circ C/cm}$

b) $\text{Avg. Temp} = \frac{1}{8}\int_0^8 T(x)\,dx$

$= \frac{1}{8}\left[\frac{100+93}{2} + (4)\frac{93+70}{2} + \frac{62+70}{2} + (2)\frac{55+62}{2}\right]$

$= \boxed{75.6875\,^\circ C}$

c) $\int_0^8 T'(x)\,dx = T(x)\Big|_0^8 = T(8)-T(0) = 55-100 = \boxed{-45\,^\circ C}$

This means that the temperature decreased by 45°C from the end where it's getting heated to the other end of the wire.

d) $T''(x) > 0$ means $T'(x)$ is always increasing and concave up

But,

$\frac{T(1)-T(0)}{1-0} = \frac{93-100}{1} = -7$

$\frac{T(5)-T(1)}{5-1} = \frac{70-93}{4} = -5.75$

$\frac{T(6)-T(5)}{6-5} = \frac{62-70}{1} = -8$

$\frac{T(8)-T(6)}{8-6} = \frac{55-62}{2} = -3.5$

These calculations imply that the average rate of change of the temperature is always decreasing. It follows that T' must decrease somewhere on intervals $(0,1)$, $(1,5)$, $(5,6)$ and $(8,6)$ because of the Mean Value Theorem.

$\boxed{\therefore\ T''(x) \text{ is not positive on the intervals } 0 \le x \le 8.}$

Question IV

a) $\int_0^{24} v(t)\,dt = \left[\frac{(4)(20)}{2}\right] + \left[(20)(12)\right] + \left[\frac{(8)(20)}{2}\right] = \boxed{360 \text{ meters}}$

b) $v'(4)$ doesn't exist because it's a corner

$v'(20) = \frac{v(24)-v(16)}{24-16} = \frac{0-20}{8} = \boxed{-\frac{5}{2}\ m^2/s}$

c) $\boxed{a(t) = \begin{cases} 5\,m^2/s, & \text{for } 0 \le t < 4 \\ 0\,m^2/s, & \text{for } 4 < t < 16 \\ -5/2\,m^2/s, & \text{for } 16 < t \le 24 \\ t \ne 4, 16 \end{cases}}$

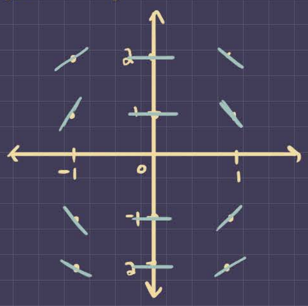
$a(t)$ is undefined when $t = 4$ and $t = 16$ because the derivative does not exist

d) $\text{Avg. rate of change} = \frac{f(20)-f(8)}{20-8} = \frac{10-20}{20-8} = \boxed{-\frac{5}{6}\ m^2/sec}$

$\boxed{\text{The MVT does not guarantee } v'(c) = -\tfrac{5}{6}\,m^2/sec}$ because it is not differentiable at $t=16$, which lies on the interval $8 \le t \le 20$. The function needs to be continuous and differentiable on two intervals, as required by the MVT.

Question V

a) $\frac{dy}{dx} = \frac{-2x}{y}$



b) $\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{-2(1)}{-1} = \frac{-2}{-1} = 2 = m$

$y - y(1) = m(x-1)$
$y - (-1) = 2(x-1)$
$y + 1 = 2(x-1)$
$y = 2(x-1) - 1$

So,

$y(1.1) = 2(1.1-1) - 1$
$= \boxed{-0.8}$

c) $\frac{dy}{dx} = \frac{-2x}{y}$

$\int y \, dy = -2 \int x \, dx$
$\frac{1}{2}y^2 = -2 \cdot \frac{1}{2}x^2 + C$
$\left.\frac{1}{2}y^2 = -x^2 + C \right|_{(1,-1)}$
$\frac{1}{2}(-1)^2 = -(1)^2 + C$
$\frac{1}{2} = -1 + C$
$C = \frac{3}{2}$

Then,

$\frac{1}{2}y^2 = -x^2 + \frac{3}{2}$
$y^2 = -2x^2 + 3$
$y = \pm\sqrt{-2x^2+3}$
$\boxed{y = -\sqrt{-2x^2+3}}$

The function needs to be negative because $\left.-2x^2+3\right|_{x=1}$ gives 1, and the y-value needs to be negative.